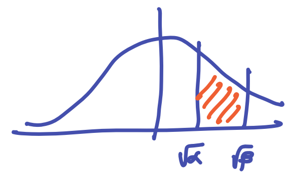
§ CHI-SQUARED DOUBLY STOCHASTIC SCALING

we want to be able to find α, β, δ such that

- $$I = \Big(\underbrace{2\cdot(\phi(\sqrt{\beta}) - \phi(\sqrt{\alpha}))}_{a}\Big)^{N^2} - \underbrace{4M \exp\left(-\frac{\delta^2 M \alpha^4}{\beta^2(\beta-\alpha)^2}\right)}_{b}$$ sufficiently close to 1

and $$\left(\frac{\beta}{\alpha}\right)^{3.5}\cdot 2 + 1 \le \frac{1}{\delta}$$

$$\Leftrightarrow \quad 1 \le \left(\frac{\beta}{\alpha}\right) \le \left(\left(\frac{1}{2}\right)\left(\frac{1}{\delta}-1\right)\right)^{\frac{2}{7}} = \boxed{\ell_\delta}$$



$$I \ge \underbrace{\left(2\, f(\sqrt{\beta})(\sqrt{\beta}-\sqrt{\alpha})\right)^{N^2}}_{\text{smaller than } a} - 4M \exp\left(\frac{-\delta^2 M}{\left(\frac{\beta}{\alpha}\right)^2\left(\left(\frac{\beta}{\alpha}\right)^2 - 1\right)}\right)$$

$$\ge \underbrace{\left(\sqrt{\frac{2}{\pi}}\, e^{-\frac{\beta}{2}}\right)^{M^2}(\sqrt{\beta}-\sqrt{\alpha})^{M^2}} - \underbrace{4M \exp\left(\frac{-\delta^2 M}{\ell_\delta^2(1-\ell_\delta)^2}\right)}_{c} = II$$

- For a fixed M : II if we wish to maximise c over δ

minimise $$\frac{\delta^2}{\ell_\delta^2(1-\ell_\delta)^2} \quad \sim \quad \text{minimise} \quad \frac{\delta^2}{\left(\frac{1}{\delta}-1\right)^{\frac{4}{7}}\left(1-\left(\left(\frac{1}{2}\right)\left(\frac{1}{\delta}-1\right)\right)^{2/7}\right)^2}$$

$$4M \exp\left(-kM\right) \le 1-P \qquad \exp(-kM) \le \frac{1}{4(1-P)M}$$

$$kM \ge \log(4-4P)M$$

$$k \ge \frac{\log(4-4P)M}{M}$$

$$\boxed{\frac{\delta^2}{\ell_\delta^2(1-\ell_\delta)^2} \ge \frac{\log 4(1-P)M}{M}}$$

Rough: $\underbrace{\exp(-k)}_{\text{larger}} \to$ smaller $\qquad dr(k)$: larger

Formulating as an optimisation problem

If we want α, β, δ at probability level p.