Image classification using topological data analysis techniques involves the construction of feature vectors representing various topological properties of the images. By associating a chain complex to each image, one can compute the topological persistence and subsequently generate a vector corresponding to this.

For instance, grayscale images can be viewed as a real valued function over a rectangular grid. This structure lends itself for the construction of cubical complexes corresponding to the level sets defined by the grayscale filtration. Feature vectors can then be generated by computing the persistent landscapes or entropy of the persistence barcodes.

In [1], different filtrations including height and radial were described for binary images. It also includes different functions for generating vectors from persistence barcodes. These can be applied to grayscale images on binarisation using a suitable threshold.

In the specific case of the MNIST data set, classification using random forest classifier on 52 feature vectors generated using height, radial, density, grayscale, line and Vietoris-Rips filtrations, followed by persistent entropy vectorisation, gives an accuracy of over 96%. For this, height and radial filtrations are seen to be the most important feature in the classification.

While the height filtration determines the birth and death of topological features along different directions in the image, the radial filtration looks at the same in correspondence to the distance from a fixed point in the image. Hence, by using these two filtrations, not only are we considering the topology of the 1-pixels in the image, but also their relative position in the rectangular grid. As a consequence of this, when the test images are subjected to rotation or translation, the accuracy of prediction drops significantly.

Ideally, we would like to perform classification using feature vectors which are invariant under isometric transformations. This would mean coming up with new filtrations or making suitable modifications to the ones previously mentioned so that they capture only the topological signatures and not the positions of the pixels

References

[1] A. Garin and G. Tauzin,"A Topological "Reading" Lesson: Classification of MNIST using TDA", https://arxiv.org/pdf/1910.08345.pdf