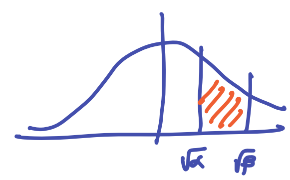
§ CHI-SQUARED DOUBLY STOCHASTIC SCALING

we want to be able to find α, β, δ such that

- $$I = \Big(\underbrace{2\cdot\big(\phi(\sqrt{\beta}) - \phi(\sqrt{\alpha})\big)}_{a}\Big)^{N^2} - \underbrace{4M \exp\left(-\frac{\delta^2 M \alpha^4}{\beta^2(\beta-\alpha)^2}\right)}_{b}$$ sufficiently close to 1

and $$\left(\frac{\beta}{\alpha}\right)^{3.5}\cdot 2 + 1 \le \frac{1}{\delta}$$

$$\Leftrightarrow \quad 1 \le \left(\frac{\beta}{\alpha}\right) \le \left(\left(\frac{1}{2}\right)\left(\frac{1}{\delta}-1\right)\right)^{\frac{2}{7}} = \boxed{\ell_\delta}$$



$$I \ge \underbrace{\left(2 f(\sqrt{\beta})(\sqrt{\beta}-\sqrt{\alpha})\right)}_{\text{smaller than } a}{}^{N^2} - 4M\exp\left(\frac{-\delta^2 M}{\left(\frac{\beta}{\alpha}\right)^2\left(\left(\frac{\beta}{\alpha}\right)^2-1\right)}\right)$$

$$\ge \underbrace{\left(\sqrt{\frac{2}{\pi}}\, e^{-\frac{\beta}{2}}\right)^{M^2}\left(\sqrt{\beta}-\sqrt{\alpha}\right)^{M^2}} - 4M\exp\underbrace{\left(\frac{-\delta^2 M}{\ell_\delta^2(1-\ell_\delta)^2}\right)}_{c} = II$$

- For a fixed M: II if we wish to maximise c over δ minimise $\dfrac{\delta^2}{\ell_\delta^2(1-\ell_\delta)^2} \sim$ minimise $\dfrac{\delta^2}{\left(\frac{1}{\delta}-1\right)^{\frac{4}{7}}\left(1-\left(\left(\frac{1}{2}\right)\left(\frac{1}{\delta}-1\right)\right)^{2/7}\right)^2}$

$$4M\exp\left(-kM\right) \le 1-P \qquad \exp(-kM) \le \frac{1}{4(1-P)M}$$

$$kM \ge \log(4-4P)M$$

$$k \ge \frac{\log(4-4P)M}{M}$$

$$\boxed{\frac{\delta^2}{\ell_\delta^2(1-\ell_\delta)^2} \ge \frac{\log 4(1-P)M}{M}}$$

Rough: $\underbrace{\exp(-k)}_{\text{larger}} \rightarrow$ smaller $\qquad$ dr(k) : larger

As $M \to \infty$ $\quad 4M \exp\left(-\dfrac{\delta^2 M \alpha^4}{\beta^2 (\beta-\alpha)^2}\right) \longrightarrow 0$

For fixed β, α, δ

Probability level $1-p$ $\qquad\qquad \beta/\alpha = \gamma$

$$4M \exp\left(-\frac{\delta^2 M}{\left(\frac{\beta}{\alpha}\right)^2\left(\frac{\beta}{\alpha}-1\right)^2}\right) = 4M \exp\left(\frac{-\delta^2 M}{\gamma^2(\gamma-1)^2}\right)$$

$$4M \exp\left(\frac{-\delta^2 M}{\gamma^2(\gamma-1)^2}\right)$$

- $\exp\left(\dfrac{-\delta^2 M}{\gamma^2(\gamma-1)^2}\right) = o\left(\dfrac{1}{M}\right)$

- $\dfrac{\delta^2 M}{\gamma^2(\gamma-1)^2} = o(\log M)$

γ growing at atleast $\boxed{o\left(\left(\dfrac{M}{\log M}\right)^{1/4}\right)}$

$$\underbrace{\left(\sqrt{\frac{2}{\pi}}\, e^{-\beta/2}\right.}_{\beta \text{ can't grow too fast}}\left.(\sqrt{\beta}-\sqrt{\alpha})\right)^{M^2}$$

$\beta = 2\gamma \log M + \log \frac{2}{\pi}$

$$\underbrace{\left[M^{-\gamma}\left((2\gamma(\log M)) - \sqrt{\alpha}\right)\right]}_{\text{Changing this?}}{}^{M^2}$$

$$\left(\sqrt{\frac{2}{\pi}}\, e^{-\beta/2} \sqrt{\alpha}\ \underbrace{\left(\sqrt{\beta/\alpha}-1\right)}_{o\left(\left(\frac{M}{\log M}\right)^{1/8}\right)}\right)^{M^2}$$

$$e^{-\beta M^2/2 + \frac{M^2}{2}\log\alpha + O\left(\frac{M^2}{8}(\log M)\right)}$$

$$\left(K(M)\, O\left(\left(\frac{M}{\log M}\right)^{1/8}\right)\right)^{M^2} \qquad O(1)$$