Real-Time Software Transactional Memory: Contention Managers, Time Bounds, and Implementations

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(ABSTRACT)

Lock-based concurrency control suffers from programmability, scalability, and composability challenges. These challenges are exacerbated in emerging multicore architectures, on which improved software performance must be achieved by exposing greater concurrency. Transactional memory (TM) is an alternative synchronization model for shared memory objects that promises to alleviate these difficulties.

In this dissertation proposal, we consider software transactional memory (STM) for concurrency control in multicore real-time software, and present a suite of real-time STM contention managers for resolving transactional conflicts. The contention managers are called RCM, ECM, LCM, and PNF. RCM and ECM resolve conflicts using fixed and dynamic priorities of real-time tasks, respectively, and are naturally intended to be used with the fixed priority (e.g., G-RMA) and dynamic priority (e.g., G-EDF) multicore real-time schedulers, respectively. LCM resolves conflicts based on task priorities as well as atomic section lengths, and can be used with G-EDF or G-RMA. Transactions under ECM, RCM and LCM can retry due to non-shared objects with higher priority tasks. PNF avoids this problem.

We establish upper bounds on transactional retry costs and task response times under all the contention managers through schedulability analysis. Since ECM and RCM conserve the semantics of the underlying real-time scheduler, their maximum transactional retry cost is double the maximum atomic section length. This is improved in the the design of LCM, which achieves shorter retry costs. However, ECM, RCM, and LCM are affected by transitive retries when transactions access multiple objects. Transitive retry causes a transaction to abort and retry due to another non-conflicting transaction. PNF avoids transitive retry, and also optimizes processor usage by lowering the priority of retrying transactions, enabling other non-conflicting transactions to proceed.

We also formally compare the proposed contention managers with lock-free synchronization. Our comparison reveals that, for most cases, ECM, RCM, G-EDF(G-RMA)/LCM achieve higher schedulability than lock-free synchronization only when the atomic section length does not exceed half of the lock-free retry loop length. Under PNF, atomic section length can equal length of retry loop. With low contention, atomic section length under ECM can equal retry loop length while still achieving better schedulabiltiy. While in RCM, atomic section length can exceed retry loop length. By adjustment of LCM design parameters, atomic section length can be of twice length of retry loop under G-EDF/LCM. While under G-RMA/LCM, atomic section length can exceed length of retry loop.

We implement the contention managers in the Rochester STM framework and conduct experimental studies using a multicore real-time Linux kernel. Our studies confirm that, the contention managers achieve orders of magnitude shorter retry costs than lock-free synchronization. Among the contention managers, PNF performs the best.

Building upon these results, we propose real-time contention managers that allow nested atomic sections – an open problem – for which STM is the only viable non-blocking synchronization solution. Optimizations of LCM and PNF to obtain improved retry costs and greater schedulability advantages are also proposed.

Dedication

To my parents, my wife, my daughter, and all my family

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Chapter 1

Introduction

Embedded systems sense physical processes and control their behavior, typically through feedback loops. Since physical processes are concurrent, computations that control them must also be concurrent, enabling them to process multiple streams of sensor input and control multiple actuators, all concurrently. Often, such computations need to concurrently read/write shared data objects. Typically, they must also process sensor input and react, satisfying application-level time constraints.

The de facto standard for programming concurrency is the threads abstraction, and the de facto synchronization abstraction is locks. Lock-based concurrency control has significant programmability, scalability, and composability challenges [61]. Coarse-grained locking (e.g., a single lock guarding a critical section) is simple to use, but permits no concurrency: the single lock forces concurrent threads to execute the critical section sequentially, in a one-at-a-time order. This is a significant limitation, especially with the emergence of multicore architectures, on which improved software performance must be achieved by exposing greater concurrency.

With fine-grained locking, a single critical section is broken down into several critical sections – e.g., each bucket of a hash table is guarded by a unique lock. Thus, threads that need to access different buckets can do so concurrently, permitting greater parallelism. However, this approach has low programmability: programmers must acquire only necessary and sufficient locks to obtain maximum concurrency without compromising safety, and must avoid deadlocks when acquiring multiple locks. Moreover, locks can lead to livelocks, lock-convoying, and priority inversion.

Perhaps, the most significant limitation of lock-based code is its non-composability. For example, atomically moving an element from one hash table to another using those tables' (lock-based) atomic methods is not possible in a straightforward manner: if the methods internally use locks, a thread cannot simultaneously acquire and hold the locks of the methods (of the two tables); if the methods were to export their locks, that will compromise safety.

Lock-free synchronization [60], which uses atomic hardware synchronization primitives (e.g., Compare And Swap [71,72], Load-Linked/Store-Conditional [108]), also permits greater concurrency, but has even lower programmability: lock-free algorithms must be custom-designed for each situation (e.g., a data structure [22,51,59,64,89]). Additionally, it is not clear how to program nested critical sections using lock-free synchronization. Most importantly, reasoning about the correctness of lock-free algorithms is significantly difficult [60].

1.1 Transactional Memory

Transactional memory (TM) is an alternative synchronization model for shared memory data objects that promises to alleviate these difficulties. With TM, programmers write concurrent code using threads, but organize code that read/write shared memory objects as *memory transactions*, which speculatively execute, while logging changes made to objects–e.g., using an undo-log or a write-buffer. Objects read and written by transactions are also monitored, in read sets and write sets, respectively. Two transactions conflict if they access the same object and one access is a write. (Conflicts are usually detected by detecting non-empty read and write set intersections.) When that happens, a contention manager (CM) resolves the conflict by aborting one and committing the other, yielding (the illusion of) atomicity. Aborted transactions are re-started, after rolling-back the changes–e.g., undoing object changes using the undo-log (eager), or discarding the write buffers (lazy).

In addition to a simple programming model (locks are excluded from the programming interface), TM provides performance comparable to lock-based synchronization [98], especially for high contention and read-dominated workloads, and is composable. TM's first implementation was proposed in hardware, called hardware transactional memory (or HTM) [63]. HTM has the lowest overhead, but HTM transactions are usually limited in space and time. Examples of HTMs include TCC [58], UTM [1], Oklahoma [110], ASF [33], and Bulk [25]. TM implementation in software, called software transactional memory (or STM) was proposed later [106]. STM transactions do not need any special hardware, are not limited in size or time, and are more flexible. However, STM has a higher overhead, and thus lower performance, than HTM. Examples of STMs include RSTM [114], TinySTM [97], Deuce [74], and AtomJava [65].

Listing 1.1: STM example

```
BEGIN_TRANSACTION;
        stm::wr_ptr<Counter> wr(m_counter);
        wr->set_value(wr->get_value(wr) + 1, wr);
END_TRANSACTION;
```

Listing 1.1 shows an example STM code written by RSTM [107]'s interface. RSTM's `BEGIN_TRANSACTION` and `END_TRANSACTION` keywords are used to enclose a critical section, which creates a transaction for the enclosed code block and guarantees its atomic execution. First line inside the transaction makes a write pointer to a variable "m_counter" of type "Counter". The second line reads the current value of the counter variable through "wr−>get_value". The counter value is incremented through "wr−>set_value" operation.

Hybrid TM (or HyTM) was subsequently proposed in [83], which combines HTM with STM, and avoids their limitations. Examples of HyTMs include SpHT [81], VTM [95], HyTM [34], LogTM [90], and LogTM-SE [119].

1.2 STM for Real-Time Software

Given the hardware-independence of STM, which is a significant advantage, we focus on STM. STM's programmability, scalability, and composability advantages are also compelling for concurrency control in multicore embedded real-time software. However, this will require bounding transactional retries, as real-time threads, which subsume transactions, must satisfy application-level time constraints. Transactional retry bounds in STM are dependent on the CM policy at hand (analogous to the way thread response time bounds are OS scheduler-dependent).

Despite the large body of work on STM contention managers, relatively few results are known on real-time contention management. STM concurrency control for real-time systems has been previously studied, but in a limited way. For example, [87] proposes a restricted version of STM for uniprocessors. Uniprocessors do not need contention management. [49] bounds response times in distributed multicore systems with STM synchronization. They consider Pfair scheduling [70], which is largely only of theoretical interest[1], limit to small atomic regions with fixed size, and limit transaction execution to span at most two quanta. [99] presents real-time scheduling of transactions and serializes transactions based on transactional deadlines. However, the work does not bound transactional retries and response times.

[102] proposes real-time HTM, which of course, requires hardware with TM support. The retry bound developed in [102] assumes that the worst case conflict between atomic sections of different tasks occurs when the sections are released at the same time. We show that, this assumption does not cover the worst case scenario (see Chapter 4). [48] presents a contention manager that resolves conflicts using task deadlines. The work also establishes upper bounds on transactional retries and task response times. However, similar to [102], [48] also assumes that the worst case conflict between atomic sections occurs when the sections are released simultaneously. Besides, [48] assumes that all transactions have equal lengths. The ideas in [48] are extended in [9], which presents three real-time CM designs. But no retry bounds or schedulability analysis techniques are presented for those CMs.

[1] This is due to Pfair class of algorithm's time quantum-driven nature of scheduling and consequent high run-time overheads.

Thus, past efforts on real-time STM are limited, and do not answer important fundamental questions:

(1) How to design "general purpose" real-time STM contention managers for multicore architectures? By general purpose, we mean those that do not impose any restrictions on transactional properties (e.g., transaction lengths, number of transactional objects, levels of transactional nestings), which are key limitations of past work.

(2) What tight upper bounds exist for transactional retries and task response times under such real-time CMs?

(3) How does the schedulability of real-time CMs compare with that of lock-free synchronization? i.e., are there upper bounds or lower bounds for transaction lengths below or above which is STM superior to lock-free synchronization?

(4) How does transactional retry costs and task response times of real-time CMs compare with that of lock-free synchronization in practice (i.e., on average)?

1.3 Research Contributions

In this dissertation proposal, we answer these questions. We present a suite of real-time STM contention managers, called RCM, ECM, LCM, and PNF. The contention managers progressively improve transactional retry and task response time upper bounds (and consequently improve STM's schedulability advantages) and also relax the underlying task models. RCM and ECM resolve conflicts using fixed and dynamic priorities of real-time tasks, respectively, and are naturally intended to be used with the fixed priority (e.g., G-RMA [23]) and dynamic priority (e.g., G-EDF [23]) multicore real-time schedulers, respectively. LCM resolves conflicts based on task priorities as well as atomic section lengths, and can be used with G-EDF or G-RMA. Transactions under ECM, RCM and LCM can restart because of other transactions that share no objects with them. This is called transitive retry. PNF solves this problem. PNF also optimizes processor usage through reducing priority of aborted transactions. So, other tasks can proceed.

We establish upper bounds on transactional retry costs and task response times under all the contention managers through schedulability analysis. Since ECM and RCM conserve the semantics of the underlying real-time scheduler, their maximum transactional retry cost is double the maximum atomic section length. This is improved in the design of LCM, which achieves shorter retry costs. However, ECM, RCM, and LCM are affected by transitive retries when transactions access multiple objects. Transitive retry causes a transaction to abort and retry due to another non-conflicting transaction. PNG avoids transitive retry, and also optimizes processor usage by lowering the priority of retrying transactions, enabling other non-conflicting transactions to proceed.

We formally compare the schedulability of the proposed contention managers with lock-free synchronization. Our comparison reveals that, for most cases, ECM and RCM achieve higher schedulability than lock-free synchronization only when the atomic section length does not exceed half of lock-free synchronization's retry loop length. However, in some cases, the atomic section length can reach the lock-free retry loop length for ECM and it can even be larger than the lock-free retry loop-length for RCM, and yet higher schedulability can be achieved with STM. This means that, STM is more advantageous with G-RMA than with G-EDF.

LCM achieves shorter retry costs and response times than ECM and RCM. Importantly, the atomic section length range for which STM's schedulability advantage holds is significantly expanded with LCM (over that under ECM and RCM): Under ECM, RCM and LCM, transactional length should not exceed half of lock-free retry loop length to achieve better schedulability. However, with low contention, transactional length can increase to retry loop length under ECM. Under RCM, transactional length can be of many orders of magnitude of retry loop length with low contention. With suitable LCM parameters, transactional length under G-EDF/LCM can be twice as retry loop length. While in G-RMA/LCM, transactional length can be of many orders of magnitude as retry loop length. PNF achieves better schedulability than lock-free as long as transactional length does not exceed length of retry loop.

Why are we concerned about expanding STM's schedulability advantage? When STM's schedulability advantage holds, programmers can reap STM's significant programmability and composability benefits in multicore real-time software. Thus, by expanding STM's schedulability advantage, we increase the range of real-time software for which those benefits can be tapped. Our results, for the first time, thus provides a fundamental understanding of when to use, and not use, STM concurrency control in multicore real-time software.

We also implement the contention managers in the RSTM framework [107] and conduct experimental studies using the ChronOS multicore real-time Linux kernel [36]. Our studies confirm that, the contention managers achieve shorter retry costs than lock-free synchronization by as much as 95% improvement (on average). Among the contention managers, PNF performs the best in case of high transitive retry. PNF achieves shorter retry costs than ECM, RCM and LCM by as much as 53% improvement (on average).

1.4 Summary of Proposed Post Preliminary Research

Based on our current research results, we propose the following work:

Supporting nested transactions. Transactions can be nested *linearly*, where each transaction has at most one pending transaction [91]. Nesting can also be done in *parallel* where transactions execute concurrently within the same parent [115]. Linear nesting can be *1) flat:* If a child transaction aborts, then the parent transaction also aborts. If a child commits, no effect is taken until the parent commits. Modifications made by the child transaction are only visible to the parent until the parent commits, after which they are externally visible. *2) Closed:* Similar to *flat nesting*, except that if a child transaction conflicts, it is aborted and retried, without aborting the parent, potentially improving concurrency over flat nesting. *3) Open:* If a child transaction commits, its modifications are immediately externally visible, releasing memory isolation of objects used by the child, thereby potentially improving concurrency over closed nesting. However, if the parent conflicts after the child commits, then compensating actions are executed to undo the actions of the child, before retrying the parent and the child. We propose to develop real-time contention managers that allow these different nesting models and establish their retry and response time upper bounds. Additionally, we propose to formally compare their schedulability with nested critical sections under lock-based synchronization. Note that, nesting is not viable under lock-free synchronization.

Combinations and optimizations of LCM and PNF contention managers. LCM is designed to reduce the retry cost of a transaction when it is interfered close to the end of its execution. In contrast, PNF is designed to avoid transitive retry when transactions access multiple objects. An interesting direction is to combine the two contention managers to obtain the benefits of both algorithms. Further design optimizations may also be possible to reduce retry costs and response times, by considering additional criteria for resolving transactional conflicts. Importantly, we must also understand what are the schedulability advantages of such a combined/optimized CM over that of LCM and PNF, and how such a combined/optimized CM behaves in practice. This will be our second research direction.

Formal and experimental comparison with real-time locking protocols. Lock-free synchronization offers numerous advantages over locking protocols, but (coarse-grain) locking protocols have had significant traction in real-time systems due to their good programmability (even though their concurrency is low). Example such real-time locking protocols include PCP and its variants [27, 73, 94, 104], multicore PCP (MPCP) [77, 93], SRP [8, 24], multicore SRP (MSRP) [52], PIP [39], FMLP [16, 17, 68], and OMLP [11]. OMLP and FMLP are similar, and FMLP has been established to be superior to other protocols [20]. How does their schedulability compare with that of the proposed contention managers? How do they compare in practice? These questions constitute our third research direction.

1.5 Proposal Organization

The rest of this dissertation proposal is organized as follows. Chapter 2 overviews past and related work on real-time concurrency control. Chapter 3 describes our task/system model and assumptions. Chapter 4 describes the ECM and RCM contention managers, derives upper bounds for their retry costs and response times, and compares their schedulability between themselves and with lock-free synchronization. Chapters 5 and 6 similarly describe the LCM and PNF contention managers, respectively. Chapter **??** describes our implementation and reports our experimental studies. We conclude in Chapter **??**.

Chapter 2

Past and Related Work

Many mechanisms appeared for concurrency control for real-time systems. These methods include locking [24, 82], lock-free [3–5, 28, 31, 37, 46, 47, 69, 76] and wait-free [2, 12, 26, 28, 29, 32, 47, 66, 96, 111–113]. In general, real-time locking protocols have disadvantages like: 1) serialized access to shared object, resulting in reduced concurrency and reduced utilization. 2) increased overhead due to context switches. 3) possibility of deadlock when lock holder crashes. 3) some protocols requires apriori knowledge of ceiling priorities of locks. This is not always available. 4) Operating system data structures must be updates with this knowledge which reduces flexibility. For real-time lock-free, the most important problem is to bound number of failed retries and reduce cost of a single loop. The general technique to access lock-free objects is "retry-loop". Retry-loop uses atomic primitives (e.g., CAS) which is repeated until success. To access a specific data structure efficiently, lock-free technique is customized to that data structure. This increases difficulty of response time analysis. Primitive operations do not access multiple objects concurrently. Although some attempts made to enable multi-word CAS [3], but it is not available in commodity hardware [88]. For real-time wait-free protocols. It has a space problem due to use of multiple buffers. This is inefficient in some applications like small-memory real-time embedded systems. Wait-free has the same problem of lock-free in handling multiple objects.

The rest of this Chapter is organized as follows, Section 2.1 summarizes previous work on real-time locking protocols. In Section 2.2, we preview related work on lock-free and wait-free methods for real-time systems. Section 2.3 provides concurrency control under real-time database systems as a predecessor and inspiration for real-time STM. Section 2.4 previews related work on contention management. Contention management policy affects response time analysis of real-time STM.

2.1 Real-Time Locking Protocols

A lot of work has been done on real-time locking protocols. Locks in real-time systems can lead to priority inversion [24, 82]. Under priority inversion, a higher priority job is not allowed to run because it needs a resource locked by a lower priority job. Meanwhile, an intermediate priority job preempts the lower priority one and runs. Thus, the higher priority job is blocked because of a lower priority one. Different locking protocols appeared to solve this problem, but exposing other problems. Most of real-time blocking protocols are based on *Priority Inheritance Protocol (PIP)* [24, 39, 104], *Priority Ceiling Protocol (PCP)* [24, 27, 39, 73, 77, 93, 94, 104] and *Stack Resource Protocol (SRP)* [8, 24, 53].

In PIP [24, 104], resource access is done in FIFO order. A resource holder inherits highest priority of jobs blocked on that resource. When resource holder releases the resource and it holds no other resources, its priority is returned to its normal priority. If it holds other resources, its priority is returned to highest priority job blocked on other resources. Under PIP, a high priority job can be blocked by lower priority jobs for at most the minimum of number of lower priority jobs and number of shared resources. PIP suffers from chained blocking, in which a higher priority task is blocked for each accessed resource. Besides, PIP suffers from deadlock where each of two jobs needs resources held by the other. So, each job is blocked because of the other. [39] provides response time analysis for PIP when used with fixed-priority preemptive scheduling on multiprocessor system.

PCP [24, 94, 104] provides concept of priority ceiling. Priority ceiling of a resource is the highest priority of any job that can access that resource. For any job to enter a critical section, its priority should be higher the priority ceiling of any currently accessed resource. Otherwise, the resource holder inherits the highest priority of any blocked job. Under PCP, a job can be blocked for at most one critical section. PCP prevents deadlocks. [27] extends PCP to dynamically scheduled systems.

Two protocols extend PCP to multiprocessor systems: 1) *Multiprcoessor PCP (M-PCP)* [77, 93, 94] discriminates between global resources and local resources. Local resources are accessed by PCP. A global resource has a base priority greater than any task normal priority. Priority ceiling of a global resource equals sum of its base priority and highest priority of any job that can access it. A job uses a global resource at the priority ceiling of that resource. Requests for global resources are enqueued in a priority queue according to normal priority of requesting job. 2) *Parallel-PCP (P-PCP)* [39] extends PCP to deal with fixed priority preemptive multiprocessor scheduling. P-PCP, in contrast to PCP, allows lower priority jobs to allocate resources when higher priority jobs already access resources. Thus, increasing parallelism. Under P-PCP, a higher priority job can be blocked multiple times by a lower priority job. With reasonable priority assignment, blocking time by lower priority jobs is small. P-PCP uses α_i parameter to specify permitted number of jobs with basic priority lower than i and effective priority higher than i. When α_i is small, parallelism is reduced, so as well blocking from lower priority tasks. Reverse is true. [39] provides response time analysis for P-PCP.

[84] extends P-PCP to provide *Limited-Blocking PCP (LB-PCP)*. LB-PCP provides more control on indirect blocking from lower priority tasks. LB-PCP specify additional counters that control number of times higher priority jobs can be indirectly blocked without the need of reasonable priority assignment as in P-PCP. [84] analyzes response time of LB-PCP and experimentally compares it to P-PCP. Results show that LB-PCP is appropriate for task sets with medium utilization.

PCP can be unfair from blocking point of view. PCP can cause unnecessary and long blocking for tasks that do not need any resources. Thus, [73] provides Intelligent PCP (IPCP) to increase fairness and to work in dynamically configured system (i.e., no a priori information about number of tasks, priorities and accessed resources). IPCP initially optimizes priorities of tasks and resources through learning. Then, IPCP tunes priorities according to system wide parameters to achieve fairness. During the tuning phase, penalties are assigned to tasks according to number of higher priority tasks that can be blocked.

SRP [8, 24, 53] extends PCP to allow multiunit resources and dynamic priority scheduling and sharing runtime stack-based resources. SRP uses *preemption level* as a static parameter assigned to each task despite its dynamic priority. Resource ceiling is modified to include number of available resources and preemption levels. System ceiling is the highest resource ceiling. A task is not allowed to preempt unless it is the highest priority ready one, and its preemption level is higher than the system ceiling. Under SRP, a job can be blocked at most for one critical section. SRP prevents deadlocks. *Multiprocessor SRT (M-SRP)* [52] extends SRP to multiprocessor systems. M-SRP, as M-PCP, discriminates between local and global resources. Local resources are accessed by SRP. Request for global resource are enqueued in a FIFO queue for that resource. Tasks with pending requests busy-wait until their requests are granted.

Another set of protocols appeared for PFair scheduling [16]. [67] provide initial attempts to synchronize tasks with short and long resources under PFair. In Pfair scheduling, each task receives a weight that corresponds to its share in system resources. Tasks are scheduled in quanta, where each quantum has a specific job on a specific processor. Each lock has a FIFO queue. Requesting tasks are ordered in this FIFO queue. If a task is preempted during critical section, then other tasks can be blocked for additional time known as *frozen time*. Critical sections requesting short resources execute at most in two quanta. By early lock-request, critical section can finish in one quanta, avoiding the additional blocking time. [67] proposes two protocols to deal with short resources: 1) *Skip Protocol (SP)* leaves any lock request in the FIFO queue during frozen interval until requesting task is scheduled again. 2) *Rollback Protocol (RP)* discards any request in the FIFO queue for the lock during frozen time. For long resources, [67] uses *Static Weight Server Protocol (SWSP)* where requests for each resource l is issued to a corresponding server S. S orders requests in a FIFO queue and has a static specific weight.

Flexible Multiprocessor Locking Protocol (FMLP) [16] is the most famous synchronization protocol for PFair scheduling. The FMLP allows non-nested and nested resources access without constraints. FMLP is used under global and partitioned deadline scheduling. Short or long resource is user defined. Resources can be grouped if they are nested by some task and have the same type. Request to a specific resource is issued to its containing group. Short groups are protected by non-preemptive FIFO queue locks, while long groups are protected by FIFO semaphore queues. Tasks busy-wait for short resources and suspend on long resources. Short request execute non-preemptively. Requests for long resources cannot be contained within requests for short resources. A job executing a long request inherits highest priority of blocked jobs on that resource's group. FMLP is deadlock free.

[18] is concerned with suspension protocols. Schedulability analysis for suspension protocols can be suspension-oblivious or suspension-aware. In suspension-oblivious, suspension time is added to task execution. While in suspension-aware, it is not. [18] provides *Optimal Multiprocessor Locking Protocol (OMLP)*. Under OMLP, each resource has a FIFO queue of length at most m, and a priority queue. Requests for each resource are enqueued in the corresponding FIFO queue. If FIFO queue is full, requests are added to the priority queue according to the requesting job's priority. The head of the FIFO queue is the resource holding task. Other queued requests are suspended until their turn come. OMLP achieves $O(m)$ priority inversion (pi) blocking per job under suspension oblivious analysis. This is why OMLP is asymptotically optimal under suspension oblivious analysis. Under suspension aware analysis, FMLP is asymptotically optimal. [19] extends work in [18] to clustered-based scheduled multiprocessor system. [19] provides concept of *priority donation* to ensure that each job is preempted at most once. In priority donation, a resource holder priority can be unconditionally increased. Thus, a resource holder can preempt another task. The preempted task is predetermined such that each job is preempted at most once. OMLP with priority donation can be integrated with k-exclusion locks (K-OMLP). Under K-exclusion locks, there are k instances of the same resource than can be allocated concurrently. K-OMLP has the same structure of OMLP except that there are K FIFO queues for each resource. Each FIFO queue corresponds to one of the k instances. K-OMLP has $O(m/k)$ bound for pi-blocking under s-oblivious analysis. [43] extends the K-OMLP in [19] to global scheduled multiprocessor systems. The new protocol is *Optimal K-Exclusion Global Locking Protocol (O-KGLP)*. Despite global scheduling is a special case of clustering, K-OMLP provides additional cost to tasks requesting no resources if K-OMLP is used with global scheduling. O-KGLP avoids this problem.

2.2 Real-Time Lock-Free and Wait-Free Synchronization

Due to locking problems (e,g,. priority inversion, high overhead and deadlock), research has been done on non-blocking synchronization using lock-free [3–5,28,31,46,47,69,76] and wait-free algorithms [2,12,26,28,29,32,47,66,96,111–113]. Lock-free iterates an atomic primitive (e.g., CAS) inside a retry loop until successfully accessing object. When used with real-time systems, number of failed retries must be bounded [3, 4]. Otherwise, tasks are highly likely to miss their deadlines. Wait-free algorithms, on the other hand, bound number of object access by any operation due to use of sized buffers. Synchronization under wait-free is concerned with: 1) single-writer/multi-readers where a number of reading operations may conflict with one writer. 2) multi-writer/multi-reader where a number of reading operations may conflict with number of writers. The problem with wait-free algorithms is its space cost. As embedded real-time systems are concerned with both time and space complexity, some work appeared trying to combine benefits of locking and wait-free.

[4] considers lock-free synchronization for hard-real time, periodic, uniprocessor systems. [4] upper bounds retry loop failures and derives schedulability conditions with Rate Monotonic (RM), and Earliest Deadline First (EDF). [4] compares, formally and experimentally, lock-free objects against locking protocols. [4] concludes that lock-free objects often require less overhead than locking-protocols. They require no information about tasks and allow addition of new tasks simply. Besides, lock-free object do not induce excessive context switches nor priority inversion. On the other hand, locking protocols allow nesting. Besides, performance of lock-free depends on the cost of "retry-loops". [3] extends [4] to generate a general framework for implementing lock-free objects in uniprcocessr real-time systems. The framework tackles the problem of multi-objects lock-free operations and transactions through multi-word compare and swap (MWCAS) implementation. [3] provides a general approach to calculate cost of operation interference based on linear programming. [3] compares the proposed framework with real-time locking protocols. Lock-free objects are prefered if cost of retry-loop is less than cost of lock-access-unlock sequence. [5] extends [3,4] to use lock-free objects in building memory-resident transactions for uniprocessor real-time systems. Lock-free transactions, in contrast to lock-based transactions, do not suffer from priority inversion, deadlocks, complicated data-logging and rolling back. Lock-free transaction do not require kernel support.

[37] presents two synchronization methods under G-EDF scheduled real-time multiprocessor systems for simple objects. The first synchronization technique uses queue-based spin locks, while the other uses lock-free. The queue lock is FIFO ordered. Each task appends an entry at the end of the queue, and spins on it. While the task is spinning, it is non-preemptive. The queue could have been priority-based but this complicates design and does not enhance worst case response time analysis. Spinning is suitable for short critical sections. Disabling preemption requires kernel support. So, second synchronization method uses lock-free objects. [37] bounds number of retries. [37] , analytically and experimentally, evaluates both synchronization techniques for soft and hard real-time analysis. [37] concludes that queue locks have a little overhead. They are suitable for small number of shared object operations per task. Queue locks are not generally suitable for nesting. Lock-free have high overhead compared with queue locks. Lock-free is suitable for small number of processors and object calls in the absence of kernel support.

[69] uses lock-free objects under PFair scheduling for multiprocessor system. [69] provides

concept of *supertasking* to reduce contention and number of failed retries. This is done by collecting jobs that need a common resource into the same supertask. Members of the same supertask run on the same processor. Thus, they cannot content together. [69] upper bounds worst case duration for lock-free object access with and without supertasking. [69] optimizes, not replaces, locks by lock-free objects. Locks are still used in situations like sharing external devices and accessing complex objects.

Lock-free objects are used with time utility models where importance and criticality of tasks are separated [31,76]. [76] presents *MK-Lock-Free Utility Accrual (MK-LFUA)* algorithm that minimizes system level energy consumption with lock-free synchronization. [31] uses lock-free synchronization for dynamic embedded real-time systems with resource overloads and arbitrary activity arrivals. Arbitrary activity arrivals are modelled with Universal Arrival Model (UAM). Lock-free retries are upper bounded. [31] identifies the conditions under which lock-free is better than lock-based sharing. [46] builds a lock-free linked-list queue on a multi-core ARM processor.

Wait-free protocols use multiple buffers for readers and writers. For single-writer/multiple-readers, each object has a number of buffers proportional to maximum number of reader's preemptions by the writer. This bounds number of reader's preemptions. Readers and writers can use different buffers without interfering each other.

[32] presents wait-free protocol for single-writer/multiple-readers in small memory embedded real-time systems. [32] proves space optimality of the proposed protocol, as it required the minimum number of buffers. The protocol is safe and orderly. [32] also proves, analytically and experimentally, that the protocol requires less space than other wait-free protocols. [29] extends [32] to present wait-free utility accrual real-time scheduling algorithms (RUA and DASA) for real-time embedded systems. [29] derives lower bounds on accrued utility compared with lock-based counterparts while minimizing additional space cost. Wait-free algorithms experimentally exhibit optimal utility for step time utility functions during underload, and higher utility than locks for non-step utility functions. [96] uses wait-free to build three types of concurrent objects for real-time systems. Built objects has persistent states even if they crash. [113] provides wait-free queue implementation for real-time Java specifications.

A number of wait-free protocols were developed to solve multi-writer/multi-reader problem in real-time systems. [112] provides m-writer/n-reader non-blocking synchronization protocol for real-time multiprocessor system. The protocol needs $n + m + 1$ slots. [112] provides schedulability analysis of the protocol. [2] presents wait-free methods for multi-writer/multi-reader in real-time multiprocessor system. The proposed algorithms are used for both priority and quantum based scheduling. For a B word buffer, the proposed algorithms exhibit $O(B)$ time complexity for reading and writing, and $\Theta(B)$ space complexity. [111] provides a space-efficient wait-free implementation for n-writer/n-reader synchronization in real-time multiprocessor system. The proposed algorithm uses timestamps to implement the shared buffer. [111] uses real-time properties to bound timestamps. [26] presents wait-free implementation of the multi-writer/multi-reader problem for real-time multiprocessor synchronization. The proposed mechanism replicates single-writer/multi-reader to solve the multi-writer/multi-reader problem. [26], as [111], uses real-time properties to ensure data coherence through timestamps.

Each synchronization technique has its benefits. So, a lot of work compares between locking, lock-free and wait-free algorithms. [47] compares building snapshot tool for real-time system using locking, lock-free and wait-free. [47] analytically and experimentally compares the three methods. [47] concludes that wait-free is better than its competitors. [28] presents synchronization techniques under LNREF [30] (an optimal real-time multiprocessor scheduler) for simple data structures. Synchronization mechanisms include lock-based, lock-free and wait-free. [28] derives minimum space cost for wait-free synchronization. [28] compares, analytically and experimentally, between lock-free and lock-based synchronization under LNREF.

Some work tried to combine different synchronization techniques to combine their benefits. [66] uses combination of lock-free and wait-free to build real-time systems. Lock-free is used only when CAS suffices. The proposed design aims at allowing good real-time properties of the system, thus better schedulability. The design also aims at reducing synchronization overhead on uni and multiprocessor systems. The proposed mechanism is used to implement a micro-kernel interface for a uni-processor system. [12] combines locking and wait-free for real-time multiprocessor synchronization. This combination aims to reduce required space cost compared to pure wait-free algorithms, and blocking time compared to pure locking algorithms. The proposed scheme is jsut an idea. No formal analysis nor implementation is provided.

2.3 Real-Time Database Concurrency Control

Real-time database systems (RTDBS) is not a synchronization technique. It is a predecessor and inspiration for real-time transactional memory. RTDBS itself uses synchronization techniques when transactions conflict together. RTDBS is concerned not only with logical data consistency, but also with temporal time constraints imposed on transactions. Temporal time constraints require transactions finish before their deadlines. External constraints require updating temporal data periodically to keep freshness of database. RTDBS allow mixed types of transactions. But a whole transaction is of one type. In real-time TM, a single task may contain atomic and non-atomic sections.

High-Priority two Phase Locking (HP-2PL) protocol [78, 79, 92, 121] and *Real-Time Optimistic Concurrency (RT-OCC)* protocol [35, 50, 78–80, 121] are the most two common protocols for RTDBS concurrency . HP-2PL works like 2PL except that when a higher priority transaction request a lock held by a lower priority transaction, lower priority transaction releases the lock in favor of the higher priority one. Then, lower priority transaction restarts. RT-OCC delays conflict resolution till transaction validation. If validating transaction cannot be serialized with conflicting transactions, a priority scheme is used to determine which transaction to restart. In *Optimistic Concurrency Control with Broadcast Commit (OCC-BC)*, all conflicting transactions with the validating one are restarted. HP-2PL may encounter deadlock and long blocking times, while transactions under RT-OCC suffer from restart time at validation point.

Other protocols were developed based on HP-2PL [78, 79, 92] and RT-OCC [7, 50, 78, 80]. HP-2PL, and its derivatives, are similar to locking protocols in real-time systems. They have the same problems in real-time locking protocols like priority inversion. So, the same solutions exist for the RTDBS locking protocols. Despite RT-OCC, and its derivatives, use locks in their implementation, their behaviour is closer to abort and retry semantics in TM. Some work integrates different protocols to handle different situations [92, 120].

[78] presents *Reduced Ceiling Protocol (RCP)* which is a combination of *Priority Ceiling Protocol (PCP)* and *Optimistic Concurrency Protocol (OCC)*. RCP targets database systems with mixed hard and soft real-time transactions (RTDBS). RCP aims at guarantee of schedulabiltiy of hard real-time transactions, and minimizing deadline miss of soft real-time transactions. Soft real-time transactions are blocked in favor of conflicting hard real-time transactions. While hard real-time transactions use PCP to synchrnonize among themselves, soft real-time transactions use OCC. Hard real-time transactions access locks in a *tow phase locking (2PL)* fashion. Seized locks are released as soon as hard real-time transaction no longer need them. This reduces blocking time of soft real-time transactions. [78] derives analytical and experimental evaluation of RCP against other synchronization protocols.

[120], like [78], deals with mixed transaction. [120] classifies mixed transactions into hard (HRT), soft (SRT) and non (NRT) real-time transactions. HRT has higher priority than SRT. SRT has higher priority than NRT. [120] aims at guranting deadlines of HRTs, minimizing miss rate of SRTs and reducing response time of NRTs. So, [120] deals with inter and intra-transaction concurrency. HRTs use PCP for concurrency control among themselves. SRTs use WAIT-50, and NRTs use 2PL. SRT and NRT are blocked or aborted in favor of HRT. If NRT requests a lock held by SRT, then NRT is blocked. If SRT requests a lock held by NRT, WAIT-50 is applied. Experimental evaluation showed effective improvement in overall system performance. Performance objectives of each transaction type was met.

[50] is concerned with semantic lock concurrency control. The semantic lock technique allows negotiation between logical and temporal constraints of data and transactions. It also controls imprecision resulting from negotiation. Thus, the semantic lock considers scheduling and concurrency of transactions. Semantic lock uses a compatibility function to determine if the release transaction is allowed to proceed or not.

Time Interval OCC protocols try to reduce number of transaction restarts by dynamic adjustment of serialization timestamps. Time interval OCC may encounter unnecessary restarts. [7] presents Timestamp Vector based OCC to resolve these unnecessary restarts. Timestamp Vector base OCC uses a timestamp vector instead of a single timestamp as in Time Interval OCC protocols. Experimental comparison between Timestamp Vector OCC and previous

Time Interval OCC shows higher performance of Timestamp Vector OCC.

[35] aims to investigate performance improvement of priority congnizant OCC over incognizant counterparts. In OCC-BC, all conflicting transactions with the validating transaction are restarted. [35] wonders if it is really worthy to sacrifice all other transactions in favor of one transaction. [35] proposes *Optimistic Concurrency Control- Adaptive PRiority (OCC-APR)* to answer this question. A validating transaction is restarted if it has sufficient time to its deadline if restarted, and higher priority transactions cannot be serialized with the conflicting transaction. Sufficient time estimate is adapted according to system feedback. System feedback is affected by contention level. [35] experimentally concludes that integrating priority into concurrency control management is not very useful. Time Interval OCC showed better performance.

WAIT-X [35, 80] is one of the optimistic concurrency control (OCC) protocols. WAIT-X is a prospective (forward validation) OCC. Prospective means it detects conflicts between a validating transaction and conflicting transaction that may commit in the future. In retrospective (backward validation) protocols, conflicts are detected between a validating transaction and already committed transactions. Retrospective validation aborts validating transaction if it cannot be serialized with already committed conflicting transactions. When WAIT-X detects a conflict, it can either abort validating transaction, or commit validating transaction and abort other conflicting transactions, or it can dealy validating a transction slightly hoping that conflicts resolve themsleves someway. Which action to take is a function of priorities of vlaidating and conflicting transactions. WAIT-X can delay validating transaction until percetage of higher priority transactions in the conflict set is lower than X%. WAIT-50 is a common implementation of WAIT-X.

[75] is concerned with concurrency control for multiprocessor RTDBS. [75] uses priority cap to modify *Reader/Write Prirority Ceiling Protocol (RWPCP)* [105] to work on multiprocessor systems. The proposed protocol, named *One Priority Inversion RWPCP (1PI-RWPCP)*, is deadlock-free and bounds number of priority inversions for any transaction to one. [75] derives feasiblity condition for any transaction under 1PI-RWPCP. [75] experimentally compares performance of 1PI-RWPCP against RWPCP.

[92] combines locking, multi-version and valid confirmation concurrency control mechanisms. The proposed method adopts different concurrency control mechanism according to idiographic situation. Experiments show lower rate of transactional restart of the proposed mechanis compared to 2PL-HP.

[79] is concerned with RTDBS containing periodically updated data and one time transactactions. [79] provides two new concurrency control protocols to balance freshness of data and transaction performance. [79] proposes *HP-2PL with Delayed Restart (HP-2PL-DR)* and *HP-2PL with Delayed Restart and Pre-declaration (HP-2PL-DRP)* based on HP-2PL. Before a transaction T restarts in HP-2PL-DR, next update time of each temporal data accessed by T is checked. If next update time starts before currently re-executing T, then T's restart time is delayed until the next udpate. Otherwise, T is restarted immediately. If T_r and

T_n are two transactions under HP-2PL-DRT. T_r is requesting a lock held by T_n. If priority of T_r is greater than priority of T_n, then T_n releases the lock in favor of T_r. Othewise, T_r fails. If T_n releases the lock and T_n is a one time transaction, then T_n restarts immediately. Otherwise, T_n lock waiting time is updated. Experiments show improved performance of HP-2PL-DR and HP-2PL-DRP over HP-2PL.

2.4 Real-Time TM Concurrency Control

Concurrency control in TM is done through contention managers. Contention managers are used to ensure progress of transactions. If one or more transactions conflict on an object, contention manager decides which transaction to commit. Other transactions abort or wait. Mostly, contention managers are *distributed* or *decentralized* [56, 57, 100, 101], in the sense that each transaction maintains its own contention manager. Contention managers may not know which objects will be needed by transactions and their duration. Past work on contention managers can be classified into two classes: 1) Contention management policy that decides which transaction commits and which do other actions [55–57, 100, 101, 109]. 2) Implementation of contention management policy in practice [15,38,54,86,100,109]. The two classes are orthogonal. The second class tries to increase the benefit of the the contention management policy in reality by considering different aspects in TM design (e.g., lazy versus eager, visible versus invisible readers). Second class suggests contention managers should be proactive instead of reactive. This can prevent conflicts before they happen. Contention managers can be supported a lot if they are integrated into system schedulers. This provides a global view of the system (due to applications feedback) and reduces overhead of the implementation of contention manager.

Contention management policy ranges from never aborting enemies to always aborting them [100, 101]. These two extremes can lead to deadlock, starvation, livelock and major loss of performance. Contention manager policy lies in between. Depending on heuristics, contention manager balances between decisions complexity against quality and overhead.

Different types of contention management policies can be found in [55–57, 100, 101, 109] like:

1. Passive and Aggressive: Passive contention manager aborts current transaction, while aggressive aborts enemy.

2. Polite: When conflicting on an object, a transaction spins exponensially for average of $2^{(n+k)}\, ns$, where n is number of times to access the object, and k is a tuning parameter. Spinning times is bounded by m. Afterwards, any enemy is aborted.

3. Karma: It assigns priorities to transaction based on the amount of work done so far. Amount of work is measured by number of opened objects by current transaction. Higher priority transaction aborts lower priority one. If lower priority transaction tries

to access an object for a number of times greater than priority difference between itself and higher priority transaction, enemy is aborted.

4. Eruption: It works like Karma except it adds priority of blocked transaction to the transaction blocking it. This way, enemy is sped-up, allowing blocked transactions to complete faster.

5. Kindergarten: A transaction maintains a hit list (initially empty) of enemies who previously caused current thread to abort. When a new enemy is encountered, current transaction backs off for a limited amount of time. The new enemy is recorded in the hit list. If the enemy is already in the hit list, it is aborted. If current transaction is still blocked afterwards, then it is aborted.

6. Timestamp: It is a fair contention manager. Each transaction gets a timestamp when it begins. Transaction with newer timestamp is aborted in favour of the older. Otherwise, transaction waits for a fixed intervals, marking the enemy flag as defunct. If the enemy is not done afterwards, it is killed. Active transaction clear their flag when they notice it is set.

7. Greedy: Each transaction is given a timestamp when it starts. The earlier the timestamp of a transaction, the higher its priority. If transaction A conflicts with transaction B, and B is of lower priority or is waiting for another transaction, then A aborts B. Otherwise, A waits for B to commit, abort or starts waiting.

8. Randomized: It aborts current transaction with some probability p, and waits with probability $1 - p$.

9. PublishedTimestamp: It works like Timestamp contention manager except it has a new definition for an "inactive" transaction. Each transaction maintains a "recency" flag. Recency flag is updated every time the transaction makes a request. Each transaction maintains its own "inactivity" threshold parameter that is doubled every time it is aborted up to a specific limit. If the enemy "recency" flag is behind the system global time by amount exceeding its "inactivity" threshold, then enemy is aborted.

10. Polka: It is a combination of Polite and Karma contention managers. Like Karma, it assigns priorities based on amount of job done so far. A transaction backs off for a number of intervals equals difference in priority between itself and its enemy. Unlike Karma, back-off length increases exponentially.

11. Prioritized version of some of the previous contention managers appeared. Prioritized contention managers include base priority of the thread holding the transaction into contention manager policy. This way, higher priority threads are more favoured.

[6] compares performance of different contention managers against an optimal, clairvoyant contention manager. The optimal contention manager knows all resources needed by each

transaction, as well as its release time and duration. Comparison is based on the "makespan" concept which is amount of time needed to finish a specific set of transactions. The ratio between makespan of analyzed contention manager and the makespan of the optimal contention manager is known as competitive ratio. [6] proves that any contention manager can be of $O(s)$ competitive ratio if the contention manager is work conserving (i.e., always lets the maximal set of non-conflicting transactions run), and satisfies pending property [56]. The paper proves that this result is asymptotically tight as no on-line work conserving contention manager can achieve better result. [6] also proves that the makespan of greedy contention manager is $O(s)$ instead of $O(s^2)$ [56]. This allows transactions of arbitrary release time and durations in contrast to what is assumed in [56]. For randomized contention managers, a lower bound of $\Omega(s)$ if transaction can modify their resource needs when they are reinvoked.

[55] analyzes different contention managers under different situations. [55] concludes that no single contention manager is suitable for all cases. Thus, [55] proposes a polymorphic contention manager that changes contention managers on the fly throughout different loads, concurrent threads of single load and even different phases of a single thread. To implement polymorphic contention manager, it is important to resolve conflicts resulting from different contention managers in the same application by different methods. The easiest way is to abort the enemy contention manager if it is of different type. [55] uses generic priorities for each transaction regardless of the transaction's contention manager. Upon conflict between different classes of contention manager, highest priority transaction is committed.

[109] provides a comprehensive contention manager attempting to achieve low overhead for low contention, and good throughput and fairness in case of high contention. The main components of comprehensive contention manager are lazy acquisition, extendable timestamp-based conflict detection, and efficient method for capturing conflicts and priorities.

[86] is concerned with implementation issues. [86] considers problems resulting from previous contention management policies like backing off and waiting for time intervals. These strategies make transactions suffer from many aborts that may lead to livelocks, and increased vulnerability to abort because of transactional preemption due to higher priority tasks. Imprecise information and unpredictable benefits resulting from handling long transactions make it difficult to make correct conflict resolution decisions. [86] discriminates between decisions for long and short transactions, as well as, number of threads larger or lower than number of cores. [86] suggests a number of user and kernel level support mechanisms for contention managers, attempting to reduce overhead in current contention managers' implementations. Instead of spin-locks and system calls, the paper uses shared memory segments for communication between kernel and STM library. It also proposes reducing priority of loser threads instead of aborting them. [86] increases time slices for transactions before they are preempted by higher priority threads. This way, long transactions can commit quickly before they are suspended, reducing abort numbers.

For high number of cores, back-off strategies perform poorly. This is due to hot spots created by small set of conflicts. These hotspots repeat in predictable manner. [15] introduces proactive contention manger that uses history to predict these hotspots and scheduler transactions around them without programmer's input. Proactive contention manager is useful in high contention, but has high cost for low contention. So, [15] uses a hybrid contention managers that begins with back-off strategy for low contention. After a specific threshold for contention level, hybrid contention manager switches to proactive manager.

Contention managers concentrate on preventing starvation through fair policies. They are not suitable for specific systems like real-time systems where stronger behavioural guarantees are required. [54] proposes user-defined priority transactions to make contention management suitable for these specific systems. It investigates the correlation between consistency checking (i.e., finding memory conflicts) and user-defined priority transactions. Transaction priority can be static or dynamic. Dynamic priority increases as abort numbers of transaction increases.

Contention managers are limited in: 1) they are reactive, and suitable only for imminent conflicts. They do not specify when aborted transaction should restart, making them conflict again easily. 2) Contention managers are decentralized because they consume a large part of traffic during high contention. Decentralization prevents global view of the system and limit contention management policy to heuristics. 3) As contention managers are user-level modules, it is difficult to integrate them in HTM. [100] tackles the previous problems by *adaptive transaction scheduling* (ATS). ATS uses contention intensity feedback from the application to adaptively decide number of concurrent transactions running within critical sections. ATS is called only when transaction starts in high contention. Thus, resulting traffic is low and scheduler can be centralized. ATS is integrated into HTM and STM.

[38] presents CAR-STM, a scheduling-based mechanism for STM collision avoidance and resolution. CAR-STM maintains a transaction queue per each core. Each transaction is assigned to a queue by a dispatcher. At the beginning of the transaction, dispatcher uses a conflict probability method to determine the suitable queue for the transaction. The queue with high contention for the current transaction is the most suitable one. All transactions in the same queue are executed by the same thread, thus they are serialized and cannot collide together. CAR-STM uses a serializing contention manager. If one transaction conflicts with another transaction, the former transaction is moved to the queue of the latter. This prevents further collision between them unless the second transaction is moved to a third queue. Thus, CAR-STM uses another serialization strategy in which the two transactions are moved to the third queue. This guarantees conflict between transactions for at most once.

[88] uses HTM to build single and double linked queue, and limited capacity queue. HTM is used as an alternative synchronization operation to CAS and locks. [88] provides worst case time analysis for the implemented data structures. It experimentally compares the implemented data structures with CAS and lock. [88] reverses the role of TM. Transactions are used to build the data structure, instead of accessing data structures inside transactions. [103] presents an implementation for HTM in a Java chip multiprocessor system (CMP).

The used processor is JOP, where worst case execution time analysis is supported.

[10] presents two steps to minimize and limit number of transactional aborts in real-time multiprocessor embedded systems. [10] assumes tasks are scheduled under partitioned EDF. Each task contains at most one transaction. [10] uses multi-versioned STM. In this method, read-only transactions use recent and consistent snapshot of their read sets. Thus, they do not conflict with other transactions and commit on first try. This reduction in abort number comes at the cost of increased memory storage for different versions. [10] uses real-time characteristics to bound maximum number of required versions for each object. Thus, required space is bounded. [10] serializes conflicting transaction in a chronological order. Ties are broken using least laxity and processor identification. [10] does not provide experimental evaluation of its work.

[13] studies the effect of eager versus lazy conflict detection on real-time schedulability. In eager validation, conflicts are detected as soon as they occur. One of the conflicting transactions should be aborted immediately. In lazy validation, conflict detection is delayed to commit time. [13] assumes each task is a complete transaction. [13] proves that synchronous release of tasks does not necessarily lead to worst case response time of tasks. [13] also proves that lazy validation will always result in a longer or equal response time than eager validation. Experiments show that this gap is quite high if higher priority tasks interfere with lower priority ones.

[85]proposes an adaptive scheme to meet deadlines of transactions. This adaptive scheme collects statistical information about execution length of transactions. A transaction can execute in any of three modes depending on its closeness to deadline. These modes are optimistic, visible read and irrevocable. The optimistic mode defers conflict detection to commit time. In visible read, other transactions are informed that a particular location has been read and subject to conflict. Irrevocable mode prevents transaction from aborting. As a transaction gets closer to its deadline, it moves from optimistic to visible read to irrevocable mode. Deadline transactions are supported by the underlying scheduler by disabling preemption for them. Experimental evaluation shows improvement in number of committed transactions without noticeable degradation in transactional throughput.

Chapter 3

Models and Assumptions

We consider a multicore system with m identical processors and n sporadic tasks $\tau_1, \tau_2, \ldots, \tau_n$. The k^{th} instance (or job) of a task τ_i is denoted τ_i^k. Each task τ_i is specified by its worst case execution time (WCET) c_i, its minimum period T_i between any two consecutive instances, and its relative deadline D_i, where $D_i = T_i$. Job τ_i^j is released at time r_i^j and must finish no later than its absolute deadline $d_i^j = r_i^j + D_i$. Under a fixed priority scheduler such as G-RMA, p_i determines τ_i's (fixed) priority and it is constant for all instances of τ_i. Under a dynamic priority scheduler such as G-EDF, τ_i^j's priority, p_i^j, is determined by its absolute deadline. A task τ_j may interfere with task τ_i for a number of times during a duration L, and this number is denoted as $G_{ij}(L)$. τ_j's workload that interferes with τ_i during L is denoted $W_{ij}(L)$.

Shared objects. A task may need to access (i.e., read, write) shared, in-memory objects while it is executing any of its atomic sections, which are synchronized using STM. The set of atomic sections of task τ_i is denoted s_i. s_i^k is the k^{th} atomic section of τ_i. Each object, θ, can be accessed by multiple tasks. The set of distinct objects accessed by τ_i is θ_i. The set of atomic sections used by τ_i to access θ is $s_i(\theta)$, and the sum of the lengths of those atomic sections is $len(s_i(\theta))$. $s_i^k(\theta)$ is the k^{th} atomic section of τ_i that accesses θ. $s_i^k(\theta_1, \theta_2, .., \theta_n)$ is the k^{th} atomic section of τ_i that accesses objects $\theta_1, \theta_2, .., \theta_n$. $s_i^k(\theta)$ executes for a duration $len(s_i^k(\theta))$.

If θ is shared by multiple tasks, then $s(\theta)$ is the set of atomic sections of all tasks accessing θ, and the set of tasks sharing θ with τ_i is denoted $\gamma_i(\theta)$. Atomic sections are non-nested. Each atomic section is assumed to access only one object; this allows a head-to-head comparison with lock-free synchronization [37]. (Allowing multiple object access per transaction is future work.) The maximum-length atomic section in τ_i that accesses θ is denoted $s_{i_{max}}(\theta)$, while the maximum one among all tasks is $s_{max}(\theta)$, and the maximum one among tasks with priorities lower than that of τ_i is $s_{max}^i(\theta)$.

STM retry cost. If two or more atomic sections conflict, the CM will commit one section and abort and retry the others, increasing the time to execute the aborted sections. The increased time that an atomic section $s_i^p(\theta)$ will take to execute due to interference with another section $s_j^k(\theta)$, is denoted $W_i^p(s_j^k(\theta))$. The total time that a task τ_i's atomic sections have to retry is denoted $RC(\tau_i)$. When this retry cost is calculated over the task period T_i or an interval L, it is denoted, respectively, as $RC(T_i)$ and $RC(L)$.

Chapter 4

The ECM and RCM Contention Managers

We consider software transactional memory (STM) for concurrency control in multicore embedded real-time software. We investigate real-time contention managers (CMs) for resolving transactional conflicts, including those based on dynamic and fixed priorities, and establish upper bounds on transactional retries and task response times. We identify the conditions under which STM (with the proposed CMs) is superior to lock-free synchronization [42] and real-time locking protocols (i.e., OMLP [18, 21] and RNLP [116]).

The rest of this Chapter is organized as follows, Section 4.1 investigates Earliest Deadline Contention Manager under G-EDF scheduling (ECM) and illustrates its behaviour. We provide computations of workload interference and retry cost analysis under ECM. Section 4.2 presents Rate Monotonic Contention Manager under G-RMA scheduling (RCM). It also includes retry cost and response time analysis under RCM. Schedulability of ECM and RCM is compared against schedulability of lock-free in Section 4.3 and real-time locking protocols in Section 4.4. We conclude the Chapter in Section 4.5.

4.1 ECM

Since only one atomic section among many that share the same object can commit at any time under STM, those atomic sections execute in sequential order. A task τ_i's atomic sections are interfered by other tasks that share the same objects with τ_i. Hereafter, we will use *ECM* to refer to a multicore system scheduled by G-EDF and resolves STM conflicts using the EDF CM. ECM was originally introduced in [48]. ECM will abort and retry an atomic section of τ_i^x, s_i^k due to a conflicting atomic section of τ_j^y, s_j^l, if the absolute deadline of τ_j^y is less than or equal to the absolute deadline of τ_i^x. ECM behaviour is shown in Algorithm 1. [48] assumes the worst case scenario for transactional retry occurs when conflicting transactions are released simultaneously. [48] also assumes all transactions have the same length. Here, we extend the analysis in [48] to a more worse conflicting scenario and consider distinct-length transactions. We also consider lower number of conflicting instances of any job τ_j^y to another job τ_i^x.

Algorithm 1: ECM

Data: $s_i^k \rightarrow$ interfered atomic section. $s_j^l \rightarrow$ interfering atomic section
Result: which atomic section aborts

```
if d_i^k < d_j^l then
    s_j^l aborts;
else
    s_i^k aborts;
end
```

4.1.1 Illustrative Example

Behaviour of ECM can be illustrated by the following example:

- Transaction $s_i^k \in \tau_i^x$ begins execution. Currently, s_i^k does not conflict with any other transaction.
- Transaction $s_j^l \in \tau_j^y$ is released while s_i^k is still running. $\Theta_i^k \cap \Theta_j^l \neq \emptyset$. $d_j^y < d_i^x$. So, $p_j^y > p_i^x$. Hence, ECM will abort and restart s_i^k in favour of s_j^l.
- Transaction $s_h^v \in \tau_h^u$ is released while s_j^l is still running. $d_h^u < d_j^y < d_i^x$. So, $p_h^u > p_j^y > p_i^x$. s_j^l and s_i^k will abort and retry until s_h^v commits.
- s_h^v commits. s_j^l executes while s_i^k aborts and retries.
- s_j^l commits. s_i^k executes.

4.1.2 Transitive Retry

With multiple objects per transaction, ECM will face transitive retry, which we illustrate with an example.

Example 1. Consider three atomic sections s_1^x, s_2^y, and s_3^z belonging to jobs τ_1^x,τ_2^y, and τ_3^z, with priorities $p_3^z > p_2^y > p_1^x$, respectively. Assume that s_1^x and s_2^y share objects, s_2^y and s_3^z share objects. s_1^x and s_3^z do not share objects. s_3^z can cause s_2^y to retry, which in turn will cause s_1^x to retry. This means that s_1^x may retry transitively because of s_3^z, which will increase the retry cost of s_1^x.

Assume another atomic section s_4^f is introduced. Priority of s_4^f is higher than priority of s_3^z. s_4^f shares objects only with s_3^z. Thus, s_4^f can make s_3^z to retry, which in turn will make s_2^y to retry, and finally, s_1^x to retry. Thus, transitive retry will move from s_4^f to s_1^x, increasing the retry cost of s_1^x. The situation gets worse as more tasks of higher priorities are added, where each task shares objects with its immediate lower priority task. τ_3^z may have atomic sections that share objects with τ_1^x, but this will not prevent the effect of transitive retry due to s_1^x.

Definition 1. ***Transitive(indirect) Retry:*** *A transaction s_i^k suffers from transitive retry when it conflicts with a higher priority transaction s_j^l, which in turn conflicts with a higher priority transaction s_z^h, but s_i^k does not conflict with s_z^h. Still, when s_j^l retries due to s_z^h, s_i^k also retries due to s_j^l. Thus, the effect of the higher priority transaction s_z^h is transitively moved to the lower priority transaction s_i^k, even when they do not conflict on common objects.*

Claim 1. *ECM suffers from transitive retry for multi-object transactions.*

Proof. ECM depends on priorities to resolve conflicts between transactions. Thus, lower priority transaction must always be aborted for a conflicting higher priority transaction. Claim follows. □

Because of transitive retry, Θ_i for any τ_i is extended to include any object $\theta \notin \Theta_i$, but θ can make at least one transaction $s_i^k \in \tau_i$ retry transitively. The new set of objects that can cause direct or indirect retry of at least one transaction in τ_i is denoted as Θ_i^{ex}. Θ_i^{ex} is obtained by being initialized to Θ_i (i.e., the set of objects that are already accessed by any transaction $s_i^k \in \tau_i$). We then cycle through all transactions belonging to all other higher priority tasks. Each transaction s_j^l that accesses at least one of the objects in Θ_i^{ex} adds all other objects accessed by s_j^l to Θ_i^{ex}. The loop over all higher priority tasks is repeated, each time with the new Θ_i^{ex}, until there are no more transactions accessing any object in Θ_i^{ex}. However, this solution may over-extend the set of conflicting objects, and may even contain all objects accessed by all tasks. Θ_i^* represent the set of objects not accessed directly by any transaction in τ_i, but any $\theta \in \Theta_i^*$ can make at least one transaction in τ_i retry transitively. Thus, $\Theta_i^{ex} = \Theta_i + \Theta_i^*$. Similarly, the distinct set of objects that can make s_i^k retry directly or indirectly(transitively) is denoted as $\Theta_i^{k^{ex}}$. γ_i is the extended to γ_i^{ex}. While γ_i is the set of tasks- other than τ_i- that access at least one object $\theta \in \Theta_i$, γ_i^{ex} is the set of tasks- other than τ_i- that access at least one object $\theta \in \Theta_i^{ex}$.

4.1.3 G-EDF Interference

Claim 2. *Regardless of the used scheduler, maximum number of jobs of τ_j that can exist in time interval L is upper bounded by*

$$\left\lceil \frac{L}{T_j} \right\rceil + 1 \tag{4.1}$$

where at most two jobs τ_j can be partially included in L. The remaining jobs of τ_j are totally included in L.

Proof. Generally, $L = aT_j + b$, $0 \leq b < T_j$. a is the maximum number of jobs of τ_j that contribute by their minimum periods T_j during L. If $b \geq T_j$, then there are more than a jobs of τ_j contributing by their minimum periods T_j during L, which contradicts definition of a. The remaining interval $b(= L - aT_j,\, b > 0)$ can be divided between at most two jobs of τ_j. If b can be divided between more than two jobs of τ_j, then there is more than a jobs of τ_j that contribute by their minimum periods T_j during L. This contradicts definition of a. So, if $b > 0$, then maximum number of jobs of τ_j that can exist during L is $a + 2 = \left\lceil \frac{L}{T_j} \right\rceil + 1$.

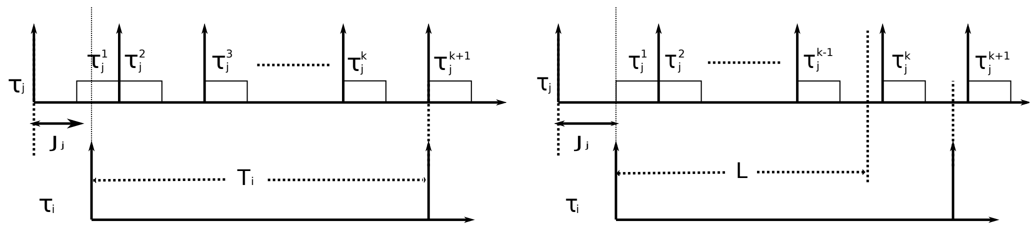
If $b = 0$, then jobs of τ_j can be shifted to the left or the right during L. This results in $a + 1$ jobs of τ_j during L. So, if $b = 0$, then maximum number of jobs of τ_j that can exist during L is $a + 1 = \left\lceil \frac{L}{T_j} \right\rceil + 1$. Claim follows. □

Claim 3. *Let $T_i = aT_j + b$, where $a = \left\lfloor \frac{T_i}{T_j} \right\rfloor$ and $0 \leq b < T_j$. Under G-EDF scheduler, maximum number of jobs of τ_j that can interfere with one job τ_i^x during time interval $L(= T_i - f,\, 0 \leq f < T_i)$ is*

$$g_{ij}^{gedf}(L) = \begin{cases} \left\lceil \frac{T_i}{T_j} \right\rceil & , f \leq b \\ \\ \left\lceil \frac{L}{T_j} \right\rceil + 1 & , Otherwise \end{cases} \tag{4.2}$$

Proof. $L = T_i - f = aT_j + b - f$. If $b - f \geq 0$, then following proof of Claim 2, $b - f$ can be devided between at most two jobs of τ_j during L. These two jobs of τ_j are: 1) *carried-in job* (i.e., τ_j^s where $r_j^s < r_i^x$ and $d_j^s < d_i^x$ [14]). 2) *carried-out job* (τ_j^e where $r_j^e > r_i^x$ and $d_j^s > d_i^x$ [14]). Under G-EDF, only jobs of τ_j with absolute deadline less than d_i^x can interfere with τ_i^x. Thus, carried-out job of τ_j cannot interfere with τ_i^x. So, $b - f$ can be the contribution of only the carried-in job. Following proof of Claim 2, maximum number of jobs of τ_j that can interfere with τ_i^x is $a + 1 = \left\lceil \frac{T_i}{T_j} \right\rceil$ if $f \leq b$. Otherwise, Claim 2 is used to determine maximum number of jobs of τ_j during L. Claim follows. □

The maximum number of times a task τ_j interferes with τ_i under G-EDF is illustrated in Figure 4.1. Upper bound on maximum interference of τ_j to τ_i (when there are no atomic sections) in $L \leq T_i$ is given in [14]. It should be noted that we consider only implicit deadline systems (i.e., $\forall \tau_i$, $T_i = D_i$). Implicit deadline system is a special case of constrained deadline system (i.e., $\forall \tau_i$, $D_i \leq T_i$) considered by [14]. The interference of τ_j to τ_i during $L = T_i - f$ where $f \leq b$ (as shown in Fig 4.1(a)), in the absence of atomic sections, is upper bounded



(a) during $L = T_i - f,\ f \leq b$ (b) during $L = T_i - f,\ f > b$

Figure 4.1: Maximum interference of jobs of τ_j to τ_i^x running on different processors, under G-EDF. $T_i = aT_j + b$

by:

$$\begin{aligned} I_{ij}^1\left(T_i\right) &\leq \left\lfloor \frac{T_i}{T_j} \right\rfloor c_j + min\left(c_j, T_i - \left\lfloor \frac{T_i}{T_j} \right\rfloor T_j\right) \\ &\leq \left\lceil \frac{T_i}{T_j} \right\rceil c_j \end{aligned} \tag{4.3}$$

The interference of τ_j to τ_i during an interval $L = T_i - f$ where $f > b$, as shown in Fig 4.1(b), in the absence of atomic sections is upper bounded by:

$$I_{ij}^2\left(L\right) \leq \left(\left\lceil \frac{L - c_j}{T_j} \right\rceil + 1\right) c_j \tag{4.4}$$

Here, τ_j^1 contributes by all its c_j, and d_j^{k-1} does not have to coincide with L, as τ_j^{k-1} has a higher priority than that of τ_i. Thus, the overall interference of τ_j to τ_i, over an interval $L \leq T_i$ is:

$$I_{ij}\left(L\right) = min\left(I_{ij}^1\left(T_i\right), I_{ij}^2\left(L\right)\right) \tag{4.5}$$

[14] upper bounds maximum response time of any job of τ_i. Upper bound on maximum response time of any job of τ_i is calculated by iteration of (4.6), starting from $R_i^{up} = c_i$.

$$R_i^{up} = c_i + \left\lfloor \frac{1}{m} \sum_{j \neq i} I_{ij}\left(R_i^{up}\right) \right\rfloor \tag{4.6}$$

where $I_{ij}\left(R_i^{up}\right)$ is calculate by (4.5).

4.1.4 Retry Cost of Atomic Sections

Claim 4. *Let s_i^k and s_j^l be two conflicting transactions. s_i^k has a lower priority than s_j^l. Let the lower priority transaction always aborts and retries due to the higher priority transaction. s_j^l interfere only once with s_i^k. s_i^k aborts and retries due to s_j^l for at most*

$$len\left(s_i^k + s_j^l\right) \tag{4.7}$$

Proof. s_j^l must start at least when s_i^k starts and not later than s_i^k finishes. Otherwise, there will be no conflict between s_i^k and s_j^l. s_i^k must retry during execution of s_j^l because of higher priority of s_j^l. The part of s_i^k that started before beginning of s_j^l will be repeated. Thus, the worst case interference between s_i^k and s_j^l occurs when s_j^l starts just when s_i^k reaches its end of execution. So, maximum retry cost of s_i^k due to s_j^l is calculated by 4.7. Claim follows. □

Claim 5. *Let conflict between transactions be resolved by priority (i.e., lower priority transaction aborts and retries due to higher priority transactions). Let $conf\left\{s_i^k\right\}$ be the set of all transactions that do not belong to any job of τ_i and are conflicting, directly or indirectly(transitively), with s_i^k. Each transaction $s_j^l \in conf\left\{s_i^k\right\}$ contributes to the retry cost of s_i^k by at most*

$$len\left(s_j^l + max\_s_{ik}^{jl}(\Theta)\right) \tag{4.8}$$

where $max\_s_{ik}^{jl}(\Theta)$ is the maximum length atomic section (transaction) in $conf\{s_i^k\}$ that accesses Θ and its priority is lower than $p(s_j^l)$ and higher than $p(s_i^k)$. $max\_s_{ik}^{jl} \notin s_i$, s_j and $\Theta \subseteq \Theta_i^{k^{ex}} \cap \Theta_j^l$.

Proof. As conflict is resolved by transactional priority, then only transactions with higher priorities than $p(s_i^k)$ will cause s_i^k to abort and retry. Also, s_j^l will abort only transactions with lower priority than $p(s_j^l)$. As transactions that belong to the same job execute sequentially, and jobs of the same task execute sequentially , so s_i^k is not aborted by other transactions that belong to τ_i. So, at any point of time after s_i^k was first released, and before the last successful run of s_i^k (i.e., the run at which s_i^k commits), one of the following cases happens:

1. s_j^l has finished before s_i^k starts. Or, s_j^l starts after s_i^k finishes. In this case, s_j^l will not cause s_i^k to abort and retry. (4.8) still upper bounds effect of s_j^l to the retry cost of s_i^k.

2. s_j^l is the only transaction that is currently aborting s_i^k. So, (4.8) follows directly from Claim 4 as $len\left(s_i^k\right) \leq len\left(max\_s_{ik}^{jl}(\Theta)\right)$.

3. A set of transactions $S \subseteq conf\{s_i^k\}$ are currently aborting s_i^k. $s_j^l \in S$ and s_j^l itself is not aborting and retrying due to any other transaction with higher priority than $p(s_j^l)$. So, s_j^l executes only once. s_j^l aborts one of the transactions with lower priority than $p(s_j^l)$ for only once. Thus, (4.8) upper bounds effect of s_j^l to the retry cost of s_i^k.

4. A set of transactions $S \subseteq conf\{s_i^k\}$ are currently aborting s_i^k. $s_j^l \in S$ and s_j^l itself is aborting and retrying due to other transactions with higher priority than $p(s_j^l)$. Without losing generality, let s_h^u be the transaction that is currently aborting s_j^l, and s_h^u is not aborting and retrying due to any other higher priority transaction. Then, s_j^l and s_i^k are both waiting for s_h^u to finish. Thus, the time of retrial of s_j^l due to s_h^u is covered by effect of s_h^u to the retry cost of s_i^k. When s_h^u finishes and s_j^l is not aborted by any other higher priority transaction, effect of s_j^l to the retry cost of s_i^k is the same as in the third case. By expanding this case to more than three transactions, then each

transaction s_j^l is either aborting one of the lower priority transactions only once (i.e., the last successful run of s_j^l), or s_i^k and s_j^l are aborted by a higher priority transaction s_h^u. When s_j^l is retrying due to the higher priority transaction s_h^u, s_j^l retrial time is not considered in retry cost of s_i^k because it is already covered by the effect of the higher priority transaction s_h^u to the retry cost of s_i^k.

Claim follows. □

Claim 6. *Under ECM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during interval $L \leq T_i$ due to direct and indirect conflict with other transactions in jobs with higher priority than τ_i^x is upper bounded by:*

$$RC_i\left(L\right) \leq \sum_{\tau_j \in \gamma_i^{ex}} \left(g_{ij}^{gedf} \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i^{ex}\right) \neq \emptyset} len\left(s_j^l + s_{max}(\Theta)\right)\right) \tag{4.9}$$

where $s_{max} \notin s_j$ and g_{ij}^{gedf} is calculated by (4.2).

Proof. ECM is used with G-EDF scheduler. Thus, $p(s_i^k)$ is a dynamic priority that depends on the absolute deadline of containing job τ_i^x. So, $conf\left\{s_i^k\right\}$ for any s_i^k includes each transaction $s_j^l \notin s_i$ where $\Theta_j^l \cap \Theta_i^{k^{ex}} \neq \emptyset$. The worst case retry cost of any s_i^k occurs when $p(s_i^k)$ is the lowest priority among all other conflicting transactions during T_i. g_{ij}^{gedf} is the maximum number of jobs of $\tau_j \in \gamma_i^{ex}$ that can interfere with one job of τ_j. Following Claims 3, 4 and 5, Claim follows. □

Claim 7. *Under ECM, upper bound on total retry cost given by (4.9) can be tightened by considering carried_in job of each τ_j (i.e., τ_j^{in} where $r_j^{in} < r_i^x$ and $d_j^{in} < d_i^x$ as defined in [14]) conflicting with τ_i^x during interval $L = T_i - f$, where $T_i = aT_j + b$, $a = \left\lfloor \frac{T_i}{T_j} \right\rfloor$ and $f \leq b$. (4.9) will be modified to*

$$RC_i(L) \leq \begin{cases} \sum_{\tau_j \in \gamma_i^{ex}} \left(\lambda_1\left(j\right) + \chi\left(i,j\right)\right) & ,f \leq b \\ \sum_{\tau_j \in \gamma_i^{ex}} \left(\left(\left\lceil \frac{L}{T_j} \right\rceil + 1\right) \sum_{\forall s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l + s_{max}(\Theta)\right)\right) & ,Otherwise \end{cases} \tag{4.10}$$

where

- $s_{max} \notin s_j$.
- $\lambda_1\left(j\right) = \sum_{\forall s_j^l \in \left[d_j^{in} - \delta, d_j^{in}\right], \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right)} len\left(s_j^{l^*} + s_{max}\left(\Theta\right)\right)$, *where $\delta = min\left(c_j, b\right)$ and $s_j^{l^*}$ is the part of s_j^l that is contained in interval $\left[d_j^{in} - \delta, d_j^{in}\right]$.*
- $\chi\left(i,j\right) = \left\lfloor \frac{T_i}{T_j} \right\rfloor \sum_{\forall s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l + s_{max}\left(\Theta\right)\right)$.

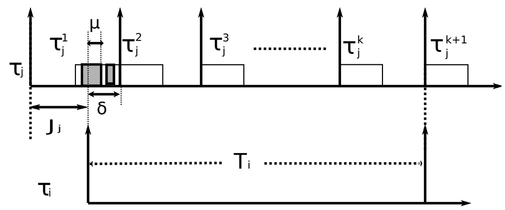


Figure 4.2: Effect of carried_in job of τ_j to retry cost of transactions in τ_i

Proof. Following proof of Claim 3, maximum number of jobs of τ_j that can interfere with τ_i^x is $\left\lceil \frac{T_i}{T_j} \right\rceil$. By definition of carried-in jobs [14] and G-EDF scheduler, there will be $\left\lfloor \frac{T_i}{T_j} \right\rfloor$ jobs of τ_j that exist by their whole periods T_j in the interval L. Carried-in job of τ_j (i.e., τ_j^{in}) will exist by at most $\delta = min\left(c_j, b\right)$ during L. τ_j^{in} is delayed by its maximum jitter to give its maximum contribution during L. Thus, τ_j^{in} starts execution at $d_j^{in} - c_j$. Consequently, only transactions of τ_j^{in} that are contained in $\left[d_j^{in} - \delta, d_j^{in}\right]$ can exist in the interval L. Also, if transaction s_j^l is partially contained in $\left[d_j^{in} - \delta, d_j^{in}\right]$, only the part of s_j^l contained in $\left[d_j^{in} - \delta, d_j^{in}\right]$ (i.e., $s_j^{l^*}$) can conflict with transactions in τ_i^x. $\lambda(j)$ stands for the retry cost of transactions in τ_i^x due to conflict with transactions of τ_j^{in}. Whereas, $\chi(i,j)$ stands for the retry cost of transactions in τ_i^x due to conflict with transactions of other jobs of τ_j (i.e., non carried-in jobs). Combining the previous notions with Claim 6, Claim follows. □

Effect of transactions in carried_in job is shown in Figure 4.2. There are two sources of retry cost for any τ_i^x under ECM. First is due to conflict between τ_i^x's transactions and transactions of other jobs. This is denoted as RC_i. Second is due to the preemption of any transaction in τ_i^x due to the release of all higher priority jobs. This is denoted as $RC_{i_{re}}$. It is up to the implementation of the contention manager to avoid RC_{re}. Here, as we are concerned with maximum total retry cost introduced by ECM, we assume that ECM does not avoid RC_{re}. Thus, we introduce RC_{re} for ECM technique.

Claim 8. *Under ECM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ due to release of jobs with higher priority than τ_i^x is upper bounded by*

$$RC_{i_{re}}(L) \leq \sum_{\forall \tau_j \in \zeta_i} \begin{cases} \left\lceil \frac{L}{T_j} \right\rceil s_{i_{max}} & , L \leq T_i - T_j \\ \\ \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{i_{max}} & , L > T_i - T_j \end{cases} \tag{4.11}$$

where $\zeta_i = \{\tau_j : (\tau_j \neq \tau_i) \wedge (D_j < D_i)\}$.

Proof. Two conditions must be satisfied for any τ_j^l to be able to preempt τ_i^x under G-EDF: $r_i^x < r_j^l < d_i^x$, and $d_j^l \leq d_i^x$. Without the first condition, τ_j^l would have been already released before τ_i^x. Thus, τ_j^l will not preempt τ_i^x. Without the second condition, τ_j^l will be of lower priority than τ_i^x and will not preempt it. If $D_j \geq D_i$, then there will be at most one instance

τ_j^l with higher priority than τ_i^x. τ_j^l must have been released at most at r_i^x, which violates the first condition. The other instance τ_j^{l+1} would have an absolute deadline greater than d_i^x. This violates the second condition. Hence, only tasks with shorter relative deadline than D_i are considered. These jobs are grouped in ζ_i.

The total number of released instances of τ_j during any interval $L \leq T_i$ is $\left\lceil \frac{L}{T_i} \right\rceil + 1$. The "carried-in" jobs (i.e., each job released before r_i^x and has an absolute deadline before d_i^x [14]) are discarded as they violate the first condition. The "carried-out" jobs (i.e., each job released after r_i^x and has an absolute deadline after d_i^x [14]) are also discarded because they violate the second condition. Thus, the number of considered higher priority instances of τ_j during the interval $L \leq T_i - T_j$ is $\left\lceil \frac{L}{T_j} \right\rceil$. The number of considered higher priority instances of τ_j during interval $L > T_i - T_j$ is $\left\lfloor \frac{T_i}{T_j} \right\rfloor$.

The worst $RC_{i_{re}}$ for τ_i^x occurs when τ_i^x is always interfered at the end of execution of its longest atomic section, $s_{i_{max}}$. τ_i^x will have to retry for $len(s_{i_{max}})$. Claim follows. □

Claim 9. *Under ECM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ is upper bounded by:*

$$RC_{i_{to}}(L) = RC_i(L) + RC_{i_{re}}(L) \tag{4.12}$$

where $RC_i(L)$ is the maximum retry cost resulting from conflict between transactions in τ_i^x and transactions of other jobs. $RC_i(L)$ is calculated by (4.9). $RC_{i_{re}}(L)$ is the maximum retry cost resulting from the release of higher priority jobs, which preempt transactions in τ_i^x. $RC_{i_{re}}(L)$ is calculated by (4.11).

Proof. Under ECM, transactions in any job $\tau_i^x \in \tau_i$ retry due to: 1) conflicting transactions of jobs with higher priority than τ_i^x. 2) release of higher priority jobs that preempt τ_i^x. Thus, (4.12) follows directly from Claims 7 and 8. Claim follows. □

4.1.5 Upper Bound on Response Time

Claim 10. *Under ECM, maximum response time of any job $\tau_i^x \in \tau_i$ is upper bounded by*

$$R_i^{up} = c_i + RC_{i_{to}}(R_i^{up}) + \left\lfloor \frac{1}{m} \sum_{j \neq i} I_{ij}(R_i^{up}) \right\rfloor \tag{4.13}$$

where:

- *R_i^{up}'s initial value is $c_i + R_i^{up}(c_i)$.*
- *$RC_{i_{to}}(R_i^{up})$ is calculated by (4.12).*

- c_j *of any job* $\tau_j^y \in \tau_j$*- contributing to the retry cost of transactions in* τ_i^x*- is modified to*

$$c_{ji} = c_j - \left(\sum_{s_j^l, \left(\Theta=\Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l\right) \right) + RC_{ji_{to}}(R_i^{up}) \tag{4.14}$$

- $RC_{ji_{to}}(R_i^{up})$ *is the same as* $RC_{j_{to}}(R_i^{up})$ *excluding atomic sections in* τ_j *that access shared objects between* τ_i *and* τ_j*.* τ_i *does not contribute to* $RC_{j_{re}}(R_i^{up})$*.*
- $I_{ij}(R_i^{up})$ *is calculated by (4.5) with* c_j *replaced by* c_{ji} *and changing (4.4) to*

$$I_{ij}(R_i^{up}) = max \begin{cases} \left(\left\lceil \frac{R_i^{up} - \left(c_{ji} + \sum_{s_j^l, \left(\Theta=\Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l)\right)}{T_j} \right\rceil + 1 \right) c_{ji} \\ \left\lceil \frac{R_i^{up} - c_j}{T_j} \right\rceil .c_{ji} + c_j - \sum_{s_j^l, \left(\Theta=\Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l) \end{cases} \tag{4.15}$$

Proof. To obtain an upper bound on the maximum response time (i.e., R_i^{up}) of any job τ_i^x of τ_i, the term $RC_{i_{to}}(R_i^{up})$ must be added to the interference of other tasks during the non-atomic execution of τ_i^x. But this requires modification of the WCET of each task as follows.

c_j of each interfering task τ_j should be inflated to accommodate the interference of each task τ_k, $k \neq j, i$. Meanwhile, atomic regions that access shared objects between τ_j and τ_i should not be considered in the inflation cost, because they have already been calculated in τ_i's retry cost. As an upper bound on R_i^{up} is calculated, then jobs of τ_j with higher priority than τ_i^x are only considered. Thus, τ_i^x has no contribution in $RC_{j_{re}}(R_i^{up})$. Thus, τ_j's inflated WCET becomes:

$$c_{ji} = c_j - \left(\sum_{s_j^l, \left(\Theta=\Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l\right) \right) + RC_{ji_{to}}(R_i^{up})$$

which is given by (4.14). c_{ji} is the new WCET of τ_j relative to τ_i. $\sum_{s_j^l, \left(\Theta=\Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l)$ is the sum of lengths of all atomic sections in τ_j that access any object $\theta \in \Theta_i^{ex}$. $\sum_{s_j^l, \left(\Theta=\Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l)$ is subtracted from c_j because $\sum_{s_j^l, \left(\Theta=\Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l)$ is already included in $RC_{i_{to}}(R_i^{up})$. $RC_{ji_{to}}(R_i^{up})$ is the $RC_{j_{to}}(R_i^{up})$ without including the shared objects between τ_i and τ_j. The calculated WCET is relative to task τ_i, as it changes from task to task. The upper bound on the response time of τ_i^x, denoted R_i^{up}, can be calculated iteratively, by modifying (4.6), as follows:

$$R_i^{up} = c_i + RC_{i_{to}}(R_i^{up}) + \left\lfloor \frac{1}{m} \sum_{j \neq i} I_{ij}(R_i^{up}) \right\rfloor$$

which is given by (4.13). R_i^{up}'s initial value is $c_i + R_i^{up}(c_i)$. $I_{ij}(R_i^{up})$ is calculated by (4.5) with c_j replaced by c_{ji}, and changing (4.4) to

$$I_{ij}(R_i^{up}) = max \begin{cases} \left(\left\lceil \frac{R_i^{up} - \left(c_{ji} + \sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l \right) \neq \emptyset} len(s_j^l) \right)}{T_j} \right\rceil + 1 \right) c_{ji} \\ \left\lceil \frac{R_i^{up} - c_j}{T_j} \right\rceil .c_{ji} + c_j - \sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l \right) \neq \emptyset} len(s_j^l) \end{cases}$$

as given by (4.15). Eq(4.4) is modified to (4.15) because there are two cases for the first job of τ_j (i.e., τ_j^1) contributing to the retry cost of τ_i^x:

Case 1. τ_j^1 (shown in Figure 4.1(b)) contributes by c_{ji}. Thus, other instances of τ_j will begin after this modified WCET, but the sum of the shared objects' atomic section lengths is removed from c_{ji}, causing other instances to start earlier. Thus, the term $\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l \right) \neq \emptyset} len(s_j^l)$ is added to c_{ji} to obtain the correct start time.

Case 2. τ_j^1 contributes by its c_j, but the sum of the shared atomic section lengths between τ_i and τ_j should be subtracted from the contribution of τ_j^1, as they are already included in the retry cost.

It should be noted that subtraction of $\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l \right) \neq \emptyset} len(s_j^l)$ is done in the first case to obtain the correct start time of other instances, while in the second case, this is done to get the correct contribution of τ_j^1. The maximum is chosen from the two terms in (4.15), because they differ in the contribution of their τ_j^1s, and the number of instances after that. Claim follows. $\square$

4.2 RCM

As G-RMA is a fixed priority scheduler, a task τ_i will be interfered by those tasks with priorities higher than τ_i (i.e., $p_j > p_i$). Upon a conflict, the RMA CM will commit the transaction that belongs to the higher priority task. Hereafter, we use *RCM* to refer to a multicore system scheduled by G-RMA and resolves STM conflicts by the RMA CM. RCM is shown in Alogrithm 2.

The same illustrative example in Section 4.1.1 is applied for RCM except that tasks' priorities are fixed.

4.2.1 Maximum Task Interference

Figure 4.3 illustrates the maximum interference caused by a task τ_j to a task τ_i under G-RMA. As τ_j is of higher priority than τ_i, τ_j^k will interfere with τ_i even if it is not totally

Algorithm 2: RCM

Data: $s_i^k \rightarrow$ interfered atomic section. $s_j^l \rightarrow$ interfering atomic section
Result: which atomic section aborts
if $T_i < T_j$ **then**
  s_j^l aborts;
else
  s_i^k aborts;
end

included in T_i. Unlike the G-EDF case shown in Figure 4.2, where only the δ part of τ_j^1 is considered, in G-RMA, τ_j^k can contribute by the whole c_j, and all atomic sections contained in τ_j^k must be considered. This is because, in G-EDF, the worst-case pattern releases τ_i^a before d_j^1 by δ time units, and τ_i^a cannot be interfered before it is released. But in G-RMA, τ_i^a is already released, and can be interfered by the whole τ_j^k, even if this makes it infeasible.

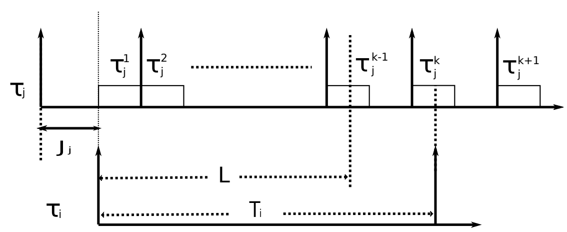


Figure 4.3: Max interference of τ_j to τ_i in G-RMA

Thus, the maximum contribution of τ_j^b to τ_i^a for any duration L is upper bounded by Claim 2, where L can extend to T_i. Note the contrast with ECM, where L cannot be extended directly to T_i, as this will have a different pattern of worst case interference from other tasks.

4.2.2 Retry Cost of Atomic Sections

Claim 11. *Under RCM, total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during interval $L \leq T_i$ due to direct and indirect conflict with other transactions in jobs with higher priorities than τ_i^x is upper bounded by:*

$$RC_i\left(L\right) \leq \sum_{\tau_j \in \gamma_i^{ex}, p_j > p_i} \left(\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} \left(\left(\left\lceil \frac{L}{T_j} \right\rceil + 1 \right) len\left(s_j^l + s_{max}\left(\Theta\right)\right) \right) \right) \tag{4.16}$$

where $s_{max}\left(\Theta\right)$ belongs to a job with lower priority than p_j.

Proof. Under G-RMA, priorities of tasks are fixed. Thus, as $p_j > p_i$, then any job of τ_j will have a higher priority than τ_i^x. So, Claim 2 gives maximum number of jobs of τ_j that interfere with τ_i^x during interval L. By definition of RCM, only transactions with lower priority than p_j can be aborted and retried due to transactions in s_j. Thus, $s_{max}(\Theta)$ cannot belong to transactions with priorities at least equal to p_j. Following proof of Claim 6, Claim follows. □

Claim 12. *Under RCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ due to release of jobs with higher priority than τ_i^x is upper bounded by*

$$RC_{i_{re}}(L) = \sum_{\forall \tau_j, p_j > p_i} \left(\left\lceil \frac{L}{T_j} \right\rceil s_{i_{max}} \right) \tag{4.17}$$

Proof. The proof is the same as that for Claim 8, except that G-RMA uses static priority. Thus, the carried-out jobs will be considered in the interference with τ_i^x. The carried-in jobs are still not considered because they are released before r_i^x. Claim follows. □

Claim 13. *Under RCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ is upper bounded by:*

$$RC_{i_{to}}(L) = RC_i(L) + RC_{i_{re}}(L) \tag{4.18}$$

where $RC_i(L)$ is the maximum retry cost resulting from conflict between transactions in τ_i^x and transactions of other jobs. $RC_i(L)$ is calculated by (4.16). $RC_{i_{re}}(L)$ is the maximum retry cost resulting from the release of higher priority jobs, which preempt transactions in τ_i^x. $RC_{i_{re}}(L)$ is calculated by (4.17).

Proof. Using Claims 11 and 12, and following proof of Claim 9, Claim follows. □

4.2.3 Upper Bound on Response Time

Claim 14. *Under RCM, maximum response time of any job $\tau_i^x \in \tau_i$ is upper bounded by*

$$R_i^{up} = c_i + RC_{i_{to}}(R_i^{up}) + \left\lfloor \frac{1}{m} \sum_{j \neq i, p_j > p_i} I_{ij}(R_i^{up}) \right\rfloor \tag{4.19}$$

where:

- *R_i^{up}'s initial value is $c_i + R_i^{up}(c_i)$.*
- *$RC_{i_{to}}(R_i^{up})$ is calculated by (4.18).*
- *c_j of any job $\tau_j^y \in \tau_j$, where $p_j > p_i$ and $\Theta_j \cap \Theta_i^{ex} \neq \emptyset$, is calculated by (4.14).*
- *$I_{ij}(R_i^{up})$ is calculated by (4.4) with c_j replaced by c_{ji}.*

Proof. Using Theorem 7 in [14], Claim 13 and following proof of Claim 10, Claim follows. □

4.3 STM versus Lock-Free

We now would like to understand when STM will be beneficial compared to lock-free synchronization. The retry-loop lock-free approach in [37] is the most relevant to our work. As lock-free instructions access only one object, then Θ_i^k for any s_i^k will be restricted to one object only (i.e., $\Theta_i^k = \theta_i^k$). Thus, transitive retry cannot happen, $\Theta_i^{ex} = \Theta_i$ and $\gamma_i^{ex} = \gamma_i$.

4.3.1 ECM versus Lock-Free

Claim 15. *For ECM's schedulability to be better or equal to that of [37]'s retry-loop lock-free approach, the size of s_{max} must not exceed one half of that of r_{max}, where r_{max} is the maximum execution cost of a single iteration of any lock-free retry loop of any task. With equal periods for conflicting tasks and high access times to each object within the same transaction, the size of s_{max} can be much larger than r_{max}.*

Proof. Using Claim 3, (4.10) can be upper bounded, during T_i, as:

$$RC_i^{max}(T_i) \leq \sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} (2.s_{max}) \right)$$

where s_{max} is the maximum length atomic section among all tasks. Similarly, (4.11) is upper bounded, during T_i, as:

$$RC_{i_{re}}^{max} \leq \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{max}$$

where $\zeta_i = \{\tau_j : (\tau_j \neq \tau_i) \wedge (D_j < D_i)\}$. Thus, $RC_{i_{to}}$ given by (4.12) can be upper bounded, during T_i, as:

$$RC_{i_{to}}^{max} \leq \left(\sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} (2.s_{max}) \right) \right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{max} \right) \tag{4.20}$$

Retry cost of τ_i during interval T_i due to conflict with other jobs under retry-loop lock-free is given in [37] as:

$$LRC \leq \sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) .\beta_{ij}.r_{max} \tag{4.21}$$

where β_{ij} is the number of retry loops of τ_j that access shared objects between τ_i and τ_j. (4.21) needs to be extended to include effect of release of any higher priority job, τ_j^l, preempting τ_i^k when τ_i^k is trying to access an object θ. Release of jobs under ECM and lock-free is independent from accessed objects. Thus, ECM and lock-free have the same pattern of jobs' release. Thus, retry cost of τ_i during T_i due to release of higher priority jobs under retry-loop lock-free is obtained directly from Claim 8 with replacing s_{max} by r_{max}. Thus, total retry cost of any job of τ_i during interval T_i due to conflict of other jobs and release of higher priority jobs is upper bounded by:

$$LRC_{to} \leq \left(\left(\sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) .\beta_{ij}\right) + \left(\sum_{\tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor\right)\right) r_{max} \tag{4.22}$$

ECM achieves equal or better schedulability than lock-free if the total utilization under ECM is less than or equal to total utilization under lock-free system:

$$\sum_{\forall \tau_i} \frac{c_i + RC_{i_{to}}^{max}}{T_i} \leq \sum_{\forall \tau_i} \frac{c_i + LRC_{to}}{T_i} \tag{4.23}$$

Eq(4.23) holds if for every task τ_i:

$$RC_{i_{to}}^{max} \leq LRC_{to} \tag{4.24}$$

Thus,

$$\begin{aligned} & \left(\left(\textstyle\sum_{\forall \tau_j \in \gamma_i} \left(2\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, (\Theta=\Theta_j^l \cap \Theta_i) \neq \emptyset}\right)\right) + \left(\textstyle\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor\right)\right) s_{max} \\ \leq & \left(\left(\textstyle\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \beta_{ij}\right) + \left(\textstyle\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor\right)\right) r_{max} \end{aligned}$$

$$\therefore \frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \beta_{ij}\right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor\right)}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, (\Theta=\Theta_j^l \cap \Theta_i) \neq \emptyset}\right)\right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor\right)} \tag{4.25}$$

Let $\sum_{\forall s_j^l, (\Theta=\Theta_j^l \cap \Theta_i) \neq \emptyset} = \beta_{ij}^*$ and $\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor = c1$. Then, (4.25) becomes

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \beta_{ij}\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2\left\lceil \frac{T_i}{T_j} \right\rceil \beta_{ij}^*\right)\right) + c1} \tag{4.26}$$

We want to get the lower bound over s_{max}/r_{max} that preserves equal or better schedulability for ECM than lock-free:

Each lock-free instruction accesses only one object once. Each transaction accesses only one object to enable comparison with lock-free. An object θ can be accessed multiple times within the same transaction. Thus, $\beta_{ij} \leq \beta_{ij}^*$.

$$\because \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil\right) \beta_{ij}^*\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2\left\lceil \frac{T_i}{T_j} \right\rceil \beta_{ij}^*\right)\right) + 2c1} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \beta_{ij}\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2\left\lceil \frac{T_i}{T_j} \right\rceil \beta_{ij}^*\right)\right) + c1}$$

Thus, (4.26) holds if

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left\lceil \frac{T_i}{T_j} \right\rceil \beta^*_{ij}\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2\left\lceil \frac{T_i}{T_j} \right\rceil \beta^*_{ij}\right)\right) + 2c1} = \frac{1}{2}$$

Thus, the lower bound over s_{max}/r_{max} that preserves equal or better schedulability for ECM than lock-free is 0.5. Now, we want to get the upper bound over s_{max}/r_{max} that preserves equal or better schedulability for ECM than lock-free:

Minimum value for $\left\lceil \frac{T_i}{T_j} \right\rceil$ is 1. So, $2\left\lceil \frac{T_i}{T_j} \right\rceil \geq \left\lceil \frac{T_i}{T_j} \right\rceil + 1, \forall i, j$. Thus, to get upper bound on s_{max}/r_{max}, $\left\lceil \frac{T_i}{T_j} \right\rceil$ assumes its minimum value (i.e., 1). Otherwise, the denominator of (4.26) gets larger than numerator, and s_{max}/r_{max} moves away from its upper bound. $\left\lceil \frac{T_i}{T_j} \right\rceil \rightarrow 1$ for any i, j if all conflicting tasks have equal periods. Thus, by substitution of $\left\lceil \frac{T_i}{T_j} \right\rceil = 1$ into (4.26), we get

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} 2\beta_{ij}\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} 2\beta^*_{ij}\right) + c1} \tag{4.27}$$

As we are looking for the upper bound over s_{max}/r_{max}, then $\beta_{ij} >> \beta^*_{ij}$. Thus, s_{max} can be much larger than r_{max} while still maintaining equal or better schedulability for ECM than lock-free. From the previous notions, Claim follows. □

4.3.2 RCM versus Lock-Free

Claim 16. *For RCM's schedulability to be better or equal to that of [37]'s retry-loop lock-free approach, the size of s_{max} must not exceed one half of that of r_{max}, where r_{max} is the maximum execution cost of a single iteration of any lock-free retry loop of any task. With equal periods for conflicting tasks and high access times to each object within the same transaction, the size of s_{max} can be much larger than r_{max}.*

Proof. Following the same steps in proof of Claim 15 with the following modifications:

Equation (4.16) is upper bounded by:

$$\sum_{\tau_j \in \gamma_i, p_j > p_i} \left(\sum_{s^l_j, \left(\Theta = \Theta^l_j \cap \Theta_i\right) \neq \emptyset} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) 2s_{max} \right) \right) \tag{4.28}$$

Equation (4.17) is upper bounded by:

$$RC_{i_{re}}(T_i) = \sum_{\forall \tau_j, p_j > p_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil s_{max} \right) \tag{4.29}$$

Thus,

$$RC_{i_{to}}^{max} \leq \sum_{\tau_j \in \gamma_i,\, p_j > p_i} \left(\sum_{s_j^l,\, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) 2 s_{max} \right) \right) + \sum_{\forall \tau_j,\, p_j > p_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil s_{max} \right) \tag{4.30}$$

As lock-free is independent from the underlying scheduler, then LRC is still calculated by (4.21). Release of jobs under RCM and lock-free is independent from accessed objects. Thus, RCM and lock-free have the same pattern for object release. Thus, retry cost of transactions in τ_i during T_i due to release of higher priority jobs under retry-loop lock-free is obtained directly from Claim 12 with replacing s_{max} by r_{max}. Thus,

$$LRC_{to} \leq \left(\left(\sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) .\beta_{ij} \right) + \left(\sum_{\tau_j,\, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil \right) \right) r_{max} \tag{4.31}$$

Similar to proof of Claim 15, RCM has equal or better schedulability than lock-free if for each τ_i

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} \right) + \left(\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil \right)}{\left(\sum_{\forall \tau_j \in \gamma_i,\, p_j > p_i} \left(2 \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} \right) \right) + \left(\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil \right)} \tag{4.32}$$

$$\because \sum_{\forall \tau_j \in \gamma_i,\, p_j > p_i} \left(2 \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} \right) \leq \sum_{\forall \tau_j \in \gamma_i} \left(2 \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} \right)$$

$\therefore$ Eq(4.32) holds if

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} \right) + \left(\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil \right)}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2 \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} \right) \right) + \left(\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil \right)} \tag{4.33}$$

Let $\sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} = \beta_{ij}^*$ and $\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil = c1$. Then (4.32) becomes

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} \right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2 \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij}^* \right) + c1 \right)} \tag{4.34}$$

We want to get lower bound over s_{max}/r_{max} that preserves equal or better schedulability for RCM than lock-free:

Similar to proof of Claim 15, β_{ij} assumes its minimum value β^*_{ij}.

$$\because \frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+c1}{\left(\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta^*_{ij}\right)+2c1\right)} \le \frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+c1}{\left(\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta^*_{ij}\right)+c1\right)} \tag{4.35}$$

Then (4.34) holds if

$$\frac{s_{max}}{r_{max}} \le \frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+c1}{\left(\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta^*_{ij}\right)+2c1\right)} = \frac{1}{2} \tag{4.36}$$

We want to get upper bound over s_{max}/r_{max} that preserves equal or better schedulability for RCM than lock-free:

Similar to proof of Claim 15, $\left\lceil\frac{T_i}{T_j}\right\rceil$ assumes its minimum value (i.e., 1), $\beta_{ij} >> \beta^*_{ij}$. Thus, s_{max} can be much larger than r_{max}. From the previous notions, Claim follows. □

4.4 STM versus Locking protocols

Schedulabilty of different CMs is compared against real-time locking protocols (i.e., OMLP [18,21] and RNLP [116]) using total utilization under G-EDF and G-RMA. In [18,21,116,117], priority inversion bound (*pi-blocking*) is considered part of each task's execution time. Thus, each task's WCET is inflated by *pi-blocking* bounds. Similarly, under different CMs, each task's WCET is inflated by its total retry cost (i.e., retry cost due to direct and indirect conflict with other tasks. Besides retry cost due to release of higher priority jobs). So, schedulability of a specific STM CM algorithm A is compared against a real-time locking protocol B as follows:

$$\sum_{\forall\tau_i}\frac{c_i+RC_A(T_i)}{T_i} \quad\le\quad \sum_{\forall\tau_i}\frac{c_i+PI_B(T_i)}{T_i} \tag{4.37}$$

Eq(4.37) holds if

$$\forall\tau_i,\ RC_A(T_i) \le PI_B(T_i) \tag{4.38}$$

If τ_i has no critical sections, then $RC_A(T_i) = PI_B(T_i) = 0$. Thus, independent tasks have the same effect in (4.37) and they will not be considered in (4.38).

4.4.1 Priority Inversion under Global OMLP

Under Global OMLP [18,21], $PI_{OMLP}(T_i)$ for any job τ^x_i is upper bounded by

$$PI_{OMLP}(T_i) \le \sum_{k=1}^{n_r} N_{i,k}(2m-1)L_{max} \tag{4.39}$$

Where n_r is total number of resources. $N_{i,k}$ is maximum number of times resource k is accessed by τ_i. L_{max} is the maximum length critical section in all tasks. Let $N_i = \sum_{k=1}^{n_r} N_{i,k}$, which is the total number of critical sections in any job τ_i^x. Thus, (4.39) becomes

$$PI_{OMLP}(T_i) \le N_i(2m-1)L_{max} \tag{4.40}$$

Let $N_{max} = max\,\{N_i\}_{\forall i}$, $N_{min} = min\,\{N_i\}_{\forall i}$, $\Phi_{max} = max \left\lceil \frac{T_i}{T_j} \right\rceil_{\forall i,\,j}$. As independent tasks are not considered in (4.38), $\therefore$ N_{max}, $N_{min} \ge 1$.

OMLP uses group locking to access multiple (i.e., nested) resources in a critical section. Thus, to enable schedulability comparison between OMLP and different CMs, it is assumed that each transaction (critical section) accesses only one resource (i.e., $\Theta_i^k = \theta_i^k$). So, there will be no transitive retry in CMs. Thus, $\Theta_i^{ex} = \Theta_i$ and $\gamma_i^{ex} = \gamma_i$. Sections 4.4.5 and 4.4.6 investigates comparison between different CMs and fine-grained locking protocols (i.e., RNLP) to access multiple resources within a critical section without group locking.

4.4.2 ECM versus Global OMLP

Claim 17. *Under globally scheduled systems, schedulability of ECM is equal or better than schedulability of Global OMLP if*

$$\frac{s_{max}}{L_{max}} \le \frac{N_{min}\,(2m-1)}{(2N_{max}+1)\,(n-1)\Phi_{max}} \tag{4.41}$$

Proof. Substitute $RC_A(T_i)$ and $PI_B(T_i)$ in (4.38) by (4.20) and (4.40) respectively. $\therefore$ (4.38) holds if $\forall \tau_i$

$$\left(\sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} (2.s_{max}) \right)\right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{max} \right) \le N_i\,(2m-1)\,L_{max} \tag{4.42}$$

Let $N_{i,j} = \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset}$. As each transaction accesses only one object to enable comparison against OMLP, then $N_{i,j}$ is number of transactions in any job of τ_j conflicting with any transaction in any job of τ_i. Thus, (4.42) becomes

$$\left(2\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil N_{i,j} \right)\right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor \right)\right) s_{max} \le N_i\,(2m-1)\,L_{max} \tag{4.43}$$

$$\therefore \frac{s_{max}}{L_{max}} \le \frac{N_i\,(2m-1)}{2\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil N_{i,j} \right)\right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor \right)} \tag{4.44}$$

Let $N_{max} = max\left\{N_i\right\}_{\forall i}$, $N_{min} = min\left\{N_i\right\}_{\forall i}$, $\Phi_{max} = max\left\lceil \frac{T_i}{T_j} \right\rceil_{\forall i,j}$. According to definition of ECM, any $\tau_j \in \gamma_i$ also belongs to ζ_i. $\therefore$ $n-1 \geq |\zeta_i| \geq |\gamma_i|$. $\because$ $N_{max} \geq N_{i,j}$, $N_{min} \leq N_i$ and $\Phi_{max} \geq \left\lceil \frac{T_i}{T_j} \right\rceil \geq \left\lfloor \frac{T_i}{T_j} \right\rfloor$. $\therefore$ Eq(4.44) holds if

$$\begin{aligned}\frac{s_{max}}{L_{max}} &\leq \frac{N_{min}\left(2m-1\right)}{2\left(\sum_{\forall \tau_j \in \zeta_i}\left(\Phi_{max}N_{max}\right)\right) + \left(\sum_{\forall \tau_j \in \zeta_i}\Phi_{max}\right)} \\ &\leq \frac{N_{min}\left(2m-1\right)}{\left(2N_{max}+1\right)\left(n-1\right)\Phi_{max}} \end{aligned} \tag{4.45}$$

Claim follows. □

4.4.3 RCM versus Global OMLP

Claim 18. *Under globally scheduled systems, schedulability of RCM is equal or better than schedulability of Global OMLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(2\left(\Phi_{max}+1\right)N_{max}+\Phi_{max}\right)\left(n-1\right)} \tag{4.46}$$

Proof. Substitute $RC_A(T_i)$ in (4.38) by (4.30) and follow the same steps in proof of Claim 17. Claim follows. □

4.4.4 Priority Inversion under RNLP

Under RNLP [116] for global scheduling and *I-KGLP* token lock (introduced as R^2DGLP in [118]), $PI_{RNLP}(T_i)$ for any job τ_i^x is upper bounded by $(2m-1)\,L_{max}$ for each outermost request, where L_{max} is the maximum length of any outermost request. Thus, if N_i is total number of outermost critical sections in any job of τ_i, then

$$PI_{RNLP}(T_i) = N_i(2m-1)L_{max} \tag{4.47}$$

Let $N_{max} = max\left\{N_i\right\}_{\forall i}$, $N_{min} = min\left\{N_i\right\}_{\forall i}$, $\Phi_{max} = max\left\lceil \frac{T_i}{T_j} \right\rceil$. As independent tasks are not considered in (4.38), $\therefore$ N_{max}, $N_{min} \geq 1$.

In contrast to OMLP, RNLP supports nesting of objects. Thus, each object can be accessed individually without being grouped with other objects in the same critical section. So, in comparison between different CMs and RNLP, each transaction can access multiple objects.

4.4.5 ECM versus RNLP

Claim 19. *Under globally scheduled systems, schedulability of ECM is equal or better than schedulability of RNLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(2N_{max}+1\right)\left(n-1\right)\Phi_{max}} \tag{4.48}$$

Proof. Substitute $RC_A(T_i)$ and $PI_B(T_i)$ in (4.38) by (4.20) and (4.47) respectively. $\therefore$ (4.38) becomes

$$\begin{aligned} &\left(\sum_{\tau_j\in\gamma_i^{ex}}\left(\left\lceil\frac{T_i}{T_j}\right\rceil\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i^{ex}\right)\neq\emptyset}\left(2s_{max}\right)\right)\right)+\left(\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor s_{max}\right)\\ \leq\ & N_i\left(2m-1\right)L_{max} \end{aligned} \tag{4.49}$$

where γ_i is replaced by γ_i^{ex} and Θ_i is replaced by Θ_i^{ex} because each transaction can access multiple objects. So, transitive retry exists. Following the same steps of proof of Claim 17, Claim follows. □

4.4.6 RCM vs. RNLP

Claim 20. *Under globally scheduled systems, schedulability of RCM is equal or better than schedulability of RNLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(2\left(\Phi_{max}+1\right)N_{max}+\Phi_{max}\right)\left(n-1\right)} \tag{4.50}$$

Proof. Substitute $RC_A(T_i)$ in (4.38) by (4.30). γ_i is replaced with γ_i^{ex} and Θ_i is replaced with Θ_i^{ex} because transitive retry exists. Following the same steps of proof of Claim 17, Claim follows. □

4.5 Conclusions

ECM and RCM use jobs' priorities to resolve conflicts between transactions. The transaction with lower priority aborts and retries due to the transaction with higher priority. As each transaction can access multiple objects, a transaction may abort indirectly due to another transaction with no shared objects between them. The indirect retrial is denoted as transitive retry. Under both ECM and RCM, a task incurs at most $2s_{max}$ retry cost for each of its atomic sections due to a conflict with another task's atomic section. Transactions can also retry due to release of higher priority jobs that preempt a transaction in a lower priority job.

The s_{max}/r_{max} ratio determines whether STM is better or as good as lock-free. ECM and RCM have equal or better schedulability than retry-loop lock-free if s_{max} does not exceed one half of r_{max}. s_{max} can exceed r_{max} with equal periods between conflicting tasks, and large access times to the same object within the same transaction.

Schedulability of ECM and RCM was compared against real-time locking protocols (i.e., Global OMLP and RNLP). ECM have equal or better schedulability than OMLP and RNLP if $\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(2N_{max}+1)(n-1)\Phi_{max}}$. RCM have equal or better schedulability than OMLP and RNLP if $\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(2(\Phi_{max}+1)N_{max}+\Phi_{max})(n-1)}$

Chapter 5

The LCM Contention Manager

RE-READ THIS INTRODUCTION AFTER FINISHING CHAPTER Under ECM and RCM, each atomic section can be aborted for at most $2.s_{max}$ by a single interfering atomic section. We present a novel contention manager (CM) for resolving transactional conflicts, called length-based CM (or LCM) [41]. LCM can reduce the abortion time of a single atomic section due to an interfering atomic section below $2.s_{max}$. We upper bound transactional retries and response times under LCM, when used with G-EDF and G-RMA schedulers. We identify the conditions under which LCM outperforms previous real-time STM CMs and lock-free synchronization.

The rest of this Chapter is organized as follows: Section 5.1 presents Length-based Contention Manager (LCM) and illustrates its behaviour. Section 5.2 derives LCM properties. Response time analysis of tasks under G-EDF/LCM is given in Section 5.3. Schedulability of G-EDF/LCM is compared to schedulability of ECM and lock-free in Section 5.4. Section 5.5 gives response time analysis for G-RMA/LCM. Schedulability of G-RMA/LCM is compared against RCM and lock-free in Section 5.6. We conclude Chapter in Section 5.7.

5.1 Length-based CM

LCM resolves conflicts based on the priority of conflicting transactions, besides the length of the interfering atomic section, and the length of the interfered atomic section. Priority of each transaction equals priority of its containing job (i.e., $p\left(s_i^k\right) = p_i^x$ where $s_i^k \in \tau_i^x$). ECM and RCM (Chapter 4) use only priorities to resolve conflicts. LCM allows lower priority jobs to retry for lesser time than that under ECM and RCM, but higher priority jobs, sometimes, wait for lower priority ones with bounded priority-inversion.

5.1.1 Design and Rationale

Algorithm 3: LCM

Data: s_i^k and s_j^l are two conflicting atomic sections.
$\psi \rightarrow$ predefined threshold $\in [0, 1]$.
$\delta_i^k \rightarrow$ remaining execution length of s_i^k.
$s\left(s_i^k\right) \rightarrow$ start time of s_i^k. $s\left(s_i^k\right)$ is updated each time s_i^k aborts and retries to the start time of the new retry.
$s\left(s_j^l\right) \rightarrow$ the same as $s\left(s_i^k\right)$ but for s_j^l.
Result: which atomic section of s_i^k or s_j^l aborts

1 **if** $s\left(s_i^k\right) < s\left(s_j^l\right)$ **then**
2   **if** $p\left(s_i^k\right) > p\left(s_j^l\right)$ **then**
3   s_j^l aborts;
4   **else**
5         $c_{ij}^{kl} = len(s_j^l)/len(s_i^k)$;
6         $\alpha_{ij}^{kl} = ln(\psi)/(ln(\psi) - c_{ij}^{kl})$;
7         $\alpha = \left(len(s_i^k) - \delta_i^k\right)/len(s_i^k)$;
8   **if** $\alpha \leq \alpha_{ij}^{kl}$ **then**
9   s_i^k aborts;
10   **else**
11   s_j^l aborts;
12   **end**
13   **end**
14 **else**
15   Swap s_i^k and s_j^l;
16 **end**

For both ECM and RCM, s_i^k can be totally repeated if s_j^l — which belongs to a higher priority job τ_j^b than τ_i^a — conflicts with s_i^k at the end of its execution, while s_i^k is just about to commit. Thus, LCM, shown in Algorithm 3, uses the remaining length of s_i^k when it is interfered, as well as $len(s_j^l)$, to decide which transaction must be aborted. If s_i^k starts before s_j^l, then s_i^k is the interfered atomic section and s_j^l is the interfering atomic section (step 1). Otherwise, s_i^k and s_j^l are swapped (step 15). If $p\left(s_i^k\right)$ was greater than $p\left(s_j^l\right)$, then s_i^k would be the one that commits, because it belongs to a higher priority job, and it started before s_j^l (step 3). Otherwise, c_{ij}^{kl} is calculated (step 5) to determine whether it is worth aborting s_i^k in favour of s_j^l, because $len(s_j^l)$ is relatively small compared to the remaining execution length of s_i^k (explained further).

We assume that:

$$c_{ij}^{kl} = len(s_j^l)/len(s_i^k) \tag{5.1}$$

where $c_{ij}^{kl} \in]0, \infty[$, to cover all possible lengths of s_j^l. Our idea is to reduce the opportunity for the abort of s_i^k if it is close to committing when interfered and $len(s_j^l)$ is large. This abort opportunity is increasingly reduced as s_i^k gets closer to the end of its execution, or $len(s_j^l)$ gets larger.

On the other hand, as s_i^k is interfered early, or $len(s_j^l)$ is small compared to s_i^k's remaining length, the abort opportunity is increased even if s_i^k is close to the end of its execution. To decide whether s_i^k must be aborted or not, we use a threshold value $\psi \in [0,1]$ that determines α_{ij}^{kl} (step 6), where α_{ij}^{kl} is the maximum percentage of $len(s_i^k)$ below which s_j^l is allowed to abort s_i^k. Thus, if the already executed part of s_i^k — when s_j^l interferes with s_i^k — does not exceed $\alpha_{ij}^{kl} len(s_i^k)$, then s_i^k is aborted (step 9). Otherwise, s_j^l is aborted (step 11).

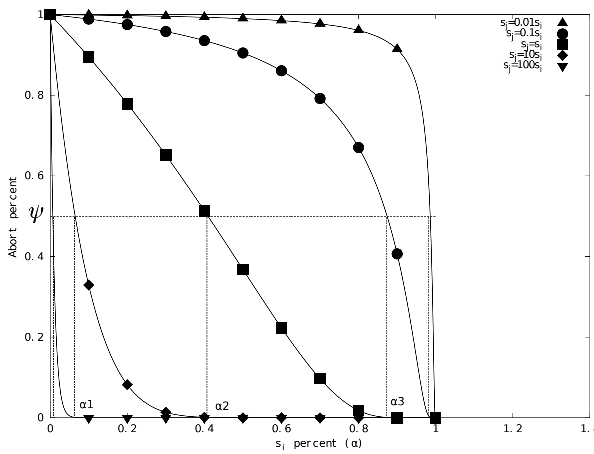


Figure 5.1: Interference of s_i^k by various lengths of s_j^l

The behavior of LCM is illustrated in Figure 5.1. In this figure, the horizontal axis corresponds to different values of α ranging from 0 to 1, and the vertical axis corresponds to different values of abort opportunities, $f(c_{ij}^{kl}, \alpha)$, ranging from 0 to 1 and calculated by (5.2):

$$f(c_{ij}^{kl}, \alpha) = e^{\frac{-c_{ij}^{kl}\alpha}{1-\alpha}} \tag{5.2}$$

where c_{ij}^{kl} is calculated by (5.1).

Figure 5.1 shows one atomic section s_i^k (whose α changes along the horizontal axis) interfered by five different lengths of s_j^l. For a predefined value of $f(c_{ij}^{kl}, \alpha)$ (denoted as ψ in Algorithm 3), there corresponds a specific value of α (which is α_{ij}^{kl} in Algorithm 3) for each curve. For example, when $len(s_j^l) = 0.1 \times len(s_i^k)$, s_j^l aborts s_i^k if the latter has not executed more than $\alpha 3$ percentage (shown in Figure 5.1) of its execution length. As $len(s_j^l)$ decreases, the corresponding α_{ij}^{kl} increases (as shown in Figure 5.1, $\alpha 3 > \alpha 2 > \alpha 1$).

Equation (5.2) achieves the desired requirement that the abort opportunity is reduced as s_i^k gets closer to the end of its execution (as $\alpha \to 1$, $f(c_{ij}^{kl}, 1) \to 0$), or as the length of the conflicting transaction increases (as $c_{ij}^{kl} \to \infty$, $f(\infty, \alpha) \to 0$). Meanwhile, this abort opportunity is increased as s_i^k is interfered closer to its release (as $\alpha \to 0$, $f(c_{ij}^{kl}, 0) \to 1$), or as the length of the conflicting transaction decreases (as $c_{ij}^{kl} \to 0$, $f(0, \alpha) \to 1$).

LCM is not a centralized CM, which means that, upon a conflict, each transactions has to decide whether it must commit or abort. LCM suffers from transitive retry (Section 4.1.2).

Claim 21. *LCM suffers from transitive retry for multi-object transactions.*

Proof. Following the proof of Claim 32, Claim follows. □

5.1.2 LCM Illustrative Example

Behaviour of LCM can be illustrated by the following example:

- Transaction $s_i^k \in \tau_i^x$ begins execution. Currently, s_i^k does not conflict with any other transaction.
- Transaction $s_j^l \in \tau_j^y$ is released while s_i^k is still running. $\Theta_i^{k^{ex}} \cap \Theta_j^l \neq \emptyset$ and $p_j^y > p_i^x$ (where priority is dynamic in G-EDF, and fixed in G-RMA). c_{ij}^{kl}, α_{ij}^{kl} and α are calculated by steps 5 to 7 in Algorithm 3. s_i^k has not reached α percentage of its execution length yet.
- $\alpha < \alpha_{ij}^{kl}$. Then, s_j^l is allowed to abort and restart s_i^k.
- s_j^l commits. s_i^k executes again.
- Transaction $s_h^v \in \tau_h^u$ is released while s_i^k is running. $\Theta_i^{k^{ex}} \cap \Theta_h^v \neq \emptyset$ and $p_h^u > p_i^x$. c_{ih}^{kv}, α_{ih}^{kv} and α are calculated by steps 5 to 7 in Algorithm 3. s_i^k has already passed α percentage of its execution length. So, s_h^v aborts and restarts in favour of s_i^k.
- Transaction $s_a^b \in \tau_a^f$ is released. $\Theta_i^{k^{ex}} \cap \Theta_a^b \neq \emptyset$ and $p_a^f > p_i^x$ but $p_a^f < p_h^u$. c_{ia}^{kb}, α_{ia}^{kb} and α are calculated by steps 5 to 7 in Algorithm 3. s_i^k has not reached α percentage of its execution length yet. So, s_a^b is allowed to abort s_i^k. Because s_a^b is just starting, LCM allows s_h^v to abort s_a^b. So, the highest priority transaction is not blocked by an intermediate priority transaction s_a^b.
- When s_h^v commits. s_a^b is allowed to execute while s_i^k is retrying.
- When s_a^b commits, s_i^k executes.
- Transaction $s_c^n \in \tau_o^z$ is released while s_i^k is running. $\Theta_i^{k^{ex}} \cap \Theta_c^n \neq \emptyset$ and $p_o^z < p_i^x$. So, s_i^k commits first, then s_c^n is allowed to proceed.

5.2 Properties

LCM properties are given by the following Lemmas. These properties are used to derive retry cost and response time of transactions and tasks under LCM.

Claim 22. *Let s_j^l interfere once with s_i^k at α_{ij}^{kl}. Then, the maximum contribution of s_j^l to s_i^k's retry cost is:*

$$W_i^k(s_j^l) \leq \alpha_{ij}^{kl} len\left(s_i^k\right) + len\left(s_j^l\right) \tag{5.3}$$

Proof. If s_j^l interferes with s_i^k at a Υ percentage, where $\Upsilon < \alpha_{ij}^{kl}$, then the retry cost of s_i^k is $\Upsilon len(s_i^k) + len(s_j^l)$, which is lower than that calculated in (5.3). Besides, if s_j^l interferes with s_i^k after α_{ij}^{kl} percentage, then s_i^k will not abort. □

Claim 23. *A higher priority transaction, s_j^l, aborts and retries due to a lower priority transaction, s_i^k, if s_j^l interferes with s_i^k after the α_{ij}^{kl} percentage. s_j^l's retry cost, due to s_i^k is upper bounded by:*

$$W_j^l(s_i^k) \leq \left(1 - \alpha_{ij}^{kl}\right) len\left(s_i^k\right) \tag{5.4}$$

Proof. It is derived directly from Claim 22, as s_j^l will have to retry for the remaining length of s_i^k. □

Claim 24. *A higher priority job, τ_j^y, suffers from priority inversion for at most number of atomic sections in τ_j^y.*

Proof. Assuming three atomic sections, $s_i^k \in \tau_i^x$, $s_j^l \in \tau_j^y$ and $s_a^b \in \tau_a^z$. $p\left(s_j^l\right) > p\left(s_i^k\right)$ and s_j^l interferes with s_i^k after α_{ij}^{kl}. Then s_j^l will have to abort and retry. At this time, if s_a^b interferes with the other two atomic sections, then $s\left(s_i^k\right) < s\left(s_j^l\right) < s\left(s_a^b\right)$, and the LCM is to decide which transaction to commit based on comparison between each two transactions. So, we have the following cases:-

- $p\left(s_a^b\right) < p\left(s_i^k\right) < p\left(s_j^l\right)$, then s_a^b will not abort any one because s_a^b starts after the other two transactions and s_a^b has the lowest priority. So. τ_j is not blocked by τ_a.

- $p\left(s_i^k\right) < p\left(s_a^b\right) < p\left(s_j^l\right)$. If s_a^b interferes with s_i^k before α_{ia}^{kb}, then s_a^b is allowed to abort s_i^k. Comparison between s_j^l and s_a^b will result in LCM choosing s_j^l to commit and abort s_a^b starts after s_j^l, and $p\left(s_j^l\right) > p\left(s_a^b\right)$. So, the higher priority job τ_j^y is not blocked by the intermediate priority job τ_a^z. If s_a^b is not allowed to abort s_i^k, the situation is still the same, because s_j^l was already retrying until s_i^k finishes. When s_i^k finishes, LCM will still choose the higher priority s_j^l.

- $p\left(s_a^b\right) > p\left(s_j^l\right) > p\left(s_i^k\right)$, then if s_a^b is chosen to commit, this is not priority inversion for τ_j^y because τ_a^z is of higher priority.

- if τ_a^z preempts τ_i^x, then LCM will compare only between s_j^l and s_a^b. If $p\left(s_a^b\right) < p\left(s_j^l\right)$, then s_j^l will commit because of its job's higher priority. Otherwise, s_j^l will retry, but this will not be priority inversion because τ_a^z is already of higher priority than τ_j^y. If τ_a^z does not access any object but it preempts τ_i^x, then LCM will choose s_j^l to commit as only already running transactions are competing together.

So, by generalizing these cases to any number of conflicting jobs, it is seen that when an atomic section, s_j^l, of a higher priority job is in conflict with a number of atomic sections belonging to lower priority jobs, s_j^l can suffer from priority inversion by only one of them. After that, LCM will choose s_j^l before other lower priority transactions. So, each higher priority job can suffer priority inversion at most its number of atomic section. Claim follows. □

Claim 25. *The maximum delay suffered by $s_j^l(\theta)$ due to lower priority jobs is caused by the maximum length atomic section accessing object θ, which belongs to a lower priority job than τ_j^b that owns $s_j^l(\theta)$.*

Proof. Assume three atomic sections, $s_i^k(\theta)$, $s_j^l(\theta)$, and $s_h^z(\theta)$, where $p_j > p_i$, $p_j > p_h$, and $len(s_i^k(\theta)) > len(s_h^z(\theta))$. Now, $\alpha_{ij}^{kl} > \alpha_{hj}^{zl}$ and $c_{ij}^{kl} < c_{hj}^{zl}$. By applying (5.4) to obtain the contribution of $s_i^k(\theta)$ and $s_h^z(\theta)$ to the priority inversion of $s_j^l(\theta)$ and dividing them, we get:

$$\frac{W_j^l(s_i^k(\theta))}{W_j^l(s_h^z(\theta))} = \frac{\left(1-\alpha_{ij}^{kl}\right) len(s_i^k(\theta))}{\left(1-\alpha_{hj}^{zl}\right) len(s_h^z(\theta))}$$

By substitution for αs from (5.2):

$$= \frac{(1-\frac{ln\psi}{ln\psi - c_{ij}^{kl}})len(s_i^k(\theta))}{(1-\frac{ln\psi}{ln\psi - c_{hj}^{zl}})len(s_h^z(\theta))} = \frac{(\frac{-c_{ij}^{kl}}{ln\psi - c_{ij}^{kl}})len(s_i^k(\theta))}{(\frac{-c_{hj}^{zl}}{ln\psi - c_{hj}^{zl}})len(s_h^z(\theta))}$$

$\because ln\psi \le 0$ and $c_{ij}^{kl}, c_{hj}^{kl} > 0, \therefore$ by substitution from (5.1)

$$= \frac{len(s_j^l(\theta))/(ln\psi - c_{ij}^{kl})}{len(s_j^l(\theta))/(ln\psi - c_{hj}^{zl})} = \frac{ln\psi - c_{hj}^{zl}}{ln\psi - c_{ij}^{kl}} > 1$$

Thus, as the length of the interfered atomic section increases, the delay suffered by the interfering atomic section increases. Claim follows. □

5.3 Response Time of G-EDF/LCM

Claim 26. *$RC(T_i)$ for a task τ_i under G-EDF/LCM is upper bounded by:*

$$\begin{aligned} RC(T_i) &= \left(\sum_{\forall \tau_h \in \gamma_i} \sum_{\forall \theta \in \theta_i \wedge \theta_h} \left(\left\lceil \frac{T_i}{T_h} \right\rceil \sum_{\forall s_h^l(\theta)} len\Big(s_h^l(\theta)\Big)\right.\right. \\ &+ \left.\left. \alpha_{max}^{hl} len\Big(s_{max}^h(\theta)\Big)\right)\right) + \sum_{\forall s_i^y(\theta)} \left(1-\alpha_{max}^{iy}\right) len\left(s_{max}^i(\theta)\right) \end{aligned} \quad (5.5)$$

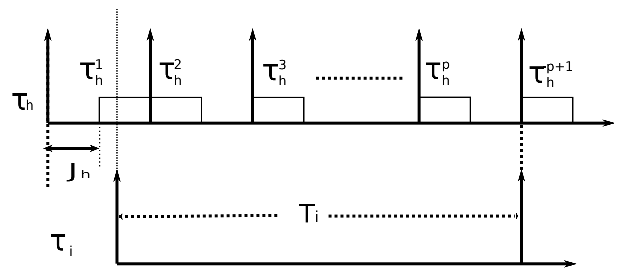


Figure 5.2: τ_h^p has a higher priority than τ_i^x

where α_{max}^{hl} is the α value that corresponds to ψ due to the interference of $s_{max}^h(\theta)$ by $s_h^l(\theta)$. α_{max}^{iy} is the α value that corresponds to ψ due to the interference of $s_{max}^i(\theta)$ by $s_i^y(\theta)$.

Proof. The maximum number of higher priority instances of τ_h that can interfere with τ_i^x is $\left\lceil \frac{T_i}{T_h} \right\rceil$, as shown in Figure 5.2, where one instance of τ_h and τ_h^p coincides with the absolute deadline of τ_i^x.

By using Claims 6, 22, 23, 24 and 25 to determine the effect of atomic sections belonging to higher and lower priority instances of interfering tasks to τ_i^x, Claim follows. □

Response time of τ_i is calculated by (4.13).

5.4 Schedulability of G-EDF/LCM

We now compare the schedulability of G-EDF/LCM with ECM (Chapter 4) to understand when G-EDF/LCM will perform better. Toward this, we compare the total utilization of ECM with that of G-EDF/LCM. For each method, we inflate the c_i of each task τ_i by adding the retry cost suffered by τ_i. Thus, if method A adds retry cost $RC_A(T_i)$ to c_i, and method B adds retry cost $RC_B(T_i)$ to c_i, then the schedulability of A and B are compared as:

$$\begin{aligned} \sum_{\forall \tau_i} \frac{c_i + RC_A(T_i)}{T_i} &\leq \sum_{\forall \tau_i} \frac{c_i + RC_B(T_i)}{T_i} \\ \sum_{\forall \tau_i} \frac{RC_A(T_i)}{T_i} &\leq \sum_{\forall \tau_i} \frac{RC_B(T_i)}{T_i} \end{aligned} \tag{5.6}$$

Thus, schedulability is compared by substituting the retry cost added by the synchronization methods in (5.6).

5.4.1 Schedulability of G-EDF/LCM and ECM

Claim 27. *Let s_{max} be the maximum length atomic section accessing any object θ. Let α_{max} and α_{min} be the maximum and minimum values of α for any two atomic sections $s_i^k(\theta)$ and $s_j^l(\theta)$. Given a threshold ψ, schedulability of G-EDF/LCM is equal or better than ECM if for any task τ_i:*

$$\frac{1-\alpha_{min}}{1-\alpha_{max}} \leq \sum_{\forall \tau_h \in \gamma_i} \left\lceil \frac{T_i}{T_h} \right\rceil \tag{5.7}$$

Proof. Under ECM, $RC(T_i)$ is upper bounded by:

$$RC(T_i) \leq \sum_{\forall \tau_h \in \gamma_i} \sum_{\forall \theta \in \ (\theta_i \wedge \theta_h)} \left(\left\lceil \frac{T_i}{T_h} \right\rceil \sum_{\forall s_h^z(\theta)} 2len(s_{max}) \right) \tag{5.8}$$

with the assumption that all lengths of atomic sections of (4.9), (**??**) and (5.5) are replaced by s_{max}. Let α_{max}^{hl} in (5.5) be replaced with α_{max}, and α_{max}^{iy} in (5.5) be replaced with α_{min}. As α_{max}, α_{min}, and $len(s_{max})$ are all constants, (5.5) is upper bounded by:

$$\begin{aligned} RC(T_i) \ \ \leq \ \ & \Bigg(\sum_{\forall \tau_h \in \gamma_i} \sum_{\forall \theta \in \theta_i \wedge \theta_h} \Bigg(\left\lceil \frac{T_i}{T_h} \right\rceil \sum_{\forall s_h^l(\theta)} (1+\alpha_{max}) \\ & len\Big(s_{max}\Big)\Bigg)\Bigg) + \sum_{\forall s_i^y(\theta)} \Big(1-\alpha_{min}\Big) len\Big(s_{max}\Big) \end{aligned} \tag{5.9}$$

If β_1^{ih} is the total number of times any instance of τ_h accesses shared objects with τ_i, then $\beta_1^{ih} = \sum_{\forall \theta \in (\theta_i \wedge \theta_h)} \sum_{\forall s_h^z(\theta)}$. Furthermore, if β_2^i is the total number of times any instance of τ_i accesses shared objects with any other instance, $\beta_2^i = \sum_{\forall s_i^y(\theta)}$, *where θ is shared with another task*. Then, $\beta_i = max\{max_{\forall \tau_h \in \gamma_i}\{\beta_1^{ih}\}, \beta_2^i\}$ is the maximum number of accesses to all shared objects by any instance of τ_i or τ_h. Thus, (5.8) becomes:

$$RC(T_i) \leq \sum_{\tau_h \in \gamma_i} 2 \left\lceil \frac{T_i}{T_h} \right\rceil \beta_i len(s_{max}) \tag{5.10}$$

and (5.9) becomes:

$$RC(T_i) \leq \beta_i len(s_{max}) \left((1-\alpha_{min}) + \sum_{\forall \tau_h \in \gamma_i} \left\lceil \frac{T_i}{T_h} \right\rceil (1+\alpha_{max}) \right) \tag{5.11}$$

We can now compare the total utilization of G-EDF/LCM with that of ECM by comparing (5.9) and (5.11) for all τ_i:

$$\sum_{\forall \tau_i} \frac{(1-\alpha_{min}) + \sum_{\forall \tau_h \in \gamma_i} \left(\left\lceil \frac{T_i}{T_h} \right\rceil (1+\alpha_{max}) \right)}{T_i}$$
$$\leq \sum_{\forall \tau_i} \frac{\sum_{\forall \tau_h \in \gamma_i} 2 \left\lceil \frac{T_i}{T_h} \right\rceil}{T_i} \tag{5.12}$$

(5.12) is satisfied if for each τ_i, the following condition is satisfied:

$$(1-\alpha_{min}) + \sum_{\forall \tau_h \in \gamma_i} \left(\left\lceil \frac{T_i}{T_h} \right\rceil (1+\alpha_{max}) \right) \leq 2 \sum_{\forall \tau_h \in \gamma_i} \left\lceil \frac{T_i}{T_h} \right\rceil$$
$$\therefore \frac{1-\alpha_{min}}{1-\alpha_{max}} \leq \sum_{\forall \tau_h \in \gamma_i} \left\lceil \frac{T_i}{T_h} \right\rceil$$

Claim follows. □

5.4.2 G-EDF/LCM versus Lock-free

We consider the retry-loop lock-free synchronization for G-EDF given in [37]. This lock-free approach is the most relevant to our work.

Claim 28. *Let s_{max} denote $len(s_{max})$ and r_{max} denote the maximum execution cost of a single iteration of any retry loop of any task in the retry-loop lock-free algorithm in [37]. Now, G-EDF/LCM achieves higher schedulability than the retry-loop lock-free approach if the upper bound on s_{max}/r_{max} under G-EDF/LCM ranges between 0.5 and 2 (which is higher than that under ECM).*

Proof. From [37], the retry-loop lock-free algorithm is upper bounded by:

$$RL(T_i) = \sum_{\tau_h \in \gamma_i} \left(\left\lceil \frac{T_i}{T_h} \right\rceil + 1 \right) \beta_i r_{max} \tag{5.13}$$

where β_i is as defined in Claim 27. The retry cost of τ_i in G-EDF/LCM is upper bounded by (5.11). By comparing G-EDF/LCM's total utilization with that of the retry-loop lock-free algorithm, we get:

$$\sum_{\forall \tau_i} \frac{\left((1-\alpha_{min}) + \sum_{\forall \tau_h \in \gamma_i} \left(\left\lceil \frac{T_i}{T_h} \right\rceil (1+\alpha_{max}) \right) \right) \beta_i s_{max}}{T_i}$$
$$\leq \sum_{\forall \tau_i} \frac{\sum_{\forall \tau_h \in \gamma_i} \left(\left\lceil \frac{T_i}{T_h} \right\rceil + 1 \right) \beta_i r_{max}}{T_i}$$

$$\therefore \frac{s_{max}}{r_{max}} \leq \frac{\sum_{\forall \tau_i} \frac{\sum_{\forall \tau_h \in \gamma_i} \left(\left\lceil \frac{T_i}{T_h} \right\rceil + 1 \right) \beta_i}{T_i}}{\sum_{\forall \tau_i} \frac{\left((1-\alpha_{min}) + \sum_{\forall \tau_h \in \gamma_i} \left(\left\lceil \frac{T_i}{T_h} \right\rceil (1+\alpha_{max}) \right) \right) \beta_i}{T_i}} \tag{5.14}$$

Let the number of tasks that have shared objects with τ_i be ω (i.e., $\sum_{\tau_h \in \gamma_i} = \omega \geq 1$ since at least one task has a shared object with τ_i; otherwise, there is no conflict between tasks). Let the total number of tasks be n, so $1 \leq \omega \leq n-1$, and $\left\lceil \frac{T_i}{T_h} \right\rceil \in [1, \infty[$. To find the minimum and maximum values for the upper bound on s_{max}/r_{max}, we consider the following cases:

- $\alpha_{min} \to 0, \alpha_{max} \to 0$

$\therefore$ (5.14) will be:

$$\frac{s_{max}}{r_{max}} \leq 1 + \frac{\sum_{\forall \tau_i} \frac{\omega - 1}{T_i}}{\sum_{\forall \tau_i} \frac{1 + \sum_{\forall \tau_h \in \gamma_i} \left\lceil \frac{T_i}{T_h} \right\rceil}{T_i}} \tag{5.15}$$

By substituting the edge values for ω and $\left\lceil \frac{T_i}{T_h} \right\rceil$ in (5.15), we derive that the upper bound on s_{max}/r_{max} lies between 1 and 2.

- $\alpha_{min} \to 0, \alpha_{max} \to 1$

(5.14) becomes

$$\frac{s_{max}}{r_{max}} \leq 0.5 + \frac{\sum_{\forall \tau_i} \frac{\omega - 0.5}{T_i}}{\sum_{\forall \tau_i} \frac{1 + 2\sum_{\forall \tau_h \in \gamma_i} \left\lceil \frac{T_i}{T_h} \right\rceil}{T_i}} \tag{5.16}$$

By applying the edge values for ω and $\left\lceil \frac{T_i}{T_h} \right\rceil$ in (5.16), we derive that the upper bound on s_{max}/r_{max} lies between 0.5 and 1.

- $\alpha_{min} \to 1, \alpha_{max} \to 0$

This case is rejected since $\alpha_{min} \leq \alpha_{max}$.

- $\alpha_{min} \to 1, \alpha_{max} \to 1$

$\therefore$ (5.14) becomes:

$$\frac{s_{max}}{r_{max}} \leq 0.5 + \frac{\sum_{\tau_i} \frac{\omega}{T_i}}{2\sum_{\tau_i} \frac{\sum_{\forall \tau_h \in \gamma_i} \left\lceil \frac{T_i}{T_h} \right\rceil}{T_i}} \tag{5.17}$$

By applying the edge values for ω and $\left\lceil \frac{T_i}{T_h} \right\rceil$ in (5.17), we derive that the upper bound on s_{max}/r_{max} lies between 0.5 and 1, which is similar to that achieved by ECM.

Summarizing from the previous cases, the upper bound on s_{max}/r_{max} lies between 0.5 and 2, whereas for ECM, it lies between 0.5 and 1. Claim follows.

□

5.5 Response Time of G-RMA/LCM

Claim 29. *Let* $\lambda_2(j,\theta) = \sum_{\forall s_j^l(\theta)} len(s_j^l(\theta)) + \alpha_{max}^{jl} len(s_{max}^j(\theta))$*, where* α_{max}^{jl} *is the* α *value corresponding to* ψ *due to the interference of* $s_{max}^j(\theta)$ *by* $s_j^l(\theta)$*. The retry cost of any task* τ_i *under G-RMA/LCM during* T_i *is given by:*

$$RC\left(T_i\right) = \sum_{\forall \tau_j^*} \left(\sum_{\theta \in (\theta_i \wedge \theta_j)} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \lambda_2(j,\theta) \right) \right) + \sum_{\forall s_i^y(\theta)} \left(1 - \alpha_{max}^{iy}\right) len\left(s_{max}^i(\theta)\right) \tag{5.18}$$

where $\tau_j^* = \{\tau_j | (\tau_j \in \gamma_i) \wedge (p_j > p_i)\}$.

Proof. Under G-RMA, all instances of a higher priority task, τ_j, can conflict with a lower priority task, τ_i, during T_i. (5.3) can be used to determine the contribution of each conflicting atomic section in τ_j to τ_i. Meanwhile, all instances of any task with lower priority than τ_i can conflict with τ_i during T_i. Claims 23 and 24 can be used to determine the contribution of conflicting atomic sections in lower priority tasks to τ_i. Using the previous notations and Claim 11, the Claim follows. □

The response time is calculated by (**??**) with replacing $RC(R_i^{up})$ with $RC(T_i)$.

5.6 Schedulability of G-RMA/LCM

5.6.1 Schedulability of G-RMA/LCM and RCM

Claim 30. *Under the same assumptions of Claims 27 and 29, G-RMA/LCM's schedulability is equal or better than RCM if:*

$$\frac{1-\alpha_{min}}{1-\alpha_{max}} \leq \sum_{\forall \tau_j^*} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \tag{5.19}$$

Proof. Under the same assumptions as that of Claims 27 and 29, (5.18) can be upper bounded as:

$$RC(T_i) \leq \sum_{\forall \tau_j^*} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) (1+\alpha_{max}) len(s_{max}) \beta_i \right) + (1-\alpha_{min}) len(s_{max}) \beta_i \tag{5.20}$$

For RCM, (4.16) for $RC(T_i)$ is upper bounded by:

$$RC(T_i) \leq \sum_{\forall \tau_j^*} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) 2\beta_i len(s_{max})$$

By comparing the total utilization of G-RMA/LCM with that of RCM, we get:

$$\sum_{\forall \tau_i} \frac{len(s_{max})\beta_i \left((1-\alpha_{min}) + \sum_{\forall \tau_j^*} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) (1+\alpha_{max}) \right) \right)}{T_i} \leq \sum_{\forall \tau_i} \frac{2len(s_{max})\beta_i \sum_{\forall \tau_j^*} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right)}{T_i} \tag{5.21}$$

(5.21) is satisfied if $\forall \tau_i$ (5.19) is satisfied. Claim follows. □

5.6.2 G-RMA/LCM versus Lock-free

Although [37] considers retry-loop lock-free synchronization for G-EDF systems, [37] also applies for G-RMA systems.

Claim 31. *Let s_{max} denote $len(s_{max})$ and r_{max} denote the maximum execution cost of a single iteration of any retry loop of any task in the retry-loop lock-free algorithm in [37]. G-RMA/LCM achieves higher schedulability than the retry-loop lock-free approach if the upper bound on s_{max}/r_{max} under G-RMA/LCM is no less than 0.5. Upper bound on s_{max}/r_{max} can extend to large values when α_{min} and α_{max} are very large.*

Proof. The retry cost for G-RMA/LCM is upper bounded by (5.18). Let $\gamma_i = \tau_j^* \cup \bar{\tau}_j$, where τ_j^* is the set of higher priority tasks than τ_i sharing objects with τ_i. $\bar{\tau}_j$ is the set of lower priority tasks than τ_i sharing objects with it. We follow the same definitions of β_i, r_{max}, and $RL(T_i)$ given in the proof of Claim (28). Schedulability of G-RMA/LCM equals or exceeds the schedulability of retry-loop lock-free algorithm if:

$$\begin{aligned}\frac{s_{max}}{r_{max}} \leq & \frac{\sum_{\forall \tau_i} \frac{\sum_{\tau_j^*}\left(\left\lceil \frac{T_i}{T_j}\right\rceil + 1\right)}{T_i}}{\sum_{\forall \tau_i} \frac{\left(1-\alpha_{min}\right) + \sum_{\tau_j^*}\left(\left\lceil \frac{T_i}{T_j}\right\rceil + 1\right)(1+\alpha_{max})}{T_i}} \\ + & \frac{2\sum_{\forall \tau_i} \frac{\sum_{\forall \bar{\tau}_j}}{T_i}}{\sum_{\forall \tau_i} \frac{\left(1-\alpha_{min}\right) + \sum_{\tau_j^*}\left(\left\lceil \frac{T_i}{T_j}\right\rceil + 1\right)(1+\alpha_{max})}{T_i}}\end{aligned} \tag{5.22}$$

If $p_j < p_i$, $\therefore \left\lceil \frac{T_i}{T_j} \right\rceil = 1$, because the system assumes implicit deadline tasks and uses the G-RMA scheduler. Let ω_1 be the size of τ_i^* and ω_2 be the size of $\bar{\tau}_i$. $\therefore \omega_1^i \geq 1$ and $\omega_2^i \geq 1$. Otherwise, there is no conflict with τ_i. To find the maximum and minimum upper bounds for s_{max}/r_{max}, the following cases are considered:

- $\alpha_{min} \to 0$, $\alpha_{max} \to 0$

  $$\therefore \frac{s_{max}}{r_{max}} \leq 1 + \frac{\sum_{\forall \tau_i} \frac{2\omega_2^i - 1}{T_i}}{\sum_{\forall \tau_i} \frac{1 + \sum_{\tau_j^*}\left(\left\lceil \frac{T_i}{T_j}\right\rceil + 1\right)}{T_i}} \tag{5.23}$$

  As the second term in (5.23) is always positive (because $\omega_2^i \geq 1$), the minimum upper bound on s_{max}/r_{max} is 1. To get the maximum upper bound on s_{max}/r_{max}, let $\left\lceil \frac{T_i}{T_j} \right\rceil$ approach its minimum value of 1, $\omega_1^i \to 0$, and $\omega_2^i \to n-1$ (the maximum and minimum values for ω_1^i and ω_2^i, respectively. n is number of tasks). Now:

  $$\therefore \frac{s_{max}}{r_{max}} \leq (2n-2)$$

  Of course, n cannot be lower than 2. Otherwise, there will be no conflicting tasks.

- $\alpha_{min} \to 0$, $\alpha_{max} \to 1$

  $$\frac{s_{max}}{r_{max}} \leq \frac{1}{2} + \frac{\sum_{\forall \tau_i} \frac{4\omega_2^i - 1}{T_i}}{2\sum_{\forall \tau_i} \frac{1 + 2\sum_{\tau_j^*}\left(\left\lceil \frac{T_i}{T_j}\right\rceil + 1\right)}{T_i}} \tag{5.24}$$

  The minimum upper bound for s_{max}/r_{max} is 0.5. This can happen when $T_i \gg T_j$. To get the maximum upper bound on s_{max}/r_{max}, let $\left\lceil \frac{T_i}{T_j} \right\rceil$ approach its minimum value 1,

$\omega_2^i \to n-1$, and $\omega_1^i \to 0$. Now:

$$\frac{s_{max}}{r_{max}} \leq 2n-2$$

- $\alpha_{min} \to 1$, $\alpha_{max} \to 0$ This case is rejected because α_{max} must be greater or equal to α_{min}.

- $\alpha_{min} \to 1$, $\alpha_{max} \to 1$

$$\frac{s_{max}}{r_{max}} \leq \frac{1}{2} + \frac{\sum_{\forall \tau_i} \frac{\omega_2^i}{T_i}}{\sum_{\forall \tau_i} \frac{\sum_{\tau_j^*} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right)}{T_i}} \tag{5.25}$$

The minimum upper bound for s_{max}/r_{max} is 0.5. This can happen when $T_i \gg T_j$. To get the maximum upper bound on s_{max}/r_{max}, let $\left\lceil \frac{T_i}{T_j} \right\rceil$ approach its minimum value 1, $\omega_2^i \to n-1$, $\omega_1^i \to 0$. Now:

$$\frac{s_{max}}{r_{max}} \to \infty$$

From the previous cases, we can derive that the upper bound on s_{max}/r_{max} extends from 0.5 to large values. Claim follows. □

5.7 Conclusions

In ECM and RCM, a task incurs at most $2s_{max}$ retry cost for each of its atomic section due to conflict with another task's atomic section. With LCM, this retry cost is reduced to $(1+\alpha_{max})s_{max}$ for each aborted atomic section. In ECM and RCM, tasks do not retry due to lower priority tasks, whereas in LCM, they do so. In G-EDF/LCM, retry due to a lower priority job is encountered only from a task τ_j's last job instance during τ_i's period. This is not the case with G-RMA/LCM, because, each higher priority task can be aborted and retried by any job instance of lower priority tasks. Schedulability of G-EDF/LCM and G-RMA/LCM is better or equal to ECM and RCM, respectively, by proper choices for α_{min} and α_{max}. Schedulability of G-EDF/LCM is better than retry-loop lock-free synchronization for G-EDF if the upper bound on s_{max}/r_{max} is between 0.5 and 2, which is higher than that achieved by ECM. G-RMA/LCM achieves higher schedulability than retry-loop lock-free synchronization if s_{max}/r_{max} is not greater than 0.5. For high values of α in G-RMA/LCM, s_{max}/r_{max} can extend to large values.

Chapter 6

The PNF Contention Manager

In this chapter, we present a novel contention manager for resolving transactional conflicts, called PNF [**?**]. We upper bound transactional retries and task response times under PNF, when used with the G-EDF and G-RMA schedulers. We formally identify the conditions under which PNF outperforms previous real-time STM contention managers and lock-free synchronization.

The rest of this Chapter is organized as follows: Section 6.1 discusses limitations of previous contention managers and the motivation to PNF. Section 6.2 give a formal description of PNF. Section 6.3 derives PNF's properties. We upper bound retry cost and response time under PNF in Section 6.4. Schedulability comparison between PNF and previous synchronization techniques is given in Section 6.5. We conclude Chapter in Section 6.6.

6.1 Limitations of ECM, RCM, and LCM

ECM, RCM and LCM [41, 42] assumes that each transaction accesses only one object. This assumption simplifies the retry cost (Claims **??**, 11, 26 and 29) and response time analysis (Sections 4.1, 4.2, 5.3 and 5.5). Besides, it enables a one-to-one comparison with lock-free synchronization in [37]. With multiple objects per transaction, ECM, RCM and LCM will face transitive retry, which we illustrate with an example.

Example 1. Consider three atomic sections s_1^x, s_2^y, and s_3^z belonging to jobs τ_1^x,τ_2^y, and τ_3^z, with priorities $p_3^z > p_2^y > p_1^x$, respectively. Assume that s_1^x and s_2^y share objects, s_2^y and s_3^z share objects. s_1^x and s_3^z do not share objects. s_3^z can cause s_2^y to retry, which in turn will cause s_1^x to retry. This means that s_1^x may retry transitively because of s_3^z, which will increase the retry cost of s_1^x.

Assume another atomic section s_4^f is introduced. Priority of s_4^f is higher than priority of s_3^z. s_4^f shares objects only with s_3^z. Thus, s_4^f can make s_3^z to retry, which in turn will make s_2^y to retry, and finally, s_1^x to retry. Thus, transitive retry will move from s_4^f to s_1^x, increasing the retry cost of s_1^x. The situation gets worse as more tasks of higher priorities are added, where each task shares objects with its immediate lower priority task. τ_3^z may have atomic sections that share objects with τ_1^x, but this will not prevent the effect of transitive retry due to s_1^x.

Definition 2. ***Transitive Retry:*** *A transaction s_i^k suffers from transitive retry when it conflicts with a higher priority transaction s_j^l, which in turn conflicts with a higher priority transaction s_z^h, but s_i^k does not conflict with s_z^h. Still, when s_j^l retries due to s_z^h, s_i^k also retries due to s_j^l. Thus, the effect of the higher priority transaction s_z^h is transitively moved to the lower priority transaction s_i^k, even when they do not conflict on common objects.*

Claim 32. *ECM, RCM and LCM suffer from transitive retry for multi-object transactions.*

Proof. ECM, RCM and LCM depend on priorities to resolve conflicts between transactions. Thus, lower priority transaction must always be aborted for a conflicting higher priority transaction in ECM and RCM. In LCM, lower priority transactions are conditionally aborted for higher priority ones. Claim follows. □

Therefore, the analysis in Chapters 4 and 5 must extend the set of objects that can cause an atomic section of a lower priority job to retry. This can be done by initializing the set of conflicting objects, γ_i, to all objects accessed by all transactions of τ_i. We then cycle through all transactions belonging to all other higher priority tasks. Each transaction s_j^l that accesses at least one of the objects in γ_i adds all other objects accessed by s_j^l to γ_i. The loop over all higher priority tasks is repeated, each time with the new γ_i, until there are no more transactions accessing any object in γ_i[1].

In addition to the *transitive retry* problem, retrying higher priority transactions can prevent lower priority tasks from running. This happens when all processors are busy with higher priority jobs. When a transaction retries, the processor time is wasted. Thus, it would be better to give the processor to some other task.

Essentially, what we present is a new contention manager that avoids the effect of transitive retry. We call it, Priority contention manager with Negative values and First access (or PNF). PNF also tries to enhance processor utilization. This is done by allocating processors to jobs with non-retrying transactions. PNF is described in Section 6.2.

6.2 The PNF Contention Manager

Algorithm 4 describes PNF. It manages two sets. The first is the m-set, which contains at most m non-conflicting transactions, where m is the number of processors, as there cannot be

[1]However, note that, this solution may over-extend the set of conflicting objects, and may even contain all objects accessed by all tasks.

more than m executing transactions (or generally, m executing jobs) at the same time. When a transaction is entered in the m-set, it executes non-preemptively and no other transaction can abort it. A transaction in the m-set is called an *executing transaction*. This means that, when a transaction is executing before the arrival of higher priority conflicting transactions, then the one that started executing first will be committed (Step 8).

Algorithm 4: PNF

Data: *Executing Transaction:* is one that cannot be aborted by any other transaction, nor preempted by a higher priority task;
m-set: m-length set that contains only non-conflicting executing transactions;
n-set: n-length set that contains retrying transactions for n tasks in non-increasing order of priority;
n(z): transaction at index z of the n-set;
s_i^k: a newly released transaction;
s_j^l: one of the executing transactions;
Result: atomic sections that will commit

1 **if** s_i^k *does not conflict with any executing transaction* **then**
2   Assign s_i^k as an executing transaction;
3   Add s_i^k to the m-set;
4   Select s_i^k to commit
5 **else**
6   Add s_i^k to the n-set according to its priority;
7   Assign temporary priority -1 to the job that owns s_i^k ;
8   Select transaction(s) conflicting with s_i^k for commit;
9 **end**
10 **if** s_j^l *commits* **then**
11   **for** *z=1 to size of n-set* **do**
12   **if** *n(z) does not conflict with any executing transaction* **then**
13   **if** *processor available*[2] **then**
14   Restore priority of task owning n(z);
15   Assign n(z) as executing transaction;
16   Add n(z) to m-set and remove it from n-set;
17   Select n(z) for commit;
18   **else**
19   Wait until processor available
20   **end**
21   **end**
22   move to the next n(z);
23   **end**
24 **end**

The second set is the n-set, which holds the transactions that are retrying because of a conflict with one or more of the executing transactions (Step 6), where n stands for the number of tasks in the system. Transactions in the n_set are known as *retrying transaction*.It also holds transactions that cannot currently execute, because processors are busy, either

[2] An idle processor or at least one that runs a non-atomic section task with priority lower than the task holding $n(z)$.

due to processing executing transactions and/or higher priority jobs. Any transaction in the n-set is assigned a temporal priority of -1 (Step 7) (hence the word "Negative" in the algorithm's name). A negative priority is considered smaller than any normal priority, and a transaction continues to hold this negative priority until it is moved to the m-set, where it is restored its normal priority.

A job holding a transaction in the n-set can be preempted by any other job with normal priority, even if that job does not have transactions conflicting with the preempted job. Hence, this set is of length n, as there can be at most n jobs. Transactions in the n-set whose jobs have been preempted are called preempted transactions. The n-set list keeps track of preempted transactions, because as it will be shown, all preempted and non-preempted transactions in the n-set are examined when any of the executing transaction commits. Then, one or more transactions are selected from the n-set to be executing transactions. If a retrying transaction is selected as an executing transaction, the task that owns the retrying transaction regains its priority.

When a new transaction is released, and if it does not conflict with any of the executing transactions (Step 1), then it will allocate a slot in the m-set and becomes an executing transaction. When this transaction is released (i.e., its containing task is already allocated to a processor), it will be able to access a processor immediately. This transaction may have a conflict with any of the transactions in the n-set. However, since transactions in the n-set have priorities of -1, they cannot prevent this new transaction from executing if it does not conflict with any of the executing transactions.

When one of the executing transactions commits (Step 10), it is time to select one of the n-set transactions to commit. The n-set is traversed from the highest priority to the lowest priority (priority here refers to the original priority of the transactions, and not -1) (Step 11). If an examined transaction in the n-set, s_h^b, does not conflict with any executing transaction (Step 12), and there is an available processor for it (Step 13) ("available" means either an idle processor, or one that is executing a job of lower priority than s_h^b), then s_h^b is moved from the n-set to the m-set as an executing transaction and its original priority is restored. If s_h^b is added to the m-set, the new m-set is compared with other transactions in the n-set with lower priority than s_h^b. Hence, if one of the transactions in the n-set, s_d^g, is of lower priority than s_h^b and conflicts with s_h^b, it will remain in the n-set.

The choice of the new transaction from the n-set depends on the original priority of transactions (hence the term "P" in the algorithm name). The algorithm avoids interrupting an already executing transaction to reduce its retry cost. In the meanwhile, it tries to avoid delaying the highest priority transaction in the n-set when it is time to select a new one to commit, even if the highest priority transaction arrives after other lower priority transactions in the n-set.

6.2.1 Illustrative Example

We illustrate PNF with an example. We use the following notions: $s_a^b \in \tau_a^k$ is transaction s_a^b in job τ_a^k. $s_a^b(\theta_1, \theta_2, \theta_3)$ means that s_a^b accesses objects $\theta_1, \theta_2, \theta_3$. $p(s_a^b)$ is the priority of transaction s_a^b. p_i^j is the priority of job τ_i^j. If $s_a^b \in \tau_a^j$, $\therefore p_o(s_a^b) = p_a^j$, where $p_o(s_a^b)$ is the original priority of s_a^b. $p(s_a^b) = -1$, if s_a^b is a retrying transaction; $p(s_a^b) = p_o(s_a^b)$ otherwise. m-set= $\{s_a^b, s_i^k\}$ means that the m-set contains transactions s_a^b and s_i^k regardless of their order. n-set= $\{s_a^b, s_i^k\}$ means that the n-set contains transactions s_a^b and s_i^k in that order, where $p_o(s_a^b) > p_o(s_i^k)$. m-set (n-set) = $\{\phi\}$ means that m-set (n-set) is empty. Assume there are five processors.

1. Initially, m-set= n-set= $\{\phi\}$. $s_a^b(\theta_1, \theta_2) \in \tau_a^b$ is released and checks m-set for conflicting transactions. As m-set is empty, s_a^b finds no conflict and becomes an executing transaction. s_a^b is added to m-set. m-set= $\{s_a^b\}$ and n-set= $\{\phi\}$. s_a^b is executing on processor 1.
2. $s_c^d(\theta_3, \theta_4) \in \tau_c^d$ is released and checks m-set for conflicting transactions. s_c^d does not conflict with s_a^b as they access different objects. s_c^d becomes an executing transaction and is added to m-set. m-set= $\{s_a^b, s_c^d\}$ and n-set= $\{\phi\}$. s_c^d is executing on processor 2.
3. $s_e^f(\theta_1, \theta_5) \in \tau_e^f$ is released and $p_o(s_e^f) < p_o(s_a^b)$. s_e^f conflicts with s_a^b when it checks m-set. s_e^f is added to n-set and becomes a retrying transaction. $p(s_e^f)$ becomes -1. m-set= $\{s_a^b, s_c^d\}$ and n-set= $\{s_e^f\}$. s_e^f is retrying on processor 3.
4. $s_g^h(\theta_1, \theta_6) \in \tau_g^h$ is released and $p_o(s_g^h) > p_o(s_a^b)$. s_g^h conflicts with s_a^b. Though s_g^h is of higher priority than s_a^b, s_a^b is an executing transaction. So s_a^b runs non-preemptively. s_g^h is added to n-set before s_e^f, because $p_o(s_g^h) > p_o(s_e^f)$. $p(s_g^h)$ becomes -1. m-set= $\{s_a^b, s_c^d\}$ and n-set= $\{s_g^h, s_e^f\}$. s_g^h is retrying on processor 4.
5. $s_i^j(\theta_5, \theta_7) \in \tau_i^j$ is released. $p_o(s_i^j) < p_o(s_e^f)$. s_i^j does not conflict with any transaction in m-set. Though s_i^j conflicts with s_e^f and $p_o(s_i^j) < p_o(s_e^f) < p_o(s_g^h)$, s_e^f and s_g^h are retrying transactions. s_i^j becomes an executing transaction and is added to m-set. m-set= $\{s_a^b, s_c^d, s_i^j\}$ and n-set= $\{s_g^h, s_e^f\}$. s_i^j is executing on processor 5.
6. τ_k^l is released. τ_k^l does not access any object. $p_k^l < p_o(s_e^f) < p_o(s_g^h)$, but $p(s_e^f) = p(s_g^h) = -1$. Since there are no more processors, τ_k^l preempts τ_e^f, because the currently assigned priority to $\tau_e^f = p(s_e^f) = -1$ and $p_o(s_g^h) > p_o(s_e^f)$. τ_k^l is running on processor 3. This way, PNF optimizes processor usage. The m-set and n-set are not changed. Although s_e^f is preempted, n-set still records it, as s_e^f might be needed (as will be shown in the following steps).
7. s_i^j commits. s_i^j is removed from m-set. Transactions in n-set are checked from the first (highest p_o) to the last (lowest p_o) for conflicts against any executing transaction. s_g^h is checked first because $p_o(s_g^h) > p_o(s_e^f)$. s_g^h conflicts with s_a^b, so s_g^h cannot be an executing transaction. Now it is time to check s_e^f, even though s_e^f is preempted in step 6. s_e^f also conflicts with s_a^b, so s_e^f cannot be an executing transaction. m-set= $\{s_a^b, s_c^d\}$ and n-set= $\{s_g^h, s_e^f\}$. Now, s_e^f can be retrying on processor 5 if τ_i^j has finished execution.

Otherwise, τ_i^j continues running on processor 5 and s_e^f is still preempted. This is because, $p(s_e^f) = -1$ and $p_i^j > p(s_e^f)$. Let us assume that τ_i^j is still running on processor 5.

8. s_a^b commits. s_a^b is removed from m-set. Transactions in n-set are checked as done in step 7. s_g^h does not conflict with any executing transaction any more. s_g^h becomes an executing transaction. s_g^h is removed from n-set and added to m-set, so m-set= $\{s_c^d, s_g^h\}$. Now, s_e^f is checked against the new m-set. s_e^f conflicts with s_g^h, so s_e^f cannot be an executing transaction. s_e^f can be retrying on processor 1 if τ_a^b has finished execution. Otherwise, s_e^f remains preempted, because $p(s_e^f) = -1$ and $p_a^b > p(s_e^f)$. n-set= $\{s_e^f\}$. Let us assume that τ_a^b is still running on processor 1.
9. s_g^h commits. s_g^h is removed from m-set. τ_g^h continues execution on processor 4. Transactions in n-set are checked again. s_e^f is the only retrying transaction in the n-set, and it does not conflict with any executing transactions. Now, the system has τ_a^b running on processor 1, s_c^d executing on processor 2, τ_k^l running on processor 3, τ_g^h running on processor 4, and τ_i^j running on processor 5. s_e^f can become an executing transaction if it can find a processor. Since p_i^j, $p_k^l < p_o(s_e^f)$, s_e^f can preempt the lowest in priority between τ_i^j and τ_k^l. s_e^f now becomes an executing transaction. s_e^f is removed from the n-set and added to the m-set. So, m-set= $\{s_c^d, s_e^f\}$ and n-set= $\{\phi\}$. If p_i^j, p_k^l were of higher priority than $p_o(s_e^f)$, then s_e^f would have remained in n-set until a processor becomes available.

The example shows that PNF avoids transitive retry. This is illustrated in step 5, where $s_i^j(\theta_5, \theta_7)$ is not affected by the retry of $s_e^f(\theta_1, \theta_5)$. The example also explains how PNF optimizes processor usage. This is illustrated in step 6, where the retrying transaction s_e^f is preempted in favor of τ_k^l.

6.3 Properties

Claim 33. *Transactions scheduled under PNF do not suffer from transitive retry.*

Proof. Proof is by contradiction. Assume that a transaction s_i^k is retrying because of a higher priority transaction s_j^l, which in turn is retrying because of another higher priority transaction s_z^h. Assume that s_i^k and s_z^h do not conflict, yet, s_i^k is transitively retrying due to s_z^h. Note that s_z^h and s_j^l cannot exit together in the m-set as they have shared objects. But they both can be in the n-set, as they can conflict with other *executing transactions*. We have three cases:

Case 1: Assume that s_z^h is an executing transaction. This means that s_j^l is in the n-set. When s_i^k arrives, by the definition of PNF, it will be compared with the m-set, which contains s_z^h. Now, it will be found that s_i^k does not conflict with s_z^h. Also, by the definition of PNF, s_i^k is not compared with transactions in the n-set. When it newly arrives, priorities of n-set transactions are lower than any normal priority. Therefore, as s_i^k does not conflict with any other executing transaction, it joins the m-set and becomes an *executing transaction*. This contradicts the assumption that s_i^k is transitively retrying because of s_z^h.

Case 2: Assume that s_z^h is in the n-set, while s_j^l is an executing transaction. When s_i^k arrives, it will conflict with s_j^l and joins the n-set. Now, s_i^k retries due to s_j^l, and not s_z^h. When s_j^l commits, the n-set is traversed from the highest priority transaction to the lowest one: if s_z^h does not conflict with any other executing transaction and there are available processors, s_z^h becomes an executing transaction. When s_i^k is compared with the m-set, it is found that it does not conflict with s_z^h. Additionally, if it also does not conflict with any other executing transaction and there are available processors, then s_i^k becomes an executing transaction. This means that s_i^k and s_z^h are executing concurrently, which violates the assumption of transitive retry.

Case 3: Assume that s_z^h and s_j^l both exist in the n-set. When s_i^k arrives, it is compared with the m-set. If s_i^k does not conflict with any executing transactions and there are available processors, then s_i^k becomes an executing transaction. Even though s_i^k has common objects with s_j^l, s_i^k is not compared with s_j^l, which is in the n-set. If s_i^k joins the n-set, it is because, it conflicts with one or more executing transactions, not because of s_z^h, which violates the transitive retry assumption. If the three transactions s_i^k, s_j^l and s_z^h exist in the n-set, and s_z^h is chosen as a new executing transaction, then s_j^l remains in the n-set. This leads to Case 1. If s_j^l is chosen, because s_z^h conflicts with another executing transaction and s_j^l does not, then this leads to Case 2. □

Claim 34. *The first access property of PNF prevents transitive retry.*

Proof. The proof is by contradiction. Assume that the retry cost of transactions in the absence of the first access property is the same as when first access exists. Now, assume that PNF is devoid of the first access property. This means that executing transactions can be aborted.

Assume three transactions s_i^k, s_j^l, and s_z^h, where s_z^h's priority is higher than s_j^l's priority, and s_j^l's priority is higher than s_i^k's priority. Assume that s_j^l conflicts with both s_i^k and s_z^h. s_i^k and s_z^h do not conflict together. If s_i^k arrives while s_z^h is an executing transaction and s_j^l exists in the n-set, then s_i^k becomes an executing transaction itself while s_j^l is retrying. If s_i^k did not commit at least when s_z^h commits, then s_j^l becomes an executing transaction. Due to the lack of the first access property, s_j^l will cause s_i^k to retry. So, the retry cost for s_i^k will be $len(s_z^h + s_j^l)$. This retry cost for s_i^k is the same if it had been transitively retrying because of s_z^h. This contradicts the first assumption. Claim follows. □

From Claims 33 and 34, PNF does not increase the retry cost of multi-object transactions. However, this is not the case for ECM and RCM as shown by Claim 32.

Claim 35. *Under PNF, any job τ_i^x is not affected by the retry cost in any other job τ_j^l.*

Proof. As explained in Section 4, PNF assigns a temporary priority of -1 to any job that includes a retrying transaction. So, retrying transactions have lower priority than any other normal priority. When τ_i^x is released and τ_j^l has a retrying transaction, τ_i^x will have a higher priority than τ_j^l. Thus, τ_i^x can run on any available processor while τ_j^l is retrying one of its transactions. Claim follows. $\square$

6.4 Retry Cost under PNF

We now derive an upper bound on the retry cost of any job τ_i^x under PNF during an interval $L \leq T_i$. Since all tasks are sporadic (i.e., each task τ_i has a minimum period T_i), T_i is the maximum study interval for each task τ_i.

Claim 36. *Under PNF, the maximum retry cost suffered by a transaction s_i^k due to a transaction s_j^l is $len(s_j^l)$.*

Proof. By PNF's definition, s_i^k cannot have started before s_j^l. Otherwise, s_i^k would have been an executing transaction and s_j^l cannot abort it. So, the earliest release time for s_i^k would have been just after s_j^l starts execution. Then, s_i^k would have to wait until s_j^l commits. Claim follows. $\square$

Claim 37. *The retry cost for any job τ_i^x due to conflicts between its transactions and transactions of other jobs under PNF during an interval $L \leq T_i$ is upper bounded by:*

$$RC(L) \leq \sum_{\tau_j \in \gamma_i} \left(\sum_{\theta \in \theta_i} \left(\left(\left\lceil \frac{L}{T_j} \right\rceil + 1 \right) \sum_{\forall \bar{s_j^l}(\theta)} len\left(\bar{s_j^l}(\theta) \right) \right) \right) \tag{6.1}$$

where $\bar{s_j^l}(\theta)$ is the same as $s_j^l(\theta)$ except for the following difference: if $\bar{s_j^l}$ accesses multiple objects in θ_i, then $\bar{s_j^l}$ is included only once in the last summation (i.e., $\bar{s_j^l}$ is not repeated for each shared object with s_i^k).

Proof. Consider a transaction s_i^k belonging to job τ_i^x. Under PNF, higher priority transactions than s_i^k can become executing transaction before s_i^k. A lower priority transaction s_v^f can also become an executing transaction before s_i^k. This happens when s_i^k conflicts with any executing transaction while s_v^f does not. The worst case scenario for s_i^k occurs when s_i^k has to wait in the n-set, while all other conflicting transactions with s_i^k are chosen to be executing transactions. Let $\bar{s_j^l}$ accesses multiple objects in θ_i. If $\bar{s_j^l}$ is an executing transaction, then $\bar{s_j^l}$ will not repeat itself for each object it accesses. Besides, $\bar{s_j^l}$ will finish before s_i^k starts execution. Consequently, $\bar{s_j^l}$ will not conflict with s_i^{k+1}. This means that an executing transaction can force no more than one transaction in a given job to retry. This is why $\bar{s_j^l}$ is included only once in (6.1) for all shared objects with s_i^k.

The maximum number of jobs of any task τ_j that can interfere with τ_i^x during interval L is $\left\lceil \frac{L}{T_j} \right\rceil + 1$. From the previous observations and Claim 36, Claim follows. $\square$

Claim 38. *The blocking time for a job τ_i^x due to lower priority jobs during an interval $L \leq T_i$ is upper bounded by:*

$$D(\tau_i^x) \leq \left\lfloor \frac{1}{m} \sum_{\forall \bar{\tau}_j^l} \left(\left(\left\lceil \frac{L}{T_j} \right\rceil + 1 \right) \sum_{\forall \ddot{s}_j^h} len\left(\ddot{s}_j^h\right) \right) \right\rfloor \tag{6.2}$$

where $D(\tau_i^x)$ is the blocking time suffered by τ_i^x due to lower priority jobs. $\bar{\tau}_j^l = \{\tau_j^l : p_j^l < p_i^x\}$ and $\ddot{s}_j^h = \{s_j^h : s_j^h$ does not conflict with any $s_i^k\}$. During this blocking time, all processors are unavailable for τ_i^x.

Proof. Under PNF, executing transactions are non preemptive. So, a lower priority executing transaction can delay a higher priority job τ_i^x if no other processors are available. Lower priority executing transactions can be conflicting or non-conflicting with any transaction in τ_i^x. They also can exist when τ_i^x is newly released, or after that. So, we have the following cases:

Lower priority conflicting transactions after τ_i^x is released: This case is already covered by the retry cost in (6.1).

Lower priority conflicting transactions when τ_i^x is newly released: Each lower priority conflicting transaction s_j^h will delay τ_i^x for $len(s_j^h)$. The effect of s_j^h is already covered by (6.1). Besides, (6.1) does not divide the retry cost by m as done in (6.2). Thus, the worst case scenario requires inclusion of s_j^h in (6.1), and not in (6.2).

Lower priority non-conflicting transactions when τ_i^x is newly released: τ_i^x is delayed if there are no available processors for it. Otherwise, τ_i^x can run in parallel with these non-conflicting lower priority transactions. Each lower priority non-conflicting transaction $\ddot{s}_j^h$ will delay τ_i^x for $len(\ddot{s}_j^h)$.

Lower priority non-conflicting transactions after τ_i^x is released: This situation can happen if τ_i^x is retrying one of its transactions s_i^k. So, τ_i^x is assigned a priority of -1. τ_i^x can be preempted by any other job. When s_i^k is checked again to be an executing transaction, all processors may be busy with lower priority non-conflicting transaction and/or higher priority jobs. Otherwise, τ_i^x can run in parallel with these lower priority non-conflicting transactions.

Each lower priority non-conflicting transaction $\ddot{s}_j^h$ will delay τ_i^x for $len(\ddot{s}_j^h)$.

From the previous cases, lower priority non-conflicting transactions act as if they were higher priority jobs interfering with τ_i^x. So, the blocking time can be calculated by the interference workload given by Theorem 7 in [14]. $\square$

Claim 39. *The response time of a job τ_i^x, during an interval $L \leq T_i$, under PNF/G-EDF is upper bounded by:*

$$R_i^{up} = c_i + RC(L) + D_{edf}(\tau_i^x) + \left\lfloor \frac{1}{m} \sum_{\forall j \neq i} W_{ij}(R_i^{up}) \right\rfloor \tag{6.3}$$

where $RC(L)$ is calculated by (6.1). $D_{edf}(\tau_i^x)$ is the same as $D(\tau_i^x)$ defined in (6.2). However, for G-EDF systems. $D_{edf}(\tau_i^x)$ is calculated as:

$$D_{edf}(\tau_i^x) \leq \left\lfloor \frac{1}{m} \sum_{\forall \bar{\tau}_j^l} \begin{cases} 0 & , L \leq T_i - T_j \\ \sum_{\forall \ddot{s}_j^h} len\left(\ddot{s}_j^h\right) & , L > T_i - T_j \end{cases} \right\rfloor \tag{6.4}$$

and $W_{ij}(R_i^{up})$ is calculated by (4.5).

Proof. Response time for τ_i^x is calculated by (4.5) with the addition of blocking time defined by Claim 38. G-EDF uses absolute deadlines for scheduling. This defines which jobs of the same task can be of lower priority than τ_i^x, and which will not. Any instance τ_j^h, released between $r_i^x - T_j$ and $d_i^x - T_j$, will be of higher priority than τ_i^x. Before $r_i^x - T_j$, τ_j^h would have finished before τ_i^x is released. After $d_i^x - T_j$, d_j^h would be greater than d_i^x. Thus, τ_j^h will be of lower priority than τ_i^x. So, during T_i, there can be only one instance τ_j^h of τ_j with lower priority than τ_i^x. τ_j^h is released between $d_i^x - T_j$ and d_i^x. Consequently, during $L < T_i - T_j$, no existing instance of τ_j is of lower priority than τ_i^x. Hence, 0 is used in the first case of (6.4). But if $L > T_i - T_j$, there can be only one instance τ_j^h of τ_j with lower priority than τ_i^x. Hence, $\left\lceil \frac{L}{T_i} \right\rceil + 1$ in (6.2) is replaced with 1 in the second case in (6.4). Claim follows. □

Claim 40. *The response time of a job τ_i^x, during an interval $L \leq T_i$, under PNF/G-RMA is upper bounded by:*

$$R_i^{up} = c_i + RC(L) + D(\tau_i^x) + \left\lfloor \frac{1}{m} \sum_{\forall j \neq i, p_j > p_i} W_{ij}(R_i^{up}) \right\rfloor \tag{6.5}$$

where $RC(L)$ is calculated by (6.1), $D(\tau_i^x)$ is calculated by (6.2), and $W_{ij}(R_i^{up})$ is calculated by (4.4).

Proof. Proof is same as of Claim 39, except that G-RMA assigns fixed priorities. Hence, (6.2) can be used directly for calculating $D(\tau_i^x)$ without modifications. Claim follows. □

6.5 PNF vs. Competitors

We now (formally) compare the schedulability of G-EDF (G-RMA) with PNF against ECM, RCM, LCM and lock-free synchronization [37, 41, 42]. Such a comparison will reveal when

PNF outperforms others. Toward this, we compare the total utilization under G-EDF (G-RMA)/PNF, with that under the other synchronization methods. Inflated execution time of each method, which is the sum of the worst-case execution time of the task and its retry cost, is used in the utilization calculation of each task.

Let $RC_A(T_i)$ denote the retry cost of any τ_i^x using the synchronization method A during T_i. $D_A(T_i)$ is the blocking time of τ_i^x during T_i by A. Let $RC_B(T_i)$ denote the retry cost of any τ_i^x using synchronization method B during T_i. $D_B(T_i)$ is the blocking time of τ_i^x during T_i by B. Then, schedulability of A is comparable to B if:

$$\begin{aligned} \sum_{\forall \tau_i} \frac{c_i + RC_A(T_i) + D_A(T_i)}{T_i} &\leq \sum_{\forall \tau_i} \frac{c_i + RC_B(T_i) + D_B(T_i)}{T_i} \\ \therefore \sum_{\forall \tau_i} \frac{RC_A(T_i) + D_A(T_i)}{T_i} &\leq \sum_{\forall \tau_i} \frac{RC_B(T_i) + D_B(T_i)}{T_i} \end{aligned} \tag{6.6}$$

As described in Section 6.1, the set of common objects needs to be extended under PNF's competitors. Toward this, we introduce a few additional notions. Let θ_i^{ex} be an extended set of distinct objects that contains all objects in θ_i. Thus, θ_i^{ex} contains all objects accessed by τ_i. θ_i^{ex} can also contain other objects that can cause any transaction in τ_i to retry, as discussed in Section 6.1. Thus, θ_i^{ex} may contain objects not accessed by τ_i. γ_i^{ex} is an extended set of tasks that access any object in θ_i^{ex}. i.e., γ_i^{ex} contains at least all tasks in γ_i.

There are two sources of retry cost for any τ_i^x under ECM, RCM, LCM and lock-free. First is due to conflict between τ_i^x's transactions and transactions of other jobs. This is denoted as RC. Second is due to the preemption of any transaction in τ_i^x due to the release of a higher priority job τ_j^h. This is denoted as RC_{re}. Retry due to the release of higher priority jobs do not occur under PNF, because executing transactions are non-preemptive. It is up to the implementation of the contention manager to safely avoid RC_{re}. Here, we assume that ECM, RCM and LCM do not avoid RC_{re}. Thus, we introduce RC_{re} for ECM, RCM and LCM first before comparing PNF with other techniques.

Claim 41. *Under ECM and G-EDF/LCM the total retry cost suffered by all transactions in any τ_i^x during an interval $L \leq T_i$ is upper bounded by:*

$$RC_{to}(L) = RC(L) + RC_{re}(L) \tag{6.7}$$

where $RC(L)$ is the retry cost resulting from conflict between transactions in τ_i^x and transactions of other jobs. $RC(L)$ is calculated by (??) for ECM and (5.5) for G-EDF/LCM. γ_i and θ_i are replaced with γ_i^{ex} and θ_i^{ex}, respectively. $RC_{re}(L)$ is the retry cost resulting from the release of higher priority jobs, which preempt τ_i^x. $RC_{re}(L)$ is:

$$RC_{re}(L) = \sum_{\forall \tau_j \in \zeta_i} \begin{cases} \left\lceil \frac{L}{T_j} \right\rceil s_{i_{max}} & , L \leq T_i - T_j \\ \\ \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{i_{max}} & , L > T_i - T_j \end{cases} \tag{6.8}$$

where $\zeta_i = \{\tau_j : (\tau_j \neq \tau_i) \wedge (D_j < D_i)\}$.

Proof. Two conditions must be satisfied for any τ_j^l to be able to preempt τ_i^x under G-EDF: $r_i^x < r_j^l < d_i^x$, and $d_j^l \leq d_i^x$. Without the first condition, τ_j^l would have been already released before τ_i^x. Thus, τ_j^l will not preempt τ_i^x. Without the second condition, τ_j^l will be of lower priority than τ_i^x and will not preempt it. If $D_j \geq D_i$, then there will be at most one instance τ_j^l with higher priority than τ_i^x. τ_j^l must have been released at most at r_i^x, which violates the first condition. The other instance τ_j^{l+1} would have an absolute deadline greater than d_i^x. This violates the second condition. Hence, only tasks with shorter relative deadline than D_i are considered. These jobs are grouped in ζ_i.

The total number of released instances of τ_j during any interval $L \leq T_i$ is $\left\lceil \frac{L}{T_i} \right\rceil + 1$. The "carried-in" jobs (i.e., each job released before r_i^x and has an absolute deadline before d_i^x [14]) are discarded as they violate the first condition. The "carried-out" jobs (i.e., each job released after r_i^x and has an absolute deadline after d_i^x [14]) are also discarded because they violate the second condition. Thus, the number of considered higher priority instances of τ_j during the interval $L \leq T_i - T_j$ is $\left\lceil \frac{L}{T_j} \right\rceil$. The number of considered higher priority instances of τ_j during interval $L > T_i - T_j$ is $\left\lfloor \frac{T_i}{T_j} \right\rfloor$.

The worst RC_{re} for τ_i^x occurs when τ_i^x is always interfered at the end of execution of its longest atomic section, $s_{i_{max}}$. τ_i^x will have to retry for $len(s_{i_{max}})$. The total retry cost suffered by τ_i^x is the combination of RC and RC_{re}. □

Claim 42. *Under RCM and G-RMA/LCM, the total retry cost suffered by all transactions in any τ_i^x during an interval $L \leq T_i$ is upper bounded by:*

$$RC_{to}(L) = RC(L) + RC_{re}(L) \tag{6.9}$$

where $RC(L)$ and $RC_{re}(L)$ are defined in Claim 41. $RC(L)$ is calculated by (4.16) for RCM, and (5.18) for G-RMA/LCM. $RC_{re}(L)$ is calculated by:

$$RC_{re}(L) = \sum_{\forall \tau_j \in \zeta_i^*} \left(\left\lceil \frac{L}{T_j} \right\rceil s_{i_{max}} \right) \tag{6.10}$$

where $\zeta_i^* = \{\tau_j : p_j > p_i\}$.

Proof. The proof is the same as that for Claim 41, except that G-RMA uses static priority. Thus, the carried-out jobs will be considered in the interference with τ_i^x. The carried-in jobs are still not considered because they are released before r_i^x. Claim follows. □

Claim 43. *Consider lock-free synchronization. Let $r_{i_{max}}$ be the maximum execution cost of a single iteration of any retry loop of τ_i. RC_{re} under G-EDF with lock-free synchronization is calculated by (6.8), where $s_{i_{max}}$ is replaced by $r_{i_{max}}$. RC_{re} under G-RMA with lock-free synchronization is calculated by (6.10), where $s_{i_{max}}$ is replaced by $r_{i_{max}}$.*

Proof. The interference pattern of higher priority jobs to lower priority jobs is the same in ECM, G-EDF/LCM, and G-EDF with lock-free. The pattern is also the same in RCM, G-RMA/LCM, and G-RMA with lock-free. Claim follows. □

6.5.1 PNF versus ECM

Claim 44. *In the absence of transitive retry, PNF/G-EDF's schedulability is better or equal to ECM's when conflicting atomic sections have equal lengths.*

Proof. Substitue $RC_A(T_i)$ and $RC_B(T_i)$ in (6.6) with (6.1) and (6.7), respectively. Let $\theta_i^{ex} = \theta_i + \theta_i^*$, where θ_i^* is the set of objects not accessed directly by τ_i but can cause transactions in τ_i to retry due to transitive retry. Let $\gamma_i^{ex} = \gamma_i + \gamma_i^*$, where γ_i^* is the set of tasks that access objects in θ_i^*. Let:

$$g(\tau_i) = \left(\sum_{\forall \tau_j \in \gamma_i^*} \sum_{\theta \in \theta_i^*} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_j^{\bar{k}}(\theta) + s_{max}(\theta) \right) \right) \right) + RC_{re}(T_i)$$

where RC_{re} is given by (6.8). $g(\tau_i)$ includes effect of transitive retry. Let:

$$\eta_1(\tau_i) = \sum_{\forall \tau_j \in \gamma_i} \sum_{\forall \theta \in \theta_i} \left(\sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_j^{\bar{k}}(\theta) \right) \right)$$

$$\eta_2(\tau_i) = \sum_{\forall \tau_j \in \gamma_i} \sum_{\forall \theta \in \theta_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_{max}^j(\theta) \right) \right)$$

$$\eta_3(\tau_i) = \sum_{\forall \tau_j \in \gamma_i} \sum_{\forall \theta \in \theta_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_j^{\bar{k}}(\theta) \right) \right)$$

By substitution of $g(\tau_i)$, $\eta_1(\tau_i)$, and $\eta_2(\tau_i)$, and subtraction of $\sum_{\forall \tau_i} \frac{\eta_3(\tau_i)}{T_i}$ from both sides of (6.6), we get:

$$\sum_{\forall \tau_i} \frac{\eta_1(\tau_i)}{T_i} \leq \sum_{\forall \tau_i} \frac{\eta_2(\tau_i) + g(\tau_i)}{T_i} \tag{6.11}$$

Assume that $g(\tau_i)_{\forall \tau_i} \rightarrow 0$. From (6.11), we note that by keeping every $len(s_j^{\bar{k}}(\theta)) \leq len(s_{max}^j(\theta))$ for each τ_i, $\tau_j \in \gamma_i$, and $\theta \in \theta_i$, (6.11) holds. Due to G-EDF's dynamic priority, $s_{max}^j(\theta)$ can belong to any task other than τ_j. By keeping $len(s_j^{\bar{k}}(\theta)) \leq len(s_{max}^j(\theta))$, then 6.11 holds. By generalizing this condition to any $s_j^k(\theta)$ and $s_{max}^j(\theta)$, then 6.11 holds if all atomic sections in all tasks have equal lengths. Claim follows. □

6.5.2 PNF versus RCM

Claim 45. *In the absence of transitive retry, PNF/G-RMA's schedulability is better or equal to RCM's schedulability when a large number of tasks heavily conflict. PNF's schedulability is improved compared with RCM's, when atomic section length increases as priority increases.*

Proof. Let $\theta_i^{ex} = \theta_i + \theta_i^*$ and $\gamma_i^{ex} = \gamma_i + \gamma_i^*$, as defined in the proof of Claim 44. Substitute $RC_A(T_i)$ and $RC_B(T_i)$ in (6.6) with (6.1) and (6.9), respectively. Let:

$$g(\tau_i) = RC_{re}(T_i) + \left(\sum_{\forall \tau_j \in (\gamma_i^* \cap \zeta_i^*)} \sum_{\forall \theta \in \theta_i^*} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \times \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_j^{\bar{k}}(\theta) + s_{max}^j(\theta) \right) \right)$$

where RC_{re} and ζ_i^* are defined by (6.10). $g(\tau_i)$ includes effect of transitive retry. Let $\gamma_i = \zeta_i^* \cup \bar{\zeta}_i$, where $\bar{\zeta}_i = \{\tau_j : (\tau_j \neq \tau_i) \wedge (p_j < p_i)\}$, thus $\zeta_i^* \cap \bar{\zeta}_i = \phi$.

Let:

$$\eta_1(\tau_i) = \sum_{\forall \tau_j \in (\gamma_i \cap \zeta_i^*)} \sum_{\forall \theta \in \theta_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_j^{\bar{k}}(\theta) \right) \right)$$

$$\eta_2(\tau_i) = \sum_{\forall \tau_j \in (\gamma_i \cap \bar{\zeta}_i)} \sum_{\forall \theta \in \theta_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_j^{\bar{k}}(\theta) \right) \right)$$

$$\eta_3(\tau_i) = \sum_{\forall \tau_j \in (\gamma_i \cap \zeta_i^*)} \sum_{\forall \theta \in \theta_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \times \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_j^{\bar{k}}(\theta) + s_{max}^j(\theta) \right) \right)$$

By substitution of $g(\tau_i)$, $\eta_1(\tau_i)$, $\eta_2(\tau_i)$, and $\eta_3(\tau_i)$ in (6.6):

$$\sum_{\forall \tau_i} \frac{\eta_1(\tau_i) + \eta_2(\tau_i)}{T_i} \leq \sum_{\forall \tau_i} \frac{\eta_3(\tau_i) + g(\tau_i)}{T_i} \tag{6.12}$$

When tasks with deadlines equal to periods are scheduled with G-RMA, $T_j > T_i$ if $p_j < p_i$. So, for each $\tau_j \in \bar{\zeta}_i$, $\left\lceil \frac{T_i}{T_j} \right\rceil = 1$. Then:

$$\eta_2(\tau_i) = 2 \sum_{\forall \tau_j \in (\gamma_i \cap \bar{\zeta}_i)} \sum_{\forall \theta \in \theta_i} \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_j^{\bar{k}}(\theta) \right) \tag{6.13}$$

Let:

$$\eta_4(\tau_i) = \sum_{\forall \tau_j \in (\gamma_i \cap \zeta_i^*)} \sum_{\forall \theta \in \theta_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^{\bar{k}}(\theta)} len\left(s_{max}^j(\theta) \right)$$

By substitution of (6.13) and subtraction of $\sum_{\forall \tau_i} \frac{\eta_1(\tau_i)}{T_i}$ from both sides of (6.12), we get:

$$2 \sum_{\forall \tau_i} \frac{\eta_2(\tau_i)}{T_i} \leq \sum_{\forall \tau_i} \frac{\eta_4(\tau_i) + g(\tau_i)}{T_i} \tag{6.14}$$

Assume that $g(\tau_i)_{\forall \tau_i} \rightarrow 0$. From (6.14), we note that when higher priority jobs increasingly conflict with lower priority jobs, (6.14) tends to hold. (6.14) also tends to hold if $len(\bar{s_{max}^{j}}(\theta))$ in the right hand side of (6.14) is larger than $len(\bar{s_j^k}(\theta))$ in the left hand side of (6.14), which means atomic section length increases as priority increases. Claim follows. □

6.5.3 PNF versus G-EDF/LCM

Claim 46. *In the absence of transitive retry, PNF/EDF's schedulability is equal or better than G-EDF/LCM's if the conflicting atomic section lengths are approximately equal and all α terms approach 1.*

Proof. Assume that $\eta_1(\tau_i)$ and $\eta_3(\tau_i)$ are the same as that defined in the proof of Claim 44. Let:

$$g(\tau_i) = \left(\sum_{\forall \tau_j \in \gamma_i^*} \sum_{\theta \in \theta_i^*} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall \bar{s_j^k}(\theta)} len\left(\bar{s_j^k}(\theta) + \alpha_{max}^{ji} s_{max}(\theta) \right) \right) \right) + RC_{re}(T_i)$$

$$\eta_2(\tau_i) = \sum_{\forall \tau_j \in \gamma_i} \sum_{\forall \theta \in \theta_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall \bar{s_j^k}(\theta)} len\left(\alpha_{max}^{jl} s_{max}^{j}(\theta) \right) \right)$$

where α_{max}^{jl} is defined in (5.5). Following the same steps in the proof of Claim 44, we get:

$$\sum_{\forall \tau_i} \frac{\eta_1(\tau_i)}{T_i} \leq \sum_{\forall \tau_i} \frac{\eta_2(\tau_i) + g(\tau_i)}{T_i} \tag{6.15}$$

Assume that $g(\tau_i)_{\forall \tau_i} \rightarrow 0$. Thus, we ignore the effect of transitive retry and retry cost due to the release of higher priority jobs. Let $len(\bar{s_j^k}(\theta)) = s_{max}^{j}(\theta) = s$, and $\alpha_{max}^{jl} = \alpha_{max}^{iy} = 1$ in (6.15). Then, PNF/EDF's schedulability equals LCM/EDF's schedulability if $\left\lceil \frac{T_i}{T_j} \right\rceil = 1, \forall \tau_i, \tau_j$ (which means equal periods for all tasks). If $\left\lceil \frac{T_i}{T_j} \right\rceil > 1, \forall \tau_i, \tau_j$, PNF/EDF's schedulability is better than LCM/EDF's. PNF/EDF's schedulability becomes more better than LCM/EDF's schedulability if $g(\tau_i)$ is not zero. Claim follows. □

6.5.4 PNF versus G-RMA/LCM

Claim 47. *In the absence of transitive retry, PNF's schedulability is equal or better than G-RMA/LCM's if: 1) lower priority tasks suffer increasing number of conflicts from higher priority tasks, 2) the lengths of the atomic sections increase as task priorities increase, and 3) α terms increase.*

Proof. Assume that $g(\tau_i)$, $\eta_1(\tau_i)$, and $\eta_2(\tau_i)$ are the same as in the proof of Claim 45. Let:

$$\eta_3(\tau_i) = \sum_{\forall \tau_j \in (\gamma_i \cap \zeta_i^*)} \sum_{\forall \theta \in \theta_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \times \sum_{\forall \bar{s_j^k}(\theta)} len \left(\bar{s_j^k}(\theta) + \alpha_{max}^{jl} s_{max}^j(\theta) \right) \right)$$

$$\eta_4(\tau_i) = \sum_{\forall \tau_j \in (\gamma_i \cap \zeta_i^*)} \sum_{\forall \theta \in \theta_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \times \sum_{\forall \bar{s_j^k}(\theta)} len \left(\alpha_{max}^{jl} s_{max}^j(\theta) \right) \right)$$

Following the steps of Claim 45's proof, $\therefore$(6.6) becomes:

$$2 \sum_{\forall \tau_i} \frac{\eta_2(\tau_i)}{T_i} \leq \sum_{\forall \tau_i} \frac{\eta_4(\tau_i) + g(\tau_i)}{T_i} \tag{6.16}$$

Assume that the effect of transitive retry and retry cost due to the release of higher priority jobs is negligible ($g(\tau_i) \to 0$). (6.16) holds if: 1) the contention from higher priority jobs to lower priority jobs increases because of the $\left\lceil \frac{T_i}{T_j} \right\rceil + 1$ term in the right hand side of (6.16); 2) α terms approach 1; and 3) the lengths of the atomic sections increase as priority increases. This makes $len(s_{max}^j(\theta))$ in (6.16)'s right side to be greater than $len(\bar{s_j^k}(\theta))$ in (6.16)'s left side. Claim follows. □

6.5.5 PNF versus Lock-free Synchronization

Lock-free synchronization [37, 42] accesses only one object. Thus, the number of accessed objects per transaction in PNF is limited to one. This allows us to compare the schedulability of PNF with the lock-free algorithm.

$RC_B(T_i)$ in (6.6) is replaced with:

$$\sum_{\forall \tau_j \in \gamma_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{i,j} r_{max} \right) + RC_{re}(T_i) \tag{6.17}$$

where $\beta_{i,j}$ is the number of retry loops of τ_j that access the same object as accessed by some retry loop of τ_i [37]. r_{max} is the maximum execution cost of a single iteration of any retry loop of any task [37]. $RC_{re}(T_i)$ is defined in Claim 43. Lock-free synchronization does not depend on priorities of tasks. Thus, (6.17) applies for both G-EDF and G-RMA systems.

Claim 48. *Let r_{max} be the maximum execution cost of a single iteration of any retry loop of any task [37]. Let s_{max} be the maximum transaction length in all tasks. Assume that each transaction under PNF accesses only one object for once. The schedulability of PNF with either G-EDF or G-RMA scheduler is better or equal to the schedulability of lock-free synchronization if $s_{max}/r_{max} \leq 1$.*

Proof. The assumption in Claim 48 is made to enable a comparison between PNF and lock-free. Let $RC_A(T_i)$ in (6.6) be replaced with (6.1) and $RC_B(T_i)$ be replaced with (6.17). To simplify comparison, (6.1) is upper bounded by:

$$RC(T_i) = \sum_{\tau_j \in \gamma_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta^*_{i,j} s_{max} \right)$$

where $\beta^*_{i,j}$ is the number of times transactions in τ_j accesses shared objects with τ_i. Thus, $\beta^*_{i,j} = \beta_{i,j}$, and (6.6) will be:

$$\sum_{\forall \tau_i} \frac{\sum_{\tau_j \in \gamma_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{i,j} s_{max} \right)}{T_i} \leq \sum_{\forall \tau_i} \frac{\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{i,j} r_{max} + RC_{re}(\tau_i)}{T_i} \tag{6.18}$$

From (6.18), we note that if $s_{max} \leq r_{max}$, then (6.18) holds. □

6.6 Conclusion

Transitive retry increases transactional retry cost under ECM, RCM, and LCM. PNF avoids transitive retry by avoiding transactional preemptions. PNF reduces the priority of aborted transactions to enable other tasks to execute, increasing processor usage. Executing transactions are not preempted due to the release of higher priority jobs. On the negative side of PNF, higher priority jobs can be blocked by executing transactions of lower priority jobs.

EDF/PNF's schedulability is equal or better than ECM's when atomic section lengths are almost equal. RMA/PNF's schedulability is equal or better than RCM's when lower priority jobs suffer greater conflicts from higher priority ones. Similar conditions hold for the schedulability comparison between PNF and LCM, in addition to the increase of α terms to 1. This is logical as LCM with G-EDF (G-RMA) defaults to ECM (RCM) with $\alpha \to 1$. For PNF's schedulability to be equal or better than lock-free, the upper bound on s_{max}/r_{max} must be 1, instead of 0.5 under ECM and RCM.

Chapter 7

The FBLT Contention Manager

In this chapter, we present a novel contention manager for resolving transactional conflicts, called FBLT [44]. We upper bound transactional retries and task response times under FBLT, when used with the G-EDF and G-RMA schedulers. We formally identify the conditions under which FBLT has better real-time schedulability than the previous best contention manager, PNF.

The rest of this Chapter is organized as follows: Section 7.1 discusses limitations of previous contention managers and the motivation to FBLT. Section 7.2 give a formal description of PNF. We upper bound retry cost and response time under FBLT in Section 7.3. Schedulability comparison between FBLT and previous synchronization techniques is given in Section 7.4. We conclude Chapter in Section 7.5.

7.1 Motivation

ECM [42], RCM [42], and LCM [41] suffer from *transitive retry*. Transitive retry is illustrated by the following example:

Consider three atomic sections s_1^x, s_2^y, and s_3^z belonging to jobs τ_1^x, τ_2^y, and τ_3^z, with priorities $p_3^z > p_2^y > p_1^x$, respectively. Assume that s_1^x and s_2^y share objects, and s_2^y and s_3^z share objects. s_1^x and s_3^z do not share objects. Now, s_3^z can cause s_2^y to retry, which in turn will cause s_1^x to retry. This means that s_1^x will retry transitively because of s_3^z, which will increase the retry cost of s_1^x. Now, consider another atomic section s_4^f with a priority higher than that of s_3^z. Suppose s_4^f shares objects only with s_3^z. Thus, s_4^f can cause s_3^z to retry, which in turn will cause s_2^y to retry, and finally, s_1^x to retry. Thus, transitive retry will move from s_4^f to s_1^x, increasing the retry cost of s_1^x. The situation gets worse as more higher priority tasks are added, where each task shares objects with its immediate lower priority task. τ_3^z may have atomic sections that share objects with τ_1^x, but this will not prevent the effect of transitive retry due to s_1^x.

Definition 3. ***Transitive retry.*** *A transaction s_i^k suffers from transitive retry when s_i^k retries due to a higher priority transaction s_z^h, and $\Theta_z^h \cap \Theta_i^k = \emptyset$.*

Therefore, the analysis in [42] and [41] extends the set of objects that can cause an atomic section of a lower priority job to retry. This is done by initializing the set of conflicting objects, γ_i, to all objects accessed by all transactions of τ_i. We then cycle through all transactions belonging to all other higher priority tasks. Each transaction s_j^l that accesses at least one of the objects in γ_i adds all other objects accessed by s_j^l to γ_i. The loop over all higher priority tasks is repeated, each time with the new γ_i, until there are no more transactions accessing any object in γ_i. The final set of objects (tasks) that can cause transactions in τ_i to retry is $\theta_i^{ex}(\gamma_i^{ex})$, respectively[1].

PNF [40,45] is designed to avoid transitive retry by concurrently executing at most m non-conflicting transactions together. These executing transactions are non-preemptive. Thus, executing transactions cannot be aborted due to direct or indirect conflict with other transactions. However, with PNF, all objects accessed by each transaction must be known a-priori. Therefore, this is not suitable with dynamic STM implementations [62]. Additionally, PNF is implemented in [40] as a centralized CM that uses locks. This increases overhead.

Thus, we propose the *First Bounded, Last Timestamp contention manager* (or FBLT) that achieves the following goals:

1. reduce the retry cost of each transaction s_i^k due to another transaction s_j^l, just as LCM [41] does compared to ECM [42] and RCM [42].
2. avoid or bound the effect of transitive retry, similar to PNF [40,45], without prior knowledge of accessed objects by each transaction, enabling dynamic STM.
3. decentralized design and avoid the use of locks, thereby reducing overhead.

7.2 The FBLT Contention Manager

Algorithm 5 illustrates FBLT. Each transaction s_i^k can be aborted during T_i for at most δ_i^k times. η_i^k records the number of times s_i^k has already been aborted up to now. If s_i^k and s_j^l have not joined the m_set yet, then they are preemptive transactions. Preemptive transactions resolve conflicts using LCM [41] (step 2). Thus, FBLT defaults to LCM when no transaction reaches its δ. If only one of the transactions is in the m_set, then the non-preemptive transaction (the one in m_set) aborts the other one (steps 15 to 26). η_i^k is incremented each time s_i^k is aborted as long as $\eta_i^k < \delta_i^k$ (steps 5 and 18). Otherwise, s_i^k is added to the m_ set and its priority is increased to *m_prio* (steps 7 to 9 and 20 to 22).

[1] However, note that, this solution may over-extend the set of conflicting objects, and may even contain all objects accessed by all tasks.

Algorithm 5: FBLT

Data: s_i^k: interfered transaction;
s_j^l: interfering transactions;
δ_i^k: the maximum number of times s_i^k can be aborted during T_i;
η_i^k: number of times s_i^k has already been aborted up to now;
m_set: contains at most m non-preemptive transactions. m is number of processors;
m_prio: priority of any transaction in m_set. *m_prio* is higher than any priority of any real-time task;
$r(s_i^k)$: time point at which s_i^k joined m_set;
Result: atomic sections that will abort

1 **if** s_i^k, $s_j^l \notin m\_set$ **then**
2   Apply LCM [41];
3   **if** s_i^k *is aborted* **then**
4   **if** $\eta_i^k < \delta_i^k$ **then**
5   Increment η_i^k by 1;
6   **else**
7   Add s_i^k to m_set;
8   Record $r(s_i^k)$;
9   Increase priority of s_i^k to *m_prio*;
10   **end**
11   **else**
12   Swap s_i^k and s_j^l;
13   Go to Step 3;
14   **end**
15 **else if** $s_j^l \in m\_set, s_i^k \notin m\_set$ **then**
16   Abort s_i^k;
17   **if** $\eta_i^k < \delta_i^k$ **then**
18   Increment η_i^k by 1;
19   **else**
20   Add s_i^k to m_set;
21   Record $r(s_i^k)$;
22   Increase priority of s_i^k to *m_prio*;
23   **end**
24 **else if** $s_i^k \in m\_set, s_j^l \notin m\_set$ **then**
25   Swap s_i^k and s_j^l;
26   Go to Step 15;
27 **else**
28   **if** $r(s_i^k) < r(s_j^l)$ **then**
29   Abort s_j^l;
30   **else**
31   Abort s_i^k;
32   **end**
33 **end**

When the priority of s_i^k is increased to m_prio, s_i^k becomes a non-preemptive transaction. Non-preemptive transactions cannot be aborted by other preemptive transactions, nor by any other real-time job. The m_set can hold at most m concurrent transactions because there are m processors in the system. $r(s_i^k)$ records the time s_i^k joined the m_set (steps 8 and 21). When non-preemptive transactions conflict together (step 27), the transaction with the smaller $r()$ commits first (steps 29 and 31). Thus, non-preemptive transactions are executed in FIFO order of the m_set.

7.3 Retry Cost and Response Time Bounds

We now derive an upper bound on the retry cost of any job τ_i^x under FBLT during an interval $L \leq T_i$. Since all tasks are sporadic (i.e., each task τ_i has a minimum period T_i), T_i is the maximum study interval for each task τ_i.

Claim 49. *The total retry cost for any job τ_i^x under FBLT due to 1) conflicts between its transactions and transactions of other jobs during an interval $L \leq T_i$ and 2) release of higher priority jobs is upper bounded by:*

$$RC_{to}(L) \leq \sum_{\forall s_i^k \in s_i} \left(\delta_i^k len(s_i^k) + \sum_{\forall s_{iz}^k \in \chi_i^k} len(s_{iz}^k) \right) + RC_{re}(L) \tag{7.1}$$

where χ_i^k is the set of at most $m-1$ maximum length transactions conflicting directly or indirectly (through transitive retry) with s_i^k. Each transaction $s_{iz}^k \in \chi_i^k$ belongs to a distinct task τ_j. $RC_{re}(L)$ is the retry cost resulting from the release of higher priority jobs which preempt τ_i^x. $RC_{re}(L)$ is calculated by (6.8) in [40] for G-EDF, and (6.10) in [40] for G-RMA schedulers.

Proof. By the definition of FBLT, $s_i^k \in \tau_i^x$ can be aborted a maximum of δ_i^k times before s_i^k joins the m_set. Before joining the m_set, s_i^k can be aborted due to higher priority transactions, or transactions in the m_set. The original priority of transactions in the m_set can be higher or lower than p_i^x. Thus, the maximum time s_i^k is aborted before joining the m_set occurs if s_i^k is aborted for δ_i^k times.

Transactions preceding s_i^k in the m_set can conflict directly with s_i^k, or indirectly through transitive retry. The worst case scenario for s_i^k after joining the m_set occurs if s_i^k is preceded by $m-1$ maximum length conflicting transactions. Hence, in the worst case, s_i^k has to wait for the previous $m-1$ transactions to commit first. The priority of s_i^k after joining the m_set is higher than any real-time job. Therefore, s_i^k is not aborted by any job. If s_i^k has not joined the m_ set yet, and a higher priority job τ_j^y is released while s_i^k is running, then s_i^k may be aborted if τ_j^y has conflicting transactions with s_i^k. τ_j^y causes only one abort in τ_i^x because τ_j^y preempts τ_i^x only once. If s_i^k has already joined the m_set, then s_i^k cannot be aborted by the release of higher priority jobs. Thus, the maximum number of times transactions in τ_i^x can be aborted due to the release of higher priority jobs is less than or equal to the number of interfering higher priority jobs to τ_i^x. Claim follows. □

Claim 50. *Under FBLT, the blocking time of a job τ_i^x due to lower priority jobs is upper bounded by:*

$$D(\tau_i^x) = \sum \left(max_m(s_{j_{max}, \forall \tau_j^l, p_j^l < p_i^x}) \right) \tag{7.2}$$

where $s_{j_{max}}$ is the maximum length transaction in any job τ_j^l with original priority lower than p_i^x. The right hand side of (7.2) is the sum of the m maximum transactional lengths in all jobs with lower priority than τ_i^x.

Proof. The worst case blocking time for τ_i^x occurs when the maximum length m transactions in lower priority jobs than τ_i^x are executing non-preemptively. After commit of each transaction in the m_set, a higher priority job τ_j^y than τ_i^x is released. So, τ_j^y allocates the released processor instead of τ_i^x. Consequently, τ_i^x has to wait for the whole maximum length m transactions of lower priority jobs. Claim follows.

□

Claim 51. *The response time of any job τ_i^x during an interval $L \leq T_i$ under FBLT is upper bounded by:*

$$R_i^{up} = c_i + RC_{to}(L) + D(\tau_i^x) + \left\lfloor \frac{1}{m} \sum_{\forall j \neq i} W_{ij}(R_i^{up}) \right\rfloor \tag{7.3}$$

where $RC_{to}(L)$ is calculated by (7.1), $D(\tau_i^x)$ is calculated by (7.2), and $W_{ij}(R_i^{up})$ is calculated by (11) in [42] for G-EDF, and (17) in [42] for G-RMA schedulers. (11) and (17) in [42] inflates c_j of any job $\tau_j^y \neq \tau_i^x$, $p_j^y > p_i^x$ by the retry cost of transactions in τ_j^y.

Proof. The response time of a job is calculated directly from FBLT's behavior. The response time of any job τ_i^x is the sum of its worst case execution time c_i, plus the retry cost of transactions in τ_i^x ($RC_{to}(L)$), plus the blocking time of τ_i^x ($D(\tau_i^x)$), and the workload interference of higher priority jobs. The workload interference of higher priority jobs scheduled by G-EDF is calculated by (11) in [42], and by (17) in [42] for G-RMA. Claim follows. □

7.4 Schedulability Comparison

We now (formally) compare the schedulability of FBLT against PNF [40, 45]. Toward this, we compare the total utilization under FBLT with that under PNF. In this comparison, we use the inflated execution time of the task, which is the sum of the worst-case execution time of the task and its retry cost, in the utilization calculation of the task.

Let $RC_A(T_i)$ and $RC_B(T_i)$ denote the retry cost of a job τ_i^x during T_i using the synchronization methods A and B, respectively. Let $D_A(\tau_i)$ and $D_B(\tau_i)$ be the maximum blocking time of any job τ_i^x due to lower priority jobs by methods A and B respectively. Now, schedulability of A is comparable to B if:

$$\begin{aligned}\sum_{\forall \tau_i} \frac{c_i + RC_A(T_i) + D_A(\tau_i)}{T_i} &\leq \sum_{\forall \tau_i} \frac{c_i + RC_B(T_i) + D_B(\tau_i)}{T_i} \\ \sum_{\forall \tau_i} \frac{RC_A(T_i) + D_A(\tau_i)}{T_i} &\leq \sum_{\forall \tau_i} \frac{RC_B(T_i) + D_B(\tau_i)}{T_i} \qquad (7.4)\end{aligned}$$

Claim 52. *Schedulability of FBLT is equal or better than PNF if: 1) For each transaction s_i^k, maximum abort times δ_i^k equals at most the ratio between difference of total length of all transactions that can conflict only with s_i^k and total length of at most $m-1$ longest transactions that can conflict directly or transitively with s_i^k to length of s_i^k. 2) For any job τ_i^x, ratio between longest transaction in τ_i^x or lower priority jobs to smallest transaction in lower priority jobs equals at most the ratio between minimum number of times τ_i^x can be blocked due to non-conflicting transactions in all lower priority jobs to maximum release time of all jobs not belonging to τ_i.*

Proof. Substitute $RC_A(T_i)$ and $RC_B(T_i)$ in (7.4) with (7.1) and (3) in [45] respectively. Substitute $D_A(\tau_i)$ and $D_B(\tau_i)$ by (7.2) and (4) in [45] respectively. Substituting $RC_{re}(T_i) = \sum_{\forall \tau_j \in \zeta_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) s_{i_{max}}$, covers $RC_{re}(T_i)$ given by (6.8) and (6.10) in [40] and maintains correctness of (7.4). $zeta_i$ is the set of higher priority tasks than any job of τ_i.

Let $\beta_i^1 = \sum_{\forall s_i^k \in s_i} \left(\delta_i^k len(s_i^k) + \sum_{s_{iz}^k \in \chi_i^k} len(s_{iz}^k) \right)$, $\beta_i^2 = \sum_{\forall \tau_j \in \zeta_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) s_{i_{max}} + \sum_{max\_m} \left\{ s_{j_{max}, \forall \bar{\tau_j^l}} \right\}$, $\beta_i^3 = \sum_{\forall \tau_j \in \gamma_i} \sum_{\theta \in \theta_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall \bar{s_j^h}(\theta)} len \left(\bar{s_j^h}(\theta) \right) \right)$ and $\beta_i^4 = \left\lfloor \frac{1}{m} \sum_{\forall \bar{\tau_j^l}} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall \ddot{s_j^h}} len \left(\ddot{s_j^h} \right) \right) \right\rfloor$.
So, (7.4) holds if

$$\sum_{\forall \tau_i} \frac{\beta_i^1 + \beta_i^2}{T_i} \leq \sum_{\forall \tau_i} \frac{\beta_i^3 + \beta_i^4}{T_i} \qquad (7.5)$$

(7.5) holds if $\forall \tau_i$

$$\beta_i^1 + \beta_i^2 \leq \beta_i^3 + \beta_i^4 \qquad (7.6)$$

or $\forall \tau_i$

$$\beta_i^1 \leq \beta_i^3 \quad and \quad \beta_i^2 \leq \beta_i^4 \qquad (7.7)$$

According to first part of (7.7)

$$\begin{aligned}&\sum_{\forall s_i^k \in s_i} \left(\delta_i^k len(s_i^k) + \sum_{s_{iz}^k \in \chi_i^k} len(s_{iz}^k) \right) \qquad (7.8) \\ \leq \;& \sum_{\forall \tau_j \in \gamma_i} \sum_{\theta \in \theta_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall \bar{s_j^h}(\theta)} len \left(\bar{s_j^h}(\theta) \right) \right)\end{aligned}$$

For each $s_i^k \in s_i$, there are a set of zero or more $\bar{s_j^h}(\Theta) \in \tau_j, \forall \tau_j \neq \tau_i$ that are conflicting with s_i^k. Assuming this set of conflicting transactions with s_i^k is denoted as $\eta_i^k(j) = \left\{\bar{s_j^h}(\Theta) \in \tau_j : (\Theta \in \theta_i) \wedge (\tau_j \neq \tau_i) \wedge \left(\bar{s_j^h}(\Theta) \notin \eta_i^l,\, l \neq k\right)\right\}$. The last condition $\bar{s_j^h}(\Theta) \notin \eta_i^l,\, l \neq k$ in definition of η_i^k ensures that common transactions $\bar{s_j^h}$ that can conflict with more than one transaction $s_i^k \in \tau_i$ are split among different $\eta_i^k(j)$, $k = 1, .., |s_i|$. This condition is necessary because in PNF, no two or more transactions of τ_i^x can be aborted by the same transaction of τ_j^h. Let γ_i^k be subset of γ_i that contains tasks with transactions conflicting directly with s_i^k. By substitution of $\eta_i^k(j)$ and γ_i^k in (7.8), (7.8) holds if $\forall\, s_i^k$:

$$\begin{aligned} \therefore\ \delta_i^k &\leq \frac{\sum_{\forall \tau_j \in \gamma_i^k} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \sum_{\forall \bar{s_j^h}(\Theta) \in \eta_i^k(j)} len\left(\bar{s_j^h}(\theta)\right)\right)}{len(s_i^k)} \\ &- \frac{\sum_{s_{iz}^k \in \chi_i^k} len(s_{iz}^k)}{len(s_i^k)} \end{aligned} \tag{7.9}$$

By definition of $\eta_i^k(j)$, if $\bar{s_j^h}(\Theta)$ can conflict with s_i^k and s_i^l, then $\bar{s_j^h}(\Theta)$ belongs either to η_i^k or η_i^l, but not both. Let $\bar{\eta_i^k}(j) = \eta_i^k(j) - \left\{\bar{s_j^h}(\Theta) | \bar{s_j^h}(\Theta)\, can\, belong\, to\, \eta_i^l,\, l \neq k\right\}$. So, $\bar{\eta_i^k}(j)$ equals $\eta_i^k(j)$ excluding any transaction that can belong to another $\eta_i^l(j)$, $l \neq k$. (7.9) holds if

$$\begin{aligned} \delta_i^k &\leq \frac{\sum_{\forall \tau_j \in \gamma_i^k} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \sum_{\forall \bar{s_j^h}(\Theta) \in \bar{\eta_i^k}(j)} len\left(\bar{s_j^h}(\theta)\right)\right)}{len(s_i^k)} \\ &- \frac{\sum_{s_{iz}^k \in \chi_i^k} len(s_{iz}^k)}{len(s_i^k)} \end{aligned} \tag{7.10}$$

Now, we consider the second part of (7.7). Let $s_{i,j_{max}} = max_{\forall \bar{\tau_j^l}}(s_{i_{max}}, s_{j_{max}})$. So, $s_{i,j_{max}}$ is the maximum transactional length in any job of τ_i or any lower priority job. Let $s_{j_{min}} = min\left\{len(s_j^h),\, \forall s_j^h \in \bar{\tau_j^l}\right\}$. So, $s_{j_{min}}$ is the smallest transactional length in any job of τ_j with lower priority than any job of τ_i. $\because\ \sum_{max\_m}\left\{s_{j_{max}, \forall \bar{\tau_j^l}}\right\} \leq \sum_{\forall \bar{\tau_j^l}}\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) s_{i,j_{max}}$, $\therefore$ the second part of (7.7) holds if

$$\begin{aligned} &\sum_{\forall \tau_j \in \zeta_i}\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) s_{i,j_{max}} + \sum_{\forall \bar{\tau_j^l}}\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) s_{i,j_{max}} \\ \leq\ &\left\lfloor \frac{1}{m} \sum_{\forall \bar{\tau_j^l}}\left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) |\ddot{s_j^h}|\, s_{j_{min}}\right)\right\rfloor \end{aligned} \tag{7.11}$$

$\because\ \zeta_i$ includes all jobs with higher priority than τ_i, and $\bar{\tau_j^l}$ includes all jobs with lower priority than τ_i, $\therefore$ (7.11) holds if $\forall\, \tau_i$

$$\therefore\ \frac{s_{i,j_{max}}}{s_{j_{min}}} \leq \frac{\left\lfloor \frac{1}{m} \sum_{\forall \bar{\tau_j^l}}\left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) |\ddot{s_j^h}|\right)\right\rfloor}{\sum_{\forall \tau_j \neq \tau_i}\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right)} \tag{7.12}$$

From (7.10) and (7.12), Claim follows.

□

7.5 Conclusions

Transitive retry increases transactional retry costs under ECM, RCM, and LCM. PNF avoids transitive retry by avoiding transactional preemptions. It avoids transitive retry cost by concurrently executing non-conflicting transactions, which are non-preemptive. However, PNF requires a-priori knowledge about objects accessed by each transaction. This is incompatible with dynamic STM implementations. Thus, we introduce the FBLT contention manager. Under FBLT, each transaction is allowed to abort for a no larger than a specified number of times. Afterwards, the transaction becomes non-preemptive. Non-preemptive transactions have higher priorities than other preemptive transactions and real-time jobs. Non-preemptive transactions resolve their conflicts using FIFO order. By proper adjustment of the maximum abort number of each transaction, we showed that FBLT's schedulability is equal to or better than PNF.

Chapter 8

Implementation and Experimental Evaluations

Chapter 9

Conclusions and Future Work

9.1 Conclusions

9.2 Future Work

From chapter4

Our work raises several questions. For example, what are the typical range of values for the different parameters that affect the retry cost (and hence the response time)? How tight is our retry and response time bounds in practice? Can real-time CMs be designed for other multicore real-time schedulers (e.g., partitioned, semi-partitioned), and those that dynamically improve application timeliness behavior? These are important directions for further work.

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