**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Since work begins 10 mins after the car is dropped, the time left to complete work is 50 mins.

Let X be the time taken to complete work.

Find Z-Scores at X=50; Z = (X - µ) / σ

Z= (50-45)/8

Z

0.625

Find probability P(X>50) = 1-stats.norm.cdf(abs(z\_score))

1-stats.norm.cdf(abs(0.625))

0.265985

Ans: 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False

q2\_lessthan\_38 = stats.norm.cdf(38, loc = mean, scale = std1)

q2\_less\_than\_44 = stats.norm.cdf(44, loc = mean,  scale = std1)

q2\_betweeen\_38\_and\_44 = (q2\_less\_than\_44 - q2\_lessthan\_38)

print('The probability of employee age between 38 and 44 is',np.round(q2\_betweeen\_38\_and\_44\*100,2),'%')

q2\_morethan\_44 = 1-stats.norm.cdf(44, loc = mean, scale = std1)

print('The probability of employee age more than 44 is',np.round(q2\_morethan\_44\*100,2),'%')

result = (q2\_morethan\_44 > q2\_betweeen\_38\_and\_44)

print('Answer:',result)

The probability of employee age between 38 and 44 is 34.13 %

The probability of employee age more than 44 is 15.87 %

Answer: False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True

q2b = stats.norm.cdf(30, loc = mean, scale = std1)

print("""A training program for employees under the age of 30 at the center would be expected to attract about"""  ,np.round((q2b\*400),0),'employees')

A training program for employees under the age of 30 at the center would be expected to attract about 36.0 employees

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: The Normaldistribution is defined by two parameters mean μ and the variance σ2 and written as X ~ N (µ, σ2).

Given *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are two independent identically distributed random variables.

From the properties of normalrandomvariables,

if X ~ N (µ1, σ1 2) and Y ~ N (µ2, σ2 2). are two independent identically distributed random variables then

the sum of normal random variables is given by

X + Y ~ N (µ1 + µ2, σ1 2 + σ2 2)

        and the difference of normal random variables is given by

X - Y ~ N (µ1 - µ2, σ1 2 - σ2 2)

When Z=aX, the product of X is given by

Z ~ N (aµ1, a2 σ1 2)

        Given to find, 2X1

Thus, following the property of multiplication, we get

2X1 ~ N (2µ, 22 σ2) => 2X1 ~ N (2µ, 4σ2)

and following the property of addition,

(X1 + X2) ~ N (μ + μ, σ2 + σ2) ~ N (2μ, 2σ2)

And the difference between the two is given by

2X1 – (X1 + X2) ~ N (2μ - 2μ, , 4σ2 - 2σ2) ~ N( 0, 2σ2)The difference between the two says that the two given variables are identically and independently distributed.

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D: 48.5, 151.5

print("""The two values of a and b, symmetric about the mean, are such that the probability of the random variable  taking a value between them is 0.99:""",np.round(stats.norm.interval(0.99, loc = 100, scale = 20),1))

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:  To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645

X= 540+(-1.645)\*(225)

print('5th percentile of profit (in Million Rupees)is',np.round(X,))

5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

0.0477903522728147

Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

0.040059156863817086