Real-Time Software Transactional Memory: Contention Managers, Time Bounds, and Implementations

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(ABSTRACT)

Lock-based concurrency control suffers from programmability, scalability, and composability challenges. These challenges are exacerbated in emerging multicore architectures, on which improved software performance must be achieved by exposing greater concurrency. Transactional memory (TM) is an alternative synchronization model for shared memory objects that promises to alleviate these difficulties.

In this dissertation proposal, we consider software transactional memory (STM) for concurrency control in multicore real-time software, and present a suite of real-time STM contention managers for resolving transactional conflicts. The contention managers are called RCM, ECM, LCM, and PNF. RCM and ECM resolve conflicts using fixed and dynamic priorities of real-time tasks, respectively, and are naturally intended to be used with the fixed priority (e.g., G-RMA) and dynamic priority (e.g., G-EDF) multicore real-time schedulers, respectively. LCM resolves conflicts based on task priorities as well as atomic section lengths, and can be used with G-EDF or G-RMA. Transactions under ECM, RCM and LCM can retry due to non-shared objects with higher priority tasks. PNF avoids this problem.

We establish upper bounds on transactional retry costs and task response times under all the contention managers through schedulability analysis. Since ECM and RCM conserve the semantics of the underlying real-time scheduler, their maximum transactional retry cost is double the maximum atomic section length. This is improved in the the design of LCM, which achieves shorter retry costs. However, ECM, RCM, and LCM are affected by transitive retries when transactions access multiple objects. Transitive retry causes a transaction to abort and retry due to another non-conflicting transaction. PNF avoids transitive retry, and also optimizes processor usage by lowering the priority of retrying transactions, enabling other non-conflicting transactions to proceed.

We also formally compare the proposed contention managers with lock-free synchronization. Our comparison reveals that, for most cases, ECM, RCM, G-EDF(G-RMA)/LCM achieve higher schedulability than lock-free synchronization only when the atomic section length does not exceed half of the lock-free retry loop length. Under PNF, atomic section length can equal length of retry loop. With low contention, atomic section length under ECM can equal retry loop length while still achieving better schedulabiltiy. While in RCM, atomic section length can exceed retry loop length. By adjustment of LCM design parameters, atomic section length can be of twice length of retry loop under G-EDF/LCM. While under G-RMA/LCM, atomic section length can exceed length of retry loop.

We implement the contention managers in the Rochester STM framework and conduct experimental studies using a multicore real-time Linux kernel. Our studies confirm that, the contention managers achieve orders of magnitude shorter retry costs than lock-free synchronization. Among the contention managers, PNF performs the best.

Building upon these results, we propose real-time contention managers that allow nested atomic sections – an open problem – for which STM is the only viable non-blocking synchronization solution. Optimizations of LCM and PNF to obtain improved retry costs and greater schedulability advantages are also proposed.

Dedication

To my parents, my wife, my daughter, and all my family

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Chapter 1

Introduction

Embedded systems sense physical processes and control their behavior, typically through feedback loops. Since physical processes are concurrent, computations that control them must also be concurrent, enabling them to process multiple streams of sensor input and control multiple actuators, all concurrently. Often, such computations need to concurrently read/write shared data objects. Typically, they must also process sensor input and react, satisfying application-level time constraints.

The de facto standard for programming concurrency is the threads abstraction, and the de facto synchronization abstraction is locks. Lock-based concurrency control has significant programmability, scalability, and composability challenges [60]. Coarse-grained locking (e.g., a single lock guarding a critical section) is simple to use, but permits no concurrency: the single lock forces concurrent threads to execute the critical section sequentially, in a one-at-a-time order. This is a significant limitation, especially with the emergence of multicore architectures, on which improved software performance must be achieved by exposing greater concurrency.

With fine-grained locking, a single critical section is broken down into several critical sections – e.g., each bucket of a hash table is guarded by a unique lock. Thus, threads that need to access different buckets can do so concurrently, permitting greater parallelism. However, this approach has low programmability: programmers must acquire only necessary and sufficient locks to obtain maximum concurrency without compromising safety, and must avoid deadlocks when acquiring multiple locks. Moreover, locks can lead to livelocks, lock-convoying, and priority inversion.

Perhaps, the most significant limitation of lock-based code is its non-composability. For example, atomically moving an element from one hash table to another using those tables' (lock-based) atomic methods is not possible in a straightforward manner: if the methods internally use locks, a thread cannot simultaneously acquire and hold the locks of the methods (of the two tables); if the methods were to export their locks, that will compromise safety.

Lock-free synchronization [59], which uses atomic hardware synchronization primitives (e.g., Compare And Swap [71,72], Load-Linked/Store-Conditional [113]), also permits greater concurrency, but has even lower programmability: lock-free algorithms must be custom-designed for each situation (e.g., a data structure [22,50,58,64,92]). Additionally, it is not clear how to program nested critical sections using lock-free synchronization. Most importantly, reasoning about the correctness of lock-free algorithms is significantly difficult [59].

1.1 Transactional Memory

Transactional memory (TM) is an alternative synchronization model for shared memory data objects that promises to alleviate these difficulties. With TM, programmers write concurrent code using threads, but organize code that read/write shared memory objects as *memory transactions*, which speculatively execute, while logging changes made to objects–e.g., using an undo-log or a write-buffer. Objects read and written by transactions are also monitored, in read sets and write sets, respectively. Two transactions conflict if they access the same object and one access is a write. (Conflicts are usually detected by detecting non-empty read and write set intersections.) When that happens, a contention manager (CM) resolves the conflict by aborting one and committing the other, yielding (the illusion of) atomicity. Aborted transactions are re-started, after rolling-back the changes–e.g., undoing object changes using the undo-log (eager), or discarding the write buffers (lazy).

In addition to a simple programming model (locks are excluded from the programming interface), TM provides performance comparable to lock-based synchronization [103], especially for high contention and read-dominated workloads, and is composable. TM's first implementation was proposed in hardware, called hardware transactional memory (or HTM) [63]. HTM has the lowest overhead, but HTM transactions are usually limited in space and time. Examples of HTMs include TCC [57], UTM [1], Oklahoma [115], ASF [33], and Bulk [25]. TM implementation in software, called software transactional memory (or STM) was proposed later [111]. STM transactions do not need any special hardware, are not limited in size or time, and are more flexible. However, STM has a higher overhead, and thus lower performance, than HTM. Examples of STMs include RSTM [121], TinySTM [101], Deuce [75], and AtomJava [65].

Listing 1.1: STM example

```
BEGIN_TRANSACTION;
        stm::wr_ptr<Counter> wr(m_counter);
        wr->set_value(wr->get_value(wr) + 1, wr);
END_TRANSACTION;
```

Listing 1.1 shows an example STM code written by RSTM [112]'s interface. RSTM's `BEGIN_TRANSACTION` and `END_TRANSACTION` keywords are used to enclose a critical section, which creates a transaction for the enclosed code block and guarantees its atomic execution. First line inside the transaction makes a write pointer to a variable "m_counter" of type "Counter". The second line reads the current value of the counter variable through "wr−>get_value". The counter value is incremented through "wr−>set_value" operation.

Hybrid TM (or HyTM) was subsequently proposed in [85], which combines HTM with STM, and avoids their limitations. Examples of HyTMs include SpHT [83], VTM [99], HyTM [34], LogTM [93], and LogTM-SE [126].

1.2 STM for Real-Time Software

Given the hardware-independence of STM, which is a significant advantage, we focus on STM. STM's programmability, scalability, and composability advantages are also compelling for concurrency control in multicore embedded real-time software. However, this will require bounding transactional retries, as real-time threads, which subsume transactions, must satisfy application-level time constraints. Transactional retry bounds in STM are dependent on the CM policy at hand (analogous to the way thread response time bounds are OS scheduler-dependent).

Despite the large body of work on STM contention managers, relatively few results are known on real-time contention management. STM concurrency control for real-time systems has been previously studied, but in a limited way. For example, [89] proposes a restricted version of STM for uniprocessors. Uniprocessors do not need contention management. [48] bounds response times in distributed multicore systems with STM synchronization. They consider Pfair scheduling [70], which is largely only of theoretical interest[1], limit to small atomic regions with fixed size, and limit transaction execution to span at most two quanta. [104] presents real-time scheduling of transactions and serializes transactions based on transactional deadlines. However, the work does not bound transactional retries and response times.

[107] proposes real-time HTM, which of course, requires hardware with TM support. The retry bound developed in [107] assumes that the worst case conflict between atomic sections of different tasks occurs when the sections are released at the same time. We show that, this assumption does not cover the worst case scenario (see Chapter 4). [47] presents a contention manager that resolves conflicts using task deadlines. The work also establishes upper bounds on transactional retries and task response times. However, similar to [107], [47] also assumes that the worst case conflict between atomic sections occurs when the sections are released simultaneously. Besides, [47] assumes that all transactions have equal lengths. The ideas in [47] are extended in [9], which presents three real-time CM designs. But no retry bounds or schedulability analysis techniques are presented for those CMs.

[1] This is due to Pfair class of algorithm's time quantum-driven nature of scheduling and consequent high run-time overheads.

Thus, past efforts on real-time STM are limited, and do not answer important fundamental questions:

(1) How to design "general purpose" real-time STM contention managers for multicore architectures? By general purpose, we mean those that do not impose any restrictions on transactional properties (e.g., transaction lengths, number of transactional objects, levels of transactional nestings), which are key limitations of past work.

(2) What tight upper bounds exist for transactional retries and task response times under such real-time CMs?

(3) How does the schedulability of real-time CMs compare with that of lock-free synchronization? i.e., are there upper bounds or lower bounds for transaction lengths below or above which is STM superior to lock-free synchronization?

(4) How does transactional retry costs and task response times of real-time CMs compare with that of lock-free synchronization in practice (i.e., on average)?

1.3 Research Contributions

In this dissertation proposal, we answer these questions. We present a suite of real-time STM contention managers, called RCM, ECM, LCM, and PNF. The contention managers progressively improve transactional retry and task response time upper bounds (and consequently improve STM's schedulability advantages) and also relax the underlying task models. RCM and ECM resolve conflicts using fixed and dynamic priorities of real-time tasks, respectively, and are naturally intended to be used with the fixed priority (e.g., G-RMA [23]) and dynamic priority (e.g., G-EDF [23]) multicore real-time schedulers, respectively. LCM resolves conflicts based on task priorities as well as atomic section lengths, and can be used with G-EDF or G-RMA. Transactions under ECM, RCM and LCM can restart because of other transactions that share no objects with them. This is called transitive retry. PNF solves this problem. PNF also optimizes processor usage through reducing priority of aborted transactions. So, other tasks can proceed.

We establish upper bounds on transactional retry costs and task response times under all the contention managers through schedulability analysis. Since ECM and RCM conserve the semantics of the underlying real-time scheduler, their maximum transactional retry cost is double the maximum atomic section length. This is improved in the design of LCM, which achieves shorter retry costs. However, ECM, RCM, and LCM are affected by transitive retries when transactions access multiple objects. Transitive retry causes a transaction to abort and retry due to another non-conflicting transaction. PNG avoids transitive retry, and also optimizes processor usage by lowering the priority of retrying transactions, enabling other non-conflicting transactions to proceed.

We formally compare the schedulability of the proposed contention managers with lock-free synchronization. Our comparison reveals that, for most cases, ECM and RCM achieve higher schedulability than lock-free synchronization only when the atomic section length does not exceed half of lock-free synchronization's retry loop length. However, in some cases, the atomic section length can reach the lock-free retry loop length for ECM and it can even be larger than the lock-free retry loop-length for RCM, and yet higher schedulability can be achieved with STM. This means that, STM is more advantageous with G-RMA than with G-EDF.

LCM achieves shorter retry costs and response times than ECM and RCM. Importantly, the atomic section length range for which STM's schedulability advantage holds is significantly expanded with LCM (over that under ECM and RCM): Under ECM, RCM and LCM, transactional length should not exceed half of lock-free retry loop length to achieve better schedulability. However, with low contention, transactional length can increase to retry loop length under ECM. Under RCM, transactional length can be of many orders of magnitude of retry loop length with low contention. With suitable LCM parameters, transactional length under G-EDF/LCM can be twice as retry loop length. While in G-RMA/LCM, transactional length can be of many orders of magnitude as retry loop length. PNF achieves better schedulability than lock-free as long as transactional length does not exceed length of retry loop.

Why are we concerned about expanding STM's schedulability advantage? When STM's schedulability advantage holds, programmers can reap STM's significant programmability and composability benefits in multicore real-time software. Thus, by expanding STM's schedulability advantage, we increase the range of real-time software for which those benefits can be tapped. Our results, for the first time, thus provides a fundamental understanding of when to use, and not use, STM concurrency control in multicore real-time software.

We also implement the contention managers in the RSTM framework [112] and conduct experimental studies using the ChronOS multicore real-time Linux kernel [36]. Our studies confirm that, the contention managers achieve shorter retry costs than lock-free synchronization by as much as 95% improvement (on average). Among the contention managers, PNF performs the best in case of high transitive retry. PNF achieves shorter retry costs than ECM, RCM and LCM by as much as 53% improvement (on average).

1.4 Summary of Proposed Post Preliminary Research

Based on our current research results, we propose the following work:

Supporting nested transactions. Transactions can be nested *linearly*, where each transaction has at most one pending transaction [94]. Nesting can also be done in *parallel* where transactions execute concurrently within the same parent [122]. Linear nesting can be *1) flat:* If a child transaction aborts, then the parent transaction also aborts. If a child commits, no effect is taken until the parent commits. Modifications made by the child transaction are only visible to the parent until the parent commits, after which they are externally visible. *2) Closed:* Similar to *flat nesting*, except that if a child transaction conflicts, it is aborted and retried, without aborting the parent, potentially improving concurrency over flat nesting. *3) Open:* If a child transaction commits, its modifications are immediately externally visible, releasing memory isolation of objects used by the child, thereby potentially improving concurrency over closed nesting. However, if the parent conflicts after the child commits, then compensating actions are executed to undo the actions of the child, before retrying the parent and the child. We propose to develop real-time contention managers that allow these different nesting models and establish their retry and response time upper bounds. Additionally, we propose to formally compare their schedulability with nested critical sections under lock-based synchronization. Note that, nesting is not viable under lock-free synchronization.

Combinations and optimizations of LCM and PNF contention managers. LCM is designed to reduce the retry cost of a transaction when it is interfered close to the end of its execution. In contrast, PNF is designed to avoid transitive retry when transactions access multiple objects. An interesting direction is to combine the two contention managers to obtain the benefits of both algorithms. Further design optimizations may also be possible to reduce retry costs and response times, by considering additional criteria for resolving transactional conflicts. Importantly, we must also understand what are the schedulability advantages of such a combined/optimized CM over that of LCM and PNF, and how such a combined/optimized CM behaves in practice. This will be our second research direction.

Formal and experimental comparison with real-time locking protocols. Lock-free synchronization offers numerous advantages over locking protocols, but (coarse-grain) locking protocols have had significant traction in real-time systems due to their good programmability (even though their concurrency is low). Example such real-time locking protocols include PCP and its variants [27, 74, 98, 109], multicore PCP (MPCP) [79, 97], SRP [8, 24], multicore SRP (MSRP) [51], PIP [39], FMLP [16, 17, 68], and OMLP [11]. OMLP and FMLP are similar, and FMLP has been established to be superior to other protocols [20]. How does their schedulability compare with that of the proposed contention managers? How do they compare in practice? These questions constitute our third research direction.

1.5 Proposal Organization

The rest of this dissertation proposal is organized as follows. Chapter 2 overviews past and related work on real-time concurrency control. Chapter 3 describes our task/system model and assumptions. Chapter 4 describes the ECM and RCM contention managers, derives upper bounds for their retry costs and response times, and compares their schedulability between themselves and with lock-free synchronization. Chapters 5 and 6 similarly describe the LCM and PNF contention managers, respectively. Chapter **??** describes our implementation and reports our experimental studies. We conclude in Chapter **??**.

Chapter 2

Past and Related Work

Many mechanisms appeared for concurrency control for real-time systems. These methods include locking [24, 84], lock-free [3–5, 28, 31, 37, 45, 46, 69, 78] and wait-free [2, 12, 26, 28, 29, 32, 46, 66, 100, 116–118]. In general, real-time locking protocols have disadvantages like: 1) serialized access to shared object, resulting in reduced concurrency and reduced utilization. 2) increased overhead due to context switches. 3) possibility of deadlock when lock holder crashes. 3) some protocols requires apriori knowledge of ceiling priorities of locks. This is not always available. 4) Operating system data structures must be updates with this knowledge which reduces flexibility. For real-time lock-free, the most important problem is to bound number of failed retries and reduce cost of a single loop. The general technique to access lock-free objects is "retry-loop". Retry-loop uses atomic primitives (e.g., CAS) which is repeated until success. To access a specific data structure efficiently, lock-free technique is customized to that data structure. This increases difficulty of response time analysis. Primitive operations do not access multiple objects concurrently. Although some attempts made to enable multi-word CAS [3], but it is not available in commodity hardware [91]. For real-time wait-free protocols. It has a space problem due to use of multiple buffers. This is inefficient in some applications like small-memory real-time embedded systems. Wait-free has the same problem of lock-free in handling multiple objects.

The rest of this Chapter is organized as follows, Section 2.1 summarizes previous work on real-time locking protocols. In Section 2.2, we preview related work on lock-free and wait-free methods for real-time systems. Section 2.3 provides concurrency control under real-time database systems as a predecessor and inspiration for real-time STM. Section 2.4 previews related work on contention management. Contention management policy affects response time analysis of real-time STM.

2.1 Real-Time Locking Protocols

A lot of work has been done on real-time locking protocols. Locks in real-time systems can lead to priority inversion [24, 84]. Under priority inversion, a higher priority job is not allowed to run because it needs a resource locked by a lower priority job. Meanwhile, an intermediate priority job preempts the lower priority one and runs. Thus, the higher priority job is blocked because of a lower priority one. Different locking protocols appeared to solve this problem, but exposing other problems. Most of real-time blocking protocols are based on *Priority Inheritance Protocol (PIP)* [24, 39, 109], *Priority Ceiling Protocol (PCP)* [24, 27, 39, 74, 79, 97, 98, 109] and *Stack Resource Protocol (SRP)* [8, 24, 52].

In PIP [24, 109], resource access is done in FIFO order. A resource holder inherits highest priority of jobs blocked on that resource. When resource holder releases the resource and it holds no other resources, its priority is returned to its normal priority. If it holds other resources, its priority is returned to highest priority job blocked on other resources. Under PIP, a high priority job can be blocked by lower priority jobs for at most the minimum of number of lower priority jobs and number of shared resources. PIP suffers from chained blocking, in which a higher priority task is blocked for each accessed resource. Besides, PIP suffers from deadlock where each of two jobs needs resources held by the other. So, each job is blocked because of the other. [39] provides response time analysis for PIP when used with fixed-priority preemptive scheduling on multiprocessor system.

PCP [24, 98, 109] provides concept of priority ceiling. Priority ceiling of a resource is the highest priority of any job that can access that resource. For any job to enter a critical section, its priority should be higher the priority ceiling of any currently accessed resource. Otherwise, the resource holder inherits the highest priority of any blocked job. Under PCP, a job can be blocked for at most one critical section. PCP prevents deadlocks. [27] extends PCP to dynamically scheduled systems.

Two protocols extend PCP to multiprocessor systems: 1) *Multiprcoessor PCP (M-PCP)* [79, 97, 98] discriminates between global resources and local resources. Local resources are accessed by PCP. A global resource has a base priority greater than any task normal priority. Priority ceiling of a global resource equals sum of its base priority and highest priority of any job that can access it. A job uses a global resource at the priority ceiling of that resource. Requests for global resources are enqueued in a priority queue according to normal priority of requesting job. 2) *Parallel-PCP (P-PCP)* [39] extends PCP to deal with fixed priority preemptive multiprocessor scheduling. P-PCP, in contrast to PCP, allows lower priority jobs to allocate resources when higher priority jobs already access resources. Thus, increasing parallelism. Under P-PCP, a higher priority job can be blocked multiple times by a lower priority job. With reasonable priority assignment, blocking time by lower priority jobs is small. P-PCP uses α_i parameter to specify permitted number of jobs with basic priority lower than i and effective priority higher than i. When α_i is small, parallelism is reduced, so as well blocking from lower priority tasks. Reverse is true. [39] provides response time analysis for P-PCP.

[86] extends P-PCP to provide *Limited-Blocking PCP (LB-PCP)*. LB-PCP provides more control on indirect blocking from lower priority tasks. LB-PCP specify additional counters that control number of times higher priority jobs can be indirectly blocked without the need of reasonable priority assignment as in P-PCP. [86] analyzes response time of LB-PCP and experimentally compares it to P-PCP. Results show that LB-PCP is appropriate for task sets with medium utilization.

PCP can be unfair from blocking point of view. PCP can cause unnecessary and long blocking for tasks that do not need any resources. Thus, [74] provides Intelligent PCP (IPCP) to increase fairness and to work in dynamically configured system (i.e., no a priori information about number of tasks, priorities and accessed resources). IPCP initially optimizes priorities of tasks and resources through learning. Then, IPCP tunes priorities according to system wide parameters to achieve fairness. During the tuning phase, penalties are assigned to tasks according to number of higher priority tasks that can be blocked.

SRP [8, 24, 52] extends PCP to allow multiunit resources and dynamic priority scheduling and sharing runtime stack-based resources. SRP uses *preemption level* as a static parameter assigned to each task despite its dynamic priority. Resource ceiling is modified to include number of available resources and preemption levels. System ceiling is the highest resource ceiling. A task is not allowed to preempt unless it is the highest priority ready one, and its preemption level is higher than the system ceiling. Under SRP, a job can be blocked at most for one critical section. SRP prevents deadlocks. *Multiprocessor SRT (M-SRP)* [51] extends SRP to multiprocessor systems. M-SRP, as M-PCP, discriminates between local and global resources. Local resources are accessed by SRP. Request for global resource are enqueued in a FIFO queue for that resource. Tasks with pending requests busy-wait until their requests are granted.

Another set of protocols appeared for PFair scheduling [16]. [67] provide initial attempts to synchronize tasks with short and long resources under PFair. In Pfair scheduling, each task receives a weight that corresponds to its share in system resources. Tasks are scheduled in quanta, where each quantum has a specific job on a specific processor. Each lock has a FIFO queue. Requesting tasks are ordered in this FIFO queue. If a task is preempted during critical section, then other tasks can be blocked for additional time known as *frozen time*. Critical sections requesting short resources execute at most in two quanta. By early lock-request, critical section can finish in one quanta, avoiding the additional blocking time. [67] proposes two protocols to deal with short resources: 1) *Skip Protocol (SP)* leaves any lock request in the FIFO queue during frozen interval until requesting task is scheduled again. 2) *Rollback Protocol (RP)* discards any request in the FIFO queue for the lock during frozen time. For long resources, [67] uses *Static Weight Server Protocol (SWSP)* where requests for each resource l is issued to a corresponding server S. S orders requests in a FIFO queue and has a static specific weight.

Flexible Multiprocessor Locking Protocol (FMLP) [16] is the most famous synchronization protocol for PFair scheduling. The FMLP allows non-nested and nested resources access without constraints. FMLP is used under global and partitioned deadline scheduling. Short or long resource is user defined. Resources can be grouped if they are nested by some task and have the same type. Request to a specific resource is issued to its containing group. Short groups are protected by non-preemptive FIFO queue locks, while long groups are protected by FIFO semaphore queues. Tasks busy-wait for short resources and suspend on long resources. Short request execute non-preemptively. Requests for long resources cannot be contained within requests for short resources. A job executing a long request inherits highest priority of blocked jobs on that resource's group. FMLP is deadlock free.

[18] is concerned with suspension protocols. Schedulability analysis for suspension protocols can be suspension-oblivious or suspension-aware. In suspension-oblivious, suspension time is added to task execution. While in suspension-aware, it is not. [18] provides *Optimal Multi-processor Locking Protocol (OMLP)*. Under OMLP, each resource has a FIFO queue of length at most m, and a priority queue. Requests for each resource are enqueued in the corresponding FIFO queue. If FIFO queue is full, requests are added to the priority queue according to the requesting job's priority. The head of the FIFO queue is the resource holding task. Other queued requests are suspended until their turn come. OMLP achieves $O(m)$ priority inversion (pi) blocking per job under suspension oblivious analysis. This is why OMLP is asymptotically optimal under suspension oblivious analysis. Under suspension aware analysis, FMLP is asymptotically optimal. [19] extends work in [18] to clustered-based scheduled multiprocessor system. [19] provides concept of *priority donation* to ensure that each job is preempted at most once. In priority donation, a resource holder priority can be unconditionally increased. Thus, a resource holder can preempt another task. The preempted task is predetermined such that each job is preempted at most once. OMLP with priority donation can be integrated with k-exclusion locks (K-OMLP). Under K-exclusion locks, there are k instances of the same resource than can be allocated concurrently. K-OMLP has the same structure of OMLP except that there are K FIFO queues for each resource. Each FIFO queue corresponds to one of the k instances. K-OMLP has $O(m/k)$ bound for pi-blocking under s-oblivious analysis. [42] extends the K-OMLP in [19] to global scheduled multiprocessor systems. The new protocol is *Optimal K-Exclusion Global Locking Protocol (O-KGLP)*. Despite global scheduling is a special case of clustering, K-OMLP provides additional cost to tasks requesting no resources if K-OMLP is used with global scheduling. O-KGLP avoids this problem.

2.2 Real-Time Lock-Free and Wait-Free Synchronization

Due to locking problems (e,g,. priority inversion, high overhead and deadlock), research has been done on non-blocking synchronization using lock-free [3–5, 28, 31, 45, 46, 69, 78] and wait-free algorithms [2, 12, 26, 28, 29, 32, 46, 66, 100, 116–118]. Lock-free iterates an atomic primitive (e.g., CAS) inside a retry loop until successfully accessing object. When used with real-time systems, number of failed retries must be bounded [3,4]. Otherwise, tasks are highly likely to miss their deadlines. Wait-free algorithms, on the other hand, bound number of object access by any operation due to use of sized buffers. Synchronization under wait-free is concerned with: 1) single-writer/multi-readers where a number of reading operations may conflict with one writer. 2) multi-writer/multi-reader where a number of reading operations may conflict with number of writers. The problem with wait-free algorithms is its space cost. As embedded real-time systems are concerned with both time and space complexity, some work appeared trying to combine benefits of locking and wait-free.

[4] considers lock-free synchronization for hard-real time, periodic, uniprocessor systems. [4] upper bounds retry loop failures and derives schedulability conditions with Rate Monotonic (RM), and Earliest Deadline First (EDF). [4] compares, formally and experimentally, lock-free objects against locking protocols. [4] concludes that lock-free objects often require less overhead than locking-protocols. They require no information about tasks and allow addition of new tasks simply. Besides, lock-free object do not induce excessive context switches nor priority inversion. On the other hand, locking protocols allow nesting. Besides, performance of lock-free depends on the cost of "retry-loops". [3] extends [4] to generate a general framework for implementing lock-free objects in uniprcocessr real-time systems. The framework tackles the problem of multi-objects lock-free operations and transactions through multi-word compare and swap (MWCAS) implementation. [3] provides a general approach to calculate cost of operation interference based on linear programming. [3] compares the proposed framework with real-time locking protocols. Lock-free objects are prefered if cost of retry-loop is less than cost of lock-access-unlock sequence. [5] extends [3,4] to use lock-free objects in building memory-resident transactions for uniprocessor real-time systems. Lock-free transactions, in contrast to lock-based transactions, do not suffer from priority inversion, deadlocks, complicated data-logging and rolling back. Lock-free transaction do not require kernel support.

[37] presents two synchronization methods under G-EDF scheduled real-time multiprocessor systems for simple objects. The first synchronization technique uses queue-based spin locks, while the other uses lock-free. The queue lock is FIFO ordered. Each task appends an entry at the end of the queue, and spins on it. While the task is spinning, it is non-preemptive. The queue could have been priority-based but this complicates design and does not enhance worst case response time analysis. Spinning is suitable for short critical sections. Disabling preemption requires kernel support. So, second synchronization method uses lock-free objects. [37] bounds number of retries. [37] , analytically and experimentally, evaluates both synchronization techniques for soft and hard real-time analysis. [37] concludes that queue locks have a little overhead. They are suitable for small number of shared object operations per task. Queue locks are not generally suitable for nesting. Lock-free have high overhead compared with queue locks. Lock-free is suitable for small number of processors and object calls in the absence of kernel support.

[69] uses lock-free objects under PFair scheduling for multiprocessor system. [69] provides

concept of *supertasking* to reduce contention and number of failed retries. This is done by collecting jobs that need a common resource into the same supertask. Members of the same supertask run on the same processor. Thus, they cannot content together. [69] upper bounds worst case duration for lock-free object access with and without supertasking. [69] optimizes, not replaces, locks by lock-free objects. Locks are still used in situations like sharing external devices and accessing complex objects.

Lock-free objects are used with time utility models where importance and criticality of tasks are separated [31,78]. [78] presents *MK-Lock-Free Utility Accrual (MK-LFUA)* algorithm that minimizes system level energy consumption with lock-free synchronization. [31] uses lock-free synchronization for dynamic embedded real-time systems with resource overloads and arbitrary activity arrivals. Arbitrary activity arrivals are modelled with Universal Arrival Model (UAM). Lock-free retries are upper bounded. [31] identifies the conditions under which lock-free is better than lock-based sharing. [45] builds a lock-free linked-list queue on a multi-core ARM processor.

Wait-free protocols use multiple buffers for readers and writers. For single-writer/multiple-readers, each object has a number of buffers proportional to maximum number of reader's preemptions by the writer. This bounds number of reader's preemptions. Readers and writers can use different buffers without interfering each other.

[32] presents wait-free protocol for single-writer/multiple-readers in small memory embedded real-time systems. [32] proves space optimality of the proposed protocol, as it required the minimum number of buffers. The protocol is safe and orderly. [32] also proves, analytically and experimentally, that the protocol requires less space than other wait-free protocols. [29] extends [32] to present wait-free utility accrual real-time scheduling algorithms (RUA and DASA) for real-time embedded systems. [29] derives lower bounds on accrued utility compared with lock-based counterparts while minimizing additional space cost. Wait-free algorithms experimentally exhibit optimal utility for step time utility functions during underload, and higher utility than locks for non-step utility functions. [100] uses wait-free to build three types of concurrent objects for real-time systems. Built objects has persistent states even if they crash. [118] provides wait-free queue implementation for real-time Java specifications.

A number of wait-free protocols were developed to solve multi-writer/multi-reader problem in real-time systems. [117] provides m-writer/n-reader non-blocking synchronization protocol for real-time multiprocessor system. The protocol needs $n + m + 1$ slots. [117] provides schedulability analysis of the protocol. [2] presents wait-free methods for multi-writer/multi-reader in real-time multiprocessor system. The proposed algorithms are used for both priority and quantum based scheduling. For a B word buffer, the proposed algorithms exhibit $O(B)$ time complexity for reading and writing, and $\Theta(B)$ space complexity. [116] provides a space-efficient wait-free implementation for n-writer/n-reader synchronization in real-time multiprocessor system. The proposed algorithm uses timestamps to implement the shared buffer. [116] uses real-time properties to bound timestamps. [26] presents wait-free implementation of the multi-writer/multi-reader problem for real-time multiprocessor synchronization. The proposed mechanism replicates single-writer/multi-reader to solve the multi-writer/multi-reader problem. [26], as [116], uses real-time properties to ensure data coherence through timestamps.

Each synchronization technique has its benefits. So, a lot of work compares between locking, lock-free and wait-free algorithms. [46] compares building snapshot tool for real-time system using locking, lock-free and wait-free. [46] analytically and experimentally compares the three methods. [46] concludes that wait-free is better than its competitors. [28] presents synchronization techniques under LNREF [30] (an optimal real-time multiprocessor scheduler) for simple data structures. Synchronization mechanisms include lock-based, lock-free and wait-free. [28] derives minimum space cost for wait-free synchronization. [28] compares, analytically and experimentally, between lock-free and lock-based synchronization under LNREF.

Some work tried to combine different synchronization techniques to combine their benefits. [66] uses combination of lock-free and wait-free to build real-time systems. Lock-free is used only when CAS suffices. The proposed design aims at allowing good real-time properties of the system, thus better schedulability. The design also aims at reducing synchronization overhead on uni and multiprocessor systems. The proposed mechanism is used to implement a micro-kernel interface for a uni-processor system. [12] combines locking and wait-free for real-time multiprocessor synchronization. This combination aims to reduce required space cost compared to pure wait-free algorithms, and blocking time compared to pure locking algorithms. The proposed scheme is jsut an idea. No formal analysis nor implementation is provided.

2.3 Real-Time Database Concurrency Control

Real-time database systems (RTDBS) is not a synchronization technique. It is a predecessor and inspiration for real-time transactional memory. RTDBS itself uses synchronization techniques when transactions conflict together. RTDBS is concerned not only with logical data consistency, but also with temporal time constraints imposed on transactions. Temporal time constraints require transactions finish before their deadlines. External constraints require updating temporal data periodically to keep freshness of database. RTDBS allow mixed types of transactions. But a whole transaction is of one type. In real-time TM, a single task may contain atomic and non-atomic sections.

High-Priority two Phase Locking (HP-2PL) protocol [80, 81, 95, 128] and *Real-Time Optimistic Concurrency (RT-OCC)* protocol [35, 49, 80–82, 128] are the most two common protocols for RTDBS concurrency . HP-2PL works like 2PL except that when a higher priority transaction request a lock held by a lower priority transaction, lower priority transaction releases the lock in favor of the higher priority one. Then, lower priority transaction restarts. RT-OCC delays conflict resolution till transaction validation. If validating transaction cannot be serialized with conflicting transactions, a priority scheme is used to determine which transaction to restart. In *Optimistic Concurrency Control with Broadcast Commit (OCC-BC)*, all conflicting transactions with the validating one are restarted. HP-2PL may encounter deadlock and long blocking times, while transactions under RT-OCC suffer from restart time at validation point.

Other protocols were developed based on HP-2PL [80, 81, 95] and RT-OCC [7, 49, 80, 82]. HP-2PL, and its derivatives, are similar to locking protocols in real-time systems. They have the same problems in real-time locking protocols like priority inversion. So, the same solutions exist for the RTDBS locking protocols. Despite RT-OCC, and its derivatives, use locks in their implementation, their behaviour is closer to abort and retry semantics in TM. Some work integrates different protocols to handle different situations [95, 127].

[80] presents *Reduced Ceiling Protocol (RCP)* which is a combination of *Priority Ceiling Protocol (PCP)* and *Optimistic Concurrency Protocol (OCC)*. RCP targets database systems with mixed hard and soft real-time transactions (RTDBS). RCP aims at guarantee of schedulabiltiy of hard real-time transactions, and minimizing deadline miss of soft real-time transactions. Soft real-time transactions are blocked in favor of conflicting hard real-time transactions. While hard real-time transactions use PCP to synchrnonize among themselves, soft real-time transactions use OCC. Hard real-time transactions access locks in a *tow phase locking (2PL)* fashion. Seized locks are released as soon as hard real-time transaction no longer need them. This reduces blocking time of soft real-time transactions. [80] derives analytical and experimental evaluation of RCP against other synchronization protocols.

[127], like [80], deals with mixed transaction. [127] classifies mixed transactions into hard (HRT), soft (SRT) and non (NRT) real-time transactions. HRT has higher priority than SRT. SRT has higher priority than NRT. [127] aims at guranting deadlines of HRTs, minimizing miss rate of SRTs and reducing response time of NRTs. So, [127] deals with inter and intra-transaction concurrency. HRTs use PCP for concurrency control among themselves. SRTs use WAIT-50, and NRTs use 2PL. SRT and NRT are blocked or aborted in favor of HRT. If NRT requests a lock held by SRT, then NRT is blocked. If SRT requests a lock held by NRT, WAIT-50 is applied. Experimental evaluation showed effective improvement in overall system performance. Performance objectives of each transaction type was met.

[49] is concerned with semantic lock concurrency control. The semantic lock technique allows negotiation between logical and temporal constraints of data and transactions. It also controls imprecision resulting from negotiation. Thus, the semantic lock considers scheduling and concurrency of transactions. Semantic lock uses a compatibility function to determine if the release transaction is allowed to proceed or not.

Time Interval OCC protocols try to reduce number of transaction restarts by dynamic adjustment of serialization timestamps. Time interval OCC may encounter unnecessary restarts. [7] presents Timestamp Vector based OCC to resolve these unnecessary restarts. Timestamp Vector base OCC uses a timestamp vector instead of a single timestamp as in Time Interval OCC protocols. Experimental comparison between Timestamp Vector OCC and previous

Time Interval OCC shows higher performance of Timestamp Vector OCC.

[35] aims to investigate performance improvement of priority congnizant OCC over incognizant counterparts. In OCC-BC, all conflicting transactions with the validating transaction are restarted. [35] wonders if it is really worthy to sacrifice all other transactions in favor of one transaction. [35] proposes *Optimistic Concurrency Control- Adaptive PRiority (OCC-APR)* to answer this question. A validating transaction is restarted if it has sufficient time to its deadline if restarted, and higher priority transactions cannot be serialized with the conflicting transaction. Sufficient time estimate is adapted according to system feedback. System feedback is affected by contention level. [35] experimentally concludes that integrating priority into concurrency control management is not very useful. Time Interval OCC showed better performance.

WAIT-X [35, 82] is one of the optimistic concurrency control (OCC) protocols. WAIT-X is a prospective (forward validation) OCC. Prospective means it detects conflicts between a validating transaction and conflicting transaction that may commit in the future. In retrospective (backward validation) protocols, conflicts are detected between a validating transaction and already committed transactions. Retrospective validation aborts validating transaction if it cannot be serialized with already committed conflicting transactions. When WAIT-X detects a conflict, it can either abort validating transaction, or commit validating transaction and abort other conflicting transactions, or it can dealy validating a transction slightly hoping that conflicts resolve themsleves someway. Which action to take is a function of priorities of vlaidating and conflicting transactions. WAIT-X can delay validating transaction until percetage of higher priority transactions in the conflict set is lower than X%. WAIT-50 is a common implementation of WAIT-X.

[77] is concerned with concurrency control for multiprocessor RTDBS. [77] uses priority cap to modify *Reader/Write Prirority Ceiling Protocol (RWPCP)* [110] to work on multiprocessor systems. The proposed protocol, named *One Priority Inversion RWPCP (1PI-RWPCP)*, is deadlock-free and bounds number of priority inversions for any transaction to one. [77] derives feasiblity condition for any transaction under 1PI-RWPCP. [77] experimentally compares performance of 1PI-RWPCP against RWPCP.

[95] combines locking, multi-version and valid confirmation concurrency control mechanisms. The proposed method adopts different concurrency control mechanism according to idiographic situation. Experiments show lower rate of transactional restart of the proposed mechanis compared to 2PL-HP.

[81] is concerned with RTDBS containing periodically updated data and one time transactactions. [81] provides two new concurrency control protocols to balance freshness of data and transaction performance. [81] proposes *HP-2PL with Delayed Restart (HP-2PL-DR)* and *HP-2PL with Delayed Restart and Pre-declaration (HP-2PL-DRP)* based on HP-2PL. Before a transaction T restarts in HP-2PL-DR, next update time of each temporal data accessed by T is checked. If next update time starts before currently re-executing T, then T's restart time is delayed until the next udpate. Otherwise, T is restarted immediately. If T_r and

T_n are two transactions under HP-2PL-DRT. T_r is requesting a lock held by T_n. If priority of T_r is greater than priority of T_n, then T_n releases the lock in favor of T_r. Othewise, T_r fails. If T_n releases the lock and T_n is a one time transaction, then T_n restarts immediately. Otherwise, T_n lock waiting time is updated. Experiments show improved performance of HP-2PL-DR and HP-2PL-DRP over HP-2PL.

2.4 Real-Time TM Concurrency Control

Concurrency control in TM is done through contention managers. Contention managers are used to ensure progress of transactions. If one or more transactions conflict on an object, contention manager decides which transaction to commit. Other transactions abort or wait. Mostly, contention managers are *distributed* or *decentralized* [55, 56, 105, 106], in the sense that each transaction maintains its own contention manager. Contention managers may not know which objects will be needed by transactions and their duration. Past work on contention managers can be classified into two classes: 1) Contention management policy that decides which transaction commits and which do other actions [54–56, 105, 106, 114]. 2) Implementation of contention management policy in practice [15,38,53,88,105,114]. The two classes are orthogonal. The second class tries to increase the benefit of the the contention management policy in reality by considering different aspects in TM design (e.g., lazy versus eager, visible versus invisible readers). Second class suggests contention managers should be proactive instead of reactive. This can prevent conflicts before they happen. Contention managers can be supported a lot if they are integrated into system schedulers. This provides a global view of the system (due to applications feedback) and reduces overhead of the implementation of contention manager.

Contention management policy ranges from never aborting enemies to always aborting them [105, 106]. These two extremes can lead to deadlock, starvation, livelock and major loss of performance. Contention manager policy lies in between. Depending on heuristics, contention manager balances between decisions complexity against quality and overhead.

Different types of contention management policies can be found in [54–56, 105, 106, 114] like:

1. Passive and Aggressive: Passive contention manager aborts current transaction, while aggressive aborts enemy.

2. Polite: When conflicting on an object, a transaction spins exponensially for average of $2^{(n+k)}\,ns$, where n is number of times to access the object, and k is a tuning parameter. Spinning times is bounded by m. Afterwards, any enemy is aborted.

3. Karma: It assigns priorities to transaction based on the amount of work done so far. Amount of work is measured by number of opened objects by current transaction. Higher priority transaction aborts lower priority one. If lower priority transaction tries

to access an object for a number of times greater than priority difference between itself and higher priority transaction, enemy is aborted.

4. Eruption: It works like Karma except it adds priority of blocked transaction to the transaction blocking it. This way, enemy is sped-up, allowing blocked transactions to complete faster.

5. Kindergarten: A transaction maintains a hit list (initially empty) of enemies who previously caused current thread to abort. When a new enemy is encountered, current transaction backs off for a limited amount of time. The new enemy is recorded in the hit list. If the enemy is already in the hit list, it is aborted. If current transaction is still blocked afterwards, then it is aborted.

6. Timestamp: It is a fair contention manager. Each transaction gets a timestamp when it begins. Transaction with newer timestamp is aborted in favour of the older. Otherwise, transaction waits for a fixed intervals, marking the enemy flag as defunct. If the enemy is not done afterwards, it is killed. Active transaction clear their flag when they notice it is set.

7. Greedy: Each transaction is given a timestamp when it starts. The earlier the timestamp of a transaction, the higher its priority. If transaction A conflicts with transaction B, and B is of lower priority or is waiting for another transaction, then A aborts B. Otherwise, A waits for B to commit, abort or starts waiting.

8. Randomized: It aborts current transaction with some probability p, and waits with probability $1 - p$.

9. PublishedTimestamp: It works like Timestamp contention manager except it has a new definition for an "inactive" transaction. Each transaction maintains a "recency" flag. Recency flag is updated every time the transaction makes a request. Each transaction maintains its own "inactivity" threshold parameter that is doubled every time it is aborted up to a specific limit. If the enemy "recency" flag is behind the system global time by amount exceeding its "inactivity" threshold, then enemy is aborted.

10. Polka: It is a combination of Polite and Karma contention managers. Like Karma, it assigns priorities based on amount of job done so far. A transaction backs off for a number of intervals equals difference in priority between itself and its enemy. Unlike Karma, back-off length increases exponentially.

11. Prioritized version of some of the previous contention managers appeared. Prioritized contention managers include base priority of the thread holding the transaction into contention manager policy. This way, higher priority threads are more favoured.

[6] compares performance of different contention managers against an optimal, clairvoyant contention manager. The optimal contention manager knows all resources needed by each

transaction, as well as its release time and duration. Comparison is based on the "makespan" concept which is amount of time needed to finish a specific set of transactions. The ratio between makespan of analyzed contention manager and the makespan of the optimal contention manager is known as competitive ratio. [6] proves that any contention manager can be of $O(s)$ competitive ratio if the contention manager is work conserving (i.e., always lets the maximal set of non-conflicting transactions run), and satisfies pending property [55]. The paper proves that this result is asymptotically tight as no on-line work conserving contention manager can achieve better result. [6] also proves that the makespan of greedy contention manager is $O(s)$ instead of $O(s^2)$ [55]. This allows transactions of arbitrary release time and durations in contrast to what is assumed in [55]. For randomized contention managers, a lower bound of $\Omega(s)$ if transaction can modify their resource needs when they are reinvoked.

[54] analyzes different contention managers under different situations. [54] concludes that no single contention manager is suitable for all cases. Thus, [54] proposes a polymorphic contention manager that changes contention managers on the fly throughout different loads, concurrent threads of single load and even different phases of a single thread. To implement polymorphic contention manager, it is important to resolve conflicts resulting from different contention managers in the same application by different methods. The easiest way is to abort the enemy contention manager if it is of different type. [54] uses generic priorities for each transaction regardless of the transaction's contention manager. Upon conflict between different classes of contention manager, highest priority transaction is committed.

[114] provides a comprehensive contention manager attempting to achieve low overhead for low contention, and good throughput and fairness in case of high contention. The main components of comprehensive contention manager are lazy acquisition, extendable timestamp-based conflict detection, and efficient method for capturing conflicts and priorities.

[88] is concerned with implementation issues. [88] considers problems resulting from previous contention management policies like backing off and waiting for time intervals. These strategies make transactions suffer from many aborts that may lead to livelocks, and increased vulnerability to abort because of transactional preemption due to higher priority tasks. Imprecise information and unpredictable benefits resulting from handling long transactions make it difficult to make correct conflict resolution decisions. [88] discriminates between decisions for long and short transactions, as well as, number of threads larger or lower than number of cores. [88] suggests a number of user and kernel level support mechanisms for contention managers, attempting to reduce overhead in current contention managers' implementations. Instead of spin-locks and system calls, the paper uses shared memory segments for communication between kernel and STM library. It also proposes reducing priority of loser threads instead of aborting them. [88] increases time slices for transactions before they are preempted by higher priority threads. This way, long transactions can commit quickly before they are suspended, reducing abort numbers.

For high number of cores, back-off strategies perform poorly. This is due to hot spots created by small set of conflicts. These hotspots repeat in predictable manner. [15] introduces proactive contention manger that uses history to predict these hotspots and scheduler transactions around them without programmer's input. Proactive contention manager is useful in high contention, but has high cost for low contention. So, [15] uses a hybrid contention managers that begins with back-off strategy for low contention. After a specific threshold for contention level, hybrid contention manager switches to proactive manager.

Contention managers concentrate on preventing starvation through fair policies. They are not suitable for specific systems like real-time systems where stronger behavioural guarantees are required. [53] proposes user-defined priority transactions to make contention management suitable for these specific systems. It investigates the correlation between consistency checking (i.e., finding memory conflicts) and user-defined priority transactions. Transaction priority can be static or dynamic. Dynamic priority increases as abort numbers of transaction increases.

Contention managers are limited in: 1) they are reactive, and suitable only for imminent conflicts. They do not specify when aborted transaction should restart, making them conflict again easily. 2) Contention managers are decentralized because they consume a large part of traffic during high contention. Decentralization prevents global view of the system and limit contention management policy to heuristics. 3) As contention managers are user-level modules, it is difficult to integrate them in HTM. [105] tackles the previous problems by *adaptive transaction scheduling* (ATS). ATS uses contention intensity feedback from the application to adaptively decide number of concurrent transactions running within critical sections. ATS is called only when transaction starts in high contention. Thus, resulting traffic is low and scheduler can be centralized. ATS is integrated into HTM and STM.

[38] presents CAR-STM, a scheduling-based mechanism for STM collision avoidance and resolution. CAR-STM maintains a transaction queue per each core. Each transaction is assigned to a queue by a dispatcher. At the beginning of the transaction, dispatcher uses a conflict probability method to determine the suitable queue for the transaction. The queue with high contention for the current transaction is the most suitable one. All transactions in the same queue are executed by the same thread, thus they are serialized and cannot collide together. CAR-STM uses a serializing contention manager. If one transaction conflicts with another transaction, the former transaction is moved to the queue of the latter. This prevents further collision between them unless the second transaction is moved to a third queue. Thus, CAR-STM uses another serialization strategy in which the two transactions are moved to the third queue. This guarantees conflict between transactions for at most once.

[91] uses HTM to build single and double linked queue, and limited capacity queue. HTM is used as an alternative synchronization operation to CAS and locks. [91] provides worst case time analysis for the implemented data structures. It experimentally compares the implemented data structures with CAS and lock. [91] reverses the role of TM. Transactions are used to build the data structure, instead of accessing data structures inside transactions. [108] presents an implementation for HTM in a Java chip multiprocessor system (CMP).

The used processor is JOP, where worst case execution time analysis is supported.

[10] presents two steps to minimize and limit number of transactional aborts in real-time multiprocessor embedded systems. [10] assumes tasks are scheduled under partitioned EDF. Each task contains at most one transaction. [10] uses multi-versioned STM. In this method, read-only transactions use recent and consistent snapshot of their read sets. Thus, they do not conflict with other transactions and commit on first try. This reduction in abort number comes at the cost of increased memory storage for different versions. [10] uses real-time characteristics to bound maximum number of required versions for each object. Thus, required space is bounded. [10] serializes conflicting transaction in a chronological order. Ties are broken using least laxity and processor identification. [10] does not provide experimental evaluation of its work.

[13] studies the effect of eager versus lazy conflict detection on real-time schedulability. In eager validation, conflicts are detected as soon as they occur. One of the conflicting transactions should be aborted immediately. In lazy validation, conflict detection is delayed to commit time. [13] assumes each task is a complete transaction. [13] proves that synchronous release of tasks does not necessarily lead to worst case response time of tasks. [13] also proves that lazy validation will always result in a longer or equal response time than eager validation. Experiments show that this gap is quite high if higher priority tasks interfere with lower priority ones.

[87]proposes an adaptive scheme to meet deadlines of transactions. This adaptive scheme collects statistical information about execution length of transactions. A transaction can execute in any of three modes depending on its closeness to deadline. These modes are optimistic, visible read and irrevocable. The optimistic mode defers conflict detection to commit time. In visible read, other transactions are informed that a particular location has been read and subject to conflict. Irrevocable mode prevents transaction from aborting. As a transaction gets closer to its deadline, it moves from optimistic to visible read to irrevocable mode. Deadline transactions are supported by the underlying scheduler by disabling preemption for them. Experimental evaluation shows improvement in number of committed transactions without noticeable degradation in transactional throughput.

Previous CMs try to enhance response time of real-time tasks using different policies for conflict resolution. Checkpointing does not require aborted transaction to restart from beginning. Thus, Checkpointing can be plugged into different CMs to further improve response time. [76] introduces checkpointing as an alternative to closed nesting transactions [119]. [76] uses boosted transactions [62] instead of closed nesting [73,96,119] to implement checkpointing. Booseted transactions are based on linearizable objects with abstract states and concrete implementation. Methods under boosted transaction have well defined semantics to transit objects from one state to another. Inverse methods are used to restore objects to previous states. Upon a conflict, a transaction does not need to revert to its beginning, but rather to a point where the conflict can be avoided. Thus, checkpointing enables partial abort. [120] applies checkpointing in distributed transactional memory using Hyflow [102].

Chapter 3

Models and Assumptions

BE SURE TO MENTION THAT ALL ANALYSIS IS DONE ASSUMING EACH JOB FINISHES WITHIN ITS DEADLINE We consider a multicore system with m identical processors and n sporadic tasks $\tau_1, \tau_2, \ldots, \tau_n$. The k^{th} instance (or job) of a task τ_i is denoted τ_i^k. Each task τ_i is specified by its worst case execution time (WCET) c_i, its minimum period T_i between any two consecutive instances, and its relative deadline D_i, where $D_i = T_i$. Job τ_i^j is released at time r_i^j and must finish no later than its absolute deadline $d_i^j = r_i^j + D_i$. Under a fixed priority scheduler such as G-RMA, p_i determines τ_i's (fixed) priority and it is constant for all instances of τ_i. Under a dynamic priority scheduler such as G-EDF, τ_i^j's priority, p_i^j, is determined by its absolute deadline. A task τ_j may interfere with task τ_i for a number of times during a duration L, and this number is denoted as $G_{ij}(L)$. τ_j's workload that interferes with τ_i during L is denoted $W_{ij}(L)$.

Shared objects. A task may need to access (i.e., read, write) shared, in-memory objects while it is executing any of its atomic sections, which are synchronized using STM. The set of atomic sections of task τ_i is denoted s_i. s_i^k is the k^{th} atomic section of τ_i. Each object, θ, can be accessed by multiple tasks. The set of distinct objects accessed by τ_i is θ_i. The set of atomic sections used by τ_i to access θ is $s_i(\theta)$, and the sum of the lengths of those atomic sections is $len(s_i(\theta))$. $s_i^k(\theta)$ is the k^{th} atomic section of τ_i that accesses θ. $s_i^k(\theta_1, \theta_2, .., \theta_n)$ is the k^{th} atomic section of τ_i that accesses objects $\theta_1, \theta_2, .., \theta_n$. $s_i^k(\theta)$ executes for a duration $len(s_i^k(\theta))$.

If θ is shared by multiple tasks, then $s(\theta)$ is the set of atomic sections of all tasks accessing θ, and the set of tasks sharing θ with τ_i is denoted $\gamma_i(\theta)$. Atomic sections are non-nested. Each atomic section is assumed to access only one object; this allows a head-to-head comparison with lock-free synchronization [37]. (Allowing multiple object access per transaction is future work.) The maximum-length atomic section in τ_i that accesses θ is denoted $s_{i_{max}}(\theta)$, while the maximum one among all tasks is $s_{max}(\theta)$, and the maximum one among tasks with priorities lower than that of τ_i is $s_{max}^i(\theta)$.

STM retry cost. If two or more atomic sections conflict, the CM will commit one section and abort and retry the others, increasing the time to execute the aborted sections. The increased time that an atomic section $s_i^p(\theta)$ will take to execute due to interference with another section $s_j^k(\theta)$, is denoted $W_i^p(s_j^k(\theta))$. The total time that a task τ_i's atomic sections have to retry is denoted $RC(\tau_i)$. When this retry cost is calculated over the task period T_i or an interval L, it is denoted, respectively, as $RC(T_i)$ and $RC(L)$.

Chapter 4

The ECM and RCM Contention Managers

We consider software transactional memory (STM) for concurrency control in multicore embedded real-time software. We investigate real-time contention managers (CMs) for resolving transactional conflicts, including those based on dynamic and fixed priorities, and establish upper bounds on transactional retries and task response times. We identify the conditions under which STM (with the proposed CMs) is superior to lock-free synchronization [37] and real-time locking protocols (i.e., OMLP [18, 21] and RNLP [123]).

The rest of this Chapter is organized as follows, Section 4.1 investigates Earliest Deadline Contention Manager under G-EDF scheduling (ECM) and illustrates its behaviour. We provide computations of workload interference and retry cost analysis under ECM. Section 4.2 presents Rate Monotonic Contention Manager under G-RMA scheduling (RCM). It also includes retry cost and response time analysis under RCM. Schedulability of ECM and RCM is compared against schedulability of lock-free in Section 4.3 and real-time locking protocols in Section 4.4. We conclude the Chapter in Section 4.5.

4.1 ECM

Since only one atomic section among many that share the same object can commit at any time under STM, those atomic sections execute in sequential order. A task τ_i's atomic sections are interfered by other tasks that share the same objects with τ_i. Hereafter, we will use *ECM* to refer to a multicore system scheduled by G-EDF and resolves STM conflicts using the EDF CM. ECM was originally introduced in [47]. ECM will abort and retry an atomic section of τ_i^x, s_i^k due to a conflicting atomic section of τ_j^y, s_j^l, if the absolute deadline of τ_j^y is less than or equal to the absolute deadline of τ_i^x. ECM behaviour is shown in Algorithm 1. [47] assumes the worst case scenario for transactional retry occurs when conflicting transactions are released simultaneously. [47] also assumes all transactions have the same length. Here, we extend the analysis in [47] to a more worse conflicting scenario and consider distinct-length transactions. We also consider lower number of conflicting instances of any job τ_j^y to another job τ_i^x.

Algorithm 1: ECM

Data: $s_i^k \rightarrow$ interfered atomic section. $s_j^l \rightarrow$ interfering atomic section
Result: which atomic section aborts
if $d_i^k < d_j^l$ **then**
  s_j^l aborts;
else
  s_i^k aborts;
end

4.1.1 Illustrative Example

Behaviour of ECM can be illustrated by the following example:

- Transaction $s_i^k \in \tau_i^x$ begins execution. Currently, s_i^k does not conflict with any other transaction.
- Transaction $s_j^l \in \tau_j^y$ is released while s_i^k is still running. $\Theta_i^k \cap \Theta_j^l \neq \emptyset$. $d_j^y < d_i^x$. So, $p_j^y > p_i^x$. Hence, ECM will abort and restart s_i^k in favour of s_j^l.
- Transaction $s_h^v \in \tau_h^u$ is released while s_j^l is still running. $d_h^u < d_j^y < d_i^x$. So, $p_h^u > p_j^y > p_i^x$. s_j^l and s_i^k will abort and retry until s_h^v commits.
- s_h^v commits. s_j^l executes while s_i^k aborts and retries.
- s_j^l commits. s_i^k executes.

4.1.2 Transitive Retry

With multiple objects per transaction, ECM will face transitive retry, which we illustrate with an example.

Example 1. Consider three atomic sections s_1^x, s_2^y, and s_3^z belonging to jobs τ_1^x,τ_2^y, and τ_3^z, with priorities $p_3^z > p_2^y > p_1^x$, respectively. Assume that s_1^x and s_2^y share objects, s_2^y and s_3^z share objects. s_1^x and s_3^z do not share objects. s_3^z can cause s_2^y to retry, which in turn will cause s_1^x to retry. This means that s_1^x may retry transitively because of s_3^z, which will increase the retry cost of s_1^x.

Assume another atomic section s_4^f is introduced. Priority of s_4^f is higher than priority of s_3^z. s_4^f shares objects only with s_3^z. Thus, s_4^f can make s_3^z to retry, which in turn will make s_2^y to retry, and finally, s_1^x to retry. Thus, transitive retry will move from s_4^f to s_1^x, increasing the retry cost of s_1^x. The situation gets worse as more tasks of higher priorities are added, where each task shares objects with its immediate lower priority task. τ_3^z may have atomic sections that share objects with τ_1^x, but this will not prevent the effect of transitive retry due to s_1^x.

Definition 1. ***Transitive(indirect) Retry:*** *A transaction s_i^k suffers from transitive retry when it conflicts with a higher priority transaction s_j^l, which in turn conflicts with a higher priority transaction s_z^h, but s_i^k does not conflict with s_z^h. Still, when s_j^l retries due to s_z^h, s_i^k also retries due to s_j^l. Thus, the effect of the higher priority transaction s_z^h is transitively moved to the lower priority transaction s_i^k, even when they do not conflict on common objects.*

Claim 1. *ECM suffers from transitive retry for multi-object transactions.*

Proof. ECM depends on priorities to resolve conflicts between transactions. Thus, lower priority transaction must always be aborted for a conflicting higher priority transaction. Claim follows. □

Because of transitive retry, Θ_i for any τ_i is extended to include any object $\theta \notin \Theta_i$, but θ can make at least one transaction $s_i^k \in \tau_i$ retry transitively. The new set of objects that can cause direct or indirect retry of at least one transaction in τ_i is denoted as Θ_i^{ex}. Θ_i^{ex} is obtained by being initialized to Θ_i (i.e., the set of objects that are already accessed by any transaction $s_i^k \in \tau_i$). We then cycle through all transactions belonging to all other higher priority tasks. Each transaction s_j^l that accesses at least one of the objects in Θ_i^{ex} adds all other objects accessed by s_j^l to Θ_i^{ex}. The loop over all higher priority tasks is repeated, each time with the new Θ_i^{ex}, until there are no more transactions accessing any object in Θ_i^{ex}. However, this solution may over-extend the set of conflicting objects, and may even contain all objects accessed by all tasks. Θ_i^* represent the set of objects not accessed directly by any transaction in τ_i, but any $\theta \in \Theta_i^*$ can make at least one transaction in τ_i retry transitively. Thus, $\Theta_i^{ex} = \Theta_i + \Theta_i^*$. Similarly, the distinct set of objects that can make s_i^k retry directly or indirectly(transitively) is denoted as $\Theta_i^{k^{ex}}$. γ_i is the extended to γ_i^{ex}. While γ_i is the set of tasks- other than τ_i- that access at least one object $\theta \in \Theta_i$, γ_i^{ex} is the set of tasks- other than τ_i- that access at least one object $\theta \in \Theta_i^{ex}$.

4.1.3 G-EDF Interference

Claim 2. *Regardless of the used scheduler, maximum number of jobs of τ_j that can exist in time interval L is upper bounded by*

$$\left\lceil \frac{L}{T_j} \right\rceil + 1 \tag{4.1}$$

where at most two jobs τ_j can be partially included in L. The remaining jobs of τ_j are totally included in L.

Proof. Generally, $L = aT_j + b$, $0 \leq b < T_j$. a is the maximum number of jobs of τ_j that contribute by their minimum periods T_j during L. If $b \geq T_j$, then there are more than a jobs of τ_j contributing by their minimum periods T_j during L, which contradicts definition of a. The remaining interval $b(= L - aT_j,\ b > 0)$ can be divided between at most two jobs of τ_j. If b can be divided between more than two jobs of τ_j, then there is more than a jobs of τ_j that contribute by their minimum periods T_j during L. This contradicts definition of a. So, if $b > 0$, then maximum number of jobs of τ_j that can exist during L is $a + 2 = \left\lceil \frac{L}{T_j} \right\rceil + 1$.

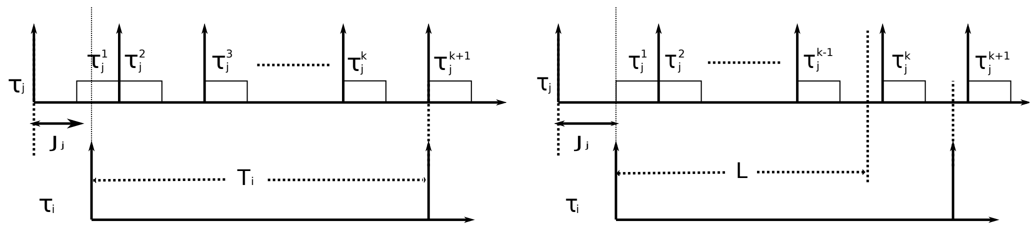
If $b = 0$, then jobs of τ_j can be shifted to the left or the right during L. This results in $a+1$ jobs of τ_j during L. So, if $b = 0$, then maximum number of jobs of τ_j that can exist during L is $a + 1 = \left\lceil \frac{L}{T_j} \right\rceil + 1$. Claim follows. □

Claim 3. *Let $T_i = aT_j + b$, where $a = \left\lfloor \frac{T_i}{T_j} \right\rfloor$ and $0 \leq b < T_j$. Under G-EDF scheduler, maximum number of jobs of τ_j that can interfere with one job τ_i^x during time interval $L(= T_i - f,\ 0 \leq f < T_i)$ is*

$$g_{ij}^{gedf}(L) = \begin{cases} \left\lceil \frac{T_i}{T_j} \right\rceil & , f \leq b \\ \\ \left\lceil \frac{L}{T_j} \right\rceil + 1 & , Otherwise \end{cases} \tag{4.2}$$

Proof. $L = T_i - f = aT_j + b - f$. If $b - f \geq 0$, then following proof of Claim 2, $b - f$ can be devided between at most two jobs of τ_j during L. These two jobs of τ_j are: 1) *carried-in job* (i.e., τ_j^s where $r_j^s < r_i^x$ and $d_j^s < d_i^x$ [14]). 2) *carried-out job* (τ_j^e where $r_j^e > r_i^x$ and $d_j^s > d_i^x$ [14]). Under G-EDF, only jobs of τ_j with absolute deadline less than d_i^x can interfere with τ_i^x. Thus, carried-out job of τ_j cannot interfere with τ_i^x. So, $b - f$ can be the contribution of only the carried-in job. Following proof of Claim 2, maximum number of jobs of τ_j that can interfere with τ_i^x is $a + 1 = \left\lceil \frac{T_i}{T_j} \right\rceil$ if $f \leq b$. Otherwise, Claim 2 is used to determine maximum number of jobs of τ_j during L. Claim follows. □

The maximum number of times a task τ_j interferes with τ_i under G-EDF is illustrated in Figure 4.1. Upper bound on maximum interference of τ_j to τ_i (when there are no atomic sections) in $L \leq T_i$ is given in [14]. It should be noted that we consider only implicit deadline systems (i.e., $\forall \tau_i$, $T_i = D_i$). Implicit deadline system is a special case of constrained deadline system (i.e., $\forall \tau_i$, $D_i \leq T_i$) considered by [14]. The interference of τ_j to τ_i during $L = T_i - f$ where $f \leq b$ (as shown in Fig 4.1(a)), in the absence of atomic sections, is upper bounded



(a) during $L = T_i - f,\ f \leq b$ (b) during $L = T_i - f,\ f > b$

Figure 4.1: Maximum interference of jobs of τ_j to τ_i^x running on different processors, under G-EDF. $T_i = aT_j + b$

by:

$$\begin{aligned} I_{ij}^1(T_i) &\leq \left\lfloor \frac{T_i}{T_j} \right\rfloor c_j + min\left(c_j, T_i - \left\lfloor \frac{T_i}{T_j} \right\rfloor T_j\right) \\ &\leq \left\lceil \frac{T_i}{T_j} \right\rceil c_j \end{aligned} \tag{4.3}$$

The interference of τ_j to τ_i during an interval $L = T_i - f$ where $f > b$, as shown in Fig 4.1(b), in the absence of atomic sections is upper bounded by:

$$I_{ij}^2(L) \leq \left(\left\lceil \frac{L - c_j}{T_j} \right\rceil + 1\right) c_j \tag{4.4}$$

Here, τ_j^1 contributes by all its c_j, and d_j^{k-1} does not have to coincide with L, as τ_j^{k-1} has a higher priority than that of τ_i. Thus, the overall interference of τ_j to τ_i, over an interval $L \leq T_i$ is:

$$I_{ij}(L) = min\left(I_{ij}^1(T_i), I_{ij}^2(L)\right) \tag{4.5}$$

[14] upper bounds maximum response time of any job of τ_i. Upper bound on maximum response time of any job of τ_i is calculated by iteration of (4.6), starting from $R_i^{up} = c_i$.

$$R_i^{up} = c_i + \left\lfloor \frac{1}{m} \sum_{j \neq i} I_{ij}\left(R_i^{up}\right) \right\rfloor \tag{4.6}$$

where $I_{ij}\left(R_i^{up}\right)$ is calculate by (4.5).

4.1.4 Retry Cost of Atomic Sections

Claim 4. *Let s_i^k and s_j^l be two conflicting transactions. s_i^k has a lower priority than s_j^l. Let the lower priority transaction always aborts and retries due to the higher priority transaction. s_j^l interfere only once with s_i^k. s_i^k aborts and retries due to s_j^l for at most*

$$len\left(s_i^k + s_j^l\right) \tag{4.7}$$

Proof. s_j^l must start at least when s_i^k starts and not later than s_i^k finishes. Otherwise, there will be no conflict between s_i^k and s_j^l. s_i^k must retry during execution of s_j^l because of higher priority of s_j^l. The part of s_i^k that started before beginning of s_j^l will be repeated. Thus, the worst case interference between s_i^k and s_j^l occurs when s_j^l starts just when s_i^k reaches its end of execution. So, maximum retry cost of s_i^k due to s_j^l is calculated by 4.7. Claim follows. □

Claim 5. *Let conflict between transactions be resolved by priority (i.e., lower priority transaction aborts and retries due to higher priority transactions). Let* $conf\left\{s_i^k\right\}$ *be the set of all transactions that do not belong to any job of* τ_i *and are conflicting, directly or indirectly(transitively), with* s_i^k*. Each transaction* $s_j^l \in conf\left\{s_i^k\right\}$ *contributes to the retry cost of* s_i^k *by at most*

$$len\left(s_j^l + max\_s_{ik}^{jl}(\Theta)\right) \tag{4.8}$$

where $max\_s_{ik}^{jl}(\Theta)$ *is the maximum length atomic section (transaction) in* $conf\{s_i^k\}$ *that accesses* Θ *and its priority is lower than* $p(s_j^l)$ *and higher than or equal to* $p(s_i^k)$*.* $max\_s_{ik}^{jl} \notin s_j$ *and* $\Theta \subseteq \Theta_i^{k^{ex}} \cap \Theta_j^l$*.*

Proof. As conflict is resolved by transactional priority, then only transactions with higher priorities than $p(s_i^k)$ will cause s_i^k to abort and retry. Also, s_j^l will abort only transactions with lower priority than $p(s_j^l)$. As transactions that belong to the same job execute sequentially, and jobs of the same task execute sequentially , so s_i^k is not aborted by other transactions that belong to τ_i. So, at any point of time after s_i^k was first released, and before the last successful run of s_i^k (i.e., the run at which s_i^k commits), one of the following cases happens:

1. s_j^l has finished before s_i^k starts. Or, s_j^l starts after s_i^k finishes. In this case, s_j^l will not cause s_i^k to abort and retry. (4.8) still upper bounds effect of s_j^l to the retry cost of s_i^k.

2. s_j^l is the only transaction that is currently aborting s_i^k. So, (4.8) follows directly from Claim 4 as $len\left(s_i^k\right) \leq len\left(max\_s_{ik}^{jl}(\Theta)\right)$.

3. A set of transactions $S \subseteq conf\{s_i^k\}$ are currently aborting s_i^k. $s_j^l \in S$ and s_j^l itself is not aborting and retrying due to any other transaction with higher priority than $p(s_j^l)$. So, s_j^l executes only once. s_j^l aborts one of the transactions with lower priority than $p(s_j^l)$ for only once. Thus, (4.8) upper bounds effect of s_j^l to the retry cost of s_i^k.

4. A set of transactions $S \subseteq conf\{s_i^k\}$ are currently aborting s_i^k. $s_j^l \in S$ and s_j^l itself is aborting and retrying due to other transactions with higher priority than $p(s_j^l)$. Without losing generality, let s_h^u be the transaction that is currently aborting s_j^l, and s_h^u is not aborting and retrying due to any other higher priority transaction. Then, s_j^l and s_i^k are both waiting for s_h^u to finish. Thus, the time of retrial of s_j^l due to s_h^u is covered by effect of s_h^u to the retry cost of s_i^k. When s_h^u finishes and s_j^l is not aborted by any other higher priority transaction, effect of s_j^l to the retry cost of s_i^k is the same as in the third case. By expanding this case to more than three transactions, then each

transaction s_j^l is either aborting one of the lower priority transactions only once (i.e., the last successful run of s_j^l), or s_i^k and s_j^l are aborted by a higher priority transaction s_h^u. When s_j^l is retrying due to the higher priority transaction s_h^u, s_j^l retrial time is not considered in retry cost of s_i^k because it is already covered by the effect of the higher priority transaction s_h^u to the retry cost of s_i^k.

Claim follows. □

Claim 6. *Under ECM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during interval $L \leq T_i$ due to direct and indirect conflict with other transactions in jobs with higher priority than τ_i^x is upper bounded by:*

$$RC_i(L) \leq \sum_{\tau_j \in \gamma_i^{ex}} \left(g_{ij}^{gedf} \sum_{\forall s_j^l,\, \left(\Theta = \Theta_j^l \cap \Theta_i^{ex}\right) \neq \emptyset} len\left(s_j^l + s_{max}(\Theta)\right) \right) \tag{4.9}$$

where $s_{max} \notin s_j$ and g_{ij}^{gedf} is calculated by (4.2).

Proof. ECM is used with G-EDF scheduler. Thus, $p(s_i^k)$ is a dynamic priority that depends on the absolute deadline of containing job τ_i^x. So, $conf\left\{s_i^k\right\}$ for any s_i^k includes each transaction $s_j^l \notin s_i$ where $\Theta_j^l \cap \Theta_i^{k^{ex}} \neq \emptyset$. The worst case retry cost of any s_i^k occurs when $p(s_i^k)$ is the lowest priority among all other conflicting transactions during T_i. g_{ij}^{gedf} is the maximum number of jobs of $\tau_j \in \gamma_i^{ex}$ that can interfere with one job of τ_j. Following Claims 3, 4 and 5, Claim follows. □

Claim 7. *Under ECM, upper bound on total retry cost given by (4.9) can be tightened by considering carried_in job of each τ_j (i.e., τ_j^{in} where $r_j^{in} < r_i^x$ and $d_j^{in} < d_i^x$ as defined in [14]) conflicting with τ_i^x during interval $L = T_i - f$, where $T_i = aT_j + b$, $a = \left\lfloor \frac{T_i}{T_j} \right\rfloor$ and $f \leq b$. (4.9) will be modified to*

$$RC_i(L) \leq \begin{cases} \sum_{\tau_j \in \gamma_i^{ex}} \left(\lambda_1(j) + \chi(i,j)\right) & ,f \leq b \\ \sum_{\tau_j \in \gamma_i^{ex}} \left(\left(\left\lceil \frac{L}{T_j} \right\rceil + 1\right) \sum_{\forall s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l + s_{max}(\Theta)\right)\right) & ,Otherwise \end{cases} \tag{4.10}$$

where

- $s_{max} \notin s_j$.
- $\lambda_1(j) = \sum_{\forall s_j^l \in \left[d_j^{in} - \delta, d_j^{in}\right], \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right)} len\left(s_j^{l^*} + s_{max}(\Theta)\right)$*, where* $\delta = min(c_j, b)$ *and* $s_j^{l^*}$ *is the part of* s_j^l *that is contained in interval* $\left[d_j^{in} - \delta, d_j^{in}\right]$.
- $\chi(i,j) = \left\lfloor \frac{T_i}{T_j} \right\rfloor \sum_{\forall s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l + s_{max}(\Theta)\right)$.

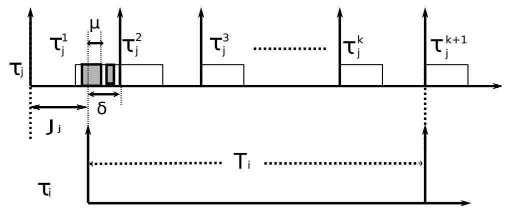


Figure 4.2: Effect of carried_in job of τ_j to retry cost of transactions in τ_i

Proof. Following proof of Claim 3, maximum number of jobs of τ_j that can interfere with τ_i^x is $\left\lceil \frac{T_i}{T_j} \right\rceil$. By definition of carried-in jobs [14] and G-EDF scheduler, there will be $\left\lfloor \frac{T_i}{T_j} \right\rfloor$ jobs of τ_j that exist by their whole periods T_j in the interval L. Carried-in job of τ_j (i.e., τ_j^{in}) will exist by at most $\delta = min\left(c_j, b\right)$ during L. τ_j^{in} is delayed by its maximum jitter to give its maximum contribution during L. Thus, τ_j^{in} starts execution at $d_j^{in} - c_j$. Consequently, only transactions of τ_j^{in} that are contained in $\left[d_j^{in} - \delta, d_j^{in}\right]$ can exist in the interval L. Also, if transaction s_j^l is partially contained in $\left[d_j^{in} - \delta, d_j^{in}\right]$, only the part of s_j^l contained in $\left[d_j^{in} - \delta, d_j^{in}\right]$ (i.e., $s_j^{l^*}$) can conflict with transactions in τ_i^x. $\lambda(j)$ stands for the retry cost of transactions in τ_i^x due to conflict with transactions of τ_j^{in}. Whereas, $\chi(i,j)$ stands for the retry cost of transactions in τ_i^x due to conflict with transactions of other jobs of τ_j (i.e., non carried-in jobs). Combining the previous notions with Claim 6, Claim follows. □

Effect of transactions in carried_in job is shown in Figure 4.2. There are two sources of retry cost for any τ_i^x under ECM. First is due to conflict between τ_i^x's transactions and transactions of other jobs. This is denoted as RC_i. Second is due to the preemption of any transaction in τ_i^x due to the release of all higher priority jobs. This is denoted as $RC_{i_{re}}$. It is up to the implementation of the contention manager to avoid RC_{re}. Here, as we are concerned with maximum total retry cost introduced by ECM, we assume that ECM does not avoid RC_{re}. Thus, we introduce RC_{re} for ECM technique.

Claim 8. *Under ECM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ due to release of jobs with higher priority than τ_i^x is upper bounded by*

$$RC_{i_{re}}(L) \leq \sum_{\forall \tau_j \in \zeta_i} \begin{cases} \left\lceil \frac{L}{T_j} \right\rceil s_{i_{max}} & , L \leq T_i - T_j \\ \\ \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{i_{max}} & , L > T_i - T_j \end{cases} \tag{4.11}$$

where $\zeta_i = \{\tau_j : (\tau_j \neq \tau_i) \wedge (D_j < D_i)\}$.

Proof. Two conditions must be satisfied for any τ_j^l to be able to preempt τ_i^x under G-EDF: $r_i^x < r_j^l < d_i^x$, and $d_j^l \leq d_i^x$. Without the first condition, τ_j^l would have been already released before τ_i^x. Thus, τ_j^l will not preempt τ_i^x. Without the second condition, τ_j^l will be of lower priority than τ_i^x and will not preempt it. If $D_j \geq D_i$, then there will be at most one instance

τ_j^l with higher priority than τ_i^x. τ_j^l must have been released at most at r_i^x, which violates the first condition. The other instance τ_j^{l+1} would have an absolute deadline greater than d_i^x. This violates the second condition. Hence, only tasks with shorter relative deadline than D_i are considered. These jobs are grouped in ζ_i.

The total number of released instances of τ_j during any interval $L \leq T_i$ is $\left\lceil \frac{L}{T_i} \right\rceil + 1$. The "carried-in" jobs (i.e., each job released before r_i^x and has an absolute deadline before d_i^x [14]) are discarded as they violate the first condition. The "carried-out" jobs (i.e., each job released after r_i^x and has an absolute deadline after d_i^x [14]) are also discarded because they violate the second condition. Thus, the number of considered higher priority instances of τ_j during the interval $L \leq T_i - T_j$ is $\left\lceil \frac{L}{T_j} \right\rceil$. The number of considered higher priority instances of τ_j during interval $L > T_i - T_j$ is $\left\lfloor \frac{T_i}{T_j} \right\rfloor$.

The worst $RC_{i_{re}}$ for τ_i^x occurs when τ_i^x is always interfered at the end of execution of its longest atomic section, $s_{i_{max}}$. τ_i^x will have to retry for $len(s_{i_{max}})$. Claim follows. □

Claim 9. *Under ECM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ is upper bounded by:*

$$RC_{i_{to}}(L) = RC_i(L) + RC_{i_{re}}(L) \tag{4.12}$$

where $RC_i(L)$ is the maximum retry cost resulting from conflict between transactions in τ_i^x and transactions of other jobs. $RC_i(L)$ is calculated by (4.9). $RC_{i_{re}}(L)$ is the maximum retry cost resulting from the release of higher priority jobs, which preempt transactions in τ_i^x. $RC_{i_{re}}(L)$ is calculated by (4.11).

Proof. Under ECM, transactions in any job $\tau_i^x \in \tau_i$ retry due to: 1) conflicting transactions of jobs with higher priority than τ_i^x. 2) release of higher priority jobs that preempt τ_i^x. Thus, (4.12) follows directly from Claims 7 and 8. Claim follows. □

4.1.5 Upper Bound on Response Time

Claim 10. *Under ECM, maximum response time of any job $\tau_i^x \in \tau_i$ is upper bounded by*

$$R_i^{up} = c_i + RC_{i_{to}}(R_i^{up}) + \left\lfloor \frac{1}{m} \sum_{j \neq i} I_{ij}(R_i^{up}) \right\rfloor \tag{4.13}$$

where:

- *R_i^{up}'s initial value is $c_i + R_i^{up}(c_i)$.*
- *$RC_{i_{to}}(R_i^{up})$ is calculated by (4.12).*

- *c_j of any job $\tau_j^y \in \tau_j$ with $p_j^y > p_i^x$ is modified to*

$$c_{ji} = c_j - \left(\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l\right) \right) + RC_{ji_{to}}(R_i^{up}) \tag{4.14}$$

- *$RC_{ji_{to}}(R_i^{up})$ is the same as $RC_{j_{to}}(R_i^{up})$ excluding atomic sections in τ_j that access shared objects between τ_i and τ_j. τ_i does not contribute to $RC_{j_{re}}(R_i^{up})$.*
- *$I_{ij}(R_i^{up})$ is calculated by (4.5) with c_j replaced by c_{ji} and changing (4.4) to*

$$I_{ij}(R_i^{up}) = max \begin{cases} \left(\left\lceil \frac{R_i^{up} - \left(c_{ji} + \sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l) \right)}{T_j} \right\rceil + 1 \right) c_{ji} \\ \left\lceil \frac{R_i^{up} - c_j}{T_j} \right\rceil .c_{ji} + c_j - \sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l) \end{cases} \tag{4.15}$$

Proof. To obtain an upper bound on the maximum response time (i.e., R_i^{up}) of any job τ_i^x of τ_i, the term $RC_{i_{to}}(R_i^{up})$ must be added to the interference of other tasks during the non-atomic execution of τ_i^x. But this requires modification of the WCET of each task as follows.

c_j of each interfering task τ_j should be inflated to accommodate the interference of each task τ_k, $k \neq j, i$. Meanwhile, atomic regions that access shared objects between τ_j and τ_i should not be considered in the inflation cost, because they have already been calculated in τ_i's retry cost. As an upper bound on R_i^{up} is calculated, then jobs of τ_j with higher priority than τ_i^x are only considered. Thus, τ_i^x has no contribution in $RC_{j_{re}}(R_i^{up})$. Thus, τ_j's inflated WCET becomes:

$$c_{ji} = c_j - \left(\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l\right) \right) + RC_{ji_{to}}(R_i^{up})$$

which is given by (4.14). c_{ji} is the new WCET of τ_j relative to τ_i. $\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l)$ is the sum of lengths of all atomic sections in τ_j that access any object $\theta \in \Theta_i^{ex}$. $\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset}$ $len(s_j^l)$ is subtracted from c_j because $\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l)$ is already included in $RC_{i_{to}}(R_i^{up})$. $RC_{ji_{to}}(R_i^{up})$ is the $RC_{j_{to}}(R_i^{up})$ without including the shared objects between τ_i and τ_j. The calculated WCET is relative to task τ_i, as it changes from task to task. The upper bound on the response time of τ_i^x, denoted R_i^{up}, can be calculated iteratively, by modifying (4.6), as follows:

$$R_i^{up} = c_i + RC_{i_{to}}(R_i^{up}) + \left\lfloor \frac{1}{m} \sum_{j \neq i} I_{ij}(R_i^{up}) \right\rfloor$$

which is given by (4.13). R_i^{up}'s initial value is $c_i + R_i^{up}(c_i)$. $I_{ij}(R_i^{up})$ is calculated by (4.5) with c_j replaced by c_{ji}, and changing (4.4) to

$$I_{ij}(R_i^{up}) = max \begin{cases} \left(\left\lceil \frac{R_i^{up} - \left(c_{ji} + \sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l) \right)}{T_j} \right\rceil + 1 \right) c_{ji} \\ \left\lceil \frac{R_i^{up} - c_j}{T_j} \right\rceil .c_{ji} + c_j - \sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l) \end{cases}$$

as given by (4.15). Eq(4.4) is modified to (4.15) because there are two cases for the first job of τ_j (i.e., τ_j^1) contributing to the retry cost of τ_i^x:

Case 1. τ_j^1 (shown in Figure 4.1(b)) contributes by c_{ji}. Thus, other instances of τ_j will begin after this modified WCET, but the sum of the shared objects' atomic section lengths is removed from c_{ji}, causing other instances to start earlier. Thus, the term $\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l)$ is added to c_{ji} to obtain the correct start time.

Case 2. τ_j^1 contributes by its c_j, but the sum of the shared atomic section lengths between τ_i and τ_j should be subtracted from the contribution of τ_j^1, as they are already included in the retry cost.

It should be noted that subtraction of $\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len(s_j^l)$ is done in the first case to obtain the correct start time of other instances, while in the second case, this is done to get the correct contribution of τ_j^1. The maximum is chosen from the two terms in (4.15), because they differ in the contribution of their τ_j^1s, and the number of instances after that. Claim follows. $\square$

4.2 RCM

As G-RMA is a fixed priority scheduler, a task τ_i will be interfered by those tasks with priorities higher than τ_i (i.e., $p_j > p_i$). Upon a conflict, the RMA CM will commit the transaction that belongs to the higher priority task. Hereafter, we use *RCM* to refer to a multicore system scheduled by G-RMA and resolves STM conflicts by the RMA CM. RCM is shown in Alogrithm 2.

The same illustrative example in Section 4.1.1 is applied for RCM except that tasks' priorities are fixed.

Claim 11. *RCM suffers from transitive retry for multi-object transactions.*

Proof. The proof is the same as proof of Claim 1. $\square$

Algorithm 2: RCM

Data: $s_i^k \rightarrow$ interfered atomic section. $s_j^l \rightarrow$ interfering atomic section
Result: which atomic section aborts

```
if T_i < T_j then
    s_j^l aborts;
else
    s_i^k aborts;
end
```

4.2.1 Maximum Task Interference

Figure 4.3 illustrates the maximum interference caused by a task τ_j to a task τ_i under G-RMA. As τ_j is of higher priority than τ_i, τ_j^k will interfere with τ_i even if it is not totally included in T_i. Unlike the G-EDF case shown in Figure 4.2, where only the δ part of τ_j^1 is considered, in G-RMA, τ_j^k can contribute by the whole c_j, and all atomic sections contained in τ_j^k must be considered. This is because, in G-EDF, the worst-case pattern releases τ_i^a before d_j^1 by δ time units, and τ_i^a cannot be interfered before it is released. But in G-RMA, τ_i^a is already released, and can be interfered by the whole τ_j^k, even if this makes it infeasible.

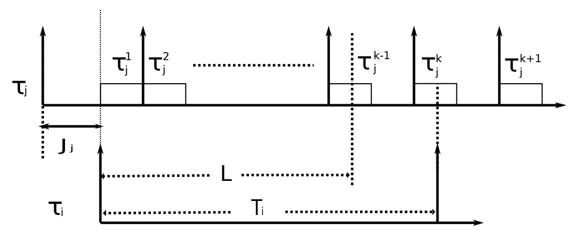


Figure 4.3: Max interference of τ_j to τ_i in G-RMA

Thus, the maximum contribution of τ_j^b to τ_i^a for any duration L is upper bounded by Claim 2, where L can extend to T_i. Note the contrast with ECM, where L cannot be extended directly to T_i, as this will have a different pattern of worst case interference from other tasks.

4.2.2 Retry Cost of Atomic Sections

Claim 12. *Under RCM, total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during interval $L \leq T_i$ due to direct and indirect conflict with other transactions in jobs with higher priorities than τ_i^x is upper bounded by:*

$$RC_i\left(L\right) \leq \sum_{\tau_j \in \gamma_i^{ex}, p_j > p_i} \left(\sum_{s_j^l, \left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} \left(\left(\left\lceil \frac{L}{T_j} \right\rceil + 1 \right) len\left(s_j^l + s_{max}\left(\Theta\right)\right) \right) \right) \qquad (4.16)$$

where $s_{max}(\Theta)$ belongs to a job with lower priority than p_j.

Proof. Under G-RMA, priorities of tasks are fixed. Thus, as $p_j > p_i$, then any job of τ_j will have a higher priority than τ_i^x. So, Claim 2 gives maximum number of jobs of τ_j that interfere with τ_i^x during interval L. By definition of RCM, only transactions with lower priority than p_j can be aborted and retried due to transactions in s_j. Thus, $s_{max}(\Theta)$ cannot belong to transactions with priorities at least equal to p_j. Following proof of Claim 6, Claim follows. □

Claim 13. *Under RCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ due to release of jobs with higher priority than τ_i^x is upper bounded by*

$$RC_{i_{re}}(L) = \sum_{\forall \tau_j,\, p_j > p_i} \left(\left\lceil \frac{L}{T_j} \right\rceil s_{i_{max}} \right) \tag{4.17}$$

Proof. The proof is the same as that for Claim 8, except that G-RMA uses static priority. Thus, the carried-out jobs will be considered in the interference with τ_i^x. The carried-in jobs are still not considered because they are released before r_i^x. Claim follows. □

Claim 14. *Under RCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ is upper bounded by:*

$$RC_{i_{to}}(L) = RC_i(L) + RC_{i_{re}}(L) \tag{4.18}$$

where $RC_i(L)$ is the maximum retry cost resulting from conflict between transactions in τ_i^x and transactions of other jobs. $RC_i(L)$ is calculated by (4.16). $RC_{i_{re}}(L)$ is the maximum retry cost resulting from the release of higher priority jobs, which preempt transactions in τ_i^x. $RC_{i_{re}}(L)$ is calculated by (4.17).

Proof. Using Claims 12 and 13, and following proof of Claim 9, Claim follows. □

4.2.3 Upper Bound on Response Time

Claim 15. *Under RCM, maximum response time of any job $\tau_i^x \in \tau_i$ is upper bounded by*

$$R_i^{up} = c_i + RC_{i_{to}}(R_i^{up}) + \left\lfloor \frac{1}{m} \sum_{j \neq i, p_j > p_i} I_{ij}(R_i^{up}) \right\rfloor \tag{4.19}$$

where:

- *R_i^{up}'s initial value is $c_i + R_i^{up}(c_i)$.*
- *$RC_{i_{to}}(R_i^{up})$ is calculated by (4.18).*

- *c_j of any job $\tau_j^y \in \tau_j$, where $p_j > p_i$ and $\Theta_j \cap \Theta_i^{ex} \neq \emptyset$, is calculated by (4.14).*
- *$I_{ij}(R_i^{up})$ is calculated by (4.4) with c_j replaced by c_{ji}.*

Proof. Using Theorem 7 in [14], Claim 14 and following proof of Claim 10, Claim follows. □

4.3 STM versus Lock-Free

We now would like to understand when STM will be beneficial compared to lock-free synchronization. The retry-loop lock-free approach in [37] is the most relevant to our work. As lock-free instructions access only one object, then Θ_i^k for any s_i^k will be restricted to one object only (i.e., $\Theta_i^k = \theta_i^k$). Thus, transitive retry cannot happen, $\Theta_i^{ex} = \Theta_i$ and $\gamma_i^{ex} = \gamma_i$.

4.3.1 ECM versus Lock-Free

Claim 16. *For ECM's schedulability to be better or equal to that of [37]'s retry-loop lock-free approach, the size of s_{max} must not exceed one half of that of r_{max}, where r_{max} is the maximum execution cost of a single iteration of any lock-free retry loop of any task. With equal periods for conflicting tasks and high access times to each object within the same transaction, the size of s_{max} can be much larger than r_{max}.*

Proof. Using Claim 3, (4.10) can be upper bounded, during T_i, as:

$$RC_i^{max}(T_i) \leq \sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} (2.s_{max}) \right)$$

where s_{max} is the maximum length atomic section among all tasks. Similarly, (4.11) is upper bounded, during T_i, as:

$$RC_{i_{re}}^{max} \leq \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{max}$$

where $\zeta_i = \{\tau_j : (\tau_j \neq \tau_i) \wedge (D_j < D_i)\}$. Thus, $RC_{i_{to}}$ given by (4.12) can be upper bounded, during T_i, as:

$$RC_{i_{to}}^{max} \leq \left(\sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} (2.s_{max}) \right) \right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{max} \right) \tag{4.20}$$

Retry cost of τ_i during interval T_i due to conflict with other jobs under retry-loop lock-free is given in [37] as:

$$LRC \leq \sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) .\beta_{ij}.r_{max} \tag{4.21}$$

where β_{ij} is the number of retry loops of τ_j that access shared objects between τ_i and τ_j. (4.21) needs to be extended to include effect of release of any higher priority job, τ_j^l, preempting τ_i^k when τ_i^k is trying to access an object θ. Release of jobs under ECM and lock-free is independent from accessed objects. Thus, ECM and lock-free have the same pattern of jobs' release. Thus, retry cost of τ_i during T_i due to release of higher priority jobs under retry-loop lock-free is obtained directly from Claim 8 with replacing s_{max} by r_{max}. Thus, total retry cost of any job of τ_i during interval T_i due to conflict of other jobs and release of higher priority jobs is upper bounded by:

$$LRC_{to} \leq \left(\left(\sum_{\tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) .\beta_{ij} \right) + \left(\sum_{\tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor \right) \right) r_{max} \tag{4.22}$$

ECM achieves equal or better schedulability than lock-free if the total utilization under ECM is less than or equal to total utilization under lock-free system:

$$\sum_{\forall \tau_i} \frac{c_i + RC_{i_{to}}^{max}}{T_i} \leq \sum_{\forall \tau_i} \frac{c_i + LRC_{to}}{T_i} \tag{4.23}$$

Eq(4.23) holds if for every task τ_i:

$$RC_{i_{to}}^{max} \leq LRC_{to} \tag{4.24}$$

Thus,

$$\begin{aligned} & \left(\left(\sum_{\forall \tau_j \in \gamma_i} \left(2 \left\lceil \tfrac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, (\Theta = \Theta_j^l \cap \Theta_i) \neq \emptyset} \right) \right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \tfrac{T_i}{T_j} \right\rfloor \right) \right) s_{max} \\ \leq & \left(\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \tfrac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} \right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \tfrac{T_i}{T_j} \right\rfloor \right) \right) r_{max} \\ \therefore \frac{s_{max}}{r_{max}} \leq & \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \tfrac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} \right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \tfrac{T_i}{T_j} \right\rfloor \right)}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2 \left\lceil \tfrac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, (\Theta = \Theta_j^l \cap \Theta_i) \neq \emptyset} \right) \right) + \left(\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \tfrac{T_i}{T_j} \right\rfloor \right)} \end{aligned} \tag{4.25}$$

Let $\sum_{\forall s_j^l, (\Theta = \Theta_j^l \cap \Theta_i) \neq \emptyset} = \beta_{ij}^*$ and $\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor = c1$. Then, (4.25) becomes

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \tfrac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} \right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2 \left\lceil \tfrac{T_i}{T_j} \right\rceil \beta_{ij}^* \right) \right) + c1} \tag{4.26}$$

We want to get the lower bound over s_{max}/r_{max} that preserves equal or better schedulability for ECM than lock-free:

Each lock-free instruction accesses only one object once. Each transaction accesses only one object to enable comparison with lock-free. An object θ can be accessed multiple times within the same transaction. Thus, $\beta_{ij} \leq \beta^*_{ij}$.

$$\because \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil\right) \beta^*_{ij}\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2 \left\lceil \frac{T_i}{T_j} \right\rceil \beta^*_{ij}\right)\right) + 2c1} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \beta_{ij}\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2 \left\lceil \frac{T_i}{T_j} \right\rceil \beta^*_{ij}\right)\right) + c1}$$

Thus, (4.26) holds if

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} \left\lceil \frac{T_i}{T_j} \right\rceil \beta^*_{ij}\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(2 \left\lceil \frac{T_i}{T_j} \right\rceil \beta^*_{ij}\right)\right) + 2c1} = \frac{1}{2}$$

Thus, the lower bound over s_{max}/r_{max} that preserves equal or better schedulability for ECM than lock-free is 0.5. Now, we want to get the upper bound over s_{max}/r_{max} that preserves equal or better schedulability for ECM than lock-free:

Minimum value for $\left\lceil \frac{T_i}{T_j} \right\rceil$ is 1. So, $2\left\lceil \frac{T_i}{T_j} \right\rceil \geq \left\lceil \frac{T_i}{T_j} \right\rceil + 1, \forall i, j$. Thus, to get upper bound on s_{max}/r_{max}, $\left\lceil \frac{T_i}{T_j} \right\rceil$ assumes its minimum value (i.e., 1). Otherwise, the denominator of (4.26) gets larger than numerator, and s_{max}/r_{max} moves away from its upper bound. $\left\lceil \frac{T_i}{T_j} \right\rceil \rightarrow 1$ for any i, j if all conflicting tasks have equal periods. Thus, by substitution of $\left\lceil \frac{T_i}{T_j} \right\rceil = 1$ into (4.26), we get

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i} 2\beta_{ij}\right) + c1}{\left(\sum_{\forall \tau_j \in \gamma_i} 2\beta^*_{ij}\right) + c1} \tag{4.27}$$

As we are looking for the upper bound over s_{max}/r_{max}, then $\beta_{ij} >> \beta^*_{ij}$. Thus, s_{max} can be much larger than r_{max} while still maintaining equal or better schedulability for ECM than lock-free. From the previous notions, Claim follows. □

4.3.2 RCM versus Lock-Free

Claim 17. *For RCM's schedulability to be better or equal to that of [37]'s retry-loop lock-free approach, the size of s_{max} must not exceed one half of that of r_{max}, where r_{max} is the maximum execution cost of a single iteration of any lock-free retry loop of any task. With equal periods for conflicting tasks and high access times to each object within the same transaction, the size of s_{max} can be much larger than r_{max}.*

Proof. Following the same steps in proof of Claim 16 with the following modifications: Equation (4.16) is upper bounded by:

$$\sum_{\tau_j\in\gamma_i,\,p_j>p_i}\left(\sum_{s_j^l,\,\left(\Theta=\Theta_j^l\cap\Theta_i\right)\neq\emptyset}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)2s_{max}\right)\right) \tag{4.28}$$

Equation (4.17) is upper bounded by:

$$RC_{i_{re}}(T_i)=\sum_{\forall\tau_j,\,p_j>p_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil s_{max}\right) \tag{4.29}$$

Thus,

$$RC_{i_{to}}^{max}\leq\sum_{\tau_j\in\gamma_i,\,p_j>p_i}\left(\sum_{s_j^l,\,\left(\Theta=\Theta_j^l\cap\Theta_i\right)\neq\emptyset}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)2s_{max}\right)\right)+\sum_{\forall\tau_j,\,p_j>p_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil s_{max}\right) \tag{4.30}$$

As lock-free is independent from the underlying scheduler, then LRC is still calculated by (4.21). Release of jobs under RCM and lock-free is independent from accessed objects. Thus, RCM and lock-free have the same pattern for object release. Thus, retry cost of transactions in τ_i during T_i due to release of higher priority jobs under retry-loop lock-free is obtained directly from Claim 13 with replacing s_{max} by r_{max}. Thus,

$$LRC_{to}\leq\left(\left(\sum_{\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right).\beta_{ij}\right)+\left(\sum_{\tau_j,\,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)\right)r_{max} \tag{4.31}$$

Similar to proof of Claim 16, RCM has equal or better schedulability than lock-free if for each τ_i

$$\frac{s_{max}}{r_{max}}\leq\frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)}{\left(\sum_{\forall\tau_j\in\gamma_i,\,p_j>p_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i\right)\neq\emptyset}\right)\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)} \tag{4.32}$$

$$\because\sum_{\forall\tau_j\in\gamma_i,\,p_j>p_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i\right)\neq\emptyset}\right)\leq\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i\right)\neq\emptyset}\right)$$

$\therefore$ Eq(4.32) holds if

$$\frac{s_{max}}{r_{max}}\leq\frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)}{\left(\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i\right)\neq\emptyset}\right)\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)} \tag{4.33}$$

Let $\sum_{\forall s_j^l,(\Theta=\Theta_j^l\cap\Theta_i)\neq\emptyset}=\beta_{ij}^*$ and $\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil=c1$. Then (4.32) becomes

$$\frac{s_{max}}{r_{max}}\leq\frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+c1}{\left(\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}^*\right)+c1\right)}\tag{4.34}$$

We want to get lower bound over s_{max}/r_{max} that preserves equal or better schedulability for RCM than lock-free:

Similar to proof of Claim 16, β_{ij} assumes its minimum value β_{ij}^*.

$$\because\frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+c1}{\left(\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}^*\right)+2c1\right)}\leq\frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+c1}{\left(\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}^*\right)+c1\right)}\tag{4.35}$$

Then (4.34) holds if

$$\frac{s_{max}}{r_{max}}\leq\frac{\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+c1}{\left(\sum_{\forall\tau_j\in\gamma_i}\left(2\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}^*\right)+2c1\right)}=\frac{1}{2}\tag{4.36}$$

We want to get upper bound over s_{max}/r_{max} that preserves equal or better schedulability for RCM than lock-free:

Similar to proof of Claim 16, $\left\lceil\frac{T_i}{T_j}\right\rceil$ assumes its minimum value (i.e., 1), $\beta_{ij}>>\beta_{ij}^*$. Thus, s_{max} can be much larger than r_{max}. From the previous notions, Claim follows. □

4.4 STM versus Locking protocols

Schedulabilty of different CMs is compared against real-time locking protocols (i.e., OMLP [18,21] and RNLP [123]) using total utilization under G-EDF and G-RMA. In [18,21,123,124], priority inversion bound (*pi-blocking*) is considered part of each task's execution time. Thus, each task's WCET is inflated by *pi-blocking* bounds. Similarly, under different CMs, each task's WCET is inflated by its total retry cost (i.e., retry cost due to direct and indirect conflict with other tasks. Besides retry cost due to release of higher priority jobs). So, schedulability of a specific STM CM algorithm A is compared against a real-time locking protocol B as follows:

$$\sum_{\forall\tau_i}\frac{c_i+RC_A(T_i)}{T_i}\quad\leq\quad\sum_{\forall\tau_i}\frac{c_i+PI_B(T_i)}{T_i}\tag{4.37}$$

Eq(4.37) holds if

$$\forall\tau_i,\ RC_A(T_i)\leq PI_B(T_i)\tag{4.38}$$

If τ_i has no critical sections, then $RC_A(T_i)=PI_B(T_i)=0$. Thus, independent tasks have the same effect in (4.37) and they will not be considered in (4.38).

4.4.1 Priority Inversion under Global OMLP

Under Global OMLP [18, 21], $PI_{OMLP}(T_i)$ for any job τ_i^x is upper bounded by

$$PI_{OMLP}(T_i) \leq \sum_{k=1}^{n_r} N_{i,k}(2m-1)L_{max} \tag{4.39}$$

Where n_r is total number of resources. $N_{i,k}$ is maximum number of times resource k is accessed by τ_i. L_{max} is the maximum length critical section in all tasks. Let $N_i = \sum_{k=1}^{n_r} N_{i,k}$, which is the total number of critical sections in any job τ_i^x. Thus, (4.39) becomes

$$PI_{OMLP}(T_i) \leq N_i(2m-1)L_{max} \tag{4.40}$$

Let $N_{max} = max\left\{N_i\right\}_{\forall i}$, $N_{min} = min\left\{N_i\right\}_{\forall i}$, $\Phi_{max} = max\left\lceil\frac{T_i}{T_j}\right\rceil_{\forall i,j}$. As independent tasks are not considered in (4.38), $\therefore$ N_{max}, $N_{min} \geq 1$.

OMLP uses group locking to access multiple (i.e., nested) objects in a critical section. Thus, all objects within the same atomic section are protected by the same lock (i.e., resource). Sections 4.4.5 and 4.4.6 investigates comparison between different CMs and fine-grained locking protocols (i.e., RNLP) to access multiple objects within a critical section without group locking.

4.4.2 ECM versus Global OMLP

Claim 18. *Schedulability of ECM is equal or better than schedulability of Global OMLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(2N_{max}+1\right)\left(n-1\right)\Phi_{max}} \tag{4.41}$$

Proof. Substitute $RC_A(T_i)$ in (4.38) by (4.20) with γ_i replaced with γ_i^{ex} and Θ_i replaced with Θ_i^{ex}. Substitute $PI_B(T_i)$ in (4.38) by (4.40). $\therefore$ (4.38) holds if $\forall\tau_i$

$$\begin{aligned}&\left(\sum_{\tau_j\in\gamma_i^{ex}}\left(\left\lceil\frac{T_i}{T_j}\right\rceil\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i^{ex}\right)\neq\emptyset}\left(2.s_{max}\right)\right)\right)+\left(\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor s_{max}\right)\\ \leq\ & N_i\left(2m-1\right)L_{max}\end{aligned} \tag{4.42}$$

Let $N_{i,j} = \sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i\right)\neq\emptyset}$. So, $N_{i,j}$ is number of transactions in any job of τ_j conflicting with any transaction in any job of τ_i. Thus, (4.42) becomes

$$\begin{aligned}&\left(2\left(\sum_{\forall\tau_j\in\gamma_i^{ex}}\left(\left\lceil\frac{T_i}{T_j}\right\rceil N_{i,j}\right)\right)+\left(\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor\right)\right)s_{max}\\ \leq\ & N_i\left(2m-1\right)L_{max}\end{aligned} \tag{4.43}$$

$$\therefore \frac{s_{max}}{L_{max}} \leq \frac{N_i\left(2m-1\right)}{2\left(\sum_{\forall \tau_j \in \gamma_i^{ex}}\left(\left\lceil \frac{T_i}{T_j}\right\rceil N_{i,j}\right)\right)+\left(\sum_{\forall \tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor\right)} \tag{4.44}$$

Let $N_{max} = max\left\{N_i\right\}_{\forall i}$, $N_{min} = min\left\{N_i\right\}_{\forall i}$, $\Phi_{max} = max\left\lceil\frac{T_i}{T_j}\right\rceil_{\forall i,j}$. By definition of γ_i^{ex} and ζ_i, $n-1 \geq |\zeta_i|, |\gamma_i^{ex}|$. $\because$ $N_{max} \geq N_{i,j}$, $N_{min} \leq N_i$ and $\Phi_{max} \geq \left\lceil\frac{T_i}{T_j}\right\rceil \geq \left\lfloor\frac{T_i}{T_j}\right\rfloor$. $\therefore$ Eq(4.44) holds if

$$\begin{aligned}\frac{s_{max}}{L_{max}} &\leq \frac{N_{min}\left(2m-1\right)}{2\left(\sum_{\forall\tau_j\in\gamma_i^{ex}}\left(\Phi_{max}N_{max}\right)\right)+\left(\sum_{\forall\tau_j\in\zeta_i}\Phi_{max}\right)}\\ &\leq \frac{N_{min}\left(2m-1\right)}{\left(2N_{max}+1\right)\left(n-1\right)\Phi_{max}}\end{aligned} \tag{4.45}$$

Claim follows. □

4.4.3 RCM versus Global OMLP

Claim 19. *Under globally scheduled systems, schedulability of RCM is equal or better than schedulability of Global OMLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(2\left(\Phi_{max}+1\right)N_{max}+\Phi_{max}\right)\left(n-1\right)} \tag{4.46}$$

Proof. Substitute $RC_A(T_i)$ in (4.38) by (4.30) with γ_i replaced with γ_i^{ex} and Θ_i replaced with Θ_i^{ex}. Following the same steps in proof of Claim 18, Claim follows. □

4.4.4 Priority Inversion under RNLP

Under RNLP [123] for global scheduling and *I-KGLP* token lock (introduced as R^2DGLP in [125]), $PI_{RNLP}(T_i)$ for any job τ_i^x is upper bounded by $(2m-1)\,L_{max}$ for each outermost request, where L_{max} is the maximum length of any outermost request. Thus, if N_i is total number of outermost critical sections in any job of τ_i, then

$$PI_{RNLP}(T_i) = N_i(2m-1)L_{max} \tag{4.47}$$

Let $N_{max} = max\left\{N_i\right\}_{\forall i}$, $N_{min} = min\left\{N_i\right\}_{\forall i}$, $\Phi_{max} = max\left\lceil\frac{T_i}{T_j}\right\rceil$. As independent tasks are not considered in (4.38), $\therefore$ $N_{max}, N_{min} \geq 1$.

In contrast to OMLP, RNLP supports nesting of objects. Thus, each object can be accessed individually without being grouped with other objects in the same critical section.

4.4.5 ECM versus RNLP

Claim 20. *Schedulability of ECM is equal or better than schedulability of RNLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(2N_{max}+1\right)\left(n-1\right)\Phi_{max}} \tag{4.48}$$

Proof. Substitute $RC_A(T_i)$ in (4.38) by (4.20) with γ_i replaced with γ_i^{ex} and Θ_i replaced with Θ_i^{ex}. Substitute $PI_B(T_i)$ in (4.38) by (4.47). $\therefore$ (4.38) becomes

$$\begin{aligned} &\left(\sum_{\tau_j\in\gamma_i^{ex}}\left(\left\lceil\frac{T_i}{T_j}\right\rceil\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i^{ex}\right)\neq\emptyset}\left(2s_{max}\right)\right)\right)+\left(\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor s_{max}\right) \\ \leq\; & N_i\left(2m-1\right)L_{max} \end{aligned} \tag{4.49}$$

Following the same steps of proof of Claim 18, Claim follows. □

4.4.6 RCM vs. RNLP

Claim 21. *Under globally scheduled systems, schedulability of RCM is equal or better than schedulability of RNLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(2\left(\Phi_{max}+1\right)N_{max}+\Phi_{max}\right)\left(n-1\right)} \tag{4.50}$$

Proof. Substitute $RC_A(T_i)$ in (4.38) by (4.30). γ_i is replaced with γ_i^{ex} and Θ_i is replaced with Θ_i^{ex}. Substitute $PI_B(T_i)$ in (4.38) by (4.47). Following the same steps of proof of Claim 18, Claim follows. □

4.5 Conclusions

ECM and RCM use jobs' priorities to resolve conflicts between transactions. The transaction with lower priority aborts and retries due to the transaction with higher priority. As each transaction can access multiple objects, a transaction may abort indirectly due to another transaction with no shared objects between them. The indirect retrial is denoted as transitive retry. Under both ECM and RCM, a task incurs at most $2s_{max}$ retry cost for each of its atomic sections due to a conflict with another task's atomic section. Transactions can also retry due to release of higher priority jobs that preempt a transaction in a lower priority job.

The s_{max}/r_{max} ratio determines whether STM is better or as good as lock-free. ECM and RCM have equal or better schedulability than retry-loop lock-free if s_{max} does not exceed one half of r_{max}. s_{max} can exceed r_{max} with equal periods between conflicting tasks, and large access times to the same object within the same transaction.

Schedulability of ECM and RCM was compared against real-time locking protocols (i.e., Global OMLP and RNLP). ECM have equal or better schedulability than OMLP and RNLP if $\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(2N_{max}+1)(n-1)\Phi_{max}}$. RCM have equal or better schedulability than OMLP and RNLP if $\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(2(\Phi_{max}+1)N_{max}+\Phi_{max})(n-1)}$

Chapter 5

The LCM Contention Manager

Under ECM and RCM (Chapter 4), each atomic section can be aborted for at most $2s_{max}$ by a single interfering atomic section. We present a novel contention manager (CM) for resolving transactional conflicts, called length-based CM (or LCM) [40]. LCM can reduce the abortion time of a single atomic section due to an interfering atomic section below $2s_{max}$. We upper bound transactional retries and response times under LCM, when used with G-EDF and G-RMA schedulers. We identify the conditions under which LCM outperforms ECM, RCM, retry-loop lock-free [37] and real-time locking protocols (i.e., OMLP [18, 21] and RNLP [123]).

The rest of this Chapter is organized as follows: Section 5.1 presents Length-based Contention Manager (LCM) and illustrates its behaviour. Section 5.2 derives LCM properties. Retry cost and response time analysis of tasks under G-EDF/LCM is given in Section 5.3. Schedulability of G-EDF/LCM is compared to schedulability of ECM, lock-free and locking protocols in Section 5.4. Section 5.5 gives retry cost and response time analysis for G-RMA/LCM. Schedulability of G-RMA/LCM is compared against RCM, lock-free and locking protocols in Section 5.6. We conclude Chapter in Section 5.7.

5.1 Length-based CM

LCM resolves conflicts based on the priority of conflicting transactions, besides the length of the interfering atomic section, and the length of the interfered atomic section. Priority of each transaction equals priority of its containing job (i.e., $p\left(s_i^k\right) = p_i^x$ where $s_i^k \in \tau_i^x$). ECM and RCM (Chapter 4) use only priorities to resolve conflicts. LCM allows lower priority jobs to retry for lesser time than that under ECM and RCM, but higher priority jobs, sometimes, wait for lower priority ones with bounded priority-inversion.

5.1.1 Design and Rationale

Algorithm 3: LCM

Data: s_i^k and s_j^l are two conflicting atomic sections.
$\psi \rightarrow$ predefined threshold $\in [0, 1]$.
$\delta_i^k \rightarrow$ remaining execution length of s_i^k.
$s\left(s_i^k\right) \rightarrow$ start time of s_i^k. $s\left(s_i^k\right)$ is updated each time s_i^k aborts and retries to the start time of the new retry.
$s\left(s_j^l\right) \rightarrow$ the same as $s\left(s_i^k\right)$ but for s_j^l.
Result: which atomic section of s_i^k or s_j^l aborts

1 **if** $s\left(s_i^k\right) < s\left(s_j^l\right)$ **then**
2   **if** $p\left(s_i^k\right) > p\left(s_j^l\right)$ **then**
3   s_j^l aborts;
4   **else**
5         $c_{ij}^{kl} = len(s_j^l)/len(s_i^k)$;
6         $\alpha_{ij}^{kl} = ln(\psi)/(ln(\psi) - c_{ij}^{kl})$;
7         $\alpha = \left(len(s_i^k) - \delta_i^k\right)/len(s_i^k)$;
8   **if** $\alpha \leq \alpha_{ij}^{kl}$ **then**
9   s_i^k aborts;
10   **else**
11   s_j^l aborts;
12   **end**
13   **end**
14 **else**
15   Swap s_i^k and s_j^l;
16 **end**

For both ECM and RCM, s_i^k can be totally repeated if s_j^l — which belongs to a higher priority job τ_j^b than τ_i^a — conflicts with s_i^k at the end of its execution, while s_i^k is just about to commit. Thus, LCM, shown in Algorithm 3, uses the remaining length of s_i^k when it is interfered, as well as $len(s_j^l)$, to decide which transaction must be aborted. If s_i^k starts before s_j^l, then s_i^k is the interfered atomic section and s_j^l is the interfering atomic section (step 4). Otherwise, s_i^k and s_j^l are swapped (step 15). If $p\left(s_i^k\right)$ was greater than $p\left(s_j^l\right)$, then s_i^k would be the one that commits, because it belongs to a higher priority job, and it started before s_j^l (step 3). Otherwise, c_{ij}^{kl} is calculated (step 5) to determine whether it is worth aborting s_i^k in favour of s_j^l, because $len(s_j^l)$ is relatively small compared to the remaining execution length of s_i^k (explained further).

We assume that:

$$c_{ij}^{kl} = len(s_j^l)/len(s_i^k) \tag{5.1}$$

where $c_{ij}^{kl} \in]0, \infty[$, to cover all possible lengths of s_j^l. Our idea is to reduce the opportunity for the abort of s_i^k if it is close to committing when interfered and $len(s_j^l)$ is large. This abort opportunity is increasingly reduced as s_i^k gets closer to the end of its execution, or $len(s_j^l)$ gets larger.

On the other hand, as s_i^k is interfered early, or $len(s_j^l)$ is small compared to s_i^k's remaining length, the abort opportunity is increased even if s_i^k is close to the end of its execution. To decide whether s_i^k must be aborted or not, we use a threshold value $\psi \in [0,1]$ that determines α_{ij}^{kl} (step 6), where α_{ij}^{kl} is the maximum percentage of $len(s_i^k)$ below which s_j^l is allowed to abort s_i^k and is calculated as

$$\alpha_{ij}^{kl} = \frac{ln\,(\Psi)}{ln\,(\Psi) - c_{ij}^{kl}} \tag{5.2}$$

Thus, if the already executed part of s_i^k — when s_j^l interferes with s_i^k — does not exceed $\alpha_{ij}^{kl} len(s_i^k)$, then s_i^k is aborted (step 9). Otherwise, s_j^l is aborted (step 11).

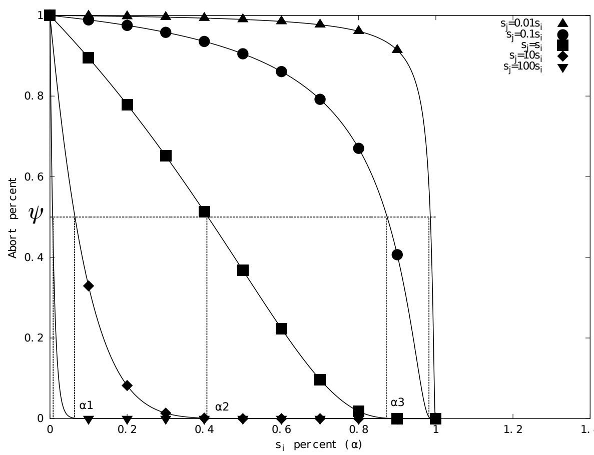


Figure 5.1: Interference of s_i^k by various lengths of s_j^l

The behaviour of LCM is illustrated in Figure 5.1. In this figure, the horizontal axis corresponds to different values of α ranging from 0 to 1, and the vertical axis corresponds to different values of abort opportunities, $f(c_{ij}^{kl}, \alpha)$, ranging from 0 to 1 and calculated by (5.3):

$$f(c_{ij}^{kl}, \alpha) = e^{\frac{-c_{ij}^{kl}\alpha}{1-\alpha}} \tag{5.3}$$

where c_{ij}^{kl} is calculated by (5.1).

Figure 5.1 shows one atomic section s_i^k (whose α changes along the horizontal axis) interfered by five different lengths of s_j^l. For a predefined value of $f(c_{ij}^{kl}, \alpha)$ (denoted as ψ in Algorithm 3), there corresponds a specific value of α (which is α_{ij}^{kl} in Algorithm 3) for each curve. For example, when $len(s_j^l) = 0.1 \times len(s_i^k)$, s_j^l aborts s_i^k if the latter has not executed more than $\alpha 3$ percentage (shown in Figure 5.1) of its execution length. As $len(s_j^l)$ decreases, the corresponding α_{ij}^{kl} increases (as shown in Figure 5.1, $\alpha 3 > \alpha 2 > \alpha 1$).

Equation (5.3) achieves the desired requirement that the abort opportunity is reduced as s_i^k gets closer to the end of its execution (as $\alpha \to 1$, $f(c_{ij}^{kl}, 1) \to 0$), or as the length of the conflicting transaction increases (as $c_{ij}^{kl} \to \infty$, $f(\infty, \alpha) \to 0$). Meanwhile, this abort opportunity is increased as s_i^k is interfered closer to its release (as $\alpha \to 0$, $f(c_{ij}^{kl}, 0) \to 1$), or as the length of the conflicting transaction decreases (as $c_{ij}^{kl} \to 0$, $f(0, \alpha) \to 1$).

LCM is not a centralized CM, which means that, upon a conflict, each transactions has to decide whether it must commit or abort. LCM suffers from transitive retry (Section 4.1.2).

Claim 22. *LCM suffers from transitive retry for multi-object transactions.*

Proof. Following the proof of Claim 1, Claim follows. □

5.1.2 LCM Illustrative Example

Behaviour of LCM can be illustrated by the following example:

- Transaction $s_i^k \in \tau_i^x$ begins execution. Currently, s_i^k does not conflict with any other transaction.
- Transaction $s_j^l \in \tau_j^y$ is released while s_i^k is still running. $\Theta_i^{k^{ex}} \cap \Theta_j^l \neq \emptyset$ and $p_j^y > p_i^x$ (where priority is dynamic in G-EDF, and fixed in G-RMA). c_{ij}^{kl}, α_{ij}^{kl} and α are calculated by steps 5 to 7 in Algorithm 3. s_i^k has not reached α percentage of its execution length yet.
- $\alpha < \alpha_{ij}^{kl}$. Then, s_j^l is allowed to abort and restart s_i^k.
- s_j^l commits. s_i^k executes again.
- Transaction $s_h^v \in \tau_h^u$ is released while s_i^k is running. $\Theta_i^{k^{ex}} \cap \Theta_h^v \neq \emptyset$ and $p_h^u > p_i^x$. c_{ih}^{kv}, α_{ih}^{kv} and α are calculated by steps 5 to 7 in Algorithm 3. s_i^k has already passed α percentage of its execution length. So, s_h^v aborts and restarts in favour of s_i^k.
- Transaction $s_a^b \in \tau_a^f$ is released. $\Theta_i^{k^{ex}} \cap \Theta_a^b \neq \emptyset$ and $p_a^f > p_i^x$ but $p_a^f < p_h^u$. c_{ia}^{kb}, α_{ia}^{kb} and α are calculated by steps 5 to 7 in Algorithm 3. s_i^k has not reached α percentage of its execution length yet. So, s_a^b is allowed to abort s_i^k. Because s_a^b is just starting, LCM allows s_h^v to abort s_a^b. So, the highest priority transaction is not blocked by an intermediate priority transaction s_a^b.
- When s_h^v commits. s_a^b is allowed to execute while s_i^k is retrying.
- When s_a^b commits, s_i^k executes.
- Transaction $s_c^n \in \tau_o^z$ is released while s_i^k is running. $\Theta_i^{k^{ex}} \cap \Theta_c^n \neq \emptyset$ and $p_o^z < p_i^x$. So, s_i^k commits first, then s_c^n is allowed to proceed.

5.2 Properties

LCM properties are given by the following Lemmas. These properties are used to derive retry cost and response time of transactions and tasks under LCM.

Claim 23. *$r\left(s_i^k\right)$ is updated each time s_i^k aborts and retries to the new start time of the new retry to avoid deadlock that can result from conflicting transactions aborting each other.*

Proof. Assume a set of transactions S that are conflicting together. Each transaction aborts and retries due to the others. So, a higher priority transaction s_j^l aborts and retries due to a lower priority transaction s_i^k. s_i^k itself aborts and retries due to another transaction. Thus, the new $r\left(s_i^k\right)$ will be at least equal to the new $r\left(s_j^l\right)$. By definition of LCM, s_j^l will be chosen to commit first because of its higher priority. By extending this result to all transactions in S, the highest priority transaction will commit. Thus, deadlock is avoided. Claim follows. □

Claim 24. *Let s_j^l interferes once with s_i^k at most at α_{ij}^{kl}. $p\left(s_j^l\right) > p\left(s_i^k\right)$. Then, the maximum contribution of s_j^l to s_i^k's retry cost is:*

$$W_i^k(s_j^l) \leq \alpha_{ij}^{kl} len\Big(s_i^k\Big) + len\Big(s_j^l\Big) \tag{5.4}$$

Proof. If s_j^l interferes with s_i^k at a Υ percentage, where $\Upsilon < \alpha_{ij}^{kl}$, then the retry cost of s_i^k is $\Upsilon len(s_i^k) + len(s_j^l)$, which is lower than that calculated in (5.4). Besides, if s_j^l interferes with s_i^k after α_{ij}^{kl} percentage, then s_i^k will not abort. □

Claim 25. *A higher priority transaction, s_j^l, aborts and retries due to a lower priority transaction, s_i^k, if s_j^l interferes with s_i^k after the α_{ij}^{kl} percentage. s_j^l's retry cost, due to s_i^k is upper bounded by:*

$$W_j^l(s_i^k) \leq \Big(1 - \alpha_{ij}^{kl}\Big) len\Big(s_i^k\Big) \tag{5.5}$$

Proof. It is derived directly from Claim 24, as s_j^l will have to retry for the remaining length of s_i^k. □

Claim 26. *As length of s_i^k- interfered by a higher priority transaction s_j^l - increases, then α_{ij}^{kl} also increases.*

Proof. As $len\left(s_i^k\right)$ increases, then c_{ij}^{kl} decreases by definition of (5.1). Noting that $ln(\Psi) \leq 0$ because $\Psi \in [0, 1]$. Thus, α_{ij}^{kl} increases as c_{ij}^{kl} decreases by definition of (5.2). Claim follows. □

Claim 27. *Let $conf\left\{s_i^k\right\}$ be the set of all transactions that do not belong to any job of τ_i and are conflicting, directly or indirectly(transitively), with s_i^k. Each transaction $s_j^l \in conf\left\{s_i^k\right\}, p\left(s_j^l\right) > p\left(s_i^k\right)$ contributes to the retry cost of s_i^k by at most*

$$len\left(s_j^l + \alpha_{max}^{jl} s_{max}(\Theta)\right) \tag{5.6}$$

where $s_{max}(\Theta)$ is the maximum length atomic section (transaction) in $conf\{s_i^k\}$ that accesses at least one object in Θ and its priority is lower than $p(s_j^l)$. $s_{max}(\Theta) \notin s_j$ and $\Theta \subseteq \Theta_i^{k^{ex}} \cap \Theta_j^l$. α_{max}^{jl} is calculated by (5.2) due to interference of $s_{max}(\Theta)$ by s_j^l.

Proof. Under ECM and RCM (Chapter 4), lower priority transactions abort and retry only due to higher priority transactions. Whereas, under LCM, a transaction s_i^k can be aborted due to higher priority transactions. s_i^k can also be delayed by lower priority transactions. Thus, proof follows proof of Claim 5 with the following modifications:

- According to Claims 24 and 25, s_j^l can cause lower priority transactions to retry and higher priority transactions to be delayed. From Claims 24 and 25, it appears that contribution of s_j^l to the retry cost of lower priority transactions is greater than delay caused by s_j^l to higher priority transactions. Thus, retry cost caused by s_j^l to lower priority transactions is taken as the contribution of s_j^l to the retry cost of s_i^k.
- By Claim 26 and definition of $s_{max}(\Theta)$, α_{max}^{jl} is the maximum α that results from interference of a lower priority transaction- accessing any object $\theta \in \Theta$ - by s_j^l.
- s_i^k can abort and retry due to higher priority transactions. Also, s_i^k can be delayed due to lower priority transactions. Thus, $p\left(s_{max}\right) < p\left(s_j^l\right)$, but $p\left(s_{max}^{jl}\right)$ does not have to be greater than $p\left(s_i^k\right)$.

Claim follows. □

Claim 28. *Let $conf\left\{s_i^k\right\}$ be the set of all transactions that do not belong to any job of τ_i and are conflicting, directly or indirectly(transitively), with s_i^k. Each transaction $s_j^l \in conf\left\{s_i^k\right\}, p\left(s_j^l\right) < p\left(s_i^k\right)$ contributes to the delay of s_i^k by at most*

$$\left(1 - \alpha_{min}^{jl}\right) len\left(s_j^l\right) \tag{5.7}$$

where α_{min}^{jl} is the minimum α_{jx}^{ly}- calculated by (5.2)- that results from delay of any higher priority transaction s_x^y by the lower priority s_j^l.

Proof. If s_j^l is to abort and retry, then the delay to s_i^k that results from each retry of s_j^l is covered by Claim 27. Thus, the delay that results from s_j^l when it does not retry is given by Claim 25 by minimizing α_{ij}^{kl} in (5.5) to its minimum value (i.e., α_{min}^{jl}). Claim follows. □

Claim 29. *Under LCM with G-EDF and G-RMA, priority inversion time for any job τ_i^x during T_i is bounded.*

Proof. Under LCM, priority of each transaction s_i^k equals priority of its containing job τ_i^x. Under G-EDF, number of lower priority jobs of τ_j that are released during T_i is upper bounded by 1. Under G-RMA, number of lower priority jobs of τ_j that are released during T_i is upper bounded by $\left\lceil \frac{T_i}{T_j} \right\rceil + 1$. Number of transactions is fixed for each job. So, by Claim 28, Claim follows. □

5.3 Retry Cost and Response Time of G-EDF/LCM

Claim 30. *Under G-EDF/LCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during interval $L \leq T_i$ due to direct and indirect conflict with other transactions is upper bounded by:*

$$RC_i\left(L\right) \leq \sum_{\tau_j \in \gamma_i^{ex}} \left(g_{ij}^{gedf} \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i^{ex}\right) \neq \emptyset} len\left(s_j^l + \alpha_{max}^{jl} s_{max}(\Theta)\right) \right) \tag{5.8}$$

where $s_{max}\left(\Theta\right) \notin s_j$ and α_{max}^{jl} is given by (5.2) due to interference of the lower priority $s_{max}\left(\Theta\right)$ by the higher priority s_j^l. g_{ij}^{gedf} is calculated by (4.2).

Proof. From Claims 24 and 25, it appears that contribution of s_j^l to the retry cost of lower priority transactions is greater than delay caused by s_j^l to higher priority transactions. Thus, retry cost caused by s_j^l to lower priority transactions is taken as the contribution of s_j^l to the retry cost of s_i^k. Under G-EDF, priorities are determined by the absolute deadline of the job. Thus, the same transaction s_j^l can be of higher or lower priority than $p\left(s_i^k\right)$ according to the absolute deadline of containing job of s_j^l. So, only jobs of $\tau_j \in \gamma_i$ that have an absolute deadline that at most coincides with d_i^x are considered. Thus, delay of lower priority transactions is ignored. Following Claim 27 and Claim 6, Claim follows □

Claim 31. *Under G-EDF/LCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ due to release of jobs with higher priority than τ_i^x is upper bounded by*

$$RC_{i_{re}}(L) \leq \sum_{\forall \tau_j \in \zeta_i} \begin{cases} \left\lceil \frac{L}{T_j} \right\rceil s_{i_{max}} & , L \leq T_i - T_j \\ \\ \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{i_{max}} & , L > T_i - T_j \end{cases} \tag{5.9}$$

where $\zeta_i = \{\tau_j : (\tau_j \neq \tau_i) \wedge (D_j < D_i)\}$.

Proof. G-EDF/LCM and ECM has the same pattern for release of jobs. Thus, proof is the same as proof of Claim 8. Claim follows. □

Claim 32. *Under G-EDF/LCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ is upper bounded by:*

$$RC_{i_{to}}(L) = RC_i(L) + RC_{i_{re}}(L) \tag{5.10}$$

where $RC_i(L)$ is the maximum retry cost resulting from conflict between transactions in τ_i^x and transactions of other jobs. $RC_i(L)$ is calculated by (5.8). $RC_{i_{re}}(L)$ is the maximum retry cost resulting from the release of higher priority jobs, which preempt transactions in τ_i^x. $RC_{i_{re}}(L)$ is calculated by (5.9).

Proof. Proof follows directly from Claims 30, 31 and proof of Claim 9. □

Claim 33. *Under G-EDF/LCM, maximum response time of any job $\tau_i^x \in \tau_i$ is upper bounded by Claim 10 where $RC_{i_{to}}(R_i^{up})$ is upper bounded by (5.10).*

Proof. Proof follows directly from Claim 32 and proof of Claim 10. □

5.4 Schedulability of G-EDF/LCM

We now compare the schedulability of G-EDF/LCM against ECM (Chapter 4), lock-free [37] and locking protocols (i.e., OMLP [18,21] and RNLP [123]) to understand when G-EDF/LCM will perform better.

5.4.1 G-EDF/LCM versus ECM

We compare the total utilization of ECM with that of G-EDF/LCM. For each method, we inflate the c_i of each task τ_i by adding the total retry cost suffered by τ_i. Thus, if method A adds total retry cost $RC_A^{to}(T_i)$ to c_i, and method B adds total retry cost $RC_B^{to}(T_i)$ to c_i, then the schedulability of A and B are compared as:

$$\sum_{\forall \tau_i} \frac{c_i + RC_A^{to}(T_i)}{T_i} \leq \sum_{\forall \tau_i} \frac{c_i + RC_B^{to}(T_i)}{T_i} \tag{5.11}$$

Equation(5.11) holds if

$$\forall \tau_i,\ RC_A^{to}(T_i) \leq RC_B^{to}(T_i) \tag{5.12}$$

Thus, schedulability is compared by comparing total retry cost added by the synchronization methods to any job of τ_i.

Claim 34. *Schedulability of G-EDF/LCM is always equal or better than ECM.*

Proof. Under ECM, $RC^{to}_{ECM}(T_i)$ is upper bounded by (4.20) with replacing γ_i by γ_i^{ex}. $RC^{to}_{G-EDF/LCM}(T_i)$ is given by (5.10) and upper bounded by

$$\begin{aligned} RC^{to}_{G-EDF/LCM}(T_i) &\leq \left(\sum_{\tau_j \in \gamma_i^{ex}} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i^{ex}\right) \neq \emptyset} len\left(s_j^l + \alpha_{max}^{jl} s_{max}(\Theta)\right)\right)\right) \\ &+ \left(\sum_{\tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{i_{max}}\right) \\ &\leq \left((1+\alpha_{max}) \sum_{\tau_j \in \gamma_i^{ex}} \left(\left\lceil \frac{T_i}{T_j} \right\rceil \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i^{ex}\right) \neq \emptyset} \right) s_{max}\right) \\ &+ \left(\sum_{\tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor s_{max}\right) \end{aligned} \tag{5.13}$$

where s_{max} is the length of the longest transaction among all tasks. α_{max} is the maximum value of α_{xy}^{kl} for any two transactions s_x^k and s_y^l. By substitution of $RC^{to}_{G-EDF/LCM}$ and $RC^{to}_{ECM}(T_i)$ into (5.12), the G-EDF/LCM has equal or better schedulability than ECM if $\alpha_{max} \leq 1$. But α_{max} is always less than or equal to 1. Claim follows. □

5.4.2 G-EDF/LCM versus Lock-free

As mentioned in Section 4.3, the retry-loop lock-free approach in [37] is the most relevant to our work. As lock-free instructions access only one object, then Θ_i^k for any s_i^k will be restricted to one object only (i.e., $\Theta_i^k = \theta_i^k$). Thus, transitive retry cannot happen, $\Theta_i^{ex} = \Theta_i$ and $\gamma_i^{ex} = \gamma_i$.

Claim 35. *For G-EDF/LCM's schedulability to be better or equal to that of [37]'s retry-loop lock-free approach, the size of s_{max} must not exceed $r_{max}/\left(1+\alpha_{max}\right)$, where s_{max} is the length of longest transaction among all tasks, r_{max} is the maximum execution cost of a single iteration of any lock-free retry loop of any task, and $\alpha_{max} = max\left\{\alpha_{xy}^{kl}\right\}_{\forall s_x^k, s_y^l}$. With equal periods for conflicting tasks and high access times to each object within the same transaction, the size of s_{max} can be much larger than r_{max}.*

Proof. Using Claim 32 and following the same steps of proof of Claim 16, Claim follows. □

5.4.3 G-EDF/LCM versus Global OMLP

Claim 36. *Following the same notations in Section 4.4.1, schedulability of G-EDF/LCM is equal or better than schedulability of Global OMLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\,(2m-1)}{\left(\left(1+\alpha_{max}\right)N_{max}+1\right)(n-1)\Phi_{max}} \tag{5.14}$$

where $\alpha_{max} = max\left\{\alpha_{xy}^{kl}\right\}_{\forall s_x^k,\, s_y^l}$.

Proof. Using Claim 32 and following the same steps of proof of Claim 18, Claim follows. □

5.4.4 G-EDF/LCM versus RNLP

Claim 37. *Following the same notations in Section 4.4.4, schedulability of G-EDF/LCM is equal or better than schedulability of RNLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\,(2m-1)}{\left(\left(1+\alpha_{max}\right)N_{max}+1\right)(n-1)\Phi_{max}} \tag{5.15}$$

where $\alpha_{max} = max\left\{\alpha_{xy}^{kl}\right\}_{\forall s_x^k,\, s_y^l}$.

Proof. Using Claim 32 and following the same steps of proof of Claim 20, Claim follows. □

5.5 Retry Cost and Response Time of G-RMA/LCM

Claim 38. *Under G-RMA/LCM, the total retry cost suffered by all transactions in any job* $\tau_i^x \in \tau_i$ *during interval* $L \leq T_i$ *due to direct and indirect conflict with other transactions is upper bounded by:*

$$\begin{aligned} RC_i\,(L) \leq & \left(\sum_{\tau_j \in \gamma_i^{ex},\, p_j > p_i} \left(\left\lceil \frac{L}{T_j} \right\rceil + 1\right) \sum_{s_j^l,\left(\Theta = \Theta_i^{ex} \cap \Theta_j^l\right) \neq \emptyset} len\left(s_j^l + \alpha_{max}^{jl} s_{max}\,(\Theta)\right)\right) \\ + & \left(\sum_{\tau_z \in \gamma_i^{ex},\, p_z < p_i} \left(\left\lceil \frac{L}{T_z} \right\rceil + 1\right) \sum_{s_z^l,\left(\Theta = \Theta_i^{ex} \cap \Theta_z^l\right) \neq \emptyset} len\left(\left(1-\alpha_{min}^{zl}\right) s_z^l\right)\right) \end{aligned} \tag{5.16}$$

where $s_{max}\,(\Theta) \notin s_j$ *and* α_{max}^{jl} *is given by (5.2) due to interference of the lower priority* $s_{max}\,(\Theta)$ *by the higher priority* s_j^l. α_{min}^{zl} *is the minimum* α_{zx}^{ly}*- calculated by (5.2)- that results from delay of a any higher priority transaction* s_x^y *by the lower priority* s_z^l.

Proof. Proof follows from Claims 27, 28 and proof of Claim 12. □

Claim 39. *Under G-RMA/LCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ due to release of jobs with higher priority than τ_i^x is upper bounded by*

$$RC_{i_{re}}(L) = \sum_{\forall \tau_j, p_j > p_i} \left(\left\lceil \frac{L}{T_j} \right\rceil s_{i_{max}} \right) \tag{5.17}$$

Proof. G-RMA/LCM and RCM has the same pattern for release of jobs. Thus, proof is the same as proof of Claim 13. Claim follows. □

Claim 40. *Under G-RMA/LCM, the total retry cost suffered by all transactions in any job $\tau_i^x \in \tau_i$ during an interval $L \leq T_i$ is upper bounded by:*

$$RC_{i_{to}}(L) = RC_i(L) + RC_{i_{re}}(L) \tag{5.18}$$

where $RC_i(L)$ is the maximum retry cost resulting from conflict between transactions in τ_i^x and transactions of other jobs. $RC_i(L)$ is calculated by (5.16). $RC_{i_{re}}(L)$ is the maximum retry cost resulting from the release of higher priority jobs, which preempt transactions in τ_i^x. $RC_{i_{re}}(L)$ is calculated by (5.17).

Proof. Using Claims 38, 39 and proof of Claim 14, Claim follows. □

Claim 41. *Under G-RMA/LCM, maximum response time of any job $\tau_i^x \in \tau_i$ is upper bounded by Claim 15 where $RC_{i_{to}}(R_i^{up})$ is upper bounded by (5.18).*

Proof. Proof follows directly from Claim 40 and proof of Claim 15. □

5.6 Schedulability of G-RMA/LCM

As in Section 5.4, we compare the schedulability of G-RMA/LCM against RCM (Chapter 4), lock-free [37] and locking protocols (i.e., OMLP [18,21] and RNLP [123]) to understand when G-RMA/LCM will perform better.

5.6.1 G-RMA/LCM versus RCM

Claim 42. *G-RMA/LCM's schedulability is equal or better than RCM if:*

$$\frac{1-\alpha_{min}}{1-\alpha_{max}} \leq \frac{\sum_{\forall \tau_j \in \gamma_i^{ex}, p_j > p_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i^{ex}\right) \neq \emptyset} \right)}{2 \sum_{\forall \tau_z \in \gamma_i^{ex}, p_z < p_i} \sum_{\forall s_z^l, \left(\Theta = \Theta_z^l \cap \Theta_i^{ex}\right) \neq \emptyset}} \tag{5.19}$$

where $\alpha_{max} = max\{\alpha_{xy}^{kl}\}_{\forall s_x^k, s_y^l}$, $\alpha_{min} = min\{\alpha_{xy}^{kl}\}_{\forall s_x^k, s_y^l}$.

Proof. Let $RC^{to}_{G-RMA/LCM}$ be the total retry cost for any job of τ_i under G-RMA/LCM. $RC^{to}_{G-RMA/LCM}$ is given by (5.18) and upper bounded by:

$$\begin{aligned} & (1+\alpha_{max}) \left(\sum_{\forall \tau_j \in \gamma_i^{ex}, p_j > p_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i^{ex}\right) \neq \emptyset}\right) s_{max}\right) \\ + & (1-\alpha_{min}) \left(\sum_{\forall \tau_z \in \gamma_i^{ex}, p_z < p_i} \left(\left(\left\lceil \frac{T_i}{T_z} \right\rceil + 1\right) \sum_{\forall s_z^l, \left(\Theta = \Theta_z^l \cap \Theta_i^{ex}\right) \neq \emptyset}\right) s_{max}\right) \\ + & \left(\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil\right) s_{max} \end{aligned} \tag{5.20}$$

Let RC^{to}_{RCM} be the total retry cost for any job of τ_i under RCM. RC^{to}_{RCM} is given by (4.18) and upper bounded by:

$$\left(\left(2 \sum_{\forall \tau_j \in \gamma_i^{ex}, p_j > p_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i^{ex}\right) \neq \emptyset}\right)\right) + \left(\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil\right)\right) s_{max} \tag{5.21}$$

$\left\lceil \frac{T_i}{T_z} \right\rceil = 1, \forall \tau_i, \tau_z$ where $p_z < p_i$ because $T_i < T_z$ by definition of G-RMA and implicit deadline tasks. By substitution of (5.20) and (5.21) into (5.12), Claim follows. □

5.6.2 G-RMA/LCM versus Lock-free

As mentioned in Section 4.3, the retry-loop lock-free approach in [37] is the most relevant to our work. As lock-free instructions access only one object, then Θ_i^k for any s_i^k will be restricted to one object only (i.e., $\Theta_i^k = \theta_i^k$). Thus, transitive retry cannot happen, $\Theta_i^{ex} = \Theta_i$ and $\gamma_i^{ex} = \gamma_i$.

Claim 43. *For G-RMA/LCM's schedulability to be better or equal to that of [37]'s retry-loop lock-free approach, the size of s_{max} must not exceed $r_{max}/\left(1+\alpha_{max}\right)$, where s_{max} is the length of longest transaction among all tasks, r_{max} is the maximum execution cost of a single iteration of any lock-free retry loop of any task, and $\alpha_{max} = max\left\{\alpha_{xy}^{kl}\right\}_{\forall s_x^k, s_y^l}$. With high access times to each object within the same transaction, the size of s_{max} can be much larger than r_{max}.*

Proof. Let $RC^{to}_{G-RMA/LCM}$ be the total retry cost for any job of τ_i under G-RMA/LCM. $RC^{to}_{G-RMA/LCM}$ is given by (5.18) and upper bounded by (5.20) where γ_i^{ex} is replaced with γ_i and Φ_i^{ex} is replaced with Φ_i. Let LRC_{to} be the total retry cost for any job of τ_i under retry-loop lock-free with G-RMA. LRC_{to} is upper bounded by (4.31). Similar to proof of Claim 17, schedulability of G-RMA/LCM is equal or better than schedulability of retry-loop lock-free if for each τ_i:

$$\begin{aligned}
& (1+\alpha_{max})\left(\sum_{\forall\tau_j\in\gamma_i,p_j>p_i}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,(\Theta=\Theta_j^l\cap\Theta_i)\neq\emptyset}\right)s_{max}\right) \\
+ \quad & (1-\alpha_{min})\left(\sum_{\forall\tau_j\in\gamma_i,p_j<p_i}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,(\Theta=\Theta_j^l\cap\Theta_i)\neq\emptyset}\right)s_{max}\right) \\
+ \quad & \left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)s_{max} \\
\leq \quad & \left(\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)\right)r_{max}
\end{aligned} \tag{5.22}$$

By definition of G-RMA and implicit deadline tasks, $\left\lceil\frac{T_i}{T_j}\right\rceil = 1$, $\forall\tau_i$, τ_j where $p_j < p_i$. So, (5.22) becomes

$$\begin{aligned}
& (1+\alpha_{max})\left(\sum_{\forall\tau_j\in\gamma_i,p_j>p_i}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,(\Theta=\Theta_j^l\cap\Theta_i)\neq\emptyset}\right)s_{max}\right) \\
+ \quad & 2\left(1-\alpha_{min}\right)\left(\sum_{\forall\tau_j\in\gamma_i,p_j<p_i}\left(\sum_{\forall s_j^l,(\Theta=\Theta_j^l\cap\Theta_i)\neq\emptyset}\right)s_{max}\right) \\
+ \quad & \left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)s_{max} \\
\leq \quad & \left(\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)\right)r_{max}
\end{aligned} \tag{5.23}$$

The set of tasks $\{\tau_j|\tau_j \neq \tau_i\}$ can be devided into four sets dependinong on priority and object sharing between τ_i and τ_j. So, $\{\tau_j|\tau_j \neq \tau_i\} = \{\tau_l\} \cup \{\tilde{\tau}_l\} \cup \{\tau_h\} \cup \{\tilde{\tau}_h\}$, where:

- $\{\tau_l\} = \{\tau_j|\,(\tau_j \neq \tau_i)\wedge(p_j < p_i)\wedge(\tau_j \in \gamma_i)\}$ is the set of tasks τ_j other than τ_i where τ_j has direct conflict with τ_i and priority of τ_j is lower than priority of τ_i. Let β_{il}^* be the number of transactions in $\tau_l \in \{\tau_l\}$ that has direct conflict with τ_i (i.e., $\beta_{il}^* = \sum_{\forall s_l^x,(\Theta=\Theta_l^x\cap\Theta_i)\neq\emptyset}$). Let β_{il} be the number of times a lower priority job of τ_l accesses shared objects with a higher priority job of τ_i using retry-loop lock-free [37]. As one object can be accessed multiple times within the same transaction, and lock-free instruction accesses one object only once, then $\beta_{il} \geq \beta_{il}^*$.
- $\{\tilde{\tau}_l\} = \{\tau_j|\,(\tau_j \neq \tau_i)\wedge(p_j < p_i)\wedge(\tau_j \notin \gamma_i)\}$ is the set of tasks τ_j other than τ_i where τ_j has no direct conflict with τ_i and priority of τ_j is lower than priority of τ_i.
- $\{\tau_h\} = \{\tau_j|\,(\tau_j \neq \tau_i)\wedge(p_j > p_i)\wedge(\tau_j \in \gamma_i)\}$ is the set of tasks τ_j other than τ_i where τ_j has direct conflict with τ_i and priority of τ_j is higher than priority of τ_i. Let β_{ih}^* be the number of transactions in $\tau_h \in \{\tau_h\}$ that has direct conflict with τ_i (i.e., $\beta_{ih}^* = \sum_{\forall s_h^x,(\Theta=\Theta_h^x\cap\Theta_i)\neq\emptyset}$). Let β_{ih} be the number of times a higher priority job of τ_h accesses shared objects with a lower priority job of τ_i using retry-loop lock-free [37]. As one object can be accessed multiple times within the same transaction, and lock-free instruction accesses one object only once, then $\beta_{ih} \geq \beta_{ih}^*$.
- $\{\tilde{\tau}_h\} = \{\tau_j|\,(\tau_j \neq \tau_i)\wedge(p_j > p_i)\wedge(\tau_j \notin \gamma_i)\}$ is the set of tasks τ_j other than τ_i where τ_j has no direct conflict with τ_i and priority of τ_j is higher than priority of τ_i.

Thus, (5.23) becomes

$$\begin{aligned}
& (1+\alpha_{max})\left(\sum_{\forall\tau_h\in\{\tau_h\}}\left(\left(\left\lceil\frac{T_i}{T_h}\right\rceil+1\right)\beta^*_{ih}\right)s_{max}\right)\\
+ & \quad 2\left(1-\alpha_{min}\right)\left(\sum_{\forall\tau_l\in\{\tau_l\}}\beta^*_{il}s_{max}\right)\\
+ & \left(\left(\sum_{\forall\tau_h\in\{\tau_h\}}\left\lceil\frac{T_i}{T_h}\right\rceil\right)+\left(\sum_{\forall\tilde{\tau}_h\in\{\tilde{\tau}_h\}}\left\lceil\frac{T_i}{\tilde{T}_h}\right\rceil\right)\right)s_{max}\\
\le & \left(\left(\sum_{\forall\tau_h\in\{\tau_h\}}\left(\left\lceil\frac{T_i}{T_h}\right\rceil+1\right)\beta_{ih}\right)+\left(2\sum_{\forall\tau_l\in\{\tau_l\}}\beta_{il}\right)\right)r_{max}\\
+ & \left(\left(\sum_{\forall\tau_h\in\{\tau_h\}}\left\lceil\frac{T_i}{T_h}\right\rceil\right)+\left(\sum_{\forall\tilde{\tau}_h\in\{\tilde{\tau}_h\}}\left\lceil\frac{T_i}{\tilde{T}_h}\right\rceil\right)\right)r_{max}
\end{aligned} \tag{5.24}$$

$$\begin{aligned}
\therefore & \sum_{\forall\tau_h\in\{\tau_h\}}\left((1+\alpha_{max})\left(\left(\left\lceil\frac{T_i}{T_h}\right\rceil+1\right)\beta^*_{ih}\right)+\left\lceil\frac{T_i}{T_h}\right\rceil\right)s_{max}\\
+ & \quad 2\left(1-\alpha_{min}\right)\left(\sum_{\forall\tau_l\in\{\tau_l\}}\beta^*_{il}s_{max}\right)\\
+ & \left(\left(\sum_{\forall\tilde{\tau}_h\in\{\tilde{\tau}_h\}}\left\lceil\frac{T_i}{\tilde{T}_h}\right\rceil\right)\right)s_{max}\\
\le & \sum_{\forall\tau_h\in\{\tau_h\}}\left(\left(\left(\left\lceil\frac{T_i}{T_h}\right\rceil+1\right)\beta_{ih}\right)+\left\lceil\frac{T_i}{T_h}\right\rceil\right)r_{max}\\
+ & \left(2\sum_{\forall\tau_l\in\{\tau_l\}}\beta_{il}\right)r_{max}\\
+ & \left(\sum_{\forall\tilde{\tau}_h\in\{\tilde{\tau}_h\}}\left\lceil\frac{T_i}{\tilde{T}_h}\right\rceil\right)r_{max}
\end{aligned} \tag{5.25}$$

(5.25) is satisfied if for each τ_i:

- $$\frac{s_{max}}{r_{max}}\le\frac{\sum_{\forall\tau_h\in\{\tau_h\}}\left(\left(\left(\left\lceil\frac{T_i}{T_h}\right\rceil+1\right)\beta_{ih}\right)+\left\lceil\frac{T_i}{T_h}\right\rceil\right)}{\sum_{\forall\tau_h\in\{\tau_h\}}\left((1+\alpha_{max})\left(\left(\left\lceil\frac{T_i}{T_h}\right\rceil+1\right)\beta^*_{ih}\right)+\left\lceil\frac{T_i}{T_h}\right\rceil\right)} \tag{5.26}$$

  To find the lower bound over s_{max}/r_{max} that satisfies (5.26), let β_{ih} assumes its minimum value (i.e., $\beta_{ih}=\beta^*_{ih}$). Thus, (5.26) is satisfied if

  $$\begin{aligned}
  \frac{s_{max}}{r_{max}} & \le \frac{\sum_{\forall\tau_h\in\{\tau_h\}}\left(\left(\left(\left\lceil\frac{T_i}{T_h}\right\rceil+1\right)\beta^*_{ih}\right)+\left\lceil\frac{T_i}{T_h}\right\rceil\right)}{\sum_{\forall\tau_h\in\{\tau_h\}}(1+\alpha_{max})\left(\left(\left(\left\lceil\frac{T_i}{T_h}\right\rceil+1\right)\beta^*_{ih}\right)+\left\lceil\frac{T_i}{T_h}\right\rceil\right)}\\
  & = \frac{1}{1+\alpha_{max}}\le\frac{1}{2}
  \end{aligned} \tag{5.27}$$

  To find the upper bound over s_{max}/r_{max} that satisfies (5.26), let $\beta_{ih}>>(1+\alpha_{max})\,\beta^*_{ih}$. Thus, s_{max} can be much larger than r_{max}.

- $$2\left(1-\alpha_{min}\right)\left(\sum_{\forall\tau_l\in\{\tau_l\}}\beta^*_{il}s_{max}\right)\le\left(2\sum_{\forall\tau_l\in\{\tau_l\}}\beta_{il}\right)r_{max}$$

$$\therefore \frac{s_{max}}{r_{max}} \leq \frac{\sum_{\forall \tau_l \in \{\tau_l\}} \beta_{il}}{(1-\alpha_{min}) \left(\sum_{\forall \tau_l \in \{\tau_l\}} \beta^*_{il}\right)} \tag{5.28}$$

To find the lower bound over s_{max}/r_{max} that satisfies (5.28), let β_{il} assumes its minimum value (i.e., $\beta_{il} = \beta^*_{il}$). Thus, (5.28) is satisfied if

$$\frac{s_{max}}{r_{max}} \leq \frac{1}{1-\alpha_{min}} \leq 1 \tag{5.29}$$

To find the upper bound over s_{max}/r_{max} that satisfies (5.28), let $\beta_{il} >> (1-\alpha_{min}) \beta^*_{il}$. Thus, s_{max} can be much larger than r_{max}.

•

$$\left(\sum_{\forall \tilde{\tau}_h \in \{\tilde{\tau}_h\}} \left\lceil \frac{T_i}{\tilde{T}_h} \right\rceil\right) s_{max} \leq \left(\sum_{\forall \tilde{\tau}_h \in \{\tilde{\tau}_h\}} \left\lceil \frac{T_i}{\tilde{T}_h} \right\rceil\right) r_{max} \tag{5.30}$$

$$\therefore \frac{s_{max}}{r_{max}} \leq 1$$

By taking the minimum lower bound and the maximum upper bound from the previous cases, Claim follows. □

5.6.3 G-RMA/LCM versus Global OMLP

Claim 44. *Following the same notations in Section 4.4.1, schedulability of G-RMA/LCM is equal or better than schedulability of Global OMLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(n-1)\left((1+\alpha_{max})\left((\Phi_{max}+1) N_{max}\right) + 2(1-\alpha_{min}) N_{max} + \Phi_{max}\right)} \tag{5.31}$$

where $\alpha_{max} = max\left\{\alpha^{kl}_{xy}\right\}_{\forall s^k_x, s^l_y}$.

Proof. Substitute $RC_A(T_i)$ in (4.38) by (5.20). Following the same steps in proof of Claim 18, Claim follows. □

5.6.4 G-RMA/LCM versus RNLP

Claim 45. *Following the same notations in Section 4.4.4, schedulability of G-RMA/LCM is equal or better than schedulability of RNLP if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(n-1)\left((1+\alpha_{max})\left((\Phi_{max}+1) N_{max}\right) + 2(1-\alpha_{min}) N_{max} + \Phi_{max}\right)} \tag{5.32}$$

where $\alpha_{max} = max\left\{\alpha^{kl}_{xy}\right\}_{\forall s^k_x, s^l_y}$.

Proof. Substitute $RC_A(T_i)$ in (4.38) by (5.20). Following the same steps of proof of Claim 20, Claim follows. □

5.7 Conclusions

In ECM and RCM, a task incurs at most $2s_{max}$ retry cost for each of its atomic section due to conflict with another task's atomic section. With LCM, this retry cost is reduced to $(1+\alpha_{max})s_{max}$ for each aborted atomic section. In ECM and RCM, higher priority tasks are not delayed due to lower priority tasks, whereas in LCM, they are. In G-EDF/LCM, delay due to a lower priority job is encountered only from a task τ_j's last job instance during τ_i's period. Contribution of a transaction s_j^l to the retry cost of a lower priority transaction is higher than delay caused by s_j^l to a higher priority transaction. Thus, under G-EDF/LCM, each transaction is assumed to contribute in the abort and retry of a lower priority transaction. Hence, delay of higher priority transactions due to lower priority transactions is ignored under G-EDF/LCM. This is not the case with G-RMA/LCM, because of fixed priority under G-RMA.

Schedulability of G-EDF/LCM is always equal or better than ECM's. Whereas, schedulability of G-RMA/LCM is equal or better than RCM's depending on α_{min} and α_{max}. Schedulability of G-EDF/LCM and G-RMA/LCM is equal or better than schedulability of retry-loop lock-free if s_{max} does not exceed $r_{max}/\left(1+\alpha_{max}\right)$. With high number of object access within each transaction, s_{max} can be much larger than r_{max} with equal or better schedulability for G-EDF/LCM (G-RMA/LCM) than schedulability of retry-loop lock-free. Schedulability of G-EDF/LCM is equal or better than Global OMLP and RNLP if $\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{((1+\alpha_{max})N_{max}+1)(n-1)\Phi_{max}}$. Schedulability of G-RMA/LCM is equal or better than Global OMLP and RNLP if $\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(n-1)((1+\alpha_{max})((\Phi_{max}+1)N_{max})+2(1-\alpha_{min})N_{max}+\Phi_{max})}$

Chapter 6

The PNF Contention Manager

In this chapter, we present a novel contention manager for resolving transactional conflicts, called PNF [44]. We upper bound transactional retries and task response times under PNF, when used with the G-EDF and G-RMA schedulers. We formally identify the conditions under which PNF outperforms previous real-time STM contention managers, lock-free and locking protocols.

The rest of this Chapter is organized as follows: Section 6.1 discusses limitations of previous contention managers and the motivation to PNF. Section 6.2 give a formal description of PNF. Section 6.3 derives PNF's properties. We upper bound retry cost and response time under PNF in Section 6.4. Schedulability comparison between PNF and other synchronization techniques is given in Section 6.5. We conclude Chapter in Section 6.6.

6.1 Limitations of ECM, RCM, and LCM

With multiple objects per transaction, ECM, RCM (Chapter 4) and LCM (Chapter 5) face transitive retry as shown by Claims 1, 11 and 22. Thus, a transaction s_i^k can abort and retry due to another transaction s_j^l where $\Theta_i^k \cap \Theta_j^l = \emptyset$. Retry cost and response time analysis-presented in Chapters 4 and 5- extend the set of objects accessed by any task τ_i to include any object that can cause direct or indirect(transitive) retry to any transaction in τ_i. However, this solution may over-extend the set of conflicting objects, and may even contain all objects accessed by all tasks.

In addition to the *transitive retry* problem, retrying higher priority transactions can prevent lower priority tasks from running. This happens when all processors are busy with higher priority jobs. When a transaction retries, the processor time is wasted. Thus, it would be better to give the processor to some other task.

Essentially, what we present is a new contention manager that avoids the effect of transitive retry. We call it, Priority contention manager with Negative values and First access (or PNF). PNF also tries to enhance processor utilization. This is done by allocating processors to jobs with non-retrying transactions. PNF is described in Section 6.2.

6.2 The PNF Contention Manager

Algorithm 4 describes PNF. It manages two sets. The first is the m-set, which contains at most m non-conflicting transactions, where m is the number of processors, as there cannot be more than m executing transactions (or generally, m executing jobs) at the same time. When a transaction is entered in the m-set, it executes non-preemptively and no other transaction can abort it. A transaction in the m-set is called an *executing transaction*. This means that, when a transaction is executing before the arrival of higher priority conflicting transactions, then the one that started executing first will be committed (Step 8) (hence the term "First access" in the algorithm's name). The second set is the n-set, which holds the transactions that are retrying because of a conflict with one or more of the executing transactions (Step 6), where n stands for the number of tasks in the system. Transactions in the n-set are known as *retrying transaction*. n-set also holds transactions that cannot currently execute, because processors are busy, either due to processing executing transactions and/or higher priority jobs. Any transaction in the n-set is assigned a temporal priority of -1 (Step 7) (hence the word "Negative" in the algorithm's name). A negative priority is considered smaller than any normal priority, and a transaction continues to hold this negative priority until it is moved to the m-set, where it restores its normal priority.

A job τ_x^y holding a transaction in the n-set can be preempted by any other job τ_z^l with normal priority, even if τ_z^l does not have transactions conflicting with τ_x^y. Hence, the n-set is of length n, as there can be at most n jobs. Transactions in the n-set whose jobs have been preempted are called *preempted transactions*. The n-set list keeps track of preempted transactions, because as it will be shown, all preempted and non-preempted transactions in the n-set are examined when any executing transaction commits. Then, one or more transactions are selected from the n-set to be executing transactions. If a retrying transaction is selected as an executing transaction, the task that owns the retrying transaction regains its priority.

When a new transaction is released, and if it does not conflict with any of the executing transactions (Step 1), then it will allocate a slot in the m-set and becomes an executing transaction. When this transaction is released (i.e., its containing task is already allocated to a processor), it will be able to access a processor immediately. This transaction may have a conflict with any of the transactions in the n-set. However, since transactions in the n-set have priorities of -1, they cannot prevent this new transaction from executing if it does not conflict with any of the executing transactions.

[1] An idle processor or at least one that runs a non-atomic section task with priority lower than the task holding $n(z)$.

Algorithm 4: PNF

Data: *Executing Transaction:* is one that cannot be aborted by any other transaction, nor preempted by a higher priority task;
m-set: m-length set that contains only non-conflicting executing transactions;
n-set: n-length set that contains retrying transactions for n tasks in non-increasing order of priority;
n(z): transaction at index z of the n-set;
s_i^k: a newly released transaction;
s_j^l: one of the executing transactions;
Result: atomic sections that will commit

1 **if** s_i^k *does not conflict with any executing transaction* **then**
2   Assign s_i^k as an executing transaction;
3   Add s_i^k to the m-set;
4   Select s_i^k to commit
5 **else**
6   Add s_i^k to the n-set according to its priority;
7   Assign temporary priority -1 to the job that owns s_i^k ;
8   Select transaction(s) conflicting with s_i^k for commit;
9 **end**
10 **if** s_j^l *commits* **then**
11   **for** *z=1 to size of n-set* **do**
12   **if** *n(z) does not conflict with any executing transaction* **then**
13   **if** *processor available*[1] **then**
14   Restore priority of task owning n(z);
15   Assign n(z) as executing transaction;
16   Add n(z) to m-set and remove it from n-set;
17   Select n(z) for commit;
18   **else**
19   Wait until processor available
20   **end**
21   **end**
22   move to the next n(z);
23   **end**
24 **end**

When one of the executing transactions commits (Step 10), it is time to select one of the n-set transactions to commit. The n-set is traversed from the highest priority to the lowest priority (priority here refers to the original priority of the transactions, and not -1) (Step 11). If an examined transaction in the n-set, s_h^b, does not conflict with any executing transaction (Step 12), and there is an available processor for it (Step 13) ("available" means either an idle processor, or one that is executing a job of lower priority than s_h^b), then s_h^b is moved from the n-set to the m-set as an executing transaction and its original priority is restored. If s_h^b is added to the m-set, the new m-set is compared with other transactions in the n-set with lower priority than s_h^b. Hence, if one of the transactions in the n-set, s_d^g, is of lower priority than s_h^b and conflicts with s_h^b, it will remain in the n-set.

The choice of the new transaction from the n-set depends on the original priority of transactions (hence the term "P" in the algorithm name). The algorithm avoids interrupting an already executing transaction to reduce its retry cost. In the meanwhile, it tries to avoid delaying the highest priority transaction in the n-set when it is time to select a new one to commit, even if the highest priority transaction arrives after other lower priority transactions in the n-set.

6.2.1 Illustrative Example

We illustrate PNF with an example. We use the following notions: $s_a^b(\theta_1, \theta_2, \theta_3)$ means that s_a^b accesses objects $\theta_1, \theta_2, \theta_3$. If $s_a^b \in \tau_a^j$, $\therefore p_o(s_a^b) = p_a^j$, where $p_o(s_a^b)$ is the original priority of s_a^b. $p(s_a^b) = -1$, if s_a^b is a retrying transaction; $p(s_a^b) = p_o(s_a^b)$ otherwise. m-set$= \{s_a^b, s_i^k\}$ means that the m-set contains transactions s_a^b and s_i^k regardless of their order. n-set$= \{s_a^b, s_i^k\}$ means that the n-set contains transactions s_a^b and s_i^k in that order, where $p_o(s_a^b) > p_o(s_i^k)$. m-set (n-set) $= \{\phi\}$ means that m-set (n-set) is empty. Assume there are five processors.

1. Initially, m-set$=$ n-set$= \{\phi\}$. $s_a^b(\theta_1, \theta_2) \in \tau_a^b$ is released and checks m-set for conflicting transactions. As m-set is empty, s_a^b finds no conflict and becomes an executing transaction. s_a^b is added to m-set. m-set$= \{s_a^b\}$ and n-set$= \{\phi\}$. s_a^b is executing on processor 1.
2. $s_c^d(\theta_3, \theta_4) \in \tau_c^d$ is released and checks m-set for conflicting transactions. s_c^d does not conflict with s_a^b as they access different objects. s_c^d becomes an executing transaction and is added to m-set. m-set$= \{s_a^b, s_c^d\}$ and n-set$= \{\phi\}$. s_c^d is executing on processor 2.
3. $s_e^f(\theta_1, \theta_5) \in \tau_e^f$ is released and $p_o(s_e^f) < p_o(s_a^b)$. s_e^f conflicts with s_a^b when it checks m-set. s_e^f is added to n-set and becomes a retrying transaction. $p(s_e^f)$ becomes -1. m-set$= \{s_a^b, s_c^d\}$ and n-set$= \{s_e^f\}$. s_e^f is retrying on processor 3.
4. $s_g^h(\theta_1, \theta_6) \in \tau_g^h$ is released and $p_o(s_g^h) > p_o(s_a^b)$. s_g^h conflicts with s_a^b. Though s_g^h is of higher priority than s_a^b, s_a^b is an executing transaction. So s_a^b runs non-preemptively. s_g^h is added to n-set before s_e^f, because $p_o(s_g^h) > p_o(s_e^f)$. $p(s_g^h)$ becomes -1. m-set$= \{s_a^b, s_c^d\}$ and n-set$= \{s_g^h, s_e^f\}$. s_g^h is retrying on processor 4.
5. $s_i^j(\theta_5, \theta_7) \in \tau_i^j$ is released. $p_o(s_i^j) < p_o(s_e^f)$. s_i^j does not conflict with any transaction in m-set. Though s_i^j conflicts with s_e^f and $p_o(s_i^j) < p_o(s_e^f) < p_o(s_g^h)$, s_e^f and s_g^h are retrying transactions. s_i^j becomes an executing transaction and is added to m-set. m-set$= \{s_a^b, s_c^d, s_i^j\}$ and n-set$= \{s_g^h, s_e^f\}$. s_i^j is executing on processor 5.
6. τ_k^l is released. τ_k^l does not access any object. $p_k^l < p_o(s_e^f) < p_o(s_g^h)$, but $p(s_e^f) = p(s_g^h) = -1$. Since there are no more processors, τ_k^l preempts τ_e^f, because the currently assigned priority to $\tau_e^f = p(s_e^f) = -1$ and $p_o(s_g^h) > p_o(s_e^f)$. τ_k^l is running on processor 3. This way, PNF optimizes processor usage. The m-set and n-set are not changed. Although s_e^f is preempted, n-set still records it, as s_e^f might be needed (as will be shown in the following steps).
7. s_i^j commits. s_i^j is removed from m-set. Transactions in n-set are checked from the first

(highest p_o) to the last (lowest p_o) for conflicts against any executing transaction. s_g^h is checked first because $p_o(s_g^h) > p_o(s_e^f)$. s_g^h conflicts with s_a^b, so s_g^h cannot be an executing transaction. Now it is time to check s_e^f, even though s_e^f is preempted in step 6. s_e^f also conflicts with s_a^b, so s_e^f cannot be an executing transaction. m-set= $\{s_a^b, s_c^d\}$ and n-set= $\{s_g^h, s_e^f\}$. Now, s_e^f can be retrying on processor 5 if τ_i^j has finished execution. Otherwise, τ_i^j continues running on processor 5 and s_e^f is still preempted. This is because, $p(s_e^f) = -1$ and $p_i^j > p(s_e^f)$. Let us assume that τ_i^j is still running on processor 5.

8. s_a^b commits. s_a^b is removed from m-set. Transactions in n-set are checked as done in step 7. s_g^h does not conflict with any executing transaction any more. s_g^h becomes an executing transaction. s_g^h is removed from n-set and added to m-set, so m-set= $\{s_c^d, s_g^h\}$. Now, s_e^f is checked against the new m-set. s_e^f conflicts with s_g^h, so s_e^f cannot be an executing transaction. s_e^f can be retrying on processor 1 if τ_a^b has finished execution. Otherwise, s_e^f remains preempted, because $p(s_e^f) = -1$ and $p_a^b > p(s_e^f)$. n-set= $\{s_e^f\}$. Let us assume that τ_a^b is still running on processor 1.
9. s_g^h commits. s_g^h is removed from m-set. τ_g^h continues execution on processor 4. Transactions in n-set are checked again. s_e^f is the only retrying transaction in the n-set, and it does not conflict with any executing transactions. Now, the system has τ_a^b running on processor 1, s_c^d executing on processor 2, τ_k^l running on processor 3, τ_g^h running on processor 4, and τ_i^j running on processor 5. s_e^f can become an executing transaction if it can find a processor. Since p_i^j, $p_k^l < p_o(s_e^f)$, s_e^f can preempt the lowest in priority between τ_i^j and τ_k^l. s_e^f now becomes an executing transaction. s_e^f is removed from the n-set and added to the m-set. So, m-set= $\{s_c^d, s_e^f\}$ and n-set= $\{\phi\}$. If p_i^j, p_k^l were of higher priority than $p_o(s_e^f)$, then s_e^f would have remained in n-set until a processor becomes available.

The example shows that PNF avoids transitive retry. This is illustrated in step 5, where $s_i^j(\theta_5, \theta_7)$ is not affected by the retry of $s_e^f(\theta_1, \theta_5)$. The example also explains how PNF optimizes processor usage. This is illustrated in step 6, where the retrying transaction s_e^f is preempted in favor of τ_k^l.

6.3 Properties

Claim 46. *Transactions scheduled under PNF do not suffer from transitive retry.*

Proof. Proof is by contradiction. Assume that a transaction s_i^k is retrying because of a higher priority transaction s_j^l, which in turn is retrying because of another higher priority transaction s_z^h. Assume that s_i^k and s_z^h do not conflict, yet, s_i^k is transitively retrying due to s_z^h. Note that s_z^h and s_j^l cannot exit together in the m-set as they have shared objects. But they both can be in the n-set, as they can conflict with other *executing transactions*. We have three cases:

Case 1: Assume that s_z^h is an executing transaction. This means that s_j^l is in the n-set. When s_i^k arrives, by the definition of PNF, it will be compared with the m-set, which contains s_z^h. Now, it will be found that s_i^k does not conflict with s_z^h. Also, by the definition of PNF, s_i^k is not compared with transactions in the n-set. When s_i^k newly arrives, priorities of n-set transactions are lower than any normal priority. Therefore, as s_i^k does not conflict with any other executing transaction, it joins the m-set and becomes an *executing transaction*. This contradicts the assumption that s_i^k is transitively retrying because of s_z^h.

Case 2: Assume that s_z^h is in the n-set, while s_j^l is an executing transaction. When s_i^k arrives, it will conflict with s_j^l and joins the n-set. Now, s_i^k retries due to s_j^l, and not s_z^h. When s_j^l commits, the n-set is traversed from the highest priority transaction to the lowest one: if s_z^h does not conflict with any other executing transaction and there are available processors, s_z^h becomes an executing transaction. When s_i^k is compared with the m-set, it is found that it does not conflict with s_z^h. Additionally, if it also does not conflict with any other executing transaction and there are available processors, then s_i^k becomes an executing transaction. This means that s_i^k and s_z^h are executing concurrently, which violates the assumption of transitive retry.

Case 3: Assume that s_z^h and s_j^l both exist in the n-set. When s_i^k arrives, it is compared with the m-set. If s_i^k does not conflict with any executing transactions and there are available processors, then s_i^k becomes an executing transaction. Even though s_i^k has common objects with s_j^l, s_i^k is not compared with s_j^l, which is in the n-set. If s_i^k joins the n-set, it is because, it conflicts with one or more executing transactions, not because of s_z^h, which violates the transitive retry assumption. If the three transactions s_i^k, s_j^l and s_z^h exist in the n-set, and s_z^h is chosen as a new executing transaction, then s_j^l remains in the n-set. This leads to Case 1. If s_j^l is chosen, because s_z^h conflicts with another executing transaction and s_j^l does not, then this leads to Case 2. □

Claim 47. *The first access property of PNF prevents transitive retry.*

Proof. The proof is by contradiction. Assume that the retry cost of transactions in the absence of the first access property is the same as when first access exists. Now, assume that PNF is devoid of the first access property. This means that executing transactions can be aborted.

Assume three transactions s_i^k, s_j^l, and s_z^h, where s_z^h's priority is higher than s_j^l's priority, and s_j^l's priority is higher than s_i^k's priority. Assume that s_j^l conflicts with both s_i^k and s_z^h. s_i^k and s_z^h do not conflict together. If s_i^k arrives while s_z^h is an executing transaction and s_j^l exists in the n-set, then s_i^k becomes an executing transaction itself while s_j^l is retrying. If s_i^k did not commit at least when s_z^h commits, then s_j^l becomes an executing transaction. Due to the lack of the first access property, s_j^l will cause s_i^k to retry. So, the retry cost for s_i^k will be $len(s_z^h + s_j^l)$. This retry cost for s_i^k is the same if it had been transitively retrying because of s_z^h. This contradicts the first assumption. Claim follows. □

From Claims 46 and 47, PNF does not increase the retry cost of multi-object transactions. However, this is not the case for ECM, RCM and LCM as shown by Claims 1, 11 and 22.

Claim 48. *Under PNF, any job τ_i^x is not affected by the retry cost in any other job τ_j^l.*

Proof. As explained in Section 4, PNF assigns a temporary priority of -1 to any job that includes a retrying transaction. So, retrying transactions have lower priority than any other normal priority for any real-time task. When τ_i^x is released and τ_j^l has a retrying transaction, τ_i^x will have a higher priority than τ_j^l. Thus, τ_i^x can run on any available processor while τ_j^l is retrying one of its transactions. Claim follows. □

6.4 Retry Cost and Response Time Under PNF

We now derive an upper bound on the retry cost of any job τ_i^x under PNF during an interval $L \leq T_i$. Since all tasks are sporadic (i.e., each task τ_i has a minimum period T_i), T_i is the maximum study interval for each task τ_i.

Claim 49. *Under PNF, the maximum retry cost suffered by a transaction s_i^k due to a transaction s_j^l is $len(s_j^l)$.*

Proof. By PNF's definition, s_i^k cannot have started before s_j^l. Otherwise, s_i^k would have been an executing transaction and s_j^l cannot abort it. So, the earliest release time for s_i^k would have been just after s_j^l starts execution. Then, s_i^k would have to wait until s_j^l commits. Claim follows. □

Claim 50. *The retry cost for any job τ_i^x due to conflicts between its transactions and transactions of other jobs under PNF during an interval $L \leq T_i$ is upper bounded by:*

$$RC_i(L) \leq \sum_{\tau_j \in \gamma_i} \left(\sum_{\forall s_j^l, \left(\Theta = \Theta_j^l \cap \Theta_i\right) \neq \emptyset} \left(\left(\left\lceil \frac{L}{T_j} \right\rceil + 1 \right) len\left(s_j^l\right) \right) \right) \tag{6.1}$$

Proof. Consider a transaction s_i^k belonging to job τ_i^x. Under PNF, higher priority transactions than s_i^k can become executing transaction before s_i^k. A lower priority transaction s_v^f can also become an executing transaction before s_i^k. This happens when s_i^k conflicts with any executing transaction while s_v^f does not. The worst case scenario for s_i^k occurs when s_i^k has to wait in the n-set, while all other conflicting transactions with s_i^k are chosen to be executing transactions. The maximum number of jobs of any task τ_j that can interfere with τ_i^x during interval L is $\left\lceil \frac{L}{T_j} \right\rceil + 1$. From the previous observations and Claim 49, Claim follows. □

Claim 51. *In contrast to ECM, RCM and LCM, release of any higher priority job τ_j^l during execution of a lower priority transaction s_i^k does not increase retry cost of s_i^k. Thus, $RC_{i_{re}}(L) = 0$ and $RC_{i_{to}}(L) = RC_i(L)$, where $L \leq T_i$ and $RC_i(L)$ is given by (6.1).*

Proof. Under PNF, executing transactions have higher priority than any other real-time task. Thus, release of a higher priority task τ_j^l will not preempt any executing transaction s_i^k. Retrying transactions are already retrying when higher priority tasks are released. When a retrying transaction s_i^k is chosen to be an executing transaction, and all processors are busy with executing transactions except the processor running τ_j^l, then τ_j^l is preempted in favour of the executing transaction s_i^k by definition of PNF. Thus, τ_j^l does not increase retry cost of s_i^k. Claim follows. □

Claim 52. *The maximum blocking time for any job in τ_i due to lower priority jobs during an interval $L \leq T_i$ is upper bounded by:*

$$D_i(L) \leq max_{\forall \tau_i^x \in \tau_i} \left(\left\lfloor \frac{1}{m} \sum_{\forall \tau_j^l, p_j^l < p_i^x} \left(\left(\left\lceil \frac{L}{T_j} \right\rceil + 1 \right) \sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset} len\left(s_j^h\right) \right) \right\rfloor \right) \tag{6.2}$$

During $D_i(L)$, all processors are unavailable for τ_i^x.

Proof. Under PNF, executing transactions are non preemptive. So, an executing transaction s_i^k can delay a higher priority job τ_i^x, where $p_o\left(s_i^k\right) < p_i^x$, if no other processors are available. Through this proof, we call an s_i^k with $p_o\left(s_i^k\right) < p_i^x$ an original lower priority transaction compared to priority of τ_i^x. An original lower priority executing transactions can be conflicting or non-conflicting with any transaction in τ_i^x. They also can exist when τ_i^x is newly released, or after that. So, we have the following cases:

Original lower priority conflicting transactions after τ_i^x is released: This case is already covered by the retry cost in (6.1).

Original lower priority conflicting transactions when τ_i^x is newly released: Each original lower priority conflicting transaction s_j^h will delay τ_i^x for $len(s_j^h)$. The effect of s_j^h is already covered by (6.1). Besides, (6.1) does not divide the retry cost by m as done in (6.2). Thus, the worst case scenario requires inclusion of s_j^h in (6.1), and not in (6.2).

Original lower priority non-conflicting transactions when τ_i^x is newly released: τ_i^x is delayed if there are no available processors for it. Otherwise, τ_i^x can run in parallel with these non-conflicting original lower priority transactions. Each original lower priority non-conflicting transaction s_j^h will delay τ_i^x for $len(s_j^h)$.

Original lower priority non-conflicting transactions after τ_i^x is released: This situation can happen if τ_i^x is not currently running any executing transaction. A retrying transaction s_i^k is chosen to be an executing transaction. All processors are busy with executing transactions except the processor running τ_i^x. Thus, τ_i^x is preempted in favour of executing transaction s_i^k. Otherwise, τ_i^x can run in parallel with these original lower priority non-conflicting transactions.

Each original lower priority non-conflicting transaction s_j^h will delay τ_i^x for $len(s_j^h)$.

From the previous cases, original lower priority non-conflicting transactions act as if they were higher priority jobs interfering with τ_i^x. So, the blocking time can be calculated by the interference workload given by Theorem 7 in [14]. Claim follows. □

Claim 53. *The response time R_i^{up} of a job τ_i^x under G-EDF/PNF is upper bounded by:*

$$R_i^{up} = c_i + RC_{i_{to}}(R_i^{up}) + D_i(R_i^{up}) + \left\lfloor \frac{1}{m} \sum_{\forall j \neq i} I_{ij}(R_i^{up}) \right\rfloor \tag{6.3}$$

where $RC_{i_{to}}(R_i^{up})$ is calculated by (6.1). $D_i(R_i^{up})$ is modified from (6.2) to fit G-EDF as follows:

$$D_i(R_i^{up}) \leq \left\lfloor \frac{1}{m} \sum_{\forall \tau_j} \begin{cases} 0 & , R_i^{up} \leq T_i - T_j \\ \sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset} len\left(s_j^h\right) & , R_i^{up} > T_i - T_j \end{cases} \right\rfloor \tag{6.4}$$

and $I_{ij}(R_i^{up})$ is calculated by (4.5).

Proof. Proof is similar to proof of Claim 10 except that: 1) Total retry cost given by (6.1) (due to Claim 51) and blocking time given by (6.2) are added to each c_i. 2) Due to Claim 48, each c_j is not changed to c_{ji}. G-EDF uses absolute deadlines for scheduling. This defines which jobs of the same task can be of lower priority than τ_i^x, and which will not. Any instance τ_j^h, released between $r_i^x - T_j$ and $d_i^x - T_j$, will be of higher priority than τ_i^x. Before $r_i^x - T_j$, τ_j^h would have finished before τ_i^x is released. After $d_i^x - T_j$, d_j^h would be greater than d_i^x. Thus, τ_j^h will be of lower priority than τ_i^x. So, during T_i, there can be only one instance τ_j^h of τ_j with lower priority than τ_i^x. τ_j^h is released between $d_i^x - T_j$ and d_i^x. Consequently, during $R_i^{up} < T_i - T_j$, no existing instance of τ_j is of lower priority than τ_i^x. Hence, 0 is used in the first case of (6.4). But if $R_i^{up} > T_i - T_j$, there can be only one instance τ_j^h of τ_j with lower priority than τ_i^x. Hence, $\left\lceil \frac{R_i^{up}}{T_i} \right\rceil + 1$ in (6.2) is replaced with 1 in the second case in (6.4). Claim follows. □

Claim 54. *The response time R_i^{up} of a job τ_i^x under G-RMA/PNF is upper bounded by:*

$$R_i^{up} = c_i + RC_i(R_i^{up}) + D_i(R_i^{up}) + \left\lfloor \frac{1}{m} \sum_{\forall j \neq i, p_j > p_i} I_{ij}(R_i^{up}) \right\rfloor \tag{6.5}$$

where $RC(R_i^{up})$ is calculated by (6.1), $D_i(R_i^{up})$ is calculated by (6.2), and $I_{ij}(R_i^{up})$ is calculated by (4.4).

Proof. Proof is same as of Claim 53, except that G-RMA assigns fixed priorities. Hence, (6.2) can be used directly for calculating $D_i(R_i^{up})$ without modifications. Claim follows. □

6.5 PNF versus Competitors

We now (formally) compare the schedulability of G-EDF (G-RMA) with PNF against ECM (Chapter 4), RCM (Chapter 4), LCM (Chapter 5), retry-loop lock-free [37] and locking protocols((i.e., OMLP [18,21] and RNLP [123]). Such a comparison will reveal when PNF outperforms others. Toward this, we compare the total utilization under G-EDF (G-RMA)/PNF, with that under the other synchronization methods. Total utilization comparison between PNF and other synchronization techniques is done as in Sections 5.4 and 5.6 with the addition of $D_i(T_i)$ - given by (6.4) under G-EDF and (6.2) under G-RMA - to the inflated execution time of any job of τ_i under PNF.

6.5.1 PNF versus ECM

Claim 55. *Schedulability of G-EDF/PNF is equal or better than ECM's if for each task τ_i total number of transactions in any task $\tau_j \neq \tau_i$ - that has no direct conflict with any transaction in τ_i - divided by number of processors is not greater than maximum number of jobs- with higher priority than current job of τ_i - that can be released during T_i.*

Proof. Proof follows from proof of Claim 34 with the following modification: Under PNF, c_i is inflated with $RC^{to}_{G-EDF/PNF}(T_i)$ given by (6.1) and $D_i(T_i)$ given by (6.4). Thus, schedulability of G-EDF/PNF is equal or better than ECM's if for each τ_i:

$$\begin{aligned}&\left(\sum_{\forall\tau_j\in\gamma_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left\lfloor\frac{\sum_{\forall\tau_j}\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}}{m}\right\rfloor\\ \leq\quad&\left(\sum_{\forall\tau_j\in\gamma_i^{ex}}\left(2\left\lceil\frac{T_i}{T_j}\right\rceil\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\right)\right)+\left(\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor\right)\end{aligned}\tag{6.6}$$

$\because\ \gamma_i\subseteq\gamma_i^{ex},\ \Theta_i\subseteq\Theta_i^{ex}$ and $2\left\lceil\frac{T_i}{T_j}\right\rceil\geq\left\lceil\frac{T_i}{T_j}\right\rceil+1,\ \therefore\ \sum_{\forall\tau_j\in\gamma_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\leq \sum_{\forall\tau_j\in\gamma_i^{ex}}\left(2\left\lceil\frac{T_i}{T_j}\right\rceil\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\right)$. So, (6.6) holds if $\left\lfloor\frac{\sum_{\forall\tau_j}\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}}{m}\right\rfloor\leq\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor$. $\sum_{\forall\tau_j}\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}$ is the total number of transactions in any task $\tau_j\neq\tau_i$ that has no direct conflict with any transaction in τ_i. $\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor$ is the maximum number of jobs- with higher priority than current job of τ_i - that can be released during T_i. Thus, Claim follows. □

6.5.2 PNF versus RCM

Claim 56. *Schedulability of G-RMA/PNF is equal or better than RCM's if for each task τ_i total number of transactions in tasks with lower priority than p_i does not exceed one half of maximum number of jobs with higher priority than p_i that can be released during T_i.*

Proof. Proof follows from proof of Claim 42 with the following modification: Under PNF, c_i is inflated with $RC^{to}_{G-RMA/PNF}(T_i)$ given by (6.1) and $D_i(T_i)$ given by (6.2). Thus, schedulability of G-RMA/PNF is equal or better than RCM's if for each τ_i:

$$\begin{aligned}
&\left(\sum_{\forall\tau_j\in\gamma_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left\lfloor\frac{2\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}\right)}{m}\right\rfloor\\
\le\;&\left(2\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)
\end{aligned}\tag{6.7}$$

$$\begin{aligned}
\therefore\;&\left(\sum_{\forall\tau_j\in\gamma_i,p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left(2\sum_{\forall\tau_j\in\gamma_i,p_j<p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\right)\right)\\
+\;&\left\lfloor\frac{2\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}\right)}{m}\right\rfloor\\
\le\;&\left(2\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)
\end{aligned}\tag{6.8}$$

Eq(6.8) holds if

$$\begin{aligned}
\therefore\;&\left(\sum_{\forall\tau_j\in\gamma_i,p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left(2\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\right)\right)\\
+\;&\left(2\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}\right)\right)\\
\le\;&\left(2\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)
\end{aligned}\tag{6.9}$$

$$\begin{aligned}
\therefore\;&\left(\sum_{\forall\tau_j\in\gamma_i,p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left(2\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^l}\right)\right)\\
\le\;&\left(2\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)
\end{aligned}\tag{6.10}$$

$\because\ \gamma_i\subseteq\gamma_i^{ex}$ and $\Theta_i\subseteq\Theta_i^{ex}$, $\therefore\ \sum_{\forall\tau_j\in\gamma_i,p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)$ is always less than $2\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)$. Thus, (6.10) holds if $\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^l}\right)$ does not exceed one half of $\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil$. $\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^l}\right)$ is total number of transactions in tasks with lower priority than τ_i. $\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil$ is maximum number of jobs with higher priority than p_i that can be released during T_i. Claim follows. □

6.5.3 PNF versus G-EDF/LCM

Claim 57. *G-EDF/PNF's schedulability is equal or better than G-EDF/LCM's if for each task τ_i:*

- *Maximum number of jobs of $\tau_j \in \gamma_i$ - with higher priority than current job of τ_i - that can exist during T_i is not less than $1/\alpha_{max}$.*
- *Total number of transactions in any task $\tau_j \neq \tau_i$ - that has no direct conflict with any transaction in τ_i - divided by number of processors is not greater than maximum number of jobs- with higher priority than current job of τ_i - that can be released during T_i.*

Proof. Proof follows from proof of Claim 55 where $RC^{to}_{G-EDF/LCM}(T_i)$ is upper bounded by (5.13). Schedulability of G-EDF/PNF is equal or better than schedulability of G-EDF/LCM if for each τ_i

$$\begin{aligned}&\left(\sum_{\forall \tau_j \in \gamma_i}\left(\sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i \neq \emptyset}\left(\left\lceil \frac{T_i}{T_j}\right\rceil + 1\right)\right)\right) + \left\lfloor \frac{\sum_{\forall \tau_j}\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}}{m}\right\rfloor \\ \leq\ & \left((1+\alpha_{max})\sum_{\forall \tau_j \in \gamma_i^{ex}}\left(\sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i^{ex} \neq \emptyset}\left\lceil \frac{T_i}{T_j}\right\rceil\right)\right) + \left(\sum_{\forall \tau_j \in \zeta_i}\left\lfloor \frac{T_i}{T_j}\right\rfloor\right)\end{aligned} \tag{6.11}$$

$\because\ \gamma_i \subseteq \gamma_i^{ex}$ and $\Theta_i \subseteq \Theta_i^{ex}$. $\therefore$ (6.11) holds if

$$\begin{aligned}&\left(\sum_{\forall \tau_j \in \gamma_i}\left(\sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i \neq \emptyset}\left(\left\lceil \frac{T_i}{T_j}\right\rceil + 1\right)\right)\right) + \left\lfloor \frac{\sum_{\forall \tau_j}\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}}{m}\right\rfloor \\ \leq\ & \left((1+\alpha_{max})\sum_{\forall \tau_j \in \gamma_i}\left(\sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i \neq \emptyset}\left\lceil \frac{T_i}{T_j}\right\rceil\right)\right) + \left(\sum_{\forall \tau_j \in \zeta_i}\left\lfloor \frac{T_i}{T_j}\right\rfloor\right)\end{aligned} \tag{6.12}$$

Eq(6.12) holds if:

1. For each τ_i and $\tau_j \in \gamma_i$

$$\sum_{\forall \tau_j \in \gamma_i}\left(\sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i \neq \emptyset}\left(\left\lceil \frac{T_i}{T_j}\right\rceil + 1\right)\right) \leq (1+\alpha_{max})\sum_{\forall \tau_j \in \gamma_i}\left(\sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i \neq \emptyset}\left\lceil \frac{T_i}{T_j}\right\rceil\right)$$

$$\therefore\ \forall \tau_j \in \gamma_i,\ \left\lceil \frac{T_i}{T_j}\right\rceil + 1 \leq (1+\alpha_{max})\left\lceil \frac{T_i}{T_j}\right\rceil$$

$$\therefore\ \forall \tau_j \in \gamma_i,\ \frac{1}{\alpha_{max}} \leq \left\lceil \frac{T_i}{T_j}\right\rceil$$

By (4.2), $\left\lceil \frac{T_i}{T_j}\right\rceil$ is maximum number of jobs of τ_j - with higher priority than current job of τ_i - that can exist during T_i.

2. $\left\lfloor \frac{\sum_{\forall \tau_j}\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}}{m}\right\rfloor \leq \sum_{\forall \tau_j \in \zeta_i}\left\lfloor \frac{T_i}{T_j}\right\rfloor$. $\sum_{\forall \tau_j}\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}$ is total number of transactions in any task $\tau_j \neq \tau_i$ that has no direct conflict with any transaction in τ_i. $\sum_{\forall \tau_j \in \zeta_i}\left\lfloor \frac{T_i}{T_j}\right\rfloor$ is the maximum number of jobs- with higher priority than current job of τ_i - that can be released during T_i.

From the previous observations, Claim follows. □

6.5.4 PNF versus G-RMA/LCM

Claim 58. *Schedulability of G-RMA/PNF is equal or better than G-RMA/LCM's if:*

- α_{min} *is small (i.e.,* $\alpha_{min\rightarrow 0}$*).*
- *For each task* τ_i*, total number of transactions in tasks with lower priority than* p_i *and have no direct conflict with any transaction in* τ_i *divided by number of processors does not exceed one half of maximum number of jobs with higher priority than* p_i *that can be released during* T_i*.*

Proof. Proof follows from proof of Claim 56 where $RC^{to}_{G-RMA/LCM}(T_i)$ is upper bounded by (5.20). Schedulability of G-RMA/PNF is equal or better than schedulability of G-RMA/LCM if for each τ_i:

$$\begin{aligned}
&\left(\sum_{\forall\tau_j\in\gamma_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left\lfloor\frac{\sum_{\forall\tau_j,p_j<p_i}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}\right)}{m}\right\rfloor\\
\leq\ & (1+\alpha_{max})\left(\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)\\
+\ & (1-\alpha_{min})\left(\sum_{\forall\tau_j\in\gamma_i^{ex},p_j<p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)\\
+\ & \left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)
\end{aligned}\tag{6.13}$$

$$\begin{aligned}
\therefore\ & \left(\sum_{\forall\tau_j\in\gamma_i,p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)+\left(2\sum_{\forall\tau_j\in\gamma_i,p_j<p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\right)\right)\\
+\ & \left\lfloor\frac{2\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}\right)}{m}\right\rfloor\\
\leq\ & (1+\alpha_{max})\left(\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)\\
+\ & (1-\alpha_{min})\left(2\sum_{\forall\tau_j\in\gamma_i^{ex},p_j<p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\right)\right)\\
+\ & \left(\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil\right)
\end{aligned}\tag{6.14}$$

$\because\ \gamma_i\subseteq\gamma_i^{ex}$, $\Theta_i\subseteq\Theta_i^{ex}$ and $\alpha_{max}\geq 0$, then $\sum_{\forall\tau_j\in\gamma_i,p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)$ is never bigger than $(1+\alpha_{max})\left(\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)\right)$. Thus, (6.14) holds if:

1. For each τ_i

$$\sum_{\forall\tau_j\in\gamma_i,p_j<p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\right)\leq(1-\alpha_{min})\left(\sum_{\forall\tau_j\in\gamma_i^{ex},p_j<p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\right)\right)\tag{6.15}$$

   Eq(6.15) holds if $\alpha_{min}\rightarrow 0$.

2. For each τ_i

$$\left\lfloor \frac{2\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}\right)}{m}\right\rfloor \leq \sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil \tag{6.16}$$

Eq(6.15) holds if

$$\frac{\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}\right)}{m} \leq \frac{\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil}{2}$$

$\sum_{\forall\tau_j,p_j<p_i}\left(\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}\right)$ is total number of transactions in tasks with lower priority than p_i that do not have direct conflict with any transaction in τ_i. $\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil$ is maximum number of jobs with higher priority than p_i that can be released during T_i.

From previous observations, Claim follows. □

6.5.5 PNF versus Lock-free Synchronization

As mentioned in Section 4.3, the retry-loop lock-free approach in [37] is the most relevant to our work. As lock-free instructions access only one object, then Θ_i^k for any s_i^k will be restricted to one object only (i.e., $\Theta_i^k = \theta_i^k$). Thus, transitive retry cannot happen, $\Theta_i^{ex} = \Theta_i$ and $\gamma_i^{ex} = \gamma_i$.

Claim 59. *If, for each task τ_i, maximum number of jobs- with higher priority than current job of τ_i - that can be released during T_i is not less than total number of transactions in any task $\tau_j \neq \tau_i$ that has no direct conflict with any transaction in τ_i, then schedulability of PNF under G-EDF is equal or better than schedulability of retry-loop lock-free [37] with $s_{max}/r_{max} \geq 1$. s_{max} is the length of longest transaction among all tasks. r_{max} is the maximum execution cost of a single iteration of any lock-free retry loop of any task.*

Proof. Following the same steps of proof Claim 35, schedulability of PNF is equal or better than schedulability of retry-loop lock-free under G-EDF if for each task τ_i

$$\begin{aligned}&\left(\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\right)\right)+\left\lfloor\frac{\sum_{\forall\tau_j}\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}}{m}\right\rfloor\right)s_{max}\\ \leq\ &\left(\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+\left(\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor\right)\right)r_{max}\end{aligned} \tag{6.17}$$

Let $\beta_{ij}^* = \sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}$. Thus, (6.17) becomes

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}\right)+\left(\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor\right)\right)}{\left(\left(\sum_{\forall\tau_j\in\gamma_i}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\beta_{ij}^*\right)\right)+\left\lfloor\frac{\sum_{\forall\tau_j}\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}}{m}\right\rfloor\right)} \tag{6.18}$$

$\because\ \beta_{ij} \geq \beta^*_{ij}$, then (6.18) holds if $\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor \geq \left\lfloor \frac{\sum_{\forall \tau_j} \sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}}{m} \right\rfloor$. $\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor$ is maximum number of jobs- with higher priority than current job of τ_i - that can be released during T_i. $\sum_{\forall \tau_j} \sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}$ is total number of transactions in any task $\tau_j \neq \tau_i$ that has no direct conflict with any transaction in τ_i. $\sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor \geq \left\lfloor \frac{\sum_{\forall \tau_j} \sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}}{m} \right\rfloor$ allows $s_{max}/r_{max} \geq 1$ with equal or better schedulability for PNF than retry-loop lock-free with G-EDF. □

Claim 60. *If, for each task τ_i, maximum number of jobs with higher priority than p_i that can be released during T_i is not less than double of total number of transactions in tasks with lower priority than p_i that have no direct conflict with any transaction in τ_i divided by number of processors, then schedulability of PNF is equal or better than schedulability of retry-loop lock-free [37] under G-RMA with $s_{max}/r_{max} \geq 1$. s_{max} is the length of longest transaction among all tasks. r_{max} is the maximum execution cost of a single iteration of any lock-free retry loop of any task.*

Proof. Following the same steps of proof Claim 43, schedulability of PNF is equal or better than schedulability of retry-loop lock-free under G-RMA if for each task τ_i

$$\begin{aligned} &\left(\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i \neq \emptyset}\right)\right) + \left\lfloor \frac{2 \sum_{\forall \tau_j, p_j < p_i} \left(\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}\right)}{m} \right\rfloor\right) s_{max} \\ \leq\ &\left(\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \beta_{ij}\right) + \left(\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil\right)\right) r_{max} \end{aligned} \tag{6.19}$$

Let $\beta^*_{ij} = \sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i \neq \emptyset}$, then (6.19) becomes

$$\frac{s_{max}}{r_{max}} \leq \frac{\left(\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \beta_{ij}\right) + \left(\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil\right)\right)}{\left(\sum_{\forall \tau_j \in \gamma_i} \left(\left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1\right) \beta^*_{ij}\right)\right) + \left\lfloor \frac{2 \sum_{\forall \tau_j, p_j < p_i} \left(\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}\right)}{m} \right\rfloor} \tag{6.20}$$

$\because\ \beta_{ij} \geq \beta^*_{ij}$, then (6.20) holds if $\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil \geq \frac{2 \sum_{\forall \tau_j, p_j < p_i} \left(\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}\right)}{m}$. $\sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil$ is maximum number of jobs with higher priority than p_i that can be released during T_i. $\sum_{\forall \tau_j, p_j < p_i} \left(\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset}\right)$ is total number of transactions in tasks with lower priority than p_i that have no direct conflict with any transaction in τ_i. □

6.5.6 PNF versus Locking Protocols

Claim 61. *Following the same notations in Section 4.4.1, schedulability of PNF is equal or better than schedulability of Global OMLP under G-EDF if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(n-1\right)N_{max}\left(\Phi_{max}+1+\frac{1}{m}\right)} \tag{6.21}$$

Proof. Use (6.1) for $RC_{i_{to}}(T_i)$ and (6.4) for $D_i(T_i)$ under G-EDF/PNF. Following the same steps of proof of Claim 18, Claim follows. □

Claim 62. *Following the same notations in Section 4.4.1, schedulability of PNF is equal or better than schedulability of Global OMLP under G-RMA if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(n-1\right)N_{max}\left(\Phi_{max}+1+\frac{2}{m}\right)} \tag{6.22}$$

Proof. Use (6.1) for $RC_{i_{to}}(T_i)$ and (6.2) for $D_i(T_i)$ under G-RMA/PNF. Following the same steps of proof of Claim 18, Claim follows. □

Claim 63. *Following the same notations in Section 4.4.4, schedulability of PNF is equal or better than schedulability of RNLP under G-EDF if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(n-1\right)N_{max}\left(\Phi_{max}+1+\frac{1}{m}\right)} \tag{6.23}$$

Proof. Use (6.1) for $RC_{i_{to}}(T_i)$ and (6.4) for $D_i(T_i)$ under G-EDF/PNF. Following the same steps of proof of Claim 20, Claim follows. □

Claim 64. *Following the same notations in Section 4.4.4, schedulability of PNF is equal or better than schedulability of Global RNLP under G-RMA if*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_{min}\left(2m-1\right)}{\left(n-1\right)N_{max}\left(\Phi_{max}+1+\frac{2}{m}\right)} \tag{6.24}$$

Proof. Use (6.1) for $RC_{i_{to}}(T_i)$ and (6.2) for $D_i(T_i)$ under G-RMA/PNF. Following the same steps of proof of Claim 20, Claim follows. □

6.6 Conclusions

Transitive retry increases transactional retry cost under ECM, RCM, and LCM. PNF avoids transitive retry by avoiding transactional preemptions. PNF reduces the priority of aborted transactions to enable other tasks to execute, increasing processor usage. Executing transactions are not preempted due to the release of higher priority jobs. On the negative side of PNF, higher priority jobs can be blocked by executing transactions of lower priority jobs.

G-EDF/PNF's schedulability is equal or better than ECM's if, for each task τ_i, total number of transactions in any task $\tau_j \neq \tau_i$ - that has no direct conflict with any transaction in τ_i - divided by number of processors is not greater than maximum number of higher priority jobs than current job of τ_i that can be released during T_i. Similar condition holds for the schedulability comparison between G-EDF/PNF and G-EDF/LCM, in addition to maintain a lower bound of $1/\alpha_{max}$ over maximum number of higher priority jobs of τ_j that can exist during T_i and have direct conflict with any transaction in τ_i.

Schedulability of G-RMA/PNF is equal or better than RCM's if, for each task τ_i, total number of transactions in tasks with lower priority than p_i does not exceed one half of maximum number of jobs with higher priority than p_i that can be released during T_i. Schedulability of G-RMA/PNF is equal or better than G-RMA/LCM's if $\alpha_{min} \to 0$ and, for each task τ_i, total number of transactions in tasks with lower priority than p_i and have no direct conflict with any transaction in τ_i divided by number of processors does not exceed one half of maximum number of jobs with higher priority than p_i that can be released during T_i.

schedulability of PNF under G-EDF and G-RMA is equal or better than schedulability of retry-loop lock-free [37] with $s_{max}/r_{max} \geq 1$ if, for each task τ_i, maximum number of higher priority jobs than current job of τ_i - that can be released during T_i - is not less than maximum number of lower priority transactions in any task $\tau_j \neq \tau_i$ that has no direct conflict with any transaction in τ_i.

Schedulability of G-EDF/PNF is equal or better than schedulability of Global OMLP and RNLP if $\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(n-1)N_{max}\left(\Phi_{max}+1+\frac{1}{m}\right)}$. Under G-RMA, schedulability of PNF is equal or better than schedulability of Global OMLP and RNLP if $\frac{s_{max}}{L_{max}} \leq \frac{N_{min}(2m-1)}{(n-1)N_{max}\left(\Phi_{max}+1+\frac{2}{m}\right)}$.

Chapter 7

The FBLT Contention Manager

In this chapter, we present a novel contention manager for resolving transactional conflicts, called FBLT [43]. We upper bound transactional retries and task response times under FBLT, when used with the G-EDF and G-RMA schedulers. We formally identify the conditions under which FBLT has better real-time schedulability than the previous previous CMs, lock-free and locking protocols.

The rest of this Chapter is organized as follows: Section 8.1 discusses limitations of previous contention managers and the motivation to FBLT. Section 7.2 give a formal description of FBLT. We upper bound retry cost and response time under FBLT in Section 7.3. Schedulability comparison between FBLT and previous synchronization techniques is given in Section 7.4. We conclude Chapter in Section 7.5.

7.1 Motivation

With multiple objects per transaction, ECM, RCM (Chapter 4) and LCM (Chapter 5) face transitive retry as shown by Claims 1, 11 and 22. PNF (Chapter 6) is designed to avoid transitive retry by concurrently executing at most m non-conflicting transactions together as shown by Claim 46. These executing transactions are non-preemptive. Thus, executing transactions cannot be aborted due to direct or indirect conflict with other transactions. However, with PNF, all objects accessed by each transaction must be known a-priori. Therefore, this is not suitable with dynamic STM implementations [61]. Additionally, PNF is a centralized CM. This implementation increases overhead.

Thus, we propose the *First Bounded, Last Timestamp contention manager* (or FBLT) that achieves the following goals:

1. Reduce the retry cost of each transaction s_i^k due to another transaction s_j^l, just as LCM does compared to ECM and RCM.

2. Avoid or bound the effect of transitive retry, similar to PNF, without prior knowledge of accessed objects by each transaction, enabling dynamic STM.
3. Decentralized design and avoid the use of locks, thereby reducing overhead.

7.2 The FBLT Contention Manager

Algorithm 5 illustrates FBLT. Each transaction s_i^k can be aborted during T_i for at most Ω_i^k times. η_i^k records the number of times s_i^k has already been aborted up to now. If s_i^k and s_j^l have not joined the m_set yet, then they are preemptive transactions. Preemptive transactions resolve conflicts using LCM (Algorithm 3) (step 2). Thus, FBLT defaults to LCM when no transaction reaches its Ω. If only one of the transactions is in the m_set, then the non-preemptive transaction (the one in m_set) aborts the other one (steps 15 to 26). η_i^k is incremented each time s_i^k is aborted as long as $\eta_i^k < \Omega_i^k$ (steps 5 and 18). Otherwise, s_i^k is added to the m_set and its priority is increased to *m_prio* (steps 7 to 9 and 20 to 22). When the priority of s_i^k is increased to *m_prio*, s_i^k becomes a non-preemptive transaction. Non-preemptive transactions cannot be aborted by other preemptive transactions, nor by any other real-time job. The m_set can hold at most m concurrent transactions because there are m processors in the system. $r(s_i^k)$ records the time s_i^k joined the m_set (steps 8 and 21). When non-preemptive transactions conflict together (step 27), the transaction that joined m_set first is the one to commit first (steps 29 and 31). Thus, non-preemptive transactions are executed in increasing order of joining the m_set.

7.2.1 Illustrative Example

We now illustrate FBLT's behavior with the following example:

1. Transaction $s_i^k(\theta_1, \theta_2)$ is released while $m\_set = \emptyset$. $\eta_i^k = 0$ and $\Omega_i^k = 3$.
2. Transaction $s_a^b(\theta_2)$ is released while $s_i^k(\theta_1, \theta_2)$ is running. $p(s_a^b) > p(s_i^k)$ and $\eta_i^k < \Omega_i^k$. Applying LCM, $s_i^k(\theta_1, \theta_2)$ is aborted in favor of s_a^b and η_i^k is incremented to 1.
3. $s_a^b(\theta_2)$ commits. $s_i^k(\theta_1, \theta_2)$ runs again. Transaction $s_c^d(\theta_2)$ is released while $s_i^k(\theta_1, \theta_2)$ is running. $p(s_c^d) > p(s_i^k)$. Applying LCM, $s_i^k(\theta_1, \theta_2)$ is aborted again in favor of $s_c^d(\theta_2)$. η_i^k is incremented to 2.
4. $s_c^d(\theta_2)$ commits. $s_e^f(\theta_2, \theta_3)$ is released. $p(s_e^f) > p(s_i^k)$ and $\Omega_e^f = 2$. $s_i^k(\theta_1, \theta_2)$ is aborted in favour of $s_e^f(\theta_2, \theta_3)$ and η_i^k is incremented to 3.
5. $s_j^l(\theta_3)$ is released. $p(s_j^l) > p(s_e^f)$. $s_e^f(\theta_2, \theta_3)$ is aborted in favor of $s_j^l(\theta_3)$ and η_e^f is incremented to 1.
6. $s_i^k(\theta_1, \theta_2)$ and $s_e^f(\theta_2, \theta_3)$ are compared again. $\because \eta_i^k = \Omega_i^k$, $\therefore$ $s_i^k(\theta_1, \theta_2)$ is added to m_set. $m\_set = \left\{s_i^k(\theta_1, \theta_2)\right\}$. $s_i^k(\theta_1, \theta_2)$ becomes a non-preemptive transaction. As $s_e^f(\theta_2, \theta_3)$ is a preemptive transaction, $\therefore$ $s_e^f(\theta_2, \theta_3)$ is aborted in favour of $s_i^k(\theta_1, \theta_2)$, despite $p(s_e^f)$ being greater than the original priority of $s_i^k(\theta_1, \theta_2)$. η_e^f is incremented to 2.

Algorithm 5: FBLT

Data: s_i^k: interfered transaction;
s_j^l: interfering transaction;
Ω_i^k: maximum number of times s_i^k can be aborted during T_i;
η_i^k: number of times s_i^k has already been aborted up to now;
m_set: contains at most m non-preemptive transactions. m is number of processors;
m_prio: priority of any transaction in m_set. *m_prio* is higher than any priority of any real-time task;
$r(s_i^k)$: time point at which s_i^k joined m_set;
Result: atomic sections that will abort

1 **if** s_i^k, $s_j^l \notin m\_set$ **then**
2   Apply LCM (Algorithm 3);
3   **if** s_i^k *is aborted* **then**
4   **if** $\eta_i^k < \Omega_i^k$ **then**
5   Increment η_i^k by 1;
6   **else**
7   Add s_i^k to m_set;
8   Record $r(s_i^k)$;
9   Increase priority of s_i^k to *m_prio*;
10   **end**
11   **else**
12   Swap s_i^k and s_j^l;
13   Go to Step 3;
14   **end**
15 **else if** $s_j^l \in m\_set, s_i^k \notin m\_set$ **then**
16   Abort s_i^k;
17   **if** $\eta_i^k < \Omega_i^k$ **then**
18   Increment η_i^k by 1;
19   **else**
20   Add s_i^k to m_set;
21   Record $r(s_i^k)$;
22   Increase priority of s_i^k to *m_prio*;
23   **end**
24 **else if** $s_i^k \in m\_set, s_j^l \notin m\_set$ **then**
25   Swap s_i^k and s_j^l;
26   Go to Step 15;
27 **else**
28   **if** $r(s_i^k) < r(s_j^l)$ **then**
29   Abort s_j^l;
30   **else**
31   Abort s_i^k;
32   **end**
33 **end**

---

7. $s_j^l(\theta_3)$ commits but $s_g^h(\theta_3)$ is released. $p(s_g^h) > p(s_e^f)$ but $\eta_e^f = \Omega_e^f$. So, $s_e^f(\theta_2, \theta_3)$ becomes a non-preemptive transaction. $m\_set = \left\{s_i^k(\theta_1, \theta_2), s_e^f(\theta_2, \theta_3)\right\}$.
8. $s_i^k(\theta_1, \theta_2)$ and $s_e^f(\theta_2, \theta_3)$ are now non-preemptive transactions. $s_i^k(\theta_1, \theta_2)$ and $s_e^f(\theta_2, \theta_3)$ still conflict together. So, they are executed according to their addition order to the m_set. So, $s_i^k(\theta_1, \theta_2)$ commits first, followed by $s_e^f(\theta_2, \theta_3)$.
9. $s_g^h(\theta_3)$ will continue to abort and retry in favour of $s_e^f(\theta_2, \theta_3)$ until $s_e^f(\theta_2, \theta_3)$ commits or $\eta_g^h = \Omega_g^h$. Even if $s_g^h(\theta_3)$ joined the m_set, $s_g^h(\theta_3)$ will still abort and retry in favour of $s_e^f(\theta_2, \theta_3)$, because $s_e^f(\theta_2, \theta_3)$ joined the m_set earlier than $s_g^h(\theta_3)$.

It is seen from steps 2 to 6 that $s_i^k(\theta_1, \theta_2)$ can be aborted due to direct conflict with other transactions, or due to transitive retry. Irrespective of the reason for the conflict, once a transaction has reached its Ω, the transaction becomes a non-preemptive one (steps 6 and 7). Non-preemptive transactions have higher priority than other preemptive transactions (steps 6 and 7). Non-preemptive transactions execute in their arrival order to the m_set.

7.3 Retry Cost and Response Time Bounds

We now derive an upper bound on the retry cost of any job τ_i^x under FBLT during an interval $L \leq T_i$. Since all tasks are sporadic (i.e., each task τ_i has a minimum period T_i), T_i is the maximum study interval for each task τ_i.

Claim 65. *The total retry cost for any job τ_i^x under FBLT with G-EDF and G-RMA due to 1) conflicts between its transactions and transactions of other jobs during an interval $L \leq T_i$ and 2) release of higher priority jobs during L is upper bounded by:*

$$RC_{i_{to}}(L) \leq \sum_{\forall s_i^k} \left(\Omega_i^k len(s_i^k) + \sum_{\forall s_{iz}^k \in \chi_i^k} len(s_{iz}^k) \right) + RC_{i_{re}}(L) \tag{7.1}$$

where χ_i^k is the set of at most $m-1$ maximum length transactions conflicting directly or indirectly (through transitive retry) with s_i^k. Each transaction $s_{iz}^k \in \chi_i^k$ belongs to a distinct task τ_j. $RC_{re}(L)$ is the retry cost resulting from the release of higher priority jobs which preempt τ_i^x. $RC_{re}(L)$ is calculated by (4.11) for G-EDF, and (4.17) for G-RMA schedulers.

Proof. By the definition of FBLT, $s_i^k \in \tau_i^x$ can be aborted a maximum of Ω_i^k times before s_i^k joins the m_set. Transactions preceding s_i^k in the m_set can conflict directly with s_i^k, or indirectly through transitive retry. The worst case scenario for s_i^k after joining the m_set occurs if s_i^k is preceded by $m-1$ maximum length conflicting transactions. Hence, in the worst case, s_i^k has to wait for the previous $m-1$ transactions to commit first. The priority of s_i^k after joining the m_set is higher than any real-time job. Therefore, the non-preemptive s_i^k is not aborted due to release of any real-time job with higher priority than original priority of s_i^k. Following proofs of Claims 8 and 13, retry cost of s_i^k - before s_i^k joins m_set- due to release of higher priority jobs is calculated by (4.11) for G-EDF, and (4.17) for G-RMA. Transactions of each task execute sequentially. Thus, the non-preemptive s_i^k cannot be preceded in the m_set by two or more transactions of the same task. So, each transaction $s_{iz}^k \in \chi_i^k$ belong to a distinct task. Claim follows. □

Claim 66. *Under FBLT with G-EDF and G-RMA, the blocking time of a job τ_i^x due to lower priority jobs is upper bounded by:*

$$D(\tau_i^x) = \sum_{max_m} \{s_{j_{max}}\}_{\forall \tau_j^l, p_j^l < p_i^x} \tag{7.2}$$

where $s_{j_{max}}$ is the maximum length transaction in any job τ_j^l with original priority lower than p_i^x. The right hand side of (7.2) is the sum of the m maximum transactional lengths in all jobs with lower priority than τ_i^x.

Proof. The worst case blocking time for τ_i^x occurs when the maximum length m transactions in lower priority jobs than τ_i^x are executing non-preemptively. The m non-preemptive transactions execute sequentially if they conflict with each other. τ_i^x is delayed by the sequential execution of non-preemptive transactions if jobs with higher priority than p_i^x are released as soon as one of the non-preemptive transactions commits. No transaction with lower priority than p_i^x can be released while τ_i^x is waiting for a processor. Claim follows. □

Claim 67. *The response time R_i^{up} of any job τ_i^x under FBLT with G-EDF and G-RMA is upper bounded by:*

$$R_i^{up} = c_i + RC_{i_{to}}(R_i^{up}) + D(\tau_i^x) + \left\lfloor \frac{1}{m} \sum_{\forall j \neq i} I_{ij}(R_i^{up}) \right\rfloor \tag{7.3}$$

where $RC_{i_{to}}(R_i^{up})$ is calculated by (7.1), $D(\tau_i^x)$ is calculated by (7.2), and $I_{ij}(R_i^{up})$ is calculated by (4.15) for G-EDF, and (4.4) for G-RMA. c_j of any job $\tau_j^y \neq \tau_i^x$, $p_j^y > p_i^x$ is inflated to c_{ji} as given by (4.14).

Proof. Using Claims 10, 15, 65 and 66, Claim follows. □

7.4 Schedulability Comparison

We now (formally) compare the schedulability of FBLT with G-EDF and G-RMA against ECM (Chapter 4), RCM (Chapter 4), LCM (Chapter 5), PNF (Chapter 6), retry-loop lock-free [37] and locking protocols((i.e., OMLP [18,21] and RNLP [123]). Such a comparison will reveal when FBLT outperforms others. Toward this, we compare the total utilization under FBLT, with that under the other synchronization methods. Total utilization comparison between FBLT and other synchronization techniques is done as in Sections 5.4 and 5.6 with the addition of $D(\tau_i^x)$ - as given by (7.2)- to the inflated execution time of any job τ_i^x under FBLT.

7.4.1 FBLT versus ECM

Claim 68. *Let* $\Omega_i^{max} = max\{\Omega_i^k\}_{\forall s_i^k}$ *be the maximum abort number for any preemptive transaction* s_i^k *in* τ_i*. Schedulability of FBLT is equal to or better than ECM's if* Ω_i^{max} *of any* τ_i *is not greater than double the difference between ratio of maximum number of transactions in all jobs with higher priority than current job of* τ_i *and have direct or indirect conflict with transactions in* τ_i *to total number of transactions in any job of* τ_i *and number of processors. Formally, schedulability of FBLT is equal or better than ECM's if for each* τ_i

$$\Omega_i^{max} \leq 2\left(\frac{\sum_{\forall \tau_j \in \gamma_i^{ex}}\left(\left\lceil\frac{T_i}{T_j}\right\rceil \sum_{\forall s_j^l,\left(\Theta=\Theta_j^l \cap \Theta_i^{ex}\right)\neq\emptyset}\right)}{|s_i|} - m\right) \tag{7.4}$$

Proof. Proof follows from proof of Claim 34 with the following modification: Under FBLT, c_i is inflated with $RC_{FBLT}^{to}(T_i)$ given by (7.1) and $D(\tau_i^x)$ given by (7.2). Thus, schedulability of FBLT is equal or better than ECM's if for each τ_i:

$$m + \sum_{\forall s_i^k}\left(\Omega_i^k + m - 1\right) \leq \sum_{\forall \tau_j \in \gamma_i^{ex}}\left(2\left\lceil\frac{T_i}{T_j}\right\rceil \sum_{\forall s_j^l,\left(\Theta=\Theta_j^l \cap \Theta_i^{ex}\right)\neq\emptyset}\right) \tag{7.5}$$

$\because$ $|s_i| = \sum_{\forall s_i^k}$, where $|s_i|$ is total number of transactions in any job of τ_i. $\because$ $\Omega_i^{max} \geq \Omega_i^k$, $\therefore$ (7.5) holds if

$$m + |s_i|\left(\Omega_i^{max} + m - 1\right) \leq \sum_{\forall \tau_j \in \gamma_i^{ex}}\left(2\left\lceil\frac{T_i}{T_j}\right\rceil \sum_{\forall s_j^l,\left(\Theta=\Theta_j^l \cap \Theta_i^{ex}\right)\neq\emptyset}\right) \tag{7.6}$$

$$\therefore\ \Omega_i^{max} \leq \frac{\left(\sum_{\forall \tau_j \in \gamma_i^{ex}}\left(2\left\lceil\frac{T_i}{T_j}\right\rceil \sum_{\forall s_j^l,\left(\Theta=\Theta_j^l \cap \Theta_i^{ex}\right)\neq\emptyset}\right)\right) - (1+|s_i|)\,m + |s_i|}{|s_i|} \tag{7.7}$$

$\because$ $|s_i| \geq 1$, $\therefore$ $\frac{1+|s_i|}{|s_i|} \leq 2$. Thus, (7.7) holds if

$$\Omega_i^{max} \leq 2\left(\frac{\sum_{\forall \tau_j \in \gamma_i^{ex}}\left(\left\lceil\frac{T_i}{T_j}\right\rceil \sum_{\forall s_j^l,\left(\Theta=\Theta_j^l \cap \Theta_i^{ex}\right)\neq\emptyset}\right)}{|s_i|} - m\right) \tag{7.8}$$

$\because$ $\sum_{\forall \tau_j \in \gamma_i^{ex}}\left(\left\lceil\frac{T_i}{T_j}\right\rceil \sum_{\forall s_j^l,\left(\Theta=\Theta_j^l \cap \Theta_i^{ex}\right)\neq\emptyset}\right)$ is the maximum number of transactions in all jobs with higher priority than current job of τ_i and have direct or indirect conflict with transactions in τ_i, Claim follows. ∎

7.4.2 FBLT versus RCM

Claim 69. *Let $\Omega_i^{max} = max\{\Omega_i^k\}_{\forall s_i^k}$ be the maximum abort number for any preemptive transaction s_i^k in τ_i. Schedulability of FBLT is equal to or better than RCM's if Ω_i^{max} of any τ_i is not greater than double the difference between ratio of maximum number of transactions in all jobs with higher priority than p_i and have direct or indirect conflict with transactions in τ_i to total number of transactions in any job of τ_i and number of processors. Formally, schedulability of FBLT is equal to or better than RCM's if for each τ_i*

$$\Omega_i^{max} \leq 2\left(\frac{\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i^{ex}\right)\neq\emptyset}\right)}{|s_i|}-m\right) \tag{7.9}$$

Proof. Proof is similar to proof of Claim 68 except that $RC_{RCM}^{to}(T_i)$ is given by (5.21). □

7.4.3 FBLT versus G-EDF/LCM

Claim 70. *Let $\Omega_i^{max} = max\{\Omega_i^k\}_{\forall s_i^k}$ be the maximum abort number for any preemptive transaction s_i^k in τ_i. Schedulability of FBLT is equal to or better than G-EDF/LCM's if double number of processors subtracted from ratio of $1+\alpha_{max}$ multiplied by maximum number of transactions in all jobs with higher priority than current job of τ_i and have direct or indirect conflict with transactions in τ_i to total number of transactions in any job of τ_i is not less than Ω_i^{max} of any τ_i. Formally, schedulability of FBLT is equal to or better than G-EDF/LCM's if for each τ_i*

$$\Omega_i^{max} \leq \frac{(1+\alpha_{max})\sum_{\forall\tau_j\in\gamma_i^{ex}}\left(\left\lceil\frac{T_i}{T_j}\right\rceil\sum_{\forall s_j^l,\left(\Theta=\Theta_j^l\cap\Theta_i^{ex}\right)\neq\emptyset}\right)}{|s_i|}-2m \tag{7.10}$$

Proof. Proof is similar to proof of Claim 68 except that $RC_{G-EDF/LCM}^{to}(T_i)$ is given by (5.13). □

7.4.4 FBLT versus G-RMA/LCM

Claim 71. *Let $\Omega_i^{max} = max\{\Omega_i^k\}_{\forall s_i^k}$ be the maximum abort number for any preemptive transaction s_i^k in τ_i. Schedulability of FBLT is equal to or better than G-RMA/LCM's if double number of processors subtracted from ratio of sum of:*

- *product of $1+\alpha_{max}$ by maximum number of transactions in all jobs with higher priority than current job of τ_i and have direct or indirect conflict with transactions in τ_i.*

- *product of $1-\alpha_{min}$ by maximum number of transactions in all jobs with lower priority than current job of τ_i and have direct or indirect conflict with transactions in τ_i*

to total number of transactions in any job of τ_i is not less than Ω_i^k of any τ_i. Formally, schedulability of FBLT is equal to or better than G-RMA/LCM's if for each τ_i

$$\begin{aligned}\Omega_i^{max} & \\ \leq & \frac{\left((1+\alpha_{max})\sum_{\forall\tau_j\in\gamma_i^{ex},p_j>p_i}\left(\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\right)\right)}{|s_i|} \\ + & \frac{\left(2(1-\alpha_{min})\sum_{\forall\tau_j\in\gamma_i^{ex},p_j<p_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i^{ex}\neq\emptyset}\right)\right)}{|s_i|}-2m\end{aligned} \tag{7.11}$$

Proof. Proof is similar to proof of Claim 68 except that $RC^{to}_{G-RMA/LCM}(T_i)$ is given by (5.20). □

7.4.5 FBLT versus G-EDF/PNF

Claim 72. *Let $\Omega_i^{max}=max\{\Omega_i^k\}_{\forall s_i^k}$ be the maximum abort number for any preemptive transaction s_i^k in τ_i. Schedulability of FBLT is equal or better than G-EDF/PNF's if sum of double number of processors and maximum number of higher priority jobs- other than current job of τ_i - that can be released during T_i subtracted from ratio of sum of:*

- *Maximum number of transactions- in any job of $\tau_j\neq\tau_i$ that can exist during T_i- that have direct conflict with any transaction in τ_i.*
- *Floor of total number of transactions in any task $\tau_j\neq\tau_i$ - that has no direct conflict with any transaction in τ_i - divided by number of processors*

to total number of transactions in any job of τ_i is not less than Ω_i^{max} of any τ_i. Formally, schedulability of FBLT is equal to or better than G-EDF/PNF's if for each τ_i

$$\Omega_i^{max}\leq\frac{\sum_{\forall\tau_j\in\gamma_i}\left(\sum_{\forall s_j^l,\Theta_j^l\cap\Theta_i\neq\emptyset}\left(\left\lceil\frac{T_i}{T_j}\right\rceil+1\right)\right)+\left\lfloor\frac{\sum_{\forall\tau_j}\sum_{\forall s_j^h,\Theta_j^h\cap\Theta_i=\emptyset}}{m}\right\rfloor}{|s_i|}-\sum_{\forall\tau_j\in\zeta_i}\left\lfloor\frac{T_i}{T_j}\right\rfloor-2m \tag{7.12}$$

Proof. Proof is similar to proof of Claim 68 except that $RC^{to}_{G-EDF/PNF}(T_i)$ is given by (6.1) and $D_i(T_i)$ given by (6.4). □

7.4.6 FBLT versus G-RMA/PNF

Claim 73. *Let $\Omega_i^{max}=max\{\Omega_i^k\}_{\forall s_i^k}$ be the maximum abort number for any preemptive transaction s_i^k in τ_i. Schedulability of FBLT is equal or better than G-RMA/PNF's if sum* *of double number of processors and maximum number of higher priority jobs- other than current job of τ_i - that can be released during T_i subtracted from ratio of sum of:*

- *Maximum number of transactions- in any job of $\tau_j \neq \tau_i$ that can exist during T_i- that have direct conflict with any transaction in τ_i.*
- *Floor of double of total number of transactions in any task τ_j with lower priority than p_i - that has no direct conflict with any transaction in τ_i - divided by number of processors*

to total number of transactions in any job of τ_i is not less than Ω_i^k of any τ_i. Formally, schedulability of FBLT is equal to or better than G-RMA/PNF's if for each τ_i

$$\Omega_i^{max} \leq \frac{\sum_{\forall \tau_j \in \gamma_i} \left(\sum_{\forall s_j^l, \Theta_j^l \cap \Theta_i \neq \emptyset} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \right) + \left\lfloor \frac{2 \sum_{\forall \tau_j, p_j < p_i} \left(\sum_{\forall s_j^h, \Theta_j^h \cap \Theta_i = \emptyset} \right)}{m} \right\rfloor}{|s_i|} - 2m - \sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil \tag{7.13}$$

Proof. Proof is similar to proof of Claim 68 except that $RC^{to}_{G-RMA/PNF}(T_i)$ is given by (6.1) and $D_i(T_i)$ given by (6.2). □

7.4.7 FBLT versus Lock-free

Claim 74. *Let $\Omega_i^{max} = max\{\Omega_i^k\}_{\forall s_i^k}$ be the maximum abort number for any preemptive transaction s_i^k in τ_i. Under G-EDF, schedulability of FBLT is equal or better than schedulability of lock-free if for each task τ_i*

$$\frac{s_{max}}{r_{max}} \leq \frac{\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor}{m + |s_i| \left(\Omega_i^{max} + m - 1 \right) + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor}$$

Proof. Using Claim 65 and following the same steps of proof of Claim 16, schedulability of FBLT is equal or better than schedulability of retry-loop lock-free under G-EDF if for each task τ_i

$$\begin{aligned} & \left(m + \sum_{\forall s_i^k} \left(\Omega_i^k + m - 1 \right) + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor \right) s_{max} \\ \leq \quad & \left(\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor \right) r_{max} \end{aligned} \tag{7.14}$$

$$\frac{s_{max}}{r_{max}} \leq \frac{\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor}{m + \sum_{\forall s_i^k} \left(\Omega_i^k + m - 1 \right) + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor} \tag{7.15}$$

$\because$ $|s_i| = \sum_{\forall s_i^k}$, where $|s_i|$ is total number of transactions in any job of τ_i. $\because$ $\Omega_i^{max} \geq \Omega_i^k$, $\therefore$ (7.15) holds if

$$\frac{s_{max}}{r_{max}} \leq \frac{\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor}{m + |s_i| \left(\Omega_i^{max} + m - 1 \right) + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor} \tag{7.16}$$

Claim follows. □

Claim 75. *Let $\Omega_i^{max} = max\{\Omega_i^k\}_{\forall s_i^k}$ be the maximum abort number for any preemptive transaction s_i^k in τ_i. Under G-RMA, schedulability of FBLT is equal or better than schedulability of lock-free if for each task τ_i*

$$\frac{s_{max}}{r_{max}} \leq \frac{\sum_{\forall \tau_j \in \gamma_i} \left(\left\lceil \frac{T_i}{T_j} \right\rceil + 1 \right) \beta_{ij} + \sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil}{m + |s_i| \left(\Omega_i^{max} + m - 1 \right) + \sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil}$$

Proof. Proof is the same as proof of Claim 74 except that $RC_{i_{re}}$ under FBLT is given by (4.17) and LRC_{to} under lock-free is given by (4.31). □

7.4.8 FBLT versus Locking Protocols

Claim 76. *Following the same notations in Section 4.4.1, schedulability of FBLT is equal or better than schedulability of Global OMLP under G-EDF if for each τ_i*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_i \left(2m - 1 \right)}{m + |s_i| \left(\Omega_i^{max} + m - 1 \right) + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor}$$

Proof. Use (7.1) for $RC_{i_{to}}(T_i)$ and (7.2) for $D_i(T_i)$ under G-EDF/FBLT. Following the same steps of proof of Claim 18, Claim follows. □

Claim 77. *Following the same notations in Section 4.4.1, schedulability of FBLT is equal or better than schedulability of Global OMLP under G-RMA if for each τ_i*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_i \left(2m - 1 \right)}{m + |s_i| \left(\Omega_i^{max} + m - 1 \right) + \sum_{\forall \tau_j, p_j > p_i} \left\lceil \frac{T_i}{T_j} \right\rceil}$$

Proof. Proof is the same as proof of Claim 76. □

Claim 78. *Following the same notations in Section 4.4.4, schedulability of FBLT is equal or better than schedulability of RNLP under G-EDF if for each τ_i*

$$\frac{s_{max}}{L_{max}} \leq \frac{N_i \left(2m - 1 \right)}{m + |s_i| \left(\Omega_i^{max} + m - 1 \right) + \sum_{\forall \tau_j \in \zeta_i} \left\lfloor \frac{T_i}{T_j} \right\rfloor}$$

Proof. Use (7.1) for $RC_{i_{to}}(T_i)$ and (7.2) for $D_i(T_i)$ under G-EDF/FBLT. Following the same steps of proof of Claim 20, Claim follows. □

Claim 79. *Following the same notations in Section 4.4.4, schedulability of FBLT is equal or better than schedulability of RNLP under G-RMA if for each* τ_i

$$\frac{s_{max}}{L_{max}} \leq \frac{N_i\left(2m-1\right)}{m+\left|s_i\right|\left(\Omega_i^{max}+m-1\right)+\sum_{\forall\tau_j,p_j>p_i}\left\lceil\frac{T_i}{T_j}\right\rceil}$$

Proof. Proof is the same as proof of Claim 78. □

7.5 Conclusions

Transitive retry increases transactional retry costs under ECM, RCM, and LCM. PNF avoids transitive retry by avoiding transactional preemptions. PNF avoids transitive retry cost by concurrently executing non-conflicting transactions, which are non-preemptive. However, PNF requires a-priori knowledge about objects accessed by each transaction. Besides, PNF is a centralized CM. This is incompatible with dynamic STM implementations. Thus, we introduce the FBLT contention manager. Under FBLT, each transaction is allowed to abort for no larger than a specified number of times. Afterwards, the transaction becomes non-preemptive. Non-preemptive transactions have higher priorities than other preemptive transactions and real-time jobs. Non-preemptive transactions resolve their conflicts according to the order they become non-preemptive (i.e., FBLT aborts the later non-preemptive transaction in favour of the earlier non-preemptive transaction).

By proper adjustment of the maximum abort number for any preemptive transaction of any task τ_i (i.e., Ω_i^{max}), FBLT's schedulability is equal to or better than schedulability of other CMs. Ratio between s_{max} for FBLT on one side and r_{max} for lock-free and L_{max} for locking protocols on the other side also depends on Ω_i^{max}. As Ω_i^{max} decreases, s_{max}/r_{max} and s_{max}/L_{max} increase and FBLT acts more like a FIFO CM.

Chapter 8

FBLT Contention Manager with Checkpointing

In this chapter, we consider checkpointing [76] with software transactional memory (STM) concurrency control for embedded multicore real-time software, and present a modified version of FBLT contention manager called *Checkpointing FBLT* (CPFBLT). We upper bound transactional retries and task response times under CPFBLT, and identify when CPFBLT is a more appropriate alternative to FBLT without checkpointing.

The rest of this Chapter is organized as follows: We introduce checkpointing FBLT (CPFBLT) that combines original FBLT with checkpointing (Section 8.2). We present the motivation for introducing checkpointing into FBLT in Section 8.1. We establish CPFBLT's retry and response time upper bounds under G-EDF and G-RMA schedulers (Section 8.3). We also identify the conditions under which CPFBLT is a better alternative to non-checkpointing FBLT (Section 8.4). We implement FBLT and CPFBLT in the Rochester STM framework [90] and conduct experimental studies (Section **??**). Our results reveal that CPFBLT has shorter response time than non-checkpointing FBLT.

8.1 Motivation

Under checkpointing [76], a transaction $s_i^k \in \tau_i$ does not need to restart from the beginning upon a conflict on object θ. s_i^k just needs to return to the first point it accessed θ. If s_i^k needs to restart from its beginning, then the time between the beginning of s_i^k and the first access to θ is wasted. Besides, restarting s_i^k from its beginning increases the chances of aborting s_i^k by other transactions. Thus, response time of τ_i can be improved by checkpointing unless s_i^k acquires all its objects at its beginning. While previous CMs (i.e., ECM, RCM (Chapter 4), LCM (Chapter 5), PNF (Chapter 6) and FBLT (Chapter 7)) without checkpointing try to resolve conflicts using proper strategies, checkpointing enhances performance by reducing aborted part of each transaction. Thus, checkpointing acts as a complementary component to different CMs to further enhance response time. Checkpointing integrated into CMs allows programmers to reap STM's significant programmability and composability benefits for multicore embedded real-time software.

Behaviour of some CMs, like PNF (Chapter 6), can make checkpointing useless. PNF requires a priori knowledge of accessed objects within transactions. Only the first m non-conflicting transactions are allowed to execute concurrently and non-preemptively. Thus, PNF makes no use of checkpointing because there is no conflict between non-preemptive transactions.

Other CMs (e. g., FBLT (Chapter 7)) allow conflicting transaction to run concurrently. So, FBLT can benefit from checkpointing. FBLT, by definition, depends on LCM. LCM, in turn, depends on ECM for G-EDF and RCM for G-RMA. Like PNF, FBLT allows any transaction s_i^k to be a non-preemptive transaction if s_i^k has been aborted for a specified number of times Ω_i^k. Non-preemptive transactions cannot be aborted by preemptive transactions, nor by non-critical sections in real-time jobs. FBLT, unlike PNF, allow non-preemptive transactions to abort each other. Non-preemptive transactions resolve conflicts using time of being a non-preemptive transaction. As FBLT tries to combine advantages of other CMs, we extend FBLT to checpointing FBLT (CPFBLT) to improve response time over original FBLT.

8.2 Checkpointing FBLT (CPFBLT)

CPFBLT depends on LCM (Chapter 5) with checkpointing. So, we initially illustrate LCM integrated with checkpointing (Section 8.2.1). Afterwards, we illustrate FBLT with checkpointing in (Section 8.2.2).

8.2.1 Checkpointing LCM (CPLCM)

Algorithm 6 presents LCM integrated with checkpointing to give CPLCM. A new checkpoint is recorded for each newly accessed object θ by any transaction s_h^u (step 2). Checkpoint is recorded when θ is accessed for the first time because any further changes to θ will be discarded upon conflict. CPLCM uses priorities of s_i^k and s_j^l, the remaining length of s_i^k when it is interfered, as well as $len(s_j^l)$, to decide which transaction must be aborted. If s_j^l starts after s_i^k and $p_i^k > p_j^l$, then s_j^l would be the transaction to abort (step 6). Otherwise, c_{ij}^{kl}, α_{ij}^{kl} and α are calculated (steps 9, 10 and 11) to determine whether it is worth aborting s_i^k in favour of s_j^l. If $len(s_j^l)$ is relatively small compared to $len(s_i^k)$, then c_{ij}^{kl} approaches its minimum value (i.e., 0), and α_{ij}^{kl} approaches its maximum value (i.e., 1) (steps 9, 10). Otherwise, c_{ij}^{kl} approaches its maximum value (i.e., ∞), and α_{ij}^{kl} approaches its minimum value (i.e., 0). Ψ is a predefined threshold that lies in $[0, 1]$. The remaining execution length of s_i^k (i.e., δ_i^k) is used to calculate α (step 11). If s_i^k still has much work to do, then δ_i^k approaches $len(s_i^k)$ and α approaches 0. Otherwise, α approaches 1. If $len(s_i^k)$ is much

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Algorithm 6: CPLCM

Data: s_i^k and s_j^l are two conflicting atomic sections.
$\psi \rightarrow$ predefined threshold $\in [0,1]$.
$\delta_i^k \rightarrow$ remaining execution length of s_i^k when interfered by s_j^l.
$s\left(s_i^k\right) \rightarrow$ start time of s_i^k. $s\left(s_i^k\right)$ is updated each time s_i^k aborts and retries to the start time of the new retry.
$s\left(s_j^l\right) \rightarrow$ the same as $s\left(s_i^k\right)$ but for s_j^l.
$cp_h^u(\theta) \rightarrow$ recorded checkpoint in transaction s_h^u for newly accessed object θ.
Result: which atomic section of s_i^k or s_j^l aborts

foreach *newly accessed* θ *requested by any transaction* s_h^u **do**
  Add a checkpoint $cp_h^u(\theta)$
end
if $s\left(s_i^k\right) < s\left(s_j^l\right)$ **then**
  if $p\left(s_i^k\right) > p\left(s_j^l\right)$ **then**
    s_j^l aborts and retreats to $cp_j^l(\theta_{ij}^{kl})$;
    Remove all checkpoints in s_j^l recorded after $cp_j^l(\theta_{ij}^{kl})$
  else
    $c_{ij}^{kl} = len(s_j^l)/len(s_i^k)$;
    $\alpha_{ij}^{kl} = ln(\psi)/(ln(\psi) - c_{ij}^{kl})$;
    $\alpha = \left(len(s_i^k) - \delta_i^k\right)/len(s_i^k)$;
    if $\alpha \leq \alpha_{ij}^{kl}$ **then**
      s_i^k aborts and retreats to $cp_i^k(\theta_{ij}^{kl})$;
      Remove all checkpoints in s_i^k recorded after $cp_i^k(\theta_{ij}^{kl})$
    else
      s_j^l aborts and retreats to $cp_j^l(\theta_{ij}^{kl})$;
      Remove all checkpoints in s_j^l recorded after $cp_j^l(\theta_{ij}^{kl})$
    end
  end
else
  Swap s_i^k and s_j^l;
end

---

longer than $len(s_j^l)$, or s_i^k still has much work to do when interfered by s_j^l, then α tends to be smaller than α_{ij}^{kl}. Consequently, s_i^k aborts in favour of s_j^l. When s_i^k aborts upon a conflict with s_j^l on object θ_{ij}^{kl}, then checkpoints in s_i^k recorded after $cp_i^k(\theta_{ij}^{kl})$ are removed (step 14). Prior checkpoints to $cp_i^k(\theta_{ij}^{kl})$ remain the same. Also, if s_j^l aborts in favour of s_i^k, then all checkpoints in s_j^l recorded after $cp_j^l(\theta_{ij}^{kl})$ are removed (steps 7, 17).

8.2.2 CPFBLT

Algorithm 7 illustrates FBLT integrated with checkpointing to give CPFBLT. A new checkpoint is recorded for each newly accessed object θ by any transaction s_h^u (step 2). Checkpoint is recorded when θ is accessed for the first time because any further changes to θ will be discarded upon conflict. Each transaction s_i^k can be aborted during T_i for at most δ_i^k times. η_i^k records the number of times s_i^k has already been aborted up to now. If s_i^k and s_j^l have not joined the m_set yet, then they are preemptive transactions. Preemptive transactions resolve conflicts using CPLCM (step 5). Thus, CPFBLT defaults to CPLCM when the conflicting transactions (s_i^k and s_j^l) have not reached their δs (δ_i^k and δ_j^l). η_i^k is incremented each time s_i^k is aborted as long as $\eta_i^k < \delta_i^k$ (steps 8 and 22). Otherwise, s_i^k is added to the $m\_$ set and priority of s_i^k is increased to m_prio (steps 10 to 12 and 24 to 26). When the priority of s_i^k is increased to m_prio, s_i^k becomes a non-preemptive transaction. Non-preemptive transactions cannot be aborted by other preemptive transactions, nor by any other real-time job (steps 18 to 30). On the other hand, non-preemptive transactions can abort each other. The m_set can hold at most m concurrent transactions because there are m processors in the system. $r(s_i^k)$ records the time s_i^k joined the m_set (steps 11 and 25). When non-preemptive transactions conflict together (step 31), the transaction that joined m_set first becomes the transaction that commits first (steps 33 and 36). When s_i^k aborts due to a conflict on θ_{ij}^{kl} with s_j^l, then s_i^k retreats to $cp_i^k(\theta_{ij}^{kl})$. All checkpoints recorded after $cp_i^k(\theta_{ij}^{kl})$ are removed (steps 20, and 37). Similarly, s_j^l removes all checkpoints recorded after $cp_j^l(\theta_{ij}^{kl})$ if aborted by s_i^k (step 34).

8.3 CPFBLT Retry Cost

Claim 80. *Assume only two transaction s_i^k and s_j^l conflicting together. Let s_i^k begins at time $S\left(s_i^k\right)$ and s_j^l begins at time $S\left(s_j^l\right)$. Let $\triangle = S\left(s_j^l\right) - S\left(s_i^k\right)$. In the absence of checkpointing, retry cost of s_i^k due to s_j^l is given by*

$$BASE\_RC_{ij}^{kl} \leq \begin{cases} len\left(s_j^l\right) + \triangle & , -len\left(s_j^l\right) \leq \triangle \leq len\left(s_i^k\right) \\ 0 & , Otherwise \end{cases} \tag{8.1}$$

$BASE\_RC_{ij}^{kl}$ is upper bounded by

$$len\left(s_j^l + s_i^k\right) \tag{8.2}$$

Algorithm 7: The CPFBLT Algorithm

Data:
s_i^k: interfered transaction.
s_j^l: interfering transaction.
δ_i^k: maximum number of times s_i^k can be aborted during T_i.
η_i^k: number of times s_i^k has already been aborted up to now.
m_set: contains at most m non-preemptive transactions. m is number of processors.
m_prio: priority of any transaction in m_set. *m_prio* is higher than any priority of any real-time task.
$r(s_i^k)$: time point at which s_i^k joined m_set.
$cp_h^u(\theta) \rightarrow$ recorded checkpoint in transaction s_h^u for newly accessed object θ
Result: which transaction, s_i^k or s_j^l, aborts

1 **foreach** *newly accessed* θ *requested by any transaction* s_h^u **do**
2   Add a checkpoint $cp_h^u(\theta)$
3 **end**
4 **if** s_i^k, $s_j^l \notin m\_set$ **then**
5   Apply CPLCM (Algorithm 6);
6   **if** s_i^k *is aborted* **then**
7   **if** $\eta_i^k < \delta_i^k$ **then**
8   Increment η_i^k by 1;
9   **else**
10   Add s_i^k to m_set;
11   Record $r(s_i^k)$;
12   Increase priority of s_i^k to *m_prio*;
13   **end**
14   **else**
15   Swap s_i^k and s_j^l;
16   Go to Step 6;
17   **end**
18 **else if** $s_j^l \in m\_set, s_i^k \notin m\_set$ **then**
19   s_i^k aborts and retreats to $cp_i^k(\theta_{ij}^{kl})$;
20   Remove all checkpoints in s_i^k recorded after $cp_i^k(\theta_{ij}^{kl})$;
21   **if** $\eta_i^k < \delta_i^k$ **then**
22   Increment η_i^k by 1;
23   **else**
24   Add s_i^k to m_set;
25   Record $r(s_i^k)$;
26   Increase priority of s_i^k to *m_prio*;
27   **end**
28 **else if** $s_i^k \in m\_set, s_j^l \notin m\_set$ **then**
29   Swap s_i^k and s_j^l;
30   Go to Step 18;
31 **else**
32   **if** $r(s_i^k) < r(s_j^l)$ **then**
33   s_j^l aborts and retreats to $cp_j^l(\theta_{ij}^{kl})$;
34   Remove all checkpoints in s_j^l recorded after $cp_j^l(\theta_{ij}^{kl})$;
35   **else**
36   s_i^k aborts and retreats to $cp_i^k(\theta_{ij}^{kl})$;
37   Remove all checkpoints in s_i^k recorded after $cp_i^k(\theta_{ij}^{kl})$;
38   **end**
39 **end**

which is the same upper bound given by Proofs of Claims 1 and 3 in [41]

Proof. Due to absence of checkpointing, s_i^k aborts and retries from its beginning due to s_j^l. So, s_i^k retries for the period starting from $S\left(s_i^k\right)$ to the end of execution of s_j^l. s_j^l ends execution at $S\left(s_j^l\right) + len\left(s_j^l\right)$. If $S\left(s_j^l\right) < S\left(s_i^k\right) - len\left(s_j^l\right)$, then s_j^l finishes execution before start of s_i^k and there will be no conflict. Also, if $S\left(s_j^l\right) > S\left(s_i^k\right) + len\left(s_i^k\right)$, then s_j^l starts execution after s_i^k finishes execution and there will be no conflict. Thus, (8.1) follows. Equation (8.2) is derived by substitution of $\triangle$ by its maximum value (i.e., $\left(s_i^k\right)$). Claim follows. □

Claim 81. *Assume only two transactions s_i^k and s_j^l conflicting on one object θ. Let ∇_j^l be the time interval between the start of s_j^l and the first access to θ. Similarly, let ∇_i^k be the time interval between the start of s_i^k and the first access to θ. Let $\triangle$ be the time difference between start of s_j^l to the start of s_i^k. So, $\triangle < 0$ if s_j^l starts before s_i^k. Under checkpointing, s_i^k aborts and retries due to s_j^l for*

$$RC0_{ij}^{kl} \leq \begin{cases} len\left(s_j^l\right) - \nabla_i^k + \triangle & , if \begin{array}{c} \triangle \geq \nabla_i^k - len\left(s_j^l\right) \\ \triangle \leq len\left(s_i^k\right) - \nabla_j^l \end{array} \\ 0 & , Otherwise \end{cases} \tag{8.3}$$

$RC0_{ij}^{kl}$ is upper bounded by

$$len\left(s_j^l + s_i^k\right) - \nabla_j^l - \nabla_i^k \tag{8.4}$$

Proof. As s_i^k and s_j^l conflict only on one object θ, there will be no conflict before both s_i^k and s_j^l access θ. Retry cost of s_i^k due to s_j^l is derived by Claim 80 excluding parts of s_i^k and s_j^l before both transactions access θ. Thus, excluding the parts of s_i^k and s_j^l that do not cause conflict. So, $len\left(s_i^k\right)$ in Claim 80 is substituted by $len\left(s_i^k\right) - \triangle_i^k$. $len\left(s_j^l\right)$ is substituted by $len\left(s_j^l\right) - \triangle_j^l$. $\triangle$ in Claim 80 is substituted by $\triangle + \nabla_j^l - \nabla_i^k$. Claim follows. □

Claim 82. *Assume only two transactions s_i^k and s_j^l conflicting on a number of objects $\theta_1, \theta_2 \ldots \theta_z$. Let ∇_{i*}^k be the time interval between start of s_i^k and the first access to the first object accessed by s_i^k and shared with s_j^l (e.g., θ_i). Let ∇_{j*}^l be the time interval between start of s_j^l and the first access to the first object accessed by s_j^l and shared with s_i^k (e.g., θ_j). θ_i and θ_j may not be the same. With checkpointing, retry cost of s_i^k due to s_j^l is upper bounded by*

$$RC1_{ij}^{kl} \leq len\left(s_i^k + s_j^l\right) - \nabla_{i*}^k - \nabla_{j*}^l \tag{8.5}$$

Proof. Proof follows directly from Claim 81 by maximizing (8.4). $len\left(s_i^k\right)$, as well as, $len\left(s_j^l\right)$ in (8.4) cannot be changed. Thus, by choosing minimum values of ∇_i^k and ∇_j^l that correspond to shared objects between s_i^k and s_j^l, (8.4) is maximized. Claim follows. □

Claim 83. *If s_j^l is conflicting indirectly (through transitive retry) with s_i^k, then it is safe to ignore ∇_{i*}^k in calculating the upper bound of retry cost of s_i^k due to s_j^l.*

Proof. If s_j^l is conflicting indirectly with s_i^k, then s_j^l is accessing an object θ that does not belong to Φ_i^k. In this case, to get an upper bound for the retry cost of s_i^k due to s_j^l, ∇_{i*}^k assumes its minimum value in (8.5). Thus, $\nabla_{i*}^k = 0$. Claim follows. □

Claim 84. *Assume only two non-preemptive transactions s_i^k and s_j^l under CPFBLT. With checkpointing, retry cost of s_i^k due to direct or indirect conflict with s_j^l is upper bounded by*

$$RC2_{ij}^{kl} \le len\left(s_j^l\right) - \nabla_{i*}^k \tag{8.6}$$

where $\nabla_{i*}^k = 0$ in case of indirect conflict.

Proof. Proof follows directly from Claims 81, 82 and 83 except that s_j^l must have become non-preemptive before s_i^k. So, s_j^l starts execution non-preemptively before s_i^k. Otherwise, by definition of CPFBLT, s_j^l will not be able to abort s_i^k. Thus, $\triangle$ must not exceed 0. Claim follows. □

Claim 85. *Let s_i^k be a non-preemptive transaction under CPFBLT. Let χ_i^k be the set of transactions conflicting (directly or indirectly) with s_i^k. Each transaction $s_j^l \in \chi_i^k$ belongs to a distinct task. Transactions in χ_i^k are organized in non-increasing order of $RC2_{ij}^{kl}$ for each $s_j^l \in \chi_i^k$. Total retry cost of non-preemptive transaction s_i^k due to other non-preemptive transactions is upper bounded by*

$$RC3_i^k \le \sum_{a=1}^{a=min\left(|\chi_i^k|,m-1\right)} RC2_i^k\left(\chi_i^k(a)\right) \tag{8.7}$$

where $\chi_i^k(a)$ is the a^{th} transaction in χ_i^k. If $\chi_i^k(a) = s_j^l$, then $RC2_i^k\left(\chi_i^k(a)\right) = RC2_{ij}^{kl}$.

Proof. By definition of CPFBLT, a transaction s_i^k can be preceded by at most $m-1$ non-preemptive transactions. As non-preemptive transactions are organized in the order they become non-preemptive, no two non-preemptive transactions can belong to the same task. Maximum retry cost of non-preemptive s_i^k occurs when: 1) s_i^k is preceded by at most $m-1$ transactions conflicting with s_i^k. 2) Each conflicting transaction s_j^l to s_i^k must have one of the highest $m-1$ values for $RC2_{ij}^{kl}$. 3) Non-preemptive transactions preceding s_i^k are executing sequentially. Thus, retry cost of non-preemptive s_i^k can be upper bounded by Claim 84 for at most the first $m-1$ transactions in χ_i^k. If the third condition is not satisfied, then (8.7) still gives a correct, but not tight, upper bound. Claim follows. □

Claim 86. *Under CPFBLT, a preemptive transaction s_i^k aborts and retries for at most*

$$RC4_i^k \le \delta_i^k\left(len\left(s_i^k\right) - min\left(\nabla_{i*}^k\right)\right) \tag{8.8}$$

where $min\left(\nabla_{i*}^k\right)$ is the minimum ∇_{i*}^k for s_i^k and any other conflicting transaction s_j^l. If there are indirectly conflicting transactions with s_i^k, then $min\left(\nabla_{i*}^k\right) = 0$.

Proof. No transaction will make preemptive s_i^k aborts and retries before $min\left(\nabla_{i*}^k\right)$. By checkpointing, s_i^k will not retreat earlier than $min\left(\nabla_{i*}^k\right)$. By definition of CPFBLT, preemptive s_i^k aborts for at most δ_i^k times before it becomes non-preemptive. Claim follows. □

Claim 87. *The total retry cost of any job τ_i^x under CPFBLT due to 1) conflicts with other transactions during an interval L. 2) release of higher priority jobs during execution of preemptive transactions is upper bounded by*

$$RC(L)_{to}^i = \sum_{s_i^k \in s_i} \left(RC4_i^k + RC3_i^k\right) + RC_{re}(L) \tag{8.9}$$

where $RC_{re}(L)$ is the retry cost resulting from the release of higher priority jobs during execution of preemptive transactions. $RC_{re}(L)$ is calculated by Claim 1 in [?].

Proof. Following Claims 83, 85, 86 and Claim 1 in [?], Claim follows. □

Any newly released task τ_i^x can be blocked by m lower priority non-preemptive transactions. Blocking time D_i of any job τ_i^x is independent of checkpointing. Thus, D_i is calculated by Claim 3 in [?]. Claim 2 in [?] is used to calculate response time under CPFBLT where $RC_{to}(T_i)$ is calculated by (8.9).

8.4 CPFBLT vs. NCPFBLT

Claim 88. *Schedulability of CPFBLT is better or equal to schedulability of NCPFBLT if shared objects within each transaction s_i^k are accessed close to the end of execution s_i^k.*

Proof. Let upper bound on retry cost of any task τ_i^x during T_i under NCPFBLT be denoted as RC_i^{ncp}. RC_i^{ncp} is calculated by Claim 1 in [?]. Let upper bound on retry cost of any task τ_i^x during T_i under CPFBTL be denoted as RC_i^{cp}. RC_i^{cp} is calculated by (8.9). Let D_i be the upper bound on blocking time of any newly released task τ_i^x during T_i due to lower priority jobs. D_i is the same for both CPFBLT and NCPFBLT. D_i is calculated by Claim 2 in [?]. For CPFBLT schedulability to be better than schedulability of NCPFBLT:

$$\sum_{\forall \tau_i} \frac{c_i + RC_i^{cp} + D_i}{T_i} \leq \sum_{\forall \tau_i} \frac{c_i + RC_i^{ncp} + D_i}{T_i} \tag{8.10}$$

$\because$ D_i and c_i are the same for each τ_i under CPFBLT and NCPFBLT, then (8.10) holds if:

$$\forall \tau_i,\ RC_i^{cp} \leq RC_i^{ncp}$$

$$\begin{aligned} & \delta_i^k \left(len\left(s_i^k\right) - min\left(\nabla_{i*}^k\right)\right) + \sum_{a=1}^{min\left(|\chi_i^k|, m-1\right)} \left(len\left(\chi_i^k(a)\right) - \nabla_{i*}^k\right) \\ \leq \quad & \delta_i^k len\left(s_i^k\right) + \sum_{a=1}^{min\left(|\gamma_i^k|, m-1\right)} \left(len\left(\gamma_i^k(a)\right)\right) \end{aligned} \tag{8.11}$$

where γ_i^k is the set of at most $m-1$ longest transactions conflicting directly or indirectly with s_i^k. Thus, $\gamma_i^k(a) \geq \chi_i^k(a), \forall a$. Thus, by increasing ∇_{i*}^k, (8.11) holds. Claim follows. $\square$

8.5 Conclusion

Past research on real-time CMs focused on devloping different conflict resultion strategis for transactions. Except for LCM [40], no policy was made to reduce the length of conflicting transactions. In this paper, we analysed effect of checkpointing over FBLT CM. Analysis shows that response time of CPFBLT can be reduced than NCPFBLT if shared objects are accessed close to the end of execution of each transaction. Experimental evaluation reveals better response time for CPFBLT than NCPFBLT. Despite retry cost of NCPFBLT is lower than retry cost of CPFBLT, but this is natural as explained previously. Some CMs make no use of checkpointing due to behaviour of that CM (e.g, under PNF, all non-preemptive transactions are non-conflicting).

Chapter 9

Implementation and Experimental Evaluations

Chapter 10

Conclusions and Future Work

10.1 Conclusions

10.2 Future Work

From chapter4

Our work raises several questions. For example, what are the typical range of values for the different parameters that affect the retry cost (and hence the response time)? How tight is our retry and response time bounds in practice? Can real-time CMs be designed for other multicore real-time schedulers (e.g., partitioned, semi-partitioned), and those that dynamically improve application timeliness behavior? These are important directions for further work.

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