Diffraction

L. Biswal 1

According to geometrical optics, light travels along a rectilinear path in a homogenous and isotropic medium. However a minute investigation of this fact reveals that light suffers some deviation from it's straight path in passing close to the edges of opaque obstacles and narrow slits. Some of the light bends into the region of geometrical shadow and it's intensity falls of rapidly. This deviation from rectilinear propagation is extremely small when the wavelength of light is small in comparision to the dimensions of obstacle or aperture and is more pronounced when dimension of slit, aperture or boundary of obstacle are comparable with the wavelength of light. This indicates that, light after passing through a narrow slit or across a sharp edge (like razor blade), it spreads and enters into regions that could be in shadow if light travelled in straightlines. This phenomenon where light spreads around the edges of a barrier is called as diffraction. Thus diffraction broadly may be defined as:-

"When light falls on obstacles or small apertures whose size is comparable with the wavelength of light, there is a departure from straightline propagation and the light bends round the corners of the obstacles or apertures and enters into the geometrical shadow. This bending of light is called as diffraction".

Under ordinary circumstances, we seldom notice the diffraction of light sources such

2 as incandescent bulbs or the Sun. These are not the monochromatic point sources, and the diffraction pattern due to different parts of the source and to different wavelengths usually overlap and obscure each other.

Diffraction limits geometrical optics, in which we represent an electromagnetic wave with a ray. If we try to form a ray by sending light through a narrow slit or through a series of narrow slits, diffraction will always defeat our effort, because it always causes light to spread. Indeed, the narrower we make the slit in the hope of producing a narrower beam, the greater the spreading is.

Fresnel attributed the phenomenon of diffraction to be due to the mutual interference of secondary wavelets, originating from various points of the wavefront, which are not blocked off by an obstacle or are allowed to pass by a slit. That is, instead of finding the new wavefront by constructing the envelope of these secondary wavelets, we must find by principle of superposition, their resultant on every point of the screen, taking due account of their relative amplitudes and phases. Fresnel thus applied Huygene's principle of secondary wavelets in conjuction with the principle of interference and calculated the position of fringes in general agreement with the observed diffraction pattern.

Here it should be emphasized that, the process responsible for the production of diffraction phenomena, is going on continiously during the propagation of every wavefront. But the diffraction effects are observed only when a portion of the wavefront is cutoff by some obstacle.

<u>Note:</u> → Every optical instrument (even eye) in fact, make use of only a limited portion of the wavefront. A telescope or a microscope for example utilises only a limited portion of the wavefront transmitted by the objective lens. Therefore, some diffraction is always present in the image. (This reduces the resolution of the optical instrument).

Two classes of diffraction phenomena: →

Diffraction phenomena which arise as a result of some limitation on the width of wavefront are divided into two classes known for historical reasons as: →

1 → Fraunhofer diffraction.

2 → Fresnel diffraction.

1 → Fraunhofer diffraction: →

In this class, the source of light and the screen are effectively at infinite distance from the obstacle or aperature causing the diffraction. This means that, the wavefront incident on the obstacle or aperture is plane wavefront. The second -ary wavelets which we consider as originating from the unblocked portion of the wavefront at the moment it just touches the aperture, all start at the same instant of time. In other words, the phase of the secondary wavelets is the same at every point of the aperture. The screen is at infinity. Therefore we have to consider the interference as taking place between parallel diffracted rays which can be brought into focus by placing a convergent lens behind the aperture. Thus, it is very conveniently observed by employing two convergent lenses (i) one to render the light from the source parallel before its incidence on the aperature and (ii) the other to unite the parallel diffracted rays in focus on the screen, an arrangement which effectively removes the source and screen to infinity.

2→ <u>Fresnel class!</u> → In this class of diffraction, the source of light on the screen or both are at finite distances from the obstacle or aperture but no lenses are employed for rendering the rays parallel or convergent. Therefore, the incident wavefront is spherical or cylindrical instead of being plane. As a consequence of this, the phase of secondary wavelets is not same at all points in the plane of aperture causing diffraction. The resultant amplitude at any point, however, is obtained by mutual interference of secondary wavelets from different element of unblocked portion of the wavefront.

Here we shall limit ourselves to the Fraunhofer class of diffraction only.

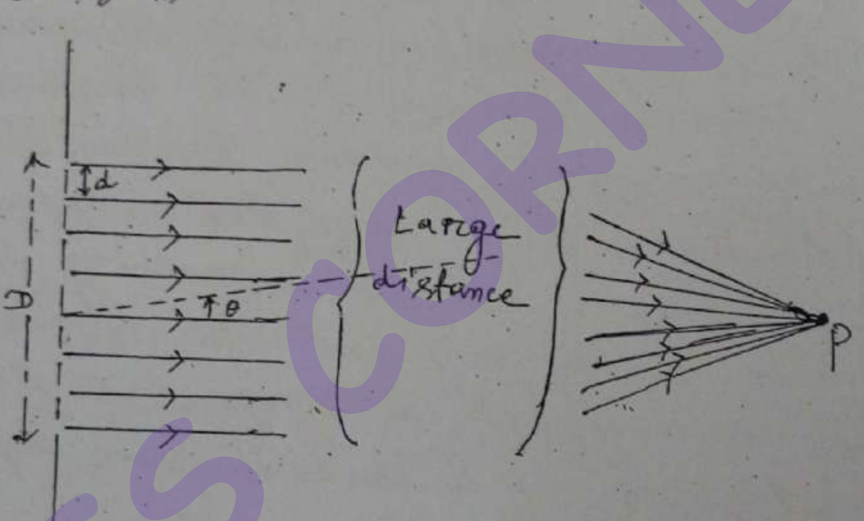
Fraunhofer class of diffraction :

In Fraunhofer class of diffraction, all the secondary waves start from the plane of aperture with same phase at any instant of time. At the point of superposition on the screen, they meet at different phases. Since, in this class of diffraction the distance between the slits and field point P is large, the variation of amplitude of successive disturbances reaching the point P can be safely neglected. So, in this class, actually the resultant wave at any point on screen is obtained due to large no of waves of same amplitude but different phases.

So let us first mathematically find out the resultant wave due to superposition of large no. of simple vibrations of equal amplitudes, same period and phases increasing in an arithmatic progression. [Such a situation arises, when we consider the diffraction pattern due to n - identical and uniformly spaced narrow slits.].

6) Superposition of large no. of simple vibrations of equal amplitudes, same period and phases increasing in an arithmatic progression:→

Consider the superposition of n $(\to \infty)$ waves coming from 'n' identical and uniformly spaced narrow slits at a point 'P' far away from the slits.



We shall suppose that, the distance between the centre lines of two consecutive slits is 'd' and that between the first and last is D. Then we can write

$$D = (n-1)d.$$

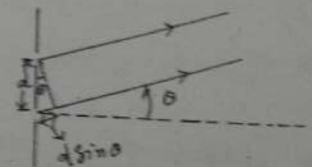
When a plane wave falls on this array of fine slits, the entire wavefront excepting the parts illuminating the n-slits is blocked. Each of the slits then become the source of secondary wavelets having same amplitude and same initial phase.

Let us find out the resultant effect due to superposition of all the secondary waves along a direction θ to the axis of the system at a distant field point P.

While superposing at 'P', the waves will have same amplitudes (Since P is at large distance the variation of amplitudes of successive disturbances reaching the point P can be safely neglected) but the phase difference between two waves from two successive slits reaching at P will have a phase difference given by: →

$\alpha = \phi_{i+1} - \phi_i$

$= \frac{2\pi}{\lambda}(d \sin\theta)$



[path difference between two consecutive waves in the direction θ is "d sinθ"].

Hence at 'P' large no. of waves of equal amplitudes but phases varying in arithmatic progression of 'α' $\left(= \frac{2\pi}{\lambda} d\sin\theta\right)$ superposes.

The resultant of all these vibrations can be obtained by phasor addition method or using the method of complex addition.

Let us assume that, secondary waves have amplitude 'a' at point of superposition 'P' and the angular frequency of vibration 'ω'. Assuming each to be sinusoidal wave, →

The resultant wave at P is :→

$$y = a\sin\omega t + a\sin(\omega t+\alpha) + a\sin(\omega t+2\alpha) + \cdots + a\sin\{\omega t+(n-1)\alpha\}$$

$$= a\left[\sin\omega t + \sin(\omega t+\alpha) + \sin(\omega t+2\alpha) + \cdots + \sin\{\omega t+(n-1)\alpha\}\right]$$

Now $\sin\omega t$, $\sin(\omega t+\alpha)$ ---- etc. can be considered as the imaginary parts of the complex functions $e^{i\omega t}$, $e^{i(\omega t+\alpha)}$ ------ etc.

Then $\sin\omega t + \sin(\omega t+\alpha) + \sin(\omega t+\alpha) + \cdots + \sin\{\omega t+(n-1)\alpha\}$

$$= I_m\left[e^{i\omega t} + e^{i(\omega t+\alpha)} + \cdots + e^{i\{\omega t+(n-1)\alpha\}}\right]$$

$$= I_m\left[e^{i\omega t}\left\{1+e^{i\alpha}+e^{2i\alpha}+\cdots+e^{i(n-1)\alpha}\right\}\right]$$

Now using the identity :-

$$1+x+x^2+\cdots+x^m = \frac{1-x^{m+1}}{1-x}, \text{ We get}$$

$$y = I_m\left[a e^{i\omega t}\cdot\frac{1-e^{in\alpha}}{1-e^{i\alpha}}\right]$$

$$= I_m\left[a e^{i\omega t}\cdot\frac{e^{in\alpha}-1}{e^{i\alpha}-1}\right]$$

$$= I_m\left[a e^{i\omega t}\cdot\frac{e^{\frac{in\alpha}{2}}\left\{e^{\frac{in\alpha}{2}}-e^{-\frac{in\alpha}{2}}\right\}}{e^{\frac{i\alpha}{2}}\left\{e^{\frac{i\alpha}{2}}-e^{-\frac{i\alpha}{2}}\right\}}\right]$$

$$= I_m\left[a e^{i\omega t}\cdot e^{i(n-1)\frac{\alpha}{2}}\cdot\frac{2i\sin\frac{n\alpha}{2}}{2i\sin\frac{\alpha}{2}}\right]$$

$$= I_m\left[a\cdot\frac{\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}\cdot e^{i\left\{\omega t+\frac{n-1}{2}\alpha\right\}}\right]$$

So

$$y = a\,\frac{\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}\cdot\sin\left[\omega t+\frac{n-1}{2}\alpha\right]$$

Substituting $a\dfrac{\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}} = R$, we get: → (1)

$$y = R\sin\left[\omega t + \frac{(n-1)}{2}\alpha\right] \quad \text{——— (2)}$$

This represents a sinusoidal wave of amplitude R and phase $\omega t + \beta$, where

$$\beta = \frac{(n-1)}{2}\alpha \quad \text{——— (3)}$$

This β is the average of the phases of the first and nth wave on arrival at point P.

putting $\alpha = \frac{2\pi}{\lambda} \cdot d\sin\theta$, eqn. (1) gives: →

$$R = a \cdot \frac{\sin\left[\frac{1}{2}n\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right]}{\sin\left[\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right]}$$

$$= a\frac{\sin\left[n\left(\frac{\pi}{\lambda}\right)d\sin\theta\right]}{\sin\left[\left(\frac{\pi}{\lambda}\right)d\sin\theta\right]} \quad \text{——— (4)}$$

Again since $D = (n-1)d$, we can write eqn (4) as: →

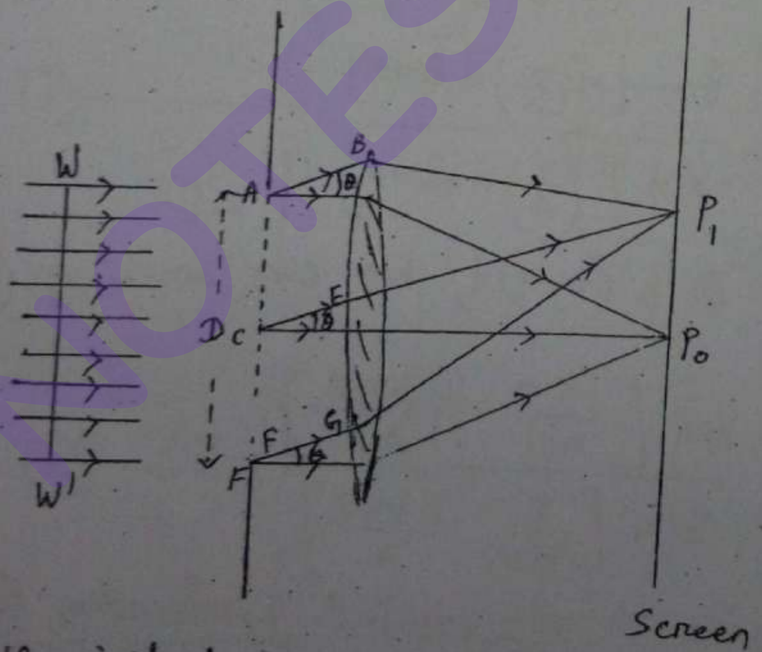
$$R = a\frac{\sin\left[\frac{n\pi}{n-1}\frac{D\sin\theta}{\lambda}\right]}{\sin\left[\frac{\pi}{n-1}\frac{D\sin\theta}{\lambda}\right]} \quad \text{——— (5)}$$

This formulation is of general application for any number of slits starting from $n=2$. We can also use it for finding the diffraction pattern of a single slit with a neat little trick.

At

Diffraction by a Single Slit

Consider a beam of parallel monochromatic light incidenting normally on an opaque plate in which there is a long narrow slit of width D. The light beam after transmitted through the slit spreads out perpendicularly to the length of the slit. Therefore when it is focussed on the screen by a lens, what is observed is a diffraction pattern. It consists of central band, much wider than the slit width, situated directly opposite to the slit and bordered by dark and bright bands of decreasing intensity. The central bright band is extremely intense and it's width is twice as great as that of the fainter sidebands.



AF → Setion of slit of width D in the plane of paper.
WW' → plane wavefront of monochromatic light of wavelen λ propagating normally to the slit.

At the instant,
when the unblocked portion of the incident plane wavefront is in the plane of slit, every point of the wavefront in the plane of slit ac ...

These secondary wavelets spread out to the right in all directions. The parts of each wavefront travelling normal to slit (i.e. in the direction of incident wave) are brought to focus at P_0 (by the lens). Those travelling at an angle θ with normal are brought to focus at P_1 and so on.

The resultant intensity at P_1 can be obtained by using eqn. (5).

Let us assume in this case that ~~[illegible]~~ n is made very large keeping the distance D fixed. Then the distance d between consecutive slits will become very small and in the limit when $n \to \infty$, the distance d will tend to zero and we shall be left with a single slit of width D.

When $n \to \infty$, we can write the numerator and denominator of eqn. (5) as: $\Rightarrow$

$$\sin\left[\frac{n\pi}{n-1} \cdot \frac{D\sin\theta}{\lambda}\right] = \sin\left(\frac{\pi D\sin\theta}{\lambda}\right)$$

and
$$\sin\left(\frac{\pi}{n-1} \cdot \frac{D\sin\theta}{\lambda}\right) = \sin\left(\frac{\pi}{n} \cdot \frac{D\sin\theta}{\lambda}\right) = \frac{\pi}{n} \cdot \frac{D\sin\theta}{\lambda}$$

[Measuring all angles in radian]

So
$$R = a \frac{\sin\left(\frac{\pi D\sin\theta}{\lambda}\right)}{\frac{\pi}{n} \times \frac{D\sin\theta}{\lambda}} \quad \text{--- (6)}$$

$$\Rightarrow \quad R = na \frac{\sin\left(\frac{\pi D \sin\theta}{\lambda}\right)}{\left(\frac{\pi D \sin\theta}{\lambda}\right)}$$

Substituting $na = A$ and $\frac{\pi D \sin\theta}{\lambda} = \beta$, we get

$$R = A \frac{\sin\beta}{\beta} \quad \text{--- (7)}$$

The resultant intensity at P_1 being proportional to the square of the resultant amplitude R, hence

$$I = KR^2 = KA^2 \frac{\sin^2\beta}{\beta^2} \quad \text{--- (8)}$$

$$= I_0 \frac{\sin^2\beta}{\beta^2} \quad \text{--- (8)}$$

(Substituting $I_0 = KA^2$)

Now let us proceed to analyse the intensity distribution on the screen.

Principal Maxima:→

The maximum value of $\frac{\sin\beta}{\beta}$ is 1. At this β, I is maximum and is equal to I_0.

But $\frac{\sin\beta}{\beta} = 1$ when $\beta \to 0$

$$\Rightarrow \frac{\pi D}{\lambda} \sin\theta = 0$$

$$\Rightarrow \theta = 0$$

The condition $\theta = 0$ simply means that this maxima is formed by parts of secondary wavelets which travel normally to the slit. The position of this maximum is therefore directly opposite to the slit. This maxima is called as principal maxima and is formed at P_0.

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Condition of Minima and Minimum Intensity positions: →

The intensity I will be zero at the positions for which $\sin\beta = 0$ but $\beta \neq 0$

$$\Rightarrow \beta = \pm\pi, \pm 2\pi, \pm 3\pi \text{ --- etc} = \pm n\pi$$

$$\Rightarrow \frac{\pi D \sin\theta}{\lambda} = \pm n\pi$$

$$\Rightarrow \sin\theta = \pm \frac{n\lambda}{D} \quad \text{——— (9)}$$

where $n = 1, 2, \text{---} \text{etc}$

[Note: → $n=0$ is not admissible as that is the condition of principal Maxima].

Eqn. (9) can be written as: →

$$D\sin\theta = \pm n\lambda \quad \text{——— (10)}$$

i.e the rays diffracted at an angle θ interfere destructively if the path difference between the extreme diffracted rays is an integral multiple of λ.

From eqn (9), the minimas are located on both sides of the principal Maxima at:-

$$\sin\theta = \pm\frac{\lambda}{D}, \pm\frac{2\lambda}{D}, \pm\frac{3\lambda}{D} \text{ --- etc.}$$

Secondary Maxima: →

With one principal Maximum and a number of minima, it is obvious that, there should also be secondary maxima in between minima. The intensity of these secondary maxima are less than that of the principal Maximum. The following procedure will tell us the position and intensity of these Seconda-ry Maximum.

I at any point is :→ $I = I_0 \frac{\sin^2\beta}{\beta^2}$

The positions of Maxima and minima corresponds to

$$\frac{dI}{d\beta} = 0 \Rightarrow \frac{d}{d\beta}\left[I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \right] = 0$$

$$\Rightarrow I_0 \, 2 \frac{\sin\beta}{\beta} \left[\frac{\beta\cos\beta - \sin\beta}{\beta^2} \right] = 0$$

This relation is satisfied if either (i) $\sin\beta = 0$

or (ii) $\beta\cos\beta - \sin\beta = 0$

The first condition, that is $\sin\beta = 0$ gives (i) the condition for principal Maximum when $\beta = 0$ and (ii) minimum when $\beta = \pm n\pi$.

Hence the Second condition $\beta\cos\beta - \sin\beta = 0$ is for Secondary Maxima. For this therefore,

$$\beta\cos\beta - \sin\beta = 0$$

$$\Rightarrow \tan\beta = \beta \quad \text{——— (11)}$$

The values of β satisfying the condition (11) are obtained graphically by plotting $y = \beta$ and $y = \tan\beta$ together. (See figure in next page).

The points of intersection of these plots are (approximately)

$$\beta = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \text{- - - -} \pm(2n+1)\frac{\pi}{2} \quad \text{or}$$

more exactly,

$$\beta = 0, \pm 1.430\pi, \pm 2.462\pi, \pm 3.471\pi, \ldots$$

$\beta = 0$ gives the principal Maximum and rest gives the condition for Secondary maximum.

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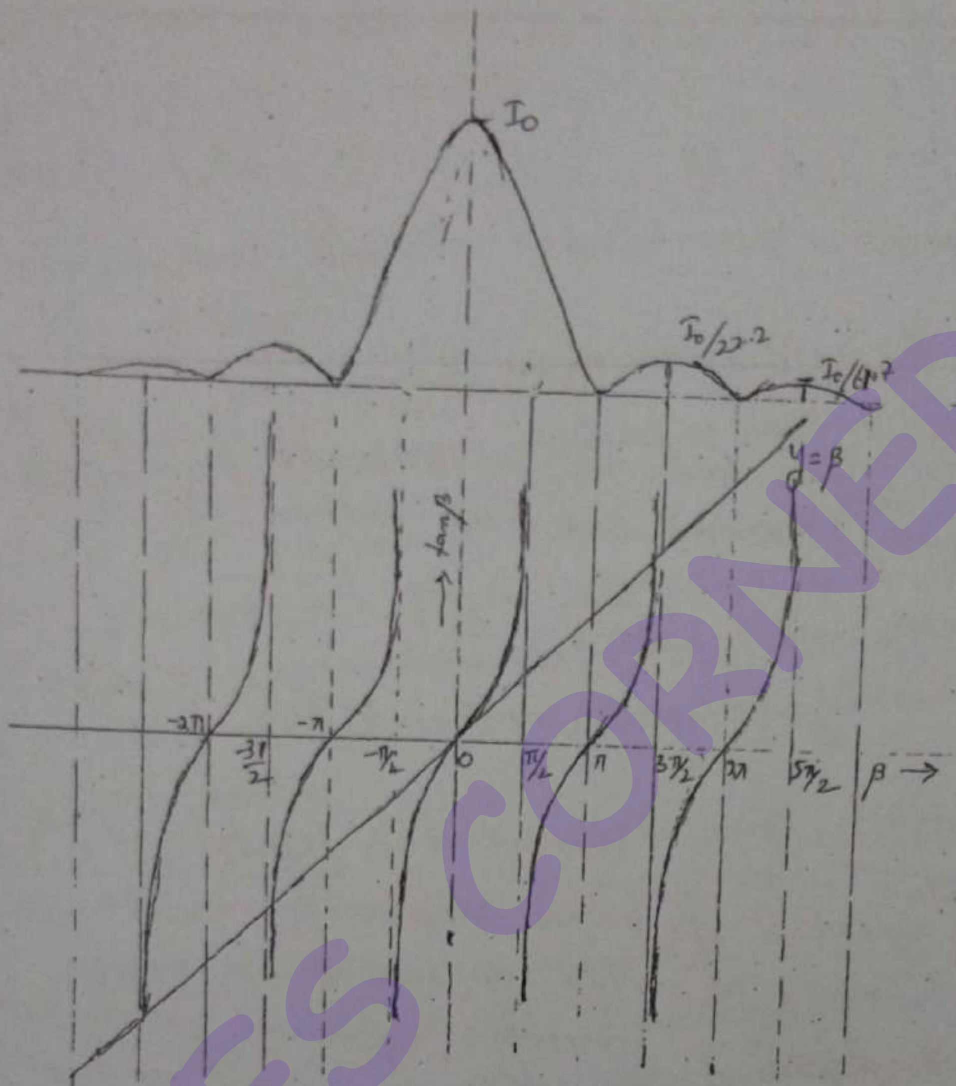

Intensity of Maxima :→

$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

putting $\beta = 0$, $\lim\limits_{\beta \to 0} \frac{\sin\beta}{\beta} = 1$

So $I = I_0$

So I_0 is the intensity of principal Maxima.

putting $\beta = \frac{3\pi}{2}$, $\frac{\sin\beta}{\beta} = \frac{(-1)}{3\pi/2}$

So $I_1 = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 = I_0 \times \frac{4}{9\pi^2} \simeq \frac{I_0}{22.2}$

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For $\beta = \frac{5\pi}{2}$, $I_2 = I_0\left(\frac{1}{5\pi/2}\right)^2 = \frac{4}{25\pi^2}\times I_0$

$\simeq \frac{I_0}{61.7}$

Similarly, $I_3 = \frac{I_0}{121}$ & so on.

So the ratio of intensities of principal maxima, 1st order maxima, 2nd order maxima etc is

$$1 : \frac{1}{22.2} : \frac{1}{61.7} : \frac{1}{121} : \dots$$

$$= 1 : 0.0469 : 0.0168 : 0.0083 : 0.0050 \text{ etc.}$$

Note: → (1) It should be pointed out that the angular spread of principal maxima (i.e. the distance between 1st minima in left to 1st minima in right $= \frac{2\lambda}{D}$) is inversely proportional to the width of the slit. Moreover if $D = \lambda$ and $n = 1$ in eqn (a) then $\sin\theta = \pm 1 \Rightarrow \theta = \pm\frac{\pi}{2}$, that is, the first minimum occurs at 90° with the normal (i.e. the incident direction). Under this condition, the length after traversing the slit spreads out in all directions with an intensity which decreases steadily as θ increases. There is no maxima and minima in this case, the slit will become just a source of secondary waves.

(2) If $D < \lambda$, then the original condition $D\sin\theta = \lambda$, for the 1st minima (i.e. $\sin\theta = \frac{\lambda}{D}$) gives $\sin\theta > 1$, which is impossible. Therefore the eqn (B) for intensity doesnot hold good when

If the slit width becomes less than one wavelength. The treatment breaks up.

Questions :→

1. Differentiate between Fresnel and Fraunhofer type of diffraction.
2. What do you mean by diffraction of light?
3. Differentiate between diffraction and interference.
4. Discuss the features of Fraunhofer diffraction pattern through a single slit when it is illuminated by a monochromatic light. Draw separately the intensity distribution in Fraunhofer single slit diffraction pattern.
5. What happens to the single slit diffraction pattern when the size of slit is :-
   (i) equal to the wavelength.?
   (ii) Smaller than the wavelength?
   (iii) Much larger than the wavelength.?
6. What is the significance of secondary Maxima in the diffraction pattern due to a single slit? Where are they located?
7. What is the effect of increase in wavelength in a Fraunhofer single slit diffraction pattern?
8. Describe the change in Fraunhofer single slit diffraction pattern, as the slit width is narrowed.
9. Describe the change in Fraunhofer single slit diffraction pattern if the slit width is gradually increased.
10. What happens to the Fraunhofer diffraction

pattern through a single slit on a screen if water replaces air in the space between the slit and screen.

11. Deduce the expression for the angular radius of the central peak of a Fraunhofer single slit diffraction pattern.

Numericals: →

1→ A single narrow slit is illuminated by monochromatic light of wavelength 6330 Å. If the angular width of the principal maxima is 1.2°, find the slit width. (Given $\sin(0.6)^\circ = 0.0105$).

2→ A monochromatic light beam of wavelength 590 is incident normally on a slit of width 0.04 cm. The diffracted beam is focussed on a screen by a convex lens of focal length 80 cm. Find the distance on the screen between the first minimum and centre of diffraction pattern.

3→ Light of wavelength 500 nm is incident on a slit having a width of 2.5 mm. Find the positions of the first minima and width of the central maxima on a screen 2m from the slit.

4→ A single slit is illuminated by light of wavelength λ_1 and λ_2 such that the first diffraction minimum of λ_1 component coincides with the second minimum of λ_2 component.

(a) what relationship exists between two wavelength

(b) Do any other minima in the two diffraction patterns coincide?

25.3.19

Magnetic Circuits

→ What is the requirement of magnetic circuit?
→ Some important terms in magnetic "
→ Comparison b/w electric & magnetic "
→ Series magnetic circuit & numericals
→ B-H Curve & Hysteresis loop.

$$E = -N\frac{d\phi}{dt} \quad , \quad H = \frac{NI}{l} \quad , \quad B = \mu H$$

$$\phi = BA \quad , \quad E_g = \frac{\phi ZN}{\varepsilon_0} \times \frac{P}{A}$$

MMF — Magneto motive Force (EMF) = N × I

Magnetic Field Intensity (H), Magnetising Force.

Reluctance (Resistance)
(S)

$$\frac{1}{S} = \text{Permeance (Conductance)}$$

→ EMF — Amt of word Done required to carry a unit magnetic pole once around a magnetic circuit

$$\Rightarrow H = \frac{NI}{l}$$ (l - length of magnetic circuit).

→ Opposition to the flow of magnetic flux is called as reluctance (S).

EMF = Current × Resistance

MMF = Flux × Resistance

Let $S = S_1 + S_2 + S_3$

$$\boxed{S = \frac{l}{A\mu}} \quad , \mu = \mu_0 \times \mu_r \quad (\mu_0 = 4\pi \times 10^{-7})$$

μ_r = relative permiability

Permeability of a material is defined as its ability of conducting magnetic lines of force or the ratio magnetic flux density by magnetic field intensity.

$$\boxed{\mu = \frac{B}{H}}$$

Relative permeability is defined as the ratio of flux density produced in the material to the flux density produced in air by the same magnetic field intensity

$$\boxed{B = \mu_r \mu_0 H}$$

Q. A coil of 600 turns and of resistance 20 ohms is wound uniformly over a steel ring of circumferance 30 cm and cross-sectional area $9 cm^2$. It is connected to supply of 20V DC. If the relative permeabity of the ring is 1600. Find the reluctance, MF Intensity, MMF, Magnetic Flux.

$N = 600$, $R = 20$

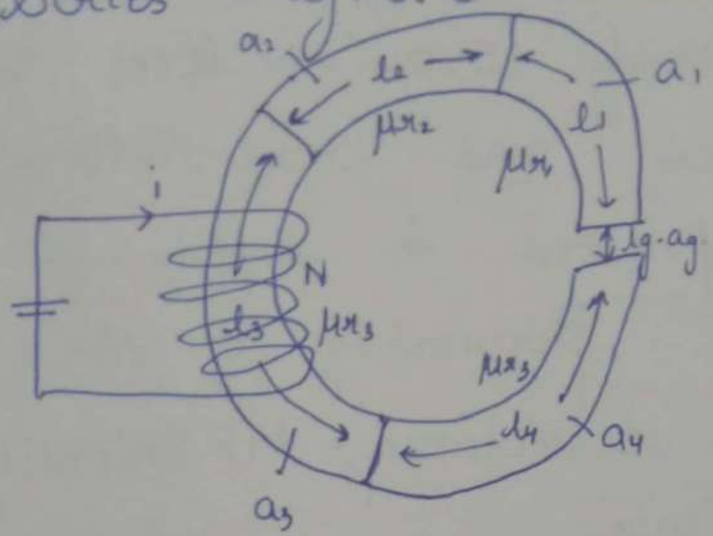
$S = \frac{l}{\mu_0 \mu_r A}$, $H = \frac{NI}{l}$

$MMF = N \times I$

Q. A steel ring having a cross-sectional area of $400 mm^2$ and mean circumferance of 400 mm has a coil of ~~400~~ 200 turns wound uniformly around it. ($\mu_r = 300$). Calculate the reluctance of the ring (ii) The current req. to produce a flux of 800 μwb in the ring.

$A = 400$, $N = 200$

⇒ Series Magnetic Circuit

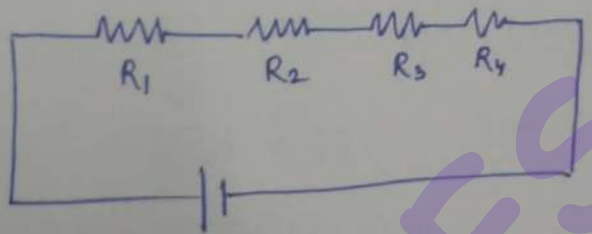


$$R = R_1 + R_2 + R_3 + R_4$$

$$E = i \cdot (R_1 + R_2 + R_3 + R_4)$$

$$S = S_1 + S_2 + S_3 + S_4 + S_g$$

$$= \frac{l_1}{\mu_0 \mu_{r_1} a_1} + \frac{l_2}{\mu_0 \mu_{r_2} a_2} + \frac{l_3}{\mu_0 \mu_{r_3} a_3} + \frac{l_4}{\mu_0 \mu_{r_4} a_4} + \frac{l_g}{\mu_0 \mu_{rg} g}$$



$$MMF = \frac{\phi l_1}{\mu_0 \mu_{r_1} a_1} + \frac{\phi l_2}{\mu_0 \mu_{a_2} a_2} + \cdots \frac{\phi l_g}{\mu_0 \cancel{\mu_{rg}} a_g}$$

$$MMF = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_4 l_4 + H_g l_g$$

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Total MMF in series Circuit.

$$MMF = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_4 l_4 + H_g l_g$$

$\downarrow$

NI

H = Magnetic field intensity

l = length of magnetic circuit

$$H = \frac{NI}{l}$$

$$MMF = \phi \left[\frac{\phi_1 l_1}{a_1 \mu_0 \mu_{r_1}} + \frac{l_2 \phi_2}{a_2 \mu_0 \mu_{r_2}} \right]$$

26.3.19

Q A iron ring of circular cross-sectional area $5 \times 10^{-4}\ m^2$ has a mean circumferance of 2 m. It has a saw cut of 2×10^{-3} m length and is wound with 800 turns of wire. Determine the exciting current when the flux in the air gap is 0.5×10^{-3} Wb. μ_r (iron) = 600 and leakage factor = 1.2. I = 5.9

$\Rightarrow$ $\phi_{in\ iron} = \phi_{in\ air\ gap} \times$ Leakage Current

$$MMF = NI = \phi\left[\frac{\phi_1 l_1}{a_1\mu_0\mu_{r_1}} + \frac{\phi_2 l_2}{a_2\mu_0\mu_{r_2}}\right]$$

$$600 \times I = \frac{0.6 \times 10^{-3} \times 2}{5 \times 10^{-4} \times 4 \times 10^{-7} \times 600 \times 3.14} + \frac{0.5 \times 10^{-3} \times 2 \times 10^{-3}}{5 \times 10^{-4} \times 4 \times 10^{-7} \times 3.14}$$

$$\left\{ \begin{aligned} \phi_1 &= 0.5 \times 10^{-3} \times 1.2 \\ &= 0.6 \times 10^{-3} \end{aligned} \right\}$$

$$= \frac{1.2 \times 10^{-3}}{12 \times 10^{3} \times 10^{-14} \times 3.14} + \frac{10^{-6}}{2 \times 10 \times 10^{-11} \times 3.14}$$

$$= \frac{0.1 \times 10^{-3} \times 10^{8}}{3.14} + \frac{0.5 \times 10^{-6} \times 10^{10}}{3.14}$$

$$= \frac{0.1 \times 10^{5}}{3.14} + \frac{0.5 \times 10^{4}}{3.14}$$

$$I = \cancel{\frac{10000 + 5000}{600}} \quad 0.03184 \times 10^{5} + 0.15924 \times 10^{5}$$

$$= \frac{3184 + 1592}{600} =$$

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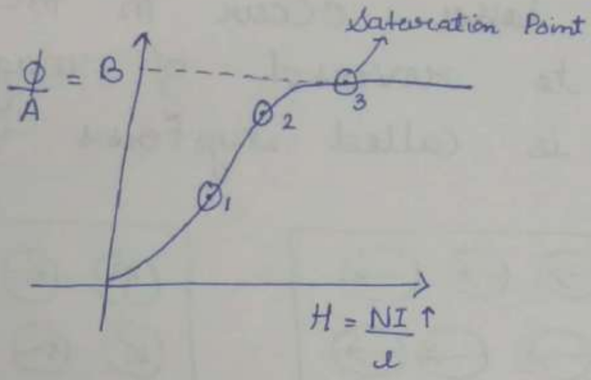
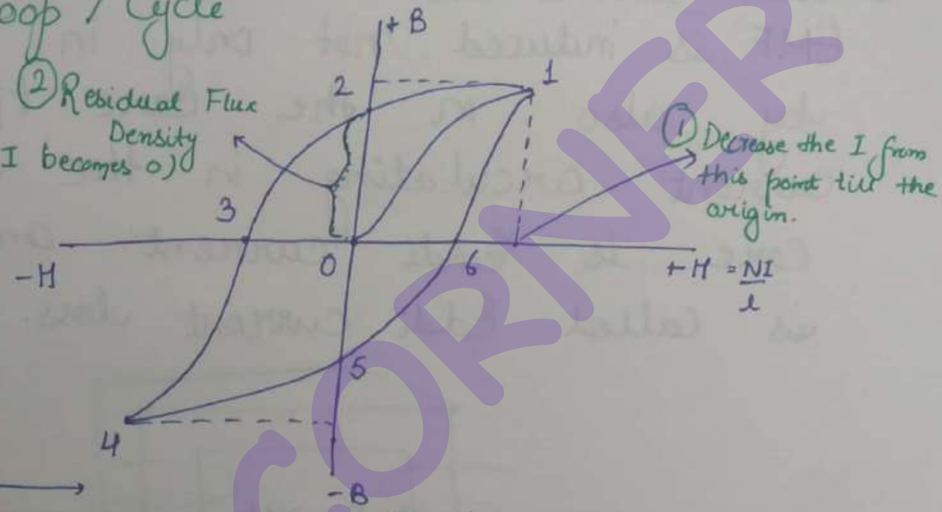
- B-H Curve : Hysteresis Loop

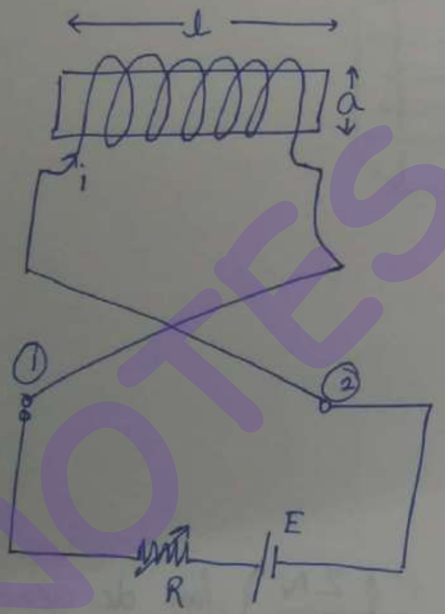
$\uparrow H = \frac{NI}{l} \uparrow$

$\uparrow B \quad , \phi \uparrow$



- Hystersis Loop / Cycle





③ Increase the I in negative direction by reversing the polarity [Coerceive Force] for the removal of residual flux.

④ After 3, saturation in negative axis is obtained, so bring the I to 0. This again increases the H.

Residual Flux Density

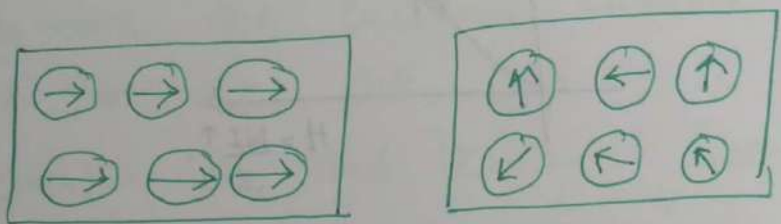
The value of magnetism, inside the magnetic material when the I supply is completely withdrawn.

Coerceive Force

It is the magnetising force required to remove the residual flux completely.

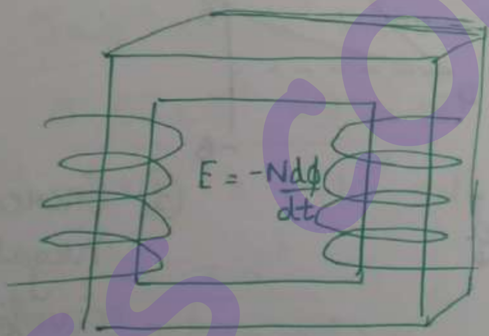
- Hysteresis Loss

Some losses occur in the transformer core due to reversal of magnetic field. That loss is called hysteresis loss.



- Eddi Current loss

EMF is induced not only in the winding by also in the core. Hence circulating current circulating in the transformer core is Eddi current and I^2R loss is called Eddi current loss.



EMF Equation.

$E_{max} = 2\pi f \phi_{max} N$ Volts

$E_{rms_1} = 4.44 . \phi_{max} f N_1$

$E_2 = 4.44 \phi_{max} f N_2$ $\left(E_g = \frac{P\phi Z N}{60 A}\right)$ for dc generator.

A = 2 = for wave winding

- P for lap winding

Z = No. of armature conductor

ϕ - Flux

→ Constructing

→ Working Principle

→ Types

→ Formula based Numericals

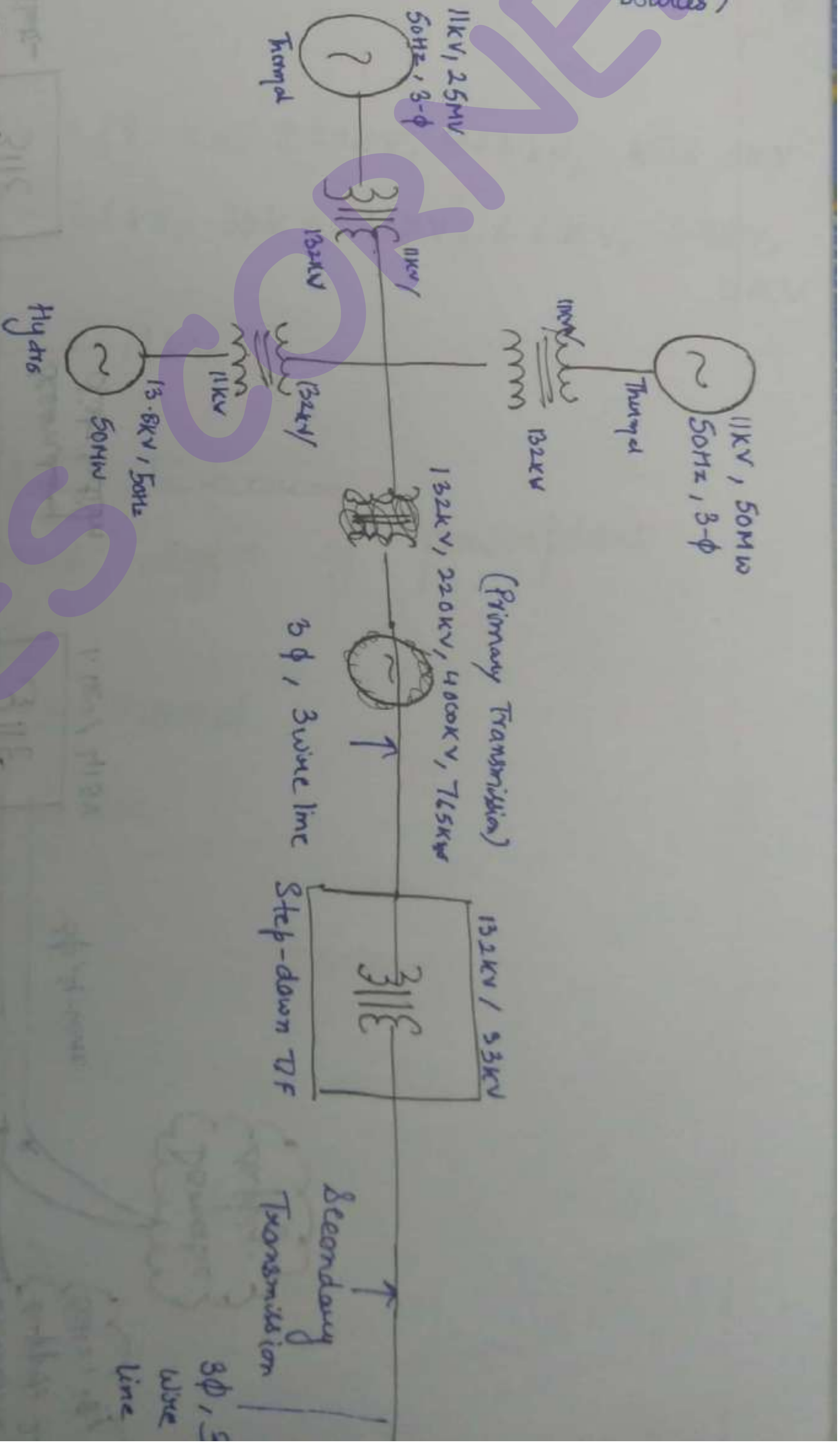
Power System

→ Generation → Different types of powerplants
→ Transmission .Only Govt. (Step-up) → Tr. line conductors, steel towers, Insulators
→ Distribution (Step-down)

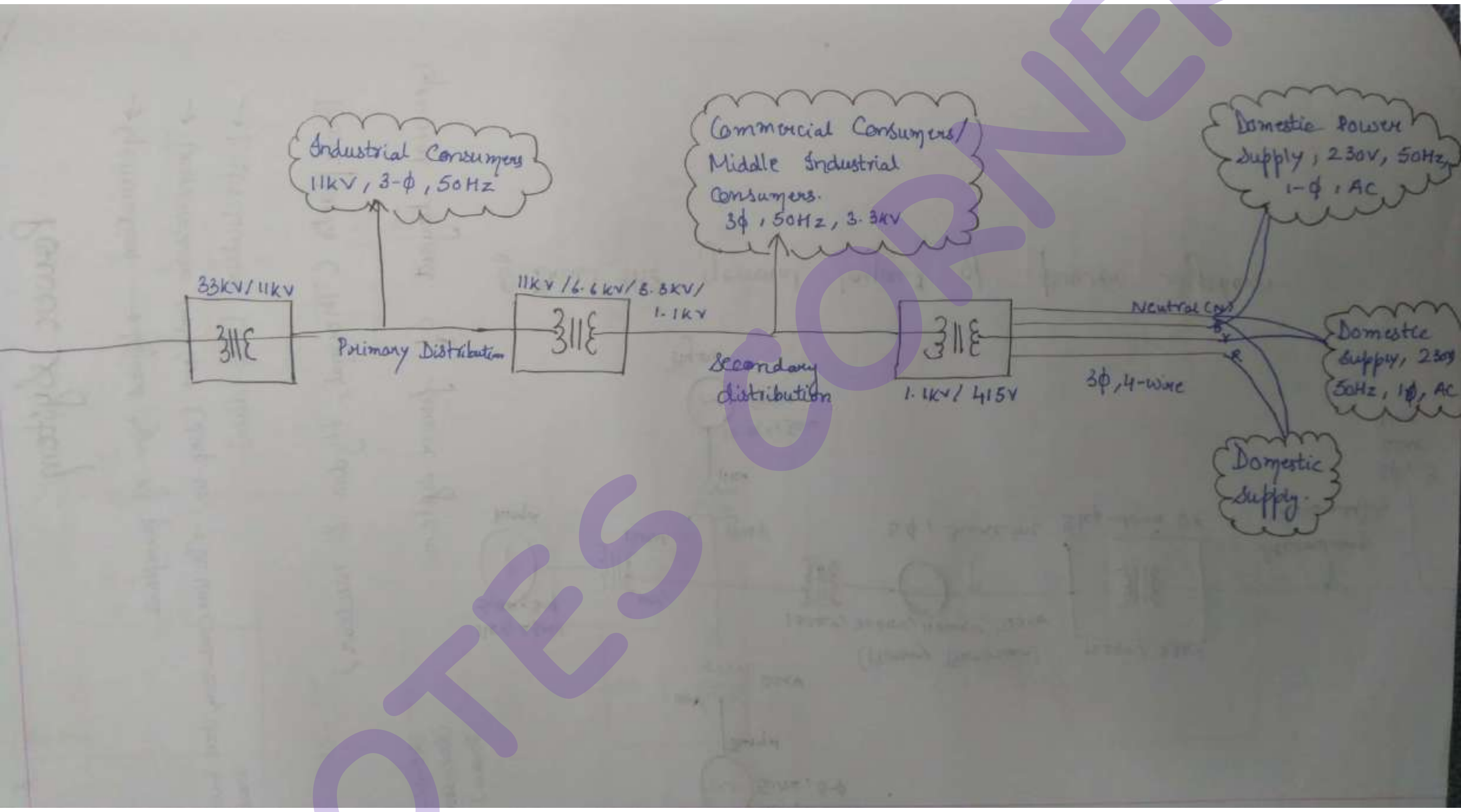
Powerplants (Thermal, Hydro & nuclear)

General layout of power system.

345mW – 30% (Renewable energy sources)



Q Draw the general layout of power system.



Industrial Consumers
11kV, 3-ϕ, 50Hz
Commercial Consumers/
Middle Industrial
Consumers.
3ϕ, 50Hz, 3.3kV
Domestic Power
Supply; 230V, 50Hz,
1-ϕ, AC
33kV/11kV
Primary Distribution
11kV/6.6kV/3.3kV/
1.1kV
Secondary
distribution
1.1kV/415V
Neutral (N)
3ϕ, 4-wire
Domestic
Supply, 230V
50Hz, 1ϕ, AC
Domestic
Supply.

Q. What are the components of powersystem?

1. Power station / Power supply
2. Step-up transformers
3. Transmission lines
4. Sub-stations
5. Step-down transformers
6. Other supplimentary equipments (switch gear and protection devices)

Q. What are the different transmission and distribution of voltages in India?

Transmission voltages — 132kV, 220kV, 400kV, 476 5kV

Distribution voltages — 66kV, 33kV, 11kV, 6.6kV, 3.3kV, 11kV

Generation voltage — 11kV

- Conductors — Copper, aluminium

Q. Comparison b/w different types of powerplant.

Initial Set-up Cost

Nuclear > Hydro > Thermal

Cleanest one ⟶ Hydro

Q. What are the different types of energy sources?

1. Renewable and non-renewable

2. Commercial and non-commercial

3. Conventional and non-Conventional

4. Primary and Secondary source of energy

Q. Comparison b/w nuclear, thermal & hydro powerplant

Q. What are the diff. types of wiring techniques?

Q. What is the requirement and different type of earthing.

Q. What are the electrical safety rules for the industry

Q. Gives the classification of different types of energy sources

Date 2 12 16

my companion

OSCILLATION & WAVES

(1) Free Oscillation

The to and fro motion of a body about a mean position, maintaining the amplitude constant with respect to time is called as free oscillation.

In free oscillation generally there is no loss of energy.

A body undergoing free oscillation experiences two type of forces

i) inertial force (F_I)

ii) Restoring force (F_R)

Inertial force

~~The~~ Inertial force is directly proportional to the acceleration.

$$F_I = M\frac{d^2y}{dt^2}$$, where, m = mass of oscillating body, y = displacement

Restoring force

Once the body is displaced from the equillibrium position it has tendency to come back to the equillibrium position.

Generally it comes with a force called restoring force. It is directly proportional to the displacement and acts in the direction opposite to the displacement.

So it is expressed by.

$$F_R = -Ky$$, where K = const (gamma/restoring)

→ In equilibrium, $F_I = F_R$

$\Rightarrow M\frac{d^2y}{...} = -Ky$

$$\Rightarrow \frac{d^2y}{dt^2} = -\frac{K}{M} y.$$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + \frac{K}{M} y = 0} \longrightarrow \text{Basic equation for free oscillation}$$

The solution of this equation becomes,

$$y = A \sin(\omega t + \phi)$$

- $A \rightarrow$ Amplitude
- $\phi \rightarrow$ phase difference
- $\omega \rightarrow$ Angular freq.

Here, $\omega^2 = \frac{k}{m}$.

So, the differential equation for free oscillation can also be written as.

$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$$

Here, ω = Natural Angular freq. of the oscillation

$$\Rightarrow \omega = 2\pi\nu = \frac{2\pi}{T}$$

ϕ = initial phase $\Rightarrow \phi = kx$.

$$\boxed{y = A \sin(\omega t + kx)}$$

(2) Damped Oscillation:

In the damped oscillation, the oscillating body continuously loses its energy to overcome the opposing forces like friction. In this case, the body experiences three type of forces.

(i) Inertial Force (F_I)
(ii) Restoring Force (F_R)
(iii) Damping Force (F_D)

Fiea.

my companion

Inertial force

Inertial force is directly proportional to acceleration

$F_I = M\frac{d^2y}{dt^2}$, where m = mass of oscillating body
y = displacement.

Restoring force

Once the body is displaced from the equillibrium position it has tendency to come back to equillibrium position.

$F_R = -ky$, k = gauss/restoring const.

Damping force

This force tries to oppose the motion. It is directly proportional to the velocity and is given by,

$F_D = -r\frac{dy}{dt}$ r = damping const.

→ In equillibrium

$F_I = F_R + F_D \Rightarrow M\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt}$

$\Rightarrow M\frac{d^2y}{dt^2} + ky + r\frac{dy}{dt} = 0$.

$\Rightarrow \boxed{\frac{d^2y}{dt^2} + \left(\frac{r}{m}\right)\frac{dy}{dt} + \left(\frac{k}{m}\right)y = 0}$

$\Rightarrow \boxed{\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = 0}$ —(1) → diff eqn for damped oscillation.

here, $2b = \frac{r}{m}$, $\omega^2 = \frac{k}{m}$

The solution to the above equation can be as follows:-

$$y = Ae^{\alpha t}$$

So

$$\frac{dy}{dt} = A\alpha e^{\alpha t}$$

& $$\frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t}$$

Putting these values in eq^n ①

$$\Rightarrow A\alpha^2 e^{\alpha t} + 2bA\alpha e^{\alpha t} + \omega^2 Ae^{\alpha t} = 0$$

$$\Rightarrow \alpha^2 + 2b\alpha + \omega^2 = 0$$

$$\Rightarrow \alpha = \frac{-2b \pm \sqrt{4b^2 - 4\omega^2}}{2}$$

$$\Rightarrow \alpha = \frac{-2b \pm 2\sqrt{b^2 - \omega^2}}{2}$$

$$\Rightarrow \boxed{\alpha = -b \pm \sqrt{b^2 - \omega^2}}$$

Sol^n for displacement is

$$\boxed{y = A_1 e^{(-b+\sqrt{b^2-\omega^2})t} + A_2 e^{(-b-\sqrt{b^2-\omega^2})t}} \quad ②$$

→ General solution of displacement for damped oscillation

Q) i) Formulate the equation for damped harmonic oscillation and find out its general solution

ii) Discuss the case of overdamping / underdamping / critical damping

The solutions depend on the different cases:-

i) Overdamping $(b > \omega)$

(ii) Critical damping $(b = \omega)$

(iii) Under damping $(b < \omega)$

(i) Overdamping $(b > \omega)$:-

In this case $\sqrt{b^2 - \omega^2}$ is a real positive number whose value is less than b.

So,

$\left(-b + \sqrt{b^2 - \omega^2}\right)$ is a negative number and $\left(-b - \sqrt{b^2 - \omega^2}\right)$ is a more negative number.

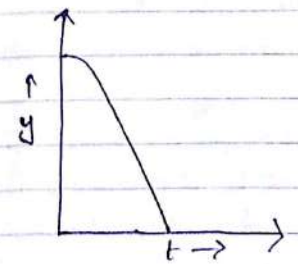
$\therefore$ The displacement is governed by the first term of eqn ②.

i.e

$$\boxed{y = A_1 e^{\left(-b + \sqrt{b^2 - \omega^2}\right)t}}$$

This eqn represent that with increase in time displacement will go on decreasing (As $-b + \sqrt{b^2 - \omega^2}$ is a $-ve$ no.)

So, in this case the displacement with respect to time can be shown in the graph as below.



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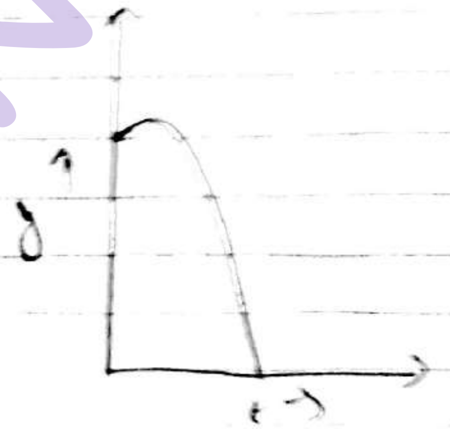
δ
t

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(iii) **Underdamping ($b<\omega$) ;-**

In this case $\sqrt{b^2-\omega^2}$ is an imaginary number

Let,

$$\sqrt{b^2-\omega^2} = i\beta \quad , \quad \beta = \sqrt{\omega^2-b^2}, \quad i = \sqrt{-1}$$

In this case, the general solution for displacement is as follow.

$$y = A_1 e^{(-b+\sqrt{b^2-\omega^2})t} + A_2 e^{(-b-\sqrt{b^2-\omega^2})t}$$

$$\Rightarrow y = A_1 e^{(-b+i\beta)t} + A_2 e^{(-b-i\beta)t}$$

$$\Rightarrow y = e^{-bt} \times \left(A_1 e^{i\beta t} + A_2 e^{-i\beta t}\right)$$

$$\Rightarrow y = e^{-bt} \times \left[A_1(\cos\beta t + i\sin\beta t) + A_2(\cos\beta t - i\sin\beta t)\right]$$

$$\Rightarrow y = e^{-bt}\left[(A_1+A_2)\cos\beta t + i(A_1-A_2)\sin\beta t\right]$$

Let $(A_1+A_2) = \sin\phi$

$i(A_1-A_2) = \cos\phi$

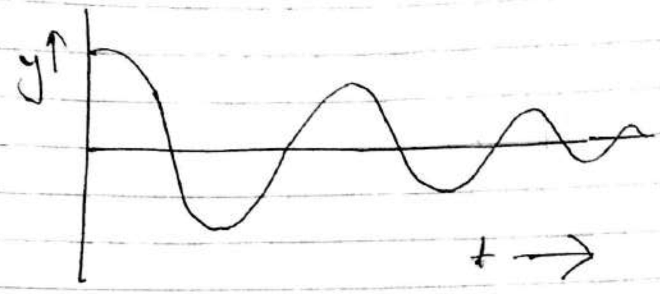
under this assumption

$$\Rightarrow y = e^{-bt}\left[\sin\phi\cos\beta t + \cos\phi\sin\beta t\right]$$

$$\Rightarrow \boxed{y = e^{-bt}\sin(\beta t+\phi)}$$ $\rightarrow$ Solⁿ for displacement in underdamping condition.

From this equation, it is clear that in the underdamping condition, the displacement will be oscillatory with decreasing amplitude.

This can be represented as follows:-



This oscillation have angular frequency $= \beta$.
Let the time period is T'

$\Rightarrow \dfrac{2\pi}{T'} = \beta$

$\Rightarrow T' = \dfrac{2\pi}{\beta}$

$$\Rightarrow \boxed{T' = \frac{2\pi}{\sqrt{\omega^2 - b^2}}}$$

From this expression it is clear that the time period of oscillation increases.

(3) Forced Oscillation :-

When a body ~~oscilla~~ capable of oscillation is subject to external periodic force then the body oscillate with the frequency other than the natural frequency. Such type of oscillation is called as forced oscillation.

When a body ~~is under forced~~ undergoes forced oscillation in presence of damping then it experi the following forces:-

(i) Internal Force
(ii) Restoring Force
(iii) Damping Force
(iv) External Force

(i) Inertial force :-

Inertial force is directly proportional to acceleration.

$F_I = M\frac{d^2y}{dt^2}$, where m = mass of oscillating body
y = displacement.

(ii) Restoring force :-

Once the body is displaced from the equilibrium position, it has tendency to come back to equilibrium position. It is opposite and proportional to displacement

$F_R = -Ky$, , K = gain/restoring constant.

(iii) Damping force :-

This force tries to oppose the motion. It is directly proportional to the velocity and is given by:-

$F_D = -r\frac{dy}{dt}$, r = damping const.

(iv) External force :-

The periodic external force can be defined as.

$F_{ex} = F \sin pt$, where
F = Amplitude of ext. force
p = Angular freq of ext. force

In equillibrium condition,

$$F_I = F_R + F_D + F_{ex}$$

$$\Rightarrow m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} + F\sin pt.$$

$$\Rightarrow M\frac{d^2y}{dt^2} + r\frac{dy}{dt} + ky = F\sin pt.$$

$$\Rightarrow \frac{d^2y}{dt^2} + \left(\frac{r}{m}\right)\frac{dy}{dt} + \left(\frac{k}{m}\right)y = \left(\frac{F}{m}\right)\sin pt.$$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f\sin pt} \qquad 2b = \frac{r}{m},\ \omega^2 = \frac{k}{m}$$

→ equation of motion for forced oscillation in presence of damping.

The solution of above equation can be as follows:

$$\boxed{y = A\sin(pt - \phi)}$$

where A = Amplitude of forced oscillation

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}$$

ϕ = phase lag betn displacement and applied force.

Generally, when the external force is there the body takes some time to follow it. This time is reflected in terms of phase lag.

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Generally, ϕ is given by,

$$\boxed{\phi = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right)}$$

Amplitude Resonance :-

This is a condition of forced oscillation where the amplitude is extremely large.

Derivation of Condition for Amplitude Resonance :-

We know in forced oscillation,

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}$$

for maximum value of A (Amplitude Resonance)

$$\frac{d}{dp}\left[(\omega^2 - p^2)^2 + 4b^2p^2\right] = 0$$

$$\boxed{p = \sqrt{\omega^2 - 2b^2}} \rightarrow \text{Cond}^n \text{ for resonance.}$$

$$\Rightarrow 2(\omega^2 - p^2) \times (-2p) + 8b^2p = 0$$

$$\Rightarrow \boxed{p = \sqrt{\omega^2 - 2b^2}} \rightarrow \text{cond}^n \text{ of resonance in absence of damping}$$

If no damping, the $b = 0$

$\Rightarrow \boxed{p = \omega} \rightarrow$ Condn of resonance in absence of damping

Expression for amplitude in resonance condition

We know

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}$$

In the resonance condn, $p^2 = \omega^2 - 2b^2$

$$A_{res} = A_{max} = \frac{f}{\sqrt{(\omega^2 - \omega^2 + 2b^2)^2 + 4b^2(\omega^2 - 2b^2)}}$$

$$= \frac{f}{\sqrt{4b^4 + 4b^2\omega^2 - 8b^4}}$$

$$= \frac{f}{\sqrt{4b^2\omega^2 - 4b^4}}$$

$$\boxed{A_{res} = A_{max} = \frac{f}{2b\sqrt{\omega^2 - b^2}}}$$

Expression for amplitude of resonance in presence of damping.

In absence of damping $(b = 0)$

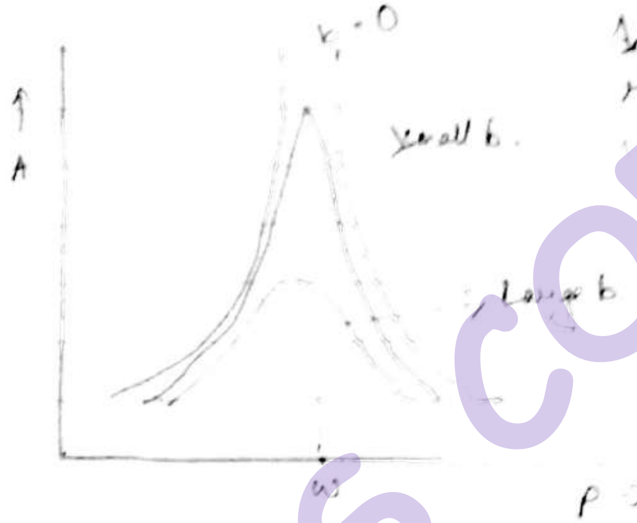
$$\boxed{A_{max} \rightarrow \infty}$$

Sharpness of Resonance :

The rate of decrease of amplitude with respect to frequency of external force around natural frequency defines the sharpness of resonance.

More the rate of decrease of amplitude sharper is the resonance (sharpness of resonance).

The resonance can be graphically as the figure below



In this case sharpness of resonance :

i) maximum : b = 0, sharpness decreases for increasing b.

ii) for large value of b resonance get flattened down. For useful application one needs sharper resonance

Q) Formulate the eqⁿ of motion for forced oscillation (damping)

(i) Write down general soln of amplitude

(ii) Discuss amplitude resonance

Q) What do you mean by sharpness of resonance

Waves

Differential form of wave equation :-

Let, a wave is represented by the equation

$$y = A\sin(\omega t + kx)$$

here A = Amplitude

$\omega = 2\pi\nu$

$\nu \rightarrow$ freq. of wave

$k = \frac{2\pi}{\lambda}$

$\lambda \rightarrow$ wavelength of wave

$V = \lambda\nu$ = velocity of wave

$$\frac{dy}{dt} = \omega A\cos(\omega t + kx)$$

$$\frac{dy}{dt} = A\cos(\omega t + kx)\left[1 + \frac{dx}{dt}\right]$$

$$\boxed{\frac{d^2y}{dt^2} = -\omega^2 y} \quad \text{--- (1)}$$

$$\frac{dy}{dx} = kA\cos(\omega t + kx)$$

$$\frac{d^2y}{dx^2} = -k^2 A\sin(\omega t + kx)$$

$$\boxed{\frac{d^2y}{dx^2} = -k^2 y} \quad \text{--- (11)}$$

$$\Rightarrow \frac{\left(\frac{d^2y}{dt^2}\right)}{\left(\frac{d^2y}{dx^2}\right)} = \left(\frac{\omega}{k}\right)^2$$

$$\Rightarrow \frac{d^2y}{dt^2} = \left(\frac{\cancel{2\pi}\nu}{\frac{\cancel{2\pi}}{\lambda}}\right)^2 \left(\frac{d^2y}{dx^2}\right)$$

$$\Rightarrow \frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}} \Rightarrow \text{diff. eq}^n \text{ of wave.}$$

v = velocity of wave.

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Interference

The redistribution of energy or intensity due to superposition of two coherent sources/waves is called as interference.

Coherent waves :-

Two waves having same wavelength and frequency and a constant phase difference are called as coherent waves or coherent wave source.

Example of coherent sources :-

→ The coherent sources can be generated from a single source of monochromatic light like Sodium lamp.

→ Two independent sources can never be coherent.

The coherent sources can be generated a single source by the principle of (i) division amplitude.

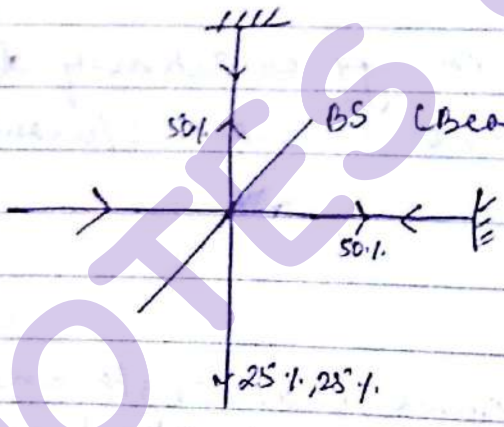
& (ii) division of wavefront

Division of amplitude :-

~~Young~~

Michaelson Interferometer, ~~based~~ Newton's Ring are on division of amplitude. In this type of experiment the two coherent sources are created by d of amplitude

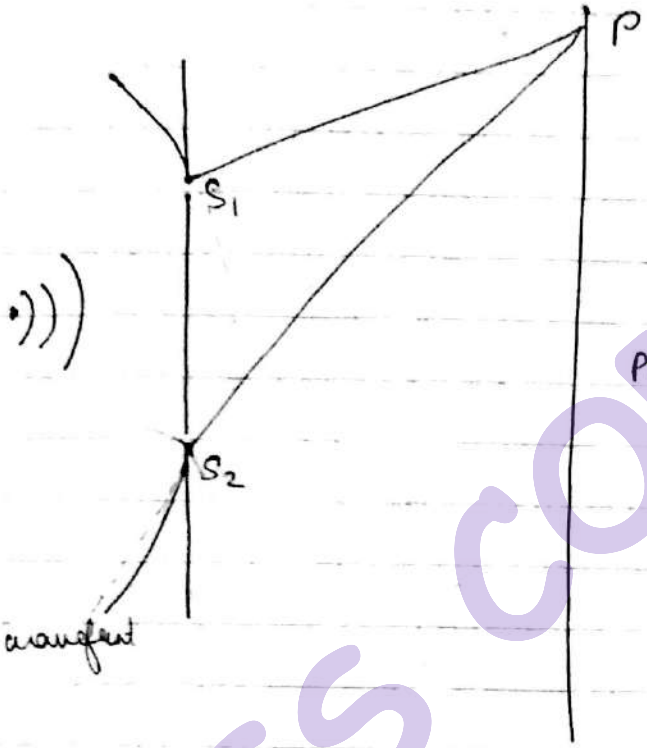
Michaelson Interferometer :-



Schematic diagram of MI.

Division of waveform :-

Young's Double Slit expt. is one example of interference due to division of waveforms.



Path diff $S_2P - S_1P$

Phase diff - $\frac{2\pi}{\lambda} \times$ path diff

* Analytical Treatment of Interference. ~~Derivation of~~

Derivation for resultant intensity,
Condition for maxima & minima,
Energy distribution curves in interference, etc.

Derivation of resultant intensity in Interference

Consider two ~~waves~~ coherent wave sources as follows.

$$y_1 = a_1 \sin(\omega t)$$
$$y_2 = a_2 \sin(\omega t + \delta)$$

δ = phase difference.

The resultant wave due to superposition of

$$y = y_1 + y_2$$ (Superposition principle)

$$y = a_1 \sin(\omega t) + a_2 \sin(\omega t + \delta)$$

$$y = a_1 \sin(\omega t) + a_2 \sin(\omega t)\cos\delta + a_2 \cos(\omega t)\sin\delta$$

$$y = \sin\omega t\,(a_1 + a_2\cos\delta) + \cos\omega t \cdot a_2 \sin\delta$$

Let $a_1 + a_2\cos\delta = R\cos\phi$ — (1)

$a_2 \sin\delta = R\sin\phi$ — (2)

$$\Rightarrow y = R(\sin\omega t\cos\phi + \cos\omega t\sin\phi)$$

$$\Rightarrow \boxed{y = R\sin(\omega t + \phi)}$$ → Resultant wave eq^n

Squaring and adding (1) & (2),

$$\Rightarrow \cancel{a_1^2 + a_2^2\cos^2\delta + a_2^2\sin^2\delta =}$$

$$\Rightarrow a_1^2 + a_2^2\cos^2\delta + 2a_1a_2\cos\delta + a_2^2\sin^2\delta = R^2$$

$$\Rightarrow a_1^2 + a_2^2 + 2a_1a_2\cos\delta = R^2$$

$$\Rightarrow \boxed{R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$$

Generally R^2 is the ~~intenti~~ intensity (I).
So resultant intensity for interference is

$$I = R^2$$

$$\Rightarrow \boxed{I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$$ → Resultant intensity interference

Scanned by CamScanner

$\Rightarrow$ $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$ here,

$I_1, I_2 \rightarrow$ Intensity of 1st and 2nd wave respectively.

Condition for Maxima :-

We know in interference the resultant intensity is given by, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$.

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta .$$

For I to be maximum, ~~cos δ~~

~~cos δ~~ $\cos\delta = 1$

$\Rightarrow$ $\boxed{\delta = 2n\pi}$ $(n = 0, \pm1, \pm2 \ldots)$

$\rightarrow$ Condition for maximum in interference in terms of phase difference.

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We know,

$$\delta = \frac{2\pi}{\lambda} \times \text{path diff.}$$

$$\Rightarrow \frac{2\pi}{\lambda} \times \text{path diff.} = 2n\pi$$

$\Rightarrow$ $\boxed{\text{path difference} = 2n \times \frac{\lambda}{2}}$ $\rightarrow$ condition for maxima in terms of path difference.

Condition for Minima :-

The intensity will be minimum for the condition $\cos\delta = -1$

$\Rightarrow$ $\boxed{\delta = (2n+1)\pi}$ $\rightarrow$ condition for minima in interference in terms of phase difference

$(n = 0, 1, 2, \ldots)$

we know;

$\Rightarrow \frac{2\pi}{\lambda} \times$ path diff $= (2n+1)\pi$.

$\Rightarrow$ path diff $= \frac{(2n+1)\lambda}{2}$

Maximum Intensity

We have

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta.$$

For maxima,

$$\cos\delta = 1$$

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2.$$

$\Rightarrow \boxed{I_{max} = (a_1 + a_2)^2} \rightarrow$ Max^m intensity.

If $a_1 = a_2 = a$.

Intensity of Minima

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos\delta.$$

For minima,

$\cos\delta = -1$

$\Rightarrow I_{min} = a_1^2 + a_2^2 - 2a_1a_2$

$\Rightarrow \boxed{I_{min} = (a_1 - a_2)^2} \rightarrow$ min^m^ intensity.

If $a_1 = a_2 = a$.

$\Rightarrow \boxed{I_{min} = 0}$

It is clear that

$$\boxed{I_{min} < I_1 + I_2}$$

Condition for sustainable interference :-

→ The interfering sources should be coherent.

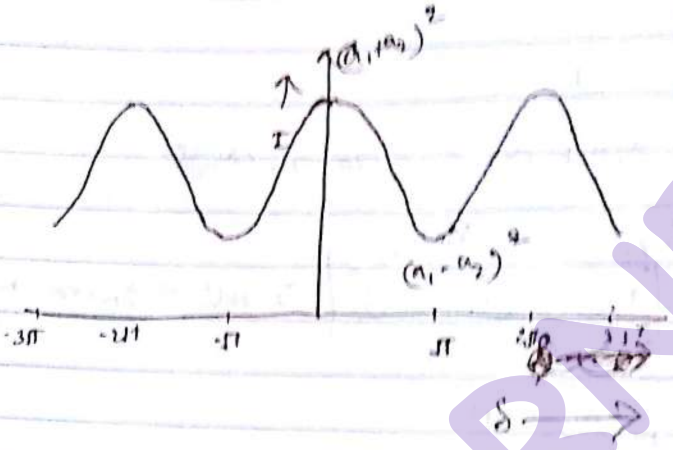
→ The polarisation of two coherent sources should be same.

Condition for good contrast :-

→ The amplitude of the interfering waves should be same for good contrast.

→ The interfering sources should be point source.

Energy / Intensity distribution curve with respect to difference in interference :-



Q) Show that energy is conserved in interference.

Ans: The average intensity in interference over a cycle of phase difference $(\delta : 0 \text{ to } 2\pi)$ is given by,

$$I_{avg} = \frac{\int_0^{2\pi} I \, d\delta}{\int_0^{2\pi} d\delta}$$

$$= \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1a_2\cos\delta)\, d\delta}{\int_0^{2\pi} d\delta} = \frac{2\pi(a_1^2+a_2^2)}{2\pi}$$

$$= \cancel{[a_1^2\delta]_0^{2\pi} + [a_2^2\delta]_0^{2\pi}}$$

$$= a_1^2 + a_2^2$$

So, the average intensity ~~both in case of interference~~ is constant (irrespective of interference or absence of interference

∴ The energy is conserved in interference.

13-12-16

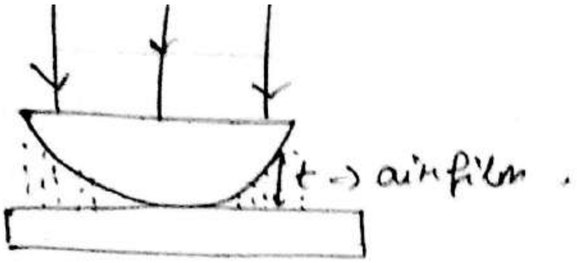
Newton's Rings

When the combination of plano convex lens and glass plate (convex side in contact with glass plate ~~Radius~~ of ~~&~~ curvature is large) is illuminated by monochromatic ~~light~~ light. Then concentric light and dark ring appears, In the air film enclosed between plano convex lens and glass slab. These concentric ~~rings~~ alternative bright and dark rings are called as Newton's Rings.

When monochromatic light fall on the ~~glass plate~~ combination of glass plate and plano convex lens, then their will be two reflected rays from the top and bottom of air film enclosed. These reflected light can superpose giving rise to Newton's ring.

The thickness of the film decide the maxima or minima i.e. bright and dark fringe. In this case locus of all the points having same thickness is a circle.

Therefore the fringes (bright/dark) are circular in nature.



CNR in reflective mode)

Condition for bright ring :-

In the air film, reflection, the path difference between the reflected rays is given by,

$$\boxed{\text{path difference} = 2\mu t\cos(\theta + r) + \frac{\lambda}{2}}$$

Here θ = Angle of wedge shaped film enclosed between plano concave lens and glass lens.
For large radius of curvature $\theta \to 0$.

r = Angle of refraction
For normal incident $r \to 0$.
So the path diff betn the two interfering waves is given by.

$$\text{path diff} = 2\mu t\cos 0^\circ + \frac{\lambda}{2}$$

$$\boxed{\text{path diff} = 2\mu t + \frac{\lambda}{2}}$$

$$\Rightarrow \boxed{2\mu t = (2n-1)\frac{\lambda}{2}}, \ n = \text{int}$$

→ Cond^n for bright ring

<u>Condition for dark ring :-</u>

For minima or dark ring condition is,

path diff $= (2n+1)\frac{\lambda}{2}$

$$\Rightarrow 2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t = 2n\frac{\lambda}{2}}$$ → Cond^n for dark ring

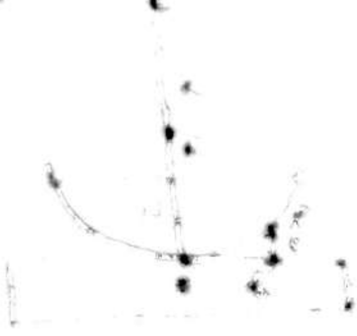
pg no 6

<u>Derivation of expression for diameter of Newton's Ring</u>

Consider a circle as shown in the figure below -

The radius of curvature of this circle is equal to radius of curvature of plano convex lens

Let r_n be the radius of n^{th} Newton ring



In this case from geometrical analysis we can prove that $AN \times NB = NP^2$

$$t(2R - t) = r_n^2$$

Here t = thickness of air film corresponding to n-th ring [illegible]

$$\Rightarrow 2Rt - t^2 = \rho_n^2$$

$$\Rightarrow 2Rt = \rho_n^2$$

$$\Rightarrow 2t = \frac{\rho_n^2}{R}$$

$$\Rightarrow \boxed{2t = \frac{D_n^2}{4R}} \quad \text{--- (1)}$$

Expression for bright ring

We know the condition for n^{th} order bright ring

$$2t = \frac{(2n-1)\lambda}{2} \quad \text{--- (2)}$$

From eq^n (1) & (2).

We have,

$$\frac{D_n^2}{4R} = \frac{(2n-1)\lambda}{2}$$

$$\Rightarrow \boxed{D_n^2 = 2(2n-1)R\lambda.}$$

$$\Rightarrow \boxed{D_n = \sqrt{2n-1} \times \sqrt{2R\lambda}}$$

$$\Rightarrow \boxed{D_n \propto \sqrt{\text{odd no.}}} \qquad \boxed{D_n \propto \sqrt{R}} \quad \boxed{D_n \propto \sqrt{\lambda}}$$

So, diameter of bright ring is directly proportional to square root of odd number. It is also directly proportional to square root of radius of curvature of plano-convex lens and also sq. root of wavelength.

Expression for dark ring :-

We condition for n^{th} dark ring is,

$$2t = \frac{2n\lambda}{2} \quad \text{———} \quad (3)$$

Comparing eqn (1) & (3),

$$\Rightarrow \frac{D_n^2}{4R} = \frac{2n\lambda}{2}$$

$$\Rightarrow \cancel{D_n^2 = \frac{2n \times 2R\lambda}{2}} \Rightarrow \boxed{D_n^2 = 4nR\lambda}$$

$$\Rightarrow \cancel{D_n = \sqrt{2n} \times \sqrt{2R\lambda}} \Rightarrow \boxed{D_n = 2\sqrt{n}\sqrt{R\lambda}}$$

$$\Rightarrow \boxed{\begin{array}{l} D_n \propto \sqrt{n} \\ D_n \propto \sqrt{R} \\ D_n \propto \sqrt{\lambda} \end{array}}$$

So, the diameter of dark Newton Ring is directly proportional to sq. root of natural no, and like bright ring these are also proportional to sq. root of R & λ.

Q) In a given experimental set up of Newton's Ring

$R = 200$ cm

$\lambda = 5893$ Å

Find out diameter of (1) 3rd bright ring

(2) 3rd dark ring

(1) $(D_n)_b = \sqrt{2n-1} \times \sqrt{2R\lambda}$

$\cancel{D_3 = \sqrt{5} \times 1535}$

$(D_3)_b =$

$(D_3)_b = 0.342$ cm

(2) $(D_n)_d = 2\sqrt{n}\sqrt{R\lambda}$

$(D_3)_d =$

$= 0.376$ cm

Q) In the Newton's Ring expt. containing plano convex lens of radius of curvature 200 cm, the diameter of 9th order bright ring is 1.5 cm. Find out the wavelength of the light source.

$(D_9)_b = \sqrt{17} \times \sqrt{2 \times 200 \times \lambda}$

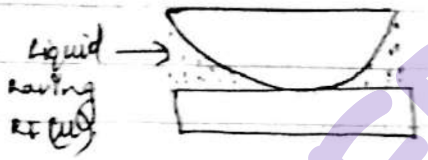
$1.5 = \sqrt{17} \times \sqrt{400\lambda}$

$2.25 = 17 \times 400\lambda \Rightarrow \lambda = 3.3 \times 10^{-4}$ cm

$\Rightarrow \lambda = 3.3 \times 10^{4}$ Å

Diameter of Newton Ring in presence of liquid film :-

In presence of liquid, the condition for bright ring is



$$2\mu t + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$\Rightarrow 2t = \frac{(2n-1)\lambda}{2\mu}$$

$$\Rightarrow \frac{D_n^2}{4R} = \frac{(2n-1)\lambda}{2\mu}$$

$$\Rightarrow D_n^2 = \frac{2(2n-1)R\lambda}{\mu}$$

$$\Rightarrow (D_n)_{liquid} = \frac{1}{\sqrt{\mu}}\sqrt{(2n-1)} \times \sqrt{2R\lambda}$$

$$\Rightarrow (D_n)_{liquid} = \frac{(D_n)_{air}}{\sqrt{\mu}}$$

In presence of liquid diameter of any order of ring is decreased by $\frac{1}{\sqrt{\mu}}$ times.

Q) By introducing liquid the diameter of Newton's ring for a given order reduced by 2 times find RI of liquid.

Ans:
$$\frac{(D_n)_{air}}{2} = \frac{(D_n)_{air}}{\sqrt{\mu}}$$

$$\Rightarrow \mu = 4$$

Q) The phase difference between two coherent sources is $60°$. What will be the resultant intensity if Amplitude of each coherent source is 1 S.I unit.

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta .$$

$$\Rightarrow I = 2 + 2\cos 60°$$

$$\Rightarrow I = 2 + 1$$

$$\Rightarrow I = 3 \text{ S.I. unit.}$$

Q) The path difference between two interfering waves is $\frac{\lambda}{4}$. What will be the resultant intensity of interference if ampl. of each wave is a.

$$\cancel{2\mu t + \frac{\lambda}{2} = \frac{\lambda}{4}}.$$

$$\Rightarrow \cancel{2\mu t = \frac{\lambda}{4} - \frac{\lambda}{2}}.$$

$$\cancel{\cos \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}}.$$

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} \Rightarrow \phi = \frac{\pi}{2}$$

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos\frac{\pi}{2} = 2a^2.$$

Q) Show that width of Newton's Ring decreases with increasing order (Tips: width of n^{th} order ring is $D_{n+1} - D_n$)

Ans: $(D_n)_b = \sqrt{2n-1} \times \sqrt{2R\lambda}$.

$(D_{n+1})_b = \sqrt{2(n+1)-1} \times \sqrt{2R\lambda}$

$= \sqrt{2n+1} \times \sqrt{2R\lambda}$.

$(D_{n+2})_b = \sqrt{2(n+2)-1} \times \sqrt{2R\lambda}$

$= \sqrt{2n+3} + \sqrt{2R\lambda}$

$(D_{n+1})_b - (D_n)_b$

$= \sqrt{2n+1} \times \sqrt{2R\lambda} - \sqrt{2n-1}\sqrt{2R\lambda}$.

$= \sqrt{2R\lambda}\left(\sqrt{2n+1} - \sqrt{2n-1}\right)$

$(D_{n+2})_b - (D_{n+1})_b$

$= \sqrt{2n+3} * \sqrt{2R\lambda} - \sqrt{2n+1} \times \sqrt{2R\lambda}$

$= \sqrt{2R\lambda}\left(\sqrt{2n+3} - \sqrt{2n+1}\right)$

$(D_{n+1}) - (D_n)$. $\sqrt{2n+1} - \sqrt{2n-1}$

Ans: Consider the width of n^{th} order dark ring. It is as follows:-

$$\beta = D_{n+1} - D_n$$

$$= 2\sqrt{R\lambda} \times (\sqrt{n+1} - \sqrt{n})$$

$$= 2\sqrt{R\lambda}\sqrt{n}\left[\left(1 + \frac{1}{n}\right)^{1/2} - 1\right]$$

$$= 2\sqrt{R\lambda} \times \sqrt{n}\left[1 + \frac{1}{2n} - 1\right]$$

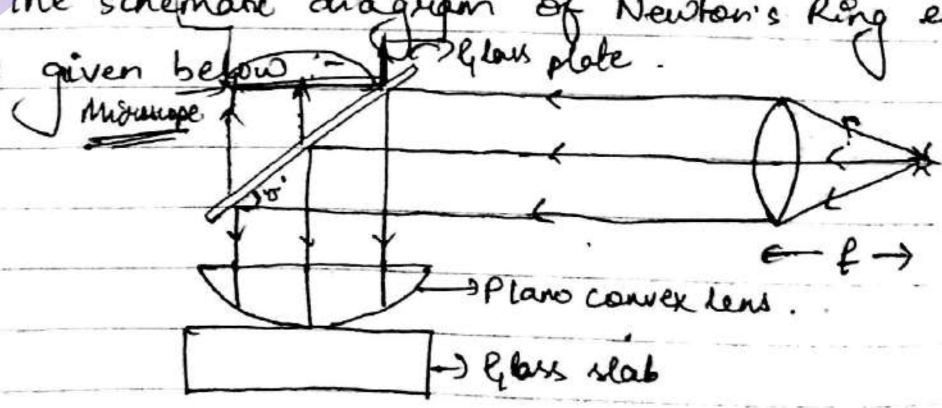
$$= \frac{2\sqrt{R\lambda}}{2\sqrt{n}}$$

$$= \frac{\sqrt{RT}}{\sqrt{n}}$$

$$\Rightarrow \boxed{\beta \propto \frac{1}{\sqrt{n}}}$$

Experimental arrangement for Newton's Ring :-

The schematic diagram of Newton's Ring experiment is given below :-



In this case the collimated beam of monochromatic light is allowed to fall on a glass plate kept at angle 45°. The reflected beam fall normally on the combination of plano convex lens and glass plate.

Two coherent beams or rays generated due to reflection from upper lower interface of air film enclosed in between plano convex.
These coherent sources interfere to give alternative dark and bright Newton's ring in air film.
These rings are viewed by a microscope.

Determination of wavelength by Newton's ring expt.

In the Newton's ring expt., the schematic diagram shown in above section is followed to form the Newton's Ring.
The diameter of n^{th} order dark ring in this case is given by

$$D_n^2 = 4Rn\lambda \quad \text{——— (I)}$$

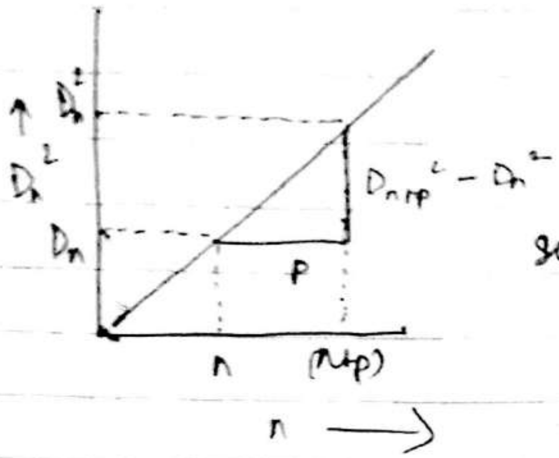
& ~~square~~ of diameter of ~~n^{th} order~~ $(n+p)^{th}$ order ring is given by

$$D_{n+p}^2 = 4R(n+p)\lambda \quad \text{——— (II)}$$

$$\Rightarrow D_{n+p}^2 - D_n^2 = 4Rp\lambda$$

$$\Rightarrow \lambda = \frac{1}{4R}\left(\frac{D_{n+p}^2 - D_n^2}{p}\right) \quad \text{——— (III)}$$

From eq^n (I), it is clear that the graph between D_n^2 and n is a straight line as show below:-



$$\text{slope} = \frac{D_{n+p}^2 - D_n^2}{p}$$

From eqn (III),

$$\boxed{\lambda = \frac{1}{4R} \times \text{slope}} \rightarrow \text{working formula.}$$

In this expt diameter of various order of rings are determined by using the travelling microscope.

For this, the position for different order ring in left & right hand side hand side are noted and the difference of this reading gives the value for diameter for different order. Finally the ~~gtat~~ graph is plotted betn D_n^2 and n to find out value of slope.

With the known value of R and measured value of slope one can measure the wavelength of the monochromatic light source.

Application of Newton's Ring expt for determination of R.I

Working formula derivation

we know,

$D_n^2 = 4nR\lambda$ -- (1) (for air)

for liquid film

$$D_n^2 = \frac{4nR\lambda}{\mu} \quad \text{-- (2)}$$

for $(n+p)^{th}$ ring in presence of liquid

$$D_{n+p}^2 = \frac{4(n+p)R\lambda}{\mu} \quad \text{-- (3)}$$

Substracting eqn (2) from (3)

$$\left(D_{n+p}^2 - D_n^2\right)_{liq} = \frac{4pR\lambda}{\mu}$$

$$= \frac{\left(D_{n+p}^2 - D_n\right)_{air}}{\mu}$$

$$\Rightarrow \boxed{\mu = \frac{\left(D_{n+p}^2 - D_n\right)_{air}}{\left(D_{n+p}^2 - D_n\right)_{liq}}} \rightarrow \text{Working formula}$$

Diffraction :-

Definition :-

Bending of light or wave ~~to~~ into the geometrical shadow region when it comes across an obstacle or slit of size comparable to wavelength is called as diffraction.

∴ The phenomena of bending of light.

This phenomena can be explained by taking Huygen's theory, which states that :-

Each point of a wavefront behaves as a source for secondary waves called wavelets. This wavelet superimpose each other to give phenomena of diffraction.

Different types of diffraction

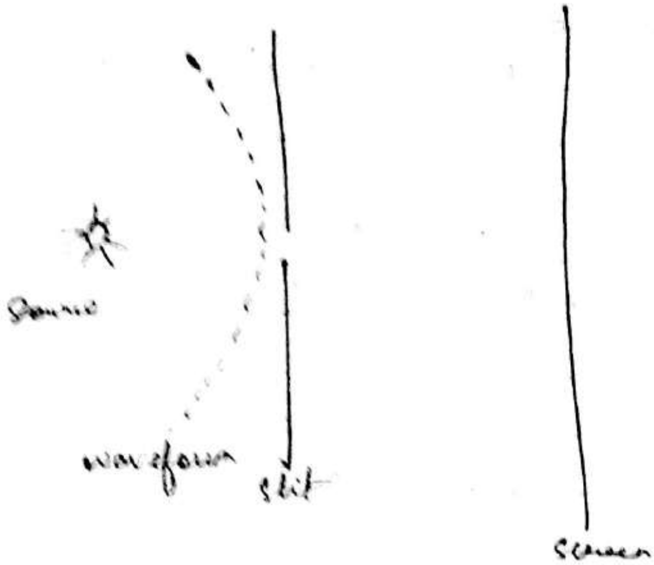
It is of two types :-

1. Fresnel diffraction
2. Fraunhofer diffraction.

(1) Fresnel diffraction :-

→ In this type of diffraction, the distance between source, slit and screen are finite.

→ In this case the wavefront hitting the slit is cylindrical or spherical in nature.



② Fraunhofer diffraction :-

→ In this type of diffraction, the distance between source, slit and screen is infinite.

→ In this case the wavefronts hitting slit is planar in nature

Q) Show that superposition of n no. of waves with amplitude a and consecutive phase difference d gives a resultant wave having amplitude

$$R = \frac{a \sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}.$$

(Th. Assignment - 2)

Q1) Distinguish between Fresnel and Fraunhoffer class diffraction.

Q2) What is diffraction and how it is different from interference of light.

Q3) Explain formation of Newton's ring. What will happen the monochromatic light used is replaced with white light.

Q4) Why Newton's Rings are circular in nature?

Q5) Show that energy is conserved in interference phenomena.

1)

Fresnel diffraction	Fraunhofer diffraction.
→ The distance betⁿ source, slit and screen are finite.	→ The distance betⁿ source, slit and screen are infinite
→ Wavefront hitting the slit is either cylindrical or spherical in nature	→ Wavefront hitting the slit is planar in nature.

2) Diffraction

Bending of light or wave to the geometrical shadow region when it comes across an obstacle or slit of size comparable to its wavelength is called diffraction

Interference of light - equal intensity.

In interference of light, the energy is redistributed due to superposition of two coherent waves/sources but in diffraction each point of a waveform acts as a source for secondary waves (called as wavelets)

~~3) When the combination of plano-convex lens and glass plate is illuminated by monochromatic light. Then concentric bright and dark rings appears in the air film enclosed between convex lens and glass slab.~~

3) when monochromatic light fall on the combination of glass plate and plano convex lens, then there will be two reflected rays from the top and bottom of air film enclosed. These reflected light can superpose giving rise to Newton's ring.

If monochromatic light is replaced by white light Newton's Rings are not formed as we know that Diameter of Newton's ring is directly proportional to square root of wavelength of light, but white light has wide range of wavelength, so Newton's ring becomes inconsistent and ~~doesn't form~~ Newton's ring doesn't form.

4) The thickness of the air film decides the maxima or minima i.e. bright or dark fringe. In ~~this~~ the combination of plano convex lens and glass slab. The locus of points having same thickness is a circle.

5) The average intensity in interference over a cycle of phase difference ($\delta = 0$ to 2π) is given by:-

$$I_{avg} = \frac{\int_0^{2\pi} I \, d\delta}{\int_0^{2\pi} d\delta}$$

$$= \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1a_2 \cos\delta) \, d\delta}{\int_0^{2\pi} d\delta}$$

$$= \frac{\cancel{2\pi}(a_1^2 + a_2^2)}{\cancel{2\pi}}$$

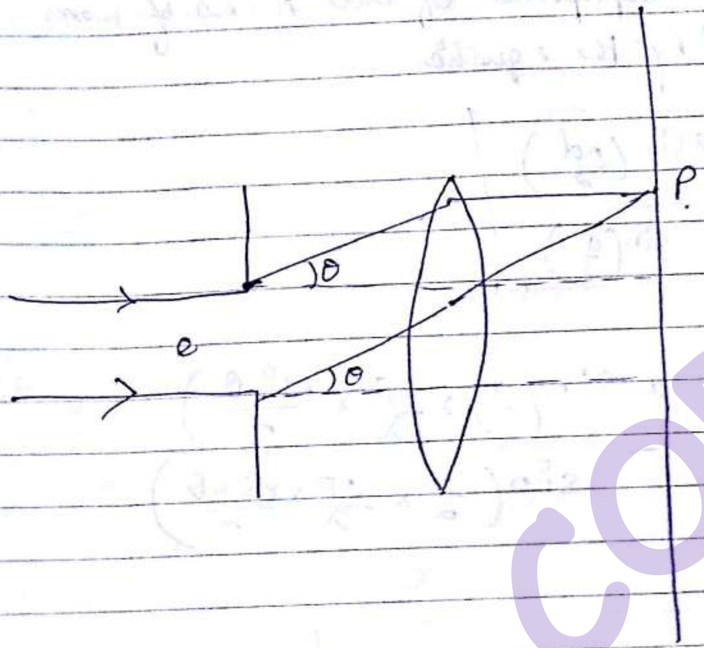
$$= a_1^2 + a_2^2$$

So, the average intensity is constant (presence of ~~absence~~ absence of interference.

$\therefore$ Energy is conserved in interference

Fraunhoffer diffraction for single slit :-

Consider a slit of dimension 'e' illuminated by ~~collimated~~ collimated beam of light of wavelength 'λ'. as shown in the figure below:-



Consider the wavefront hitting the slit is having n number of points. From these points n number of secondary wavelets will emerge and superpose at point P which is at an angle θ with respect to angle of incident.

In this case the path difference between the wavelets coming from two extreme points of the slit is $e\sin\theta$.

As there are no. of points the ~~phase~~ path difference between the consecutive wavelets will be $\frac{1}{n}(e\sin\theta)$

So, the phase difference between two consecutive waves is $\phi = \frac{2\pi}{\lambda} \times$ path difference path diff $(d) = \frac{2\pi}{\lambda} \times \frac{e\sin\theta}{n}$

We know, that superposition of these n. no. of waves is represented by the equation

$$\boxed{R = \frac{a\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}}$$

$$\Rightarrow R = \frac{a\sin\left(\frac{n}{2} \times \frac{2\pi}{\lambda} \times \frac{e\sin\theta}{n}\right)}{\sin\left(\frac{1}{2} \times \frac{2\pi}{\lambda} \times \frac{e\sin\theta}{n}\right)}$$

$$\Rightarrow \boxed{R = \frac{a\sin\left(\frac{\pi e\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e\sin\theta}{\lambda n}\right)}}$$

let

$\frac{\pi e\sin\theta}{\lambda} = \alpha$

$$R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

$$R = \frac{a\sin\alpha}{\left(\frac{\alpha}{n}\right)} \quad \left(\because \frac{\alpha}{n} = \text{small no.}, n \to \text{large}\right)$$

$$\boxed{R = \frac{na\sin\alpha}{\alpha}}$$

$$\boxed{R = A\left(\frac{\sin\alpha}{\alpha}\right)}$$, where $A = na$

This is the expression for resultant amplitude in single slit Fraunhoffer diffraction.

Now, the resultant intensity in this case is

$$I = R^2$$

$$\Rightarrow \boxed{I = A^2 \frac{\sin^2\alpha}{\alpha^2}}$$

Condition for maxima:-

For maxima,

$$\cancel{I} \quad \cancel{\frac{\sin\alpha}{\alpha} = 1} \quad \cancel{\alpha = \frac{n\pi}{2}}.$$

$$I = \frac{A^2}{\frac{n^2\pi^2}{4}}$$

$$\Rightarrow I = \frac{4A^2}{n^2\pi^2}$$

~~For maxima~~

We know the resultant amplitude is given by:-

$$R = A\left(\frac{\sin\alpha}{\alpha}\right)$$

This funct^n is maxima when $\alpha \to 0$

As $R_{max} = \lim_{\alpha\to 0} A\left(\frac{\sin\alpha}{\alpha}\right) = A$

$$\boxed{R_{max} = A}$$

So we can take condition for maxima is $\alpha = 0$. (in terms of phase)

$$\alpha = \frac{\pi e \sin\theta}{\lambda}$$

$$\frac{\pi e \sin\theta}{\lambda} = 0.$$

$$\Rightarrow \sin\theta = 0.$$

$$\Rightarrow \boxed{\theta = 0}$$

So, the maxima is there along the direction of incident (i.e. $\theta = 0$).

This maxima is called as primary maxima or principal maxima.

Condition for minima :-

$$R = 0$$

$$\Rightarrow \frac{\sin\alpha}{\alpha} = 0$$

$$\Rightarrow \sin\alpha = 0 \text{ as } \alpha \neq 0$$

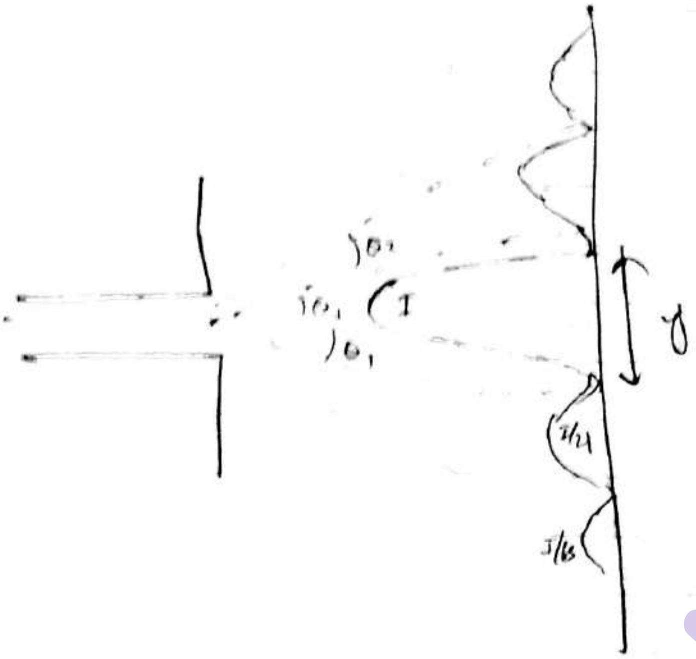
$$\Rightarrow \alpha = \pm n\pi \quad (n = 1, 2, 3, \ldots)$$

$$\frac{\pi e \sin\theta}{\lambda} = \pm n\pi$$

$$\Rightarrow \boxed{e \sin\theta = \pm n\lambda}$$ → Condn for minima in terms of physical parameter

$n = 1$ (1st order minima)

$$\boxed{e \sin\theta = \lambda}$$



From this eqn it is clear that there is present no. of minima defined as 1st order minima, 2nd order minima etc.

In between minima, there exist maxima. This means other than the principal maxima there exist other maxima called secondary maxima.

Q) In a Fraunhoffer single slit expt. ~~side~~ size of the slit is ~~1×10^-5~~ 1×10^{-3} cm. Find out the (i) angular width of the primary maxima if the slit is iluminated by Sodium vapour Lamp ($\lambda = 5893\,\text{Å}$)

Ans: $e\sin\theta_1 = \lambda$

$$\sin\theta_1 = \frac{5893 \times 10^{-8}}{1 \times 10^{-3}}$$

$$\sin\theta_1 = 5893 \times 10^{-5}$$

$$\theta_1 = 3.37^\circ$$

$$\Rightarrow 2\theta_1 = 6.74^\circ$$

So the angular width of central maxima is equal to $2\theta_1 = 6.74^\circ$.

(ii) If the diffraction pattern is observed on screen kept 1 m apart from the slit. Then what will be the linear width of the 1st order maxima.

Ans Linear width is $2y$, where y satisfy the relation

$$\tan\theta_1 = \frac{y}{1}$$

$$y = \tan\theta_1$$

$$2y = 2\tan\theta_1$$

$$2y = 2\tan 3.27$$

$$2y = 5.8\,cm$$

Secondary Maxima :-

The condition for secondary maxima can be found out from the following relation.

$$\frac{dI}{d\alpha} = 0$$

$$\Rightarrow \frac{d}{d\alpha}\left[\frac{A^2\sin^2\alpha}{\alpha^2}\right] = 0$$

$$\Rightarrow \frac{d}{d\alpha}\left[\frac{\sin^2\alpha}{\alpha^2}\right] = 0$$

$$\Rightarrow \frac{2\alpha^2\sin\alpha\cos\alpha - 2\alpha\sin^2\alpha}{\alpha^4} = 0$$

$\Rightarrow 2\alpha^2 \sin\alpha \cos\alpha - 2\alpha \sin^2\alpha = 0$

$\Rightarrow 2\alpha^2 \sin\alpha \cos\alpha = 2\alpha \sin^2\alpha$

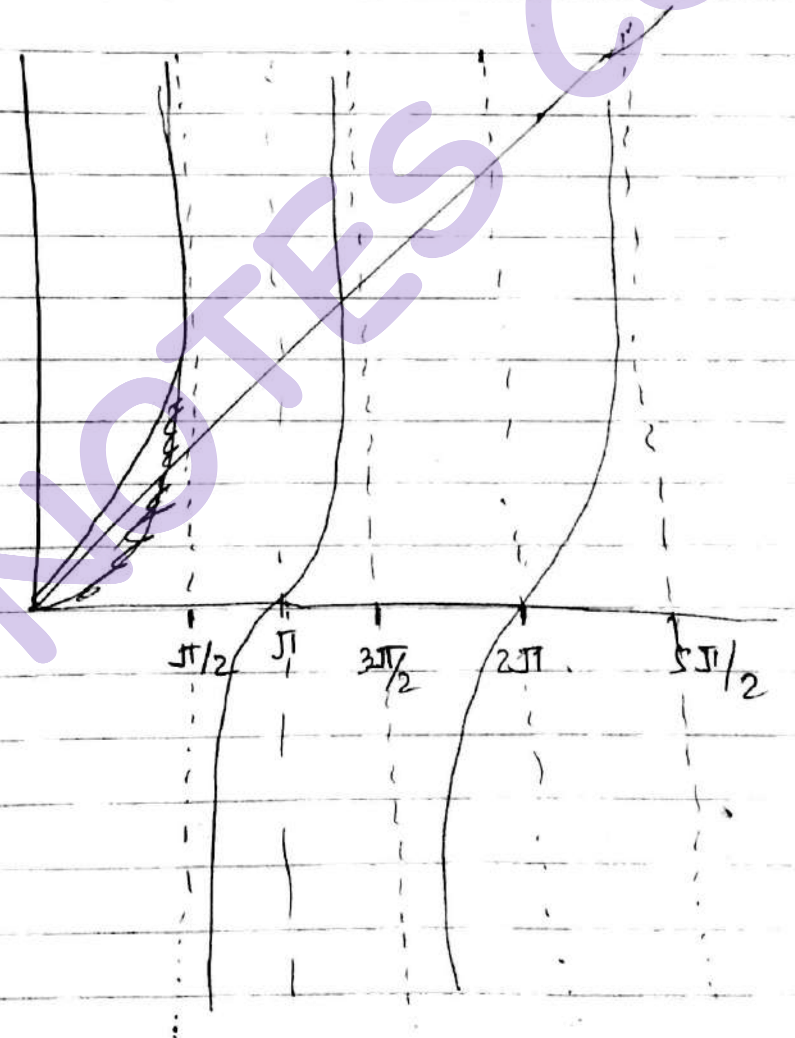
$\Rightarrow \alpha \cos\alpha = \sin\alpha$

$\Rightarrow \boxed{\alpha = \tan\alpha}$ $\longrightarrow$ Cond^n for secondary maxima in Fraunhoffer single slit diffraction

The above eq^n can be solved graphically by taking two equation

$y = \alpha$

and $y = \tan\alpha$



From the graph it is clear that value of α for various secondary maxima are

$$\alpha = \frac{n\pi}{2}, \; n = \pm 1, \pm 3, \pm 5$$

So, there are several secondary maxima.

So, ~~$\alpha = \pm \pi/2$ ($n = 1$, 1st order secondary maxima)~~

$\alpha = \pm 3\pi/2$ ($n = 3$, 1st order secondary maxima)

~~Intensity of 1st order secondary maxima~~

~~We know~~

~~$$I = \frac{A^2 \sin^2 \alpha}{\alpha^2}$$~~

~~$$I_{sm} = \frac{A^2 \sin^2 (\pi/2)}{(\pi/2)^2}$$~~

~~$$I_{sm} = \frac{4A^2}{\pi^2}$$~~

Intensity of 1st order secondary maxima

Similarly

$$I_{SM_3} = \frac{4A^2}{25\pi^2}$$

So,

~~$I_{PM} \times I_{SM_1} \times I_S$.~~

$I_{PM} : I_{SM_1} : I_{SM_2} \; \cancel{I_{SM_3}}$

$$= 1 : \frac{4}{\pi^2} : \frac{4}{9\pi^2} : \frac{4}{25\pi^2}$$

Plane Diffraction Grating :-

A plane diffraction grating is a optical element consisting of large no. of slits of size comparable to the wavelength of the light incident on it. The slits are equispaced and equidimensional. The space between the slits is opaque.

The plane diffraction grating is generally fabricated by drawing large no. of parallel lines on the surface of a glass slide using diamond cutter. The lines in this case behaves like opaque region and the region between two lines behaves like a slit.

If 'e' and 'd' are the dimension (width) of slit and opaque region respectively then (e+d) is called as grating element.

Grating element is also defined as distance between two consecutive slits or two consecutive opaque regions or consecutive equivalent points in the grating.

If there are 'N' no. of lines per ~~each~~ inch in a plane diffraction grating then its grating element is ~~e+d=~~

$$e + d = \frac{1}{N} \text{ inch}$$

$$\Rightarrow \boxed{e + d = \frac{2.54}{N} \text{ cm}}$$

Q) In a plane diffraction grating there are 15000 lines per inch. What will be the

(i) grating element (in unit of cm).

(ii) if this grating is illuminated by sodium light ($\lambda = 5893 \mathring{A}$). What will be the angle for 1st order minima

Ans: (i) Grating element $= \frac{2.54}{15000} = 1.69 \times 10^{-4}$ cm

(ii)

Q) If the size of a single slit is 5mm what will be the angle for 1st order minima (wavelength of Na light is used for diffraction expt.)

Ans: $e \sin\theta = n\lambda$ $\quad e = 0.5cm$

$$\sin\theta = \frac{\lambda}{e}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{e}\right)$$

$$= \sin^{-1}\left(\frac{5893\times10^{-8}}{5\times10^{-1}}\right)$$

$$= \sin^{-1}\left(\frac{5893\times10^{-7}}{5}\right)$$

$$= \sin^{-1}\left(1.17\times10^{-4}\right)$$

$$= \cancel{0.0006}\; 6.75\times10^{-3\,\circ}$$

Q) What is the value of angular width and linear width of central Maxima (diffraction seen in a screen kept 1m apart)

Ans: Ang. Width $= 2\theta = \underline{13.5\times10^{-3\,\circ}}$

~~Q~~ Linear Width

$$\tan\theta = \frac{y}{100}$$

$$y = 100\tan\theta$$

$$y = 100\tan(6.75\times10^{-3})$$

$$y = \cancel{0.0117}\; 0.0117 = 1.17\times10^{-2}\,cm$$

$$2y = \cancel{0.0234}\quad 2y = \underline{2.34\times10^{-2}\,cm}$$

$$R = \frac{R' \sin\left(\frac{Nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}$$

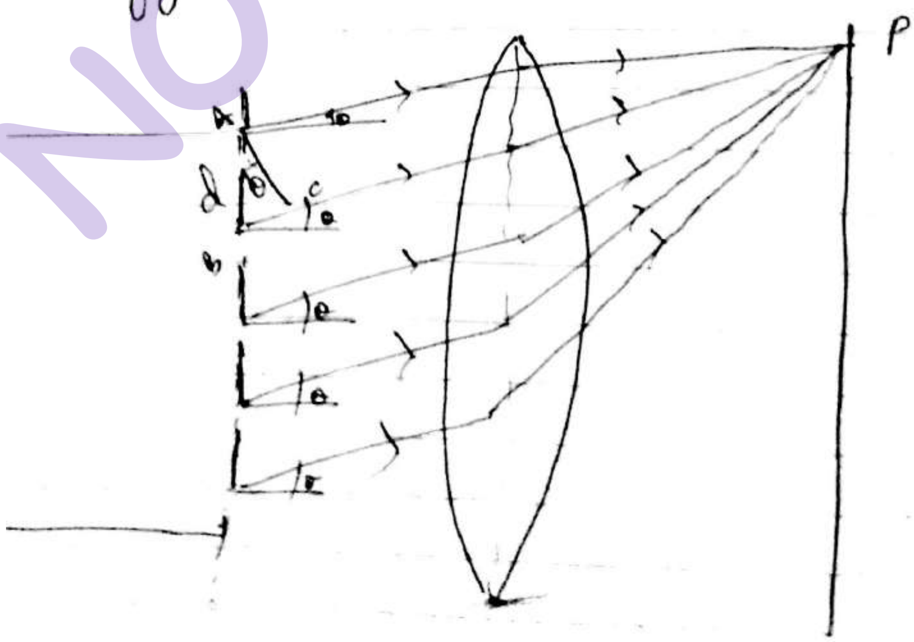
$$\boxed{R = A\left(\frac{\sin\alpha}{\alpha}\right)\frac{\sin\left(\frac{Nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}}$$

$$d = (e+d)\sin\theta \times \frac{2\pi}{\lambda}$$

$$R = A\,\frac{\sin\alpha}{\alpha} \times \frac{\sin N\beta}{\sin\beta}$$

Analytical treatment of plane - diffraction grating:

Consider a plane diffraction grating as shown in the figure below



In this plane diffraction grating slit of width e are separated by opaque region of width d.

let a parallel beam is incident on it and n no of slits are illuminated.

Due to diffraction at single slit each and every slit will give a resultant wave of amplitude

$$R' = \frac{A\sin\alpha}{\alpha}$$

For N no. of slits there will N no of waves which will again super-pose at point P. So the resultant amplitude of the

These ~~N no of~~ the resultant wave of these superposition will be given by ~~...~~

$$R = \frac{R'\sin\left(\frac{N\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}$$

Here, δ = phase difference bet^n the two consecutive waves coming from two consecutive slits.

These phase difference can be given by

$$\text{phase diff}: \delta = \frac{2\pi}{\lambda} \times \text{path diff}$$

$$\delta = \frac{2\pi}{\lambda} \times BC$$

$$= \frac{2\pi}{\lambda} \times (e+d)\sin\theta = 2\beta$$

Putting the value of R' and δ,
we get

$$\boxed{R = \left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right)}$$

This is the resultant amplitude due to Fraunhoffer diffraction in diffraction grating.

where
$\alpha = \frac{\pi e\sin\theta}{\lambda}$
$\beta = \frac{\pi(e+d)\sin\theta}{\lambda}$

So, the resultant intensity is given by.

$$I = R^2$$

$$\Rightarrow I = A^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}.$$

Derivation of condition of maxima:-

Condⁿ for primary maxima is

$$\sin\beta = 0$$

$$\Rightarrow \beta = n\pi$$

$$\Rightarrow N\beta = N \times n\pi$$

$$\Rightarrow N\beta = p\pi$$

$$\Rightarrow \sin(N\beta) = \sin(p\pi) = 0.$$

So, the condⁿ for primary maxima is

$$\boxed{\beta = n\pi}$$

$$\Rightarrow \frac{\pi(e+d)\sin\theta}{\lambda} = n\pi$$

$$\Rightarrow \boxed{(e+d)\sin\theta = n\lambda} \rightarrow$$ condition for primary maxima

$n = 0 \Rightarrow$ zeroth primary maxima

$n = \pm 1 \Rightarrow$ 1st order ~~secondary~~ primary maxima

So,
Intensity of primary maxima can be found out from the following limiting condition

$$\lim_{\beta \to n\pi} \left(\frac{\sin N\beta}{\sin \beta} \right)$$

$$= \lim_{\beta \to n\pi} \frac{N \cos N\beta}{\cos \beta}$$

$$= N$$

So, amplitude of primary maxima is given by

$$\boxed{R = \frac{A \sin \alpha}{\alpha} \times N}$$

Now,

$$\boxed{I = A^2 \frac{\sin^2 \alpha}{\alpha^2} \times N^2}$$

Minima

For minima, $I = 0$

$$\Rightarrow A^2 \frac{\sin^2 \alpha}{\alpha^2} \times \frac{\sin^2 N\beta}{\sin^2 \beta} = 0$$

$$\Rightarrow \frac{\sin^2 N\beta}{\sin^2 \beta} = 0$$

$$\Rightarrow \frac{\sin N\beta}{\sin \beta} = 0$$

$\Rightarrow \sin N\beta = 0$ but $\sin \beta \neq 0$

So, the condition for minima is

$$\boxed{\sin N\beta = 0 \quad \text{but} \quad \sin \beta \neq 0}$$

$\Rightarrow \sin N\beta = 0$

$$\Rightarrow N \frac{\pi(e+d)\sin\theta}{\lambda} = n\pi$$

$$\Rightarrow \boxed{N(e+d)\sin\theta = n\lambda} \rightarrow \text{Cond}^n \text{ for minima in terms of physical parameters}$$

$$e\sin\theta = n\lambda \rightarrow \text{Fraunhofer minima (single slit)}$$

$$(e+d)\sin\theta = n\lambda \rightarrow \text{P. Maxima} \quad \text{--- (i)}$$

$$N(e+d)\sin\theta = n\lambda \rightarrow \text{Minima} \quad \text{--- (ii)}$$

From the condition of principal maxima and principal minima it is clear that (eqn (i) & (ii)) it is clear that in between two primary maxima there will be $(N-1)$ no. of minima. This also indicates that there will be $(N-2)$ secondary maxima in between two consecutive primary maxima.

Secondary Maxima:-

The condition for secondary maxima can be derived from the following relation

$$\frac{dI}{d\beta} = 0$$

$$\Rightarrow \frac{d}{d\beta}\left(A^2\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}\right) = 0$$

$$\Rightarrow \frac{d}{d\beta}\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right) = 0$$

$$\Rightarrow 2N\sin N\beta \sin\beta -$$

$$\Rightarrow \frac{2N\sin N\beta \cos N\beta \sin^2\beta - \cos\beta \sin^2 N\beta}{\sin^4\beta} = 0$$

$$\Rightarrow \frac{2N\sin N\beta \cos N\beta}{\sin^2\beta} - \cos\beta = 0$$

$$\Rightarrow 2N\sin N\beta \cos N\beta = \cos\beta \sin^2\beta$$

$$\Rightarrow$$

$$\Rightarrow \frac{\sin^2\beta \times 2N\sin N\beta \cos N\beta - \sin^2 N\beta \, 2\sin\beta\cos\beta}{\sin^4\beta} = 0$$

$$\Rightarrow 2\sin\beta \sin N\beta \left(N\cos N\beta \sin\beta - \sin N\beta \cos\beta\right) = 0$$

$$\Rightarrow N\cos N\beta \sin\beta - \sin N\beta \cos\beta = 0$$

$$\Rightarrow N\cos N\beta \sin\beta = \sin N\beta \cos\beta$$

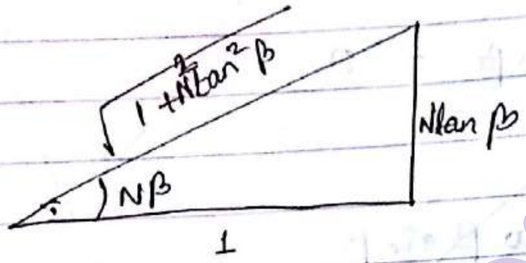
$$\Rightarrow N\frac{\sin\beta}{\cos\beta} = \frac{\sin N\beta}{\cos N\beta}$$

$$\Rightarrow \boxed{N\tan\beta = \tan N\beta}$$ $\rightarrow$ Condition for Secondary maxima.

Intensity of secondary maxima :-

Cond^n for secondary maxima is $(N\tan\beta = \tan N\beta)$

This cond^n can be geometrically shown as follows :-



In this triangle,

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

Squaring,

$$\sin^2 N\beta = \frac{N^2\tan^2\beta}{1+N^2\tan^2\beta}$$

$$\Rightarrow \sin^2 N\beta = \frac{N^2}{(\cot^2\beta + N^2)}$$

Dividing bts by $\sin^2\beta$

$$\Rightarrow \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(\cot^2\beta + N^2)\sin^2\beta}$$

$$\Rightarrow \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{\cos^2\beta + N^2\sin^2\beta}$$

$$\Rightarrow \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1-\sin^2\beta + N^2\sin^2\beta}$$

$$\Rightarrow \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1+\sin^2\beta(N^2-1)}.$$

So, the secondary maxima intensity is :-

$$I_{\substack{\text{secondary}\\ \text{maxima}}} = A^2 \frac{\sin^2\alpha}{\alpha^2} \times \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$I_{\substack{\text{sec}\\ \text{max}}} = \frac{A^2 \sin^2\alpha}{\alpha^2} \times \frac{N^2}{1+\sin^2\beta(N^2-1)}$$

$$\Rightarrow \boxed{I_{\substack{\text{sec}\\ \text{max}}} = \frac{I_{\substack{\text{prim}\\ \text{max}}}}{1+\sin^2\beta(N^2-1)}}$$ → Intensity of secondary maxima

$$\left(\text{As; } I_{\substack{\text{prim}\\ \text{max}}} = A^2 \frac{\sin^2\alpha}{\alpha^2} \times N^2\right)$$

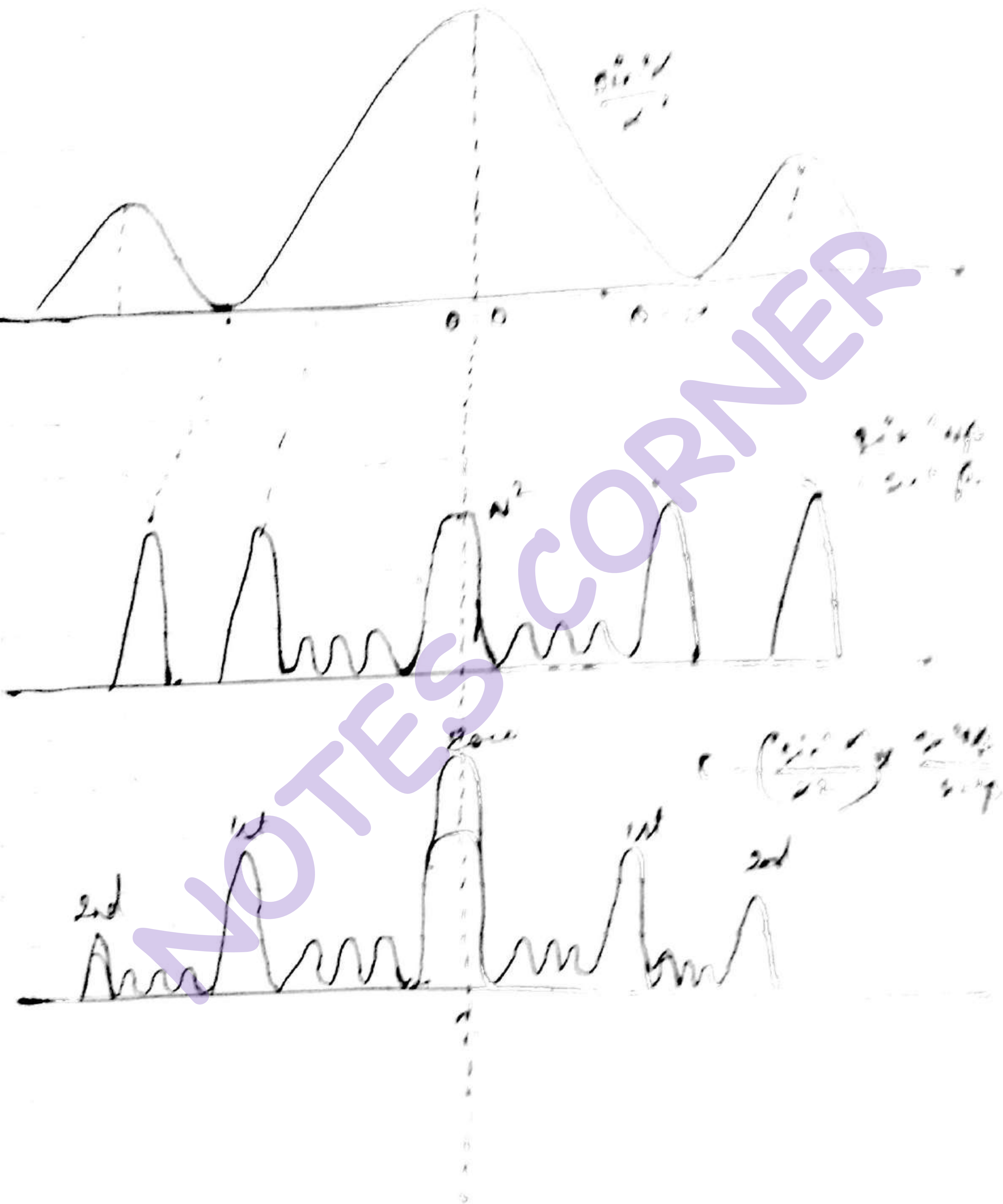
Generally, N is large, So, the secondary maxima intensity is very small as compared to primary maxima intensity.

<u>Intensity distribution curve for Fraunhoffer diffraction in plane diffraction grating</u> :-

The intensity in Fraunhoffer due to plane diffraction grating is given by. $I = A^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$.

Here the ~~diff~~ α term is called as diffraction term whereas β term is called as interference term.

These terms individually as well as the resultant intensity (I) can be shown graphically as figure below :-


2nd
1st
1st
2nd

Missing Order

$(e+d)\sin\theta = n\lambda$ → Maxima of β term

$e\sin\theta = m\lambda$ → minima of α term

$\boxed{\dfrac{e+d}{e} = \dfrac{n}{m}}$ → Condⁿ for missing order

→ In Fraunhoffer diffraction due to plane diffraction grating, for certain angle the minima of α term and maxima of β term coincides, in these angles some order of primary maxima will not be visible. These orders are called as missing order.

→ We known in Fraunhoffer diffraction due to plane diffraction grating, the minima of α term satisfy the relation

$$e\sin\theta = m\lambda \quad \text{— (i)}$$

Similarly we know maxima of β term ~~equation~~ satisfy the relation

$$(e+d)\sin\theta = n\lambda \quad \text{— (ii)}$$

Dividing eqⁿ (ii) by (i),
we get :

$$\boxed{\frac{e+d}{e} = \frac{n}{m}} \rightarrow \text{Cond}^n \text{ of missing order}$$

n, m → natural nos.

Case-1

If $e=d$.

$$\frac{e+d}{e} = \frac{n}{m}$$

$$\Rightarrow \frac{2e}{e} = \frac{n}{m}$$

$$\Rightarrow \boxed{n = 2m} \quad m = 1, 2, 3, \ldots$$

$\Rightarrow n = 2, 4, 6, 8, \ldots$ orders will be missing.

Case 2

If $d = 6e$.

$$\Rightarrow \frac{e+d}{e} = \frac{n}{m}$$

$$\Rightarrow \frac{7e}{e} = \frac{n}{m}$$

$$\Rightarrow \boxed{n = 7m}, \quad m = 1, 2, 3$$

$\Rightarrow n = 7, 14, \ldots$

LASER

Laser is an acronym for the physical phenomena Light Amplification by Stimulated Emission of Radiation.

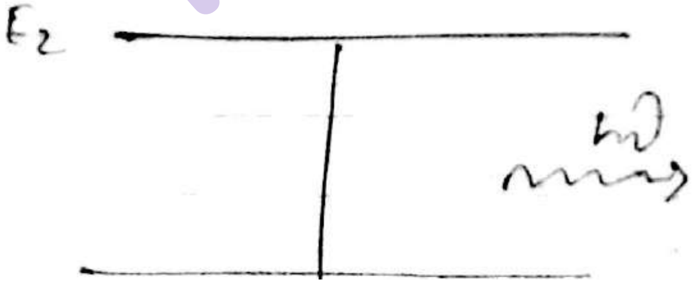
Interaction of light with matter

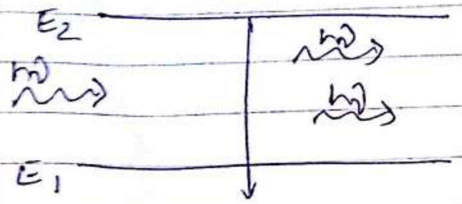
Interaction of light with matter can be described by the following physical phenomena

(i) Absorption
(ii) Spontaneous Emission
(iii) Stimulated emission

Spontaneous emmision:-

The atoms in the excited state are not stable. They loose their energy in form of radiation. In this process, the electrons at the higher energy level jump back to the lower energy level by releasing its extra energy in form of photon or radiation, this process is called as spontaneous emission. This is a natural process. This is governed by the life time of the electron in the excited state. ~~The photon emitted~~ In this emission process the radiation is omni-directional that means photon move in all possible direction. This process can be schematically shown as figure below.





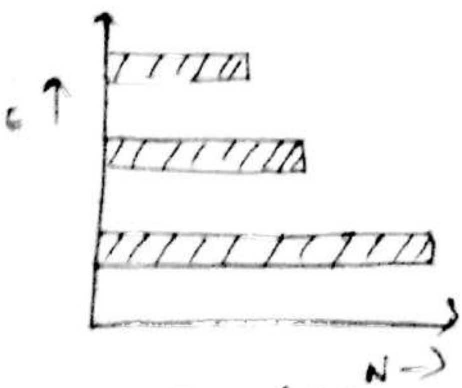
$$A^* + h\nu \rightarrow A + 2h\nu$$

Difference between spontaneous emission and stimulated emission :-

Spontaneous Emmision	Stimulated Emmision
→ It is a natural process	→ It is a forced process
→ It is not uncontrollable process	→ It is a controllable process
→ The emission follows the lifetime of the electron in excited state.	→ This emission is instantaneous
→ In this emission, the photons have arbitrary direction (omni-directional emission)	→ In this emission the photons have same emission (directional emission)
→ All the emitted photons do not have same nature	→ All the emitted photons have same nature i.e. same wavelength, direction, polarisation, momentum etc.

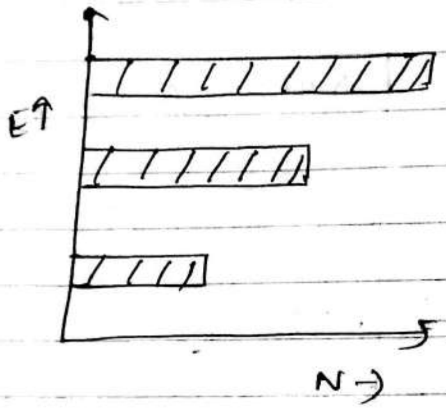
Population Inversion:-

In the normal state or ground state generally more no of atoms of a system are there in the lower energy level. With increasing As its energy level, the no. of atoms decreases As shown in fig



Normal state

By giving external energy to the system more and more no. of atoms can be switched to higher energy level as shown in the figure below:-



This condition where more no. of atoms are there in higher energy level is called as population inversion.

For lasing action to take place population inversion condition is mandatory.

Pumping :-

The process of giving external energy to the system to build up the population inversion is called as pumping.

Pumping can be done by

(i) optical means

(ii) electrical discharge

(iii) collision

(iv) carrier injection

Component of LASER

A LASER should have atleast 3 components.

(i) Gain medium
(ii) Laser / Resonator Cavity
(iii) Pumping system

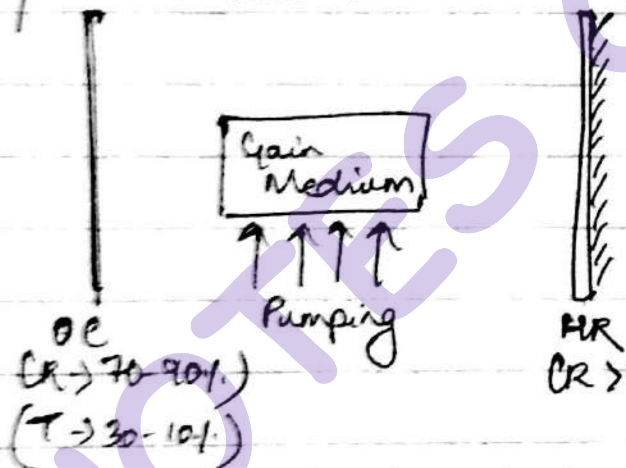
Gain Medium :-

It is the substance in which lasing action take place i.e. in this medium, where absorption, Population inversion, stimulated emission takes place.

The gain medium can be solid, liquid or gas. Accordingly the lasers are named as solid state laser, gas laser or liquid state laser. Ruby (a crystal) laser is a solid state laser whereas CO_2 laser is a gas laser.

Resonator cavity / Laser cavity :-

This consist of atleast two mirrors one of them is having reflectivity $R > 99\%$ at the lasing wavelength. This mirror is called as back mirror. The other mirror has also very high reflectivity but it has also some finite value of transmittance



The output of the laser comes out of this mirror. Therefore this mirror is called as output coupler.

Let L is the length of the cavity.

Then the various longitudinal modes of laser are having wavelength given by

$$\lambda_1/2 = L$$

$$\Rightarrow \lambda_1 = 2L$$

$$\lambda_2 = L$$

2λ

Scanned by CamScanner

[illegible]

Q) Let the length of [illegible] longitudinal modes.

A:

$\lambda_1 = 2L$

$\lambda_2 = L$

$\lambda_3 = \frac{2L}{3}$

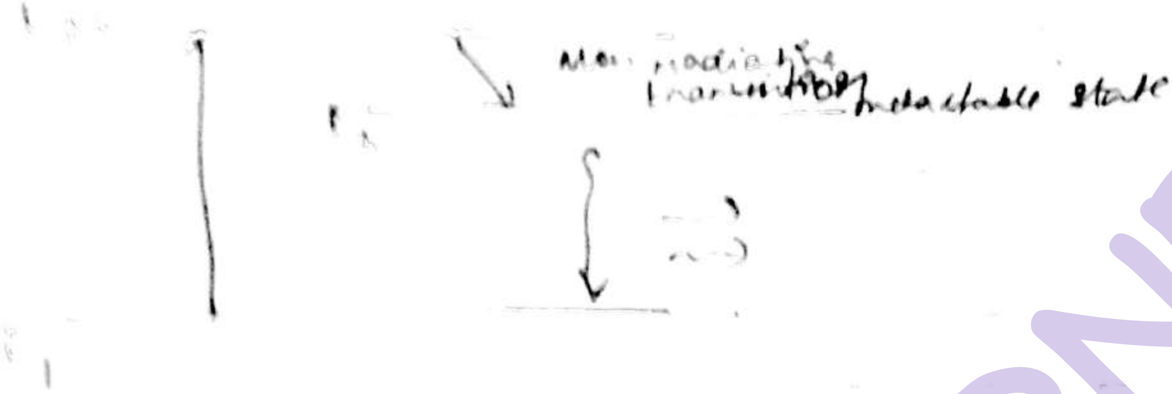
[illegible]

Pumping System :-

As the given notes:

Three level laser :-

The three level lasers have the [illegible] energy level involved in the lasing process. These energy levels are shown schematically in the figure [illegible] —



In this case the atoms transit from energy level E_1 to E_2 through absorption process. These atoms are not stable in this state, they quickly transit to energy level E_3 by non radiative fast process. ~~This type of scheme help~~ The energy level E_3 has very high life time therefore the atoms can stay there for long time. Under continuous pumping more and more no of electrons used to be there in energy level E_3. This help in building up the population inversion. The energy level E_3 is called as meta stable state. Now the stimulated emission takes place between the energy level E_3 and E_1.

Ruby Laser

It is a solid state laser its working principle, structure / construction is given below.

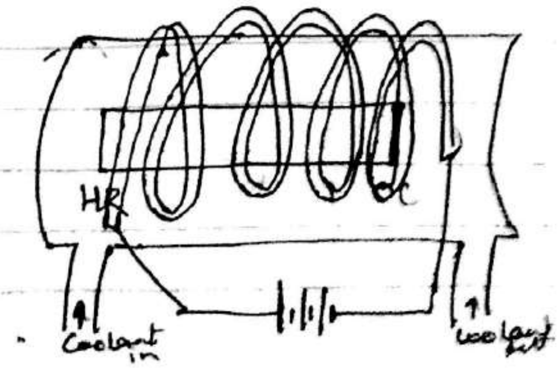
→ Ruby is a crystal of Al_2O_3 doped with Cr. This dopant generally exist in ion form. The concentration of this dopant is 0.05% by weight.

→ The ~~atomic~~ concentration of the Cr ion with this type of doping is 1.6×10^{25} per cubic meter (m^{-3}). This ruby crystal is the gain medium. The chromium ion has three energy levels that is suitable for lasing action.

→ The shape of the crystal is generally cylindrical having length 4cm and width 0.5cm.

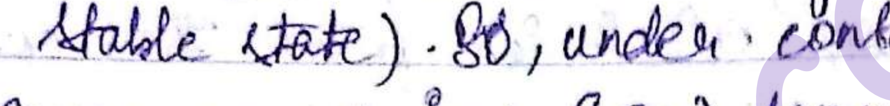
→ One of the end of the rod is polished in a such a way that it can reflect fully (high reflection). This end behaves like back mirror. The other mirror (end) is polished to have partial transmission along along with high reflectivity. This end behaves like output coupler.

This crystal is generally pumped by a helical flash lamp filled with Xenon. The 5500 Å radiation of this lamp is observed by the chromium ions. The crystal is cooled by flowing some coolant.



Working Principle

When chromium ion absorb the 5600Å radiation, it transit to higher energy level E_2 from E_1, the ion is not stable there it quickly transit to an level E_3, where the lifetime is much higher (meta stable state). So, under continuous pumping more and more no. of ions (Cr) transit to energy level E_3 from E_1 through E_2. The transition from E_2 to E_3 is generally non radiative. In this process, population inversion takes place and the stimulated emission take place between the energy levels. The emission wavelength is 6943Å.



(iv) Laser Spectroscopy
(v) Laser Gun

Oscillation

LQ

1) Establish or derive the eqn of motion for damped oscillation, find out general soln and discuss the case of overdamping or critical damping or underdamping.

2) Establish or derive the eqn for forced oscillation; write down the general soln of amplitude and discuss about condition of resonance.

3) Derive the eq wave eqn in differential form. (with some numericals)

4) Derivation of expression for resultant intensity, condition for maxima/minima, Maxima and minima intensity. expression, energy distribution curve

or

(Analytical treatment of interference)

5) Show that energy remains conserved in interference. (10 marks)

6) Analytical treatment of Newton's Ring.

or

What do you mean by Newton's Ring, derive the condn for bright and dark ring and derive their diameter expression.

7) With the experimental description discuss about Newton Ring and show how it can be used to measure / estimate wavelength and R.I.

Order of ring (n)	Diameter (D_n)	D_n^2	$D_{n+1}^2 - D_n^2$	Mean $D_{n+1}^2 - D_n^2$
1	x_1	x_1^2		
2	x_2	x_2^2	$x_2^2 - x_1^2 = y_1$	
3	x_3	x_3^2		
4	x_4	x_4^2		
5	x_5	x_5^2		

8) Analytical treatment for Fraunhoffer diffraction due to single slit.

OR

What do you mean by Fraunhoffer diffraction due to single slit.

Derive resultant intensity expression, cond^n for primary maxima, minima and secondary maxima. Show that the intensity distribution follows the relation $1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \ldots$

$1 : \frac{1}{22} : \frac{1}{63} : \ldots$

$1 : 0.045 : \ldots$

Date / /
Page

Analytical Ques

1) Show that the width of Newton's Ring decreases with order.
(2) If the glass plate is replaced by a mirror. What will happen in Newton's ring expt.
(3) What will happen if white light source is used in Newton's ~~expt~~ ring expt.
(4) What will happen if white light is used in single slit expt.

(5) What is the condition for sustainable and good contrast in interference.

(6) Difference between interference and diffraction.

(7) Write down the condn for maximum order of diffraction in single slit diffraction.
~~The max^m angle is $\pi/2$.~~ $(\theta = \pi/2)$.

8) The slit size in a Fraunhoffer single slit expt is 4λ. where λ is the wavelength of the light used. Find out (i) Angle for 1st order and 2nd order minima.
(ii) Maximum order of diffraction.

Ans:

$$\cancel{\alpha = \frac{\pi e \sin\theta}{\lambda}}$$

$$\cancel{\Rightarrow \theta = \frac{\pi\, 4\lambda \sin\theta}{\lambda}}$$

$$\cancel{\Rightarrow \sin\theta = .}$$

$$e\sin\theta = \pm n\lambda$$

$$\Rightarrow \sin\theta = \pm\frac{1}{4}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{4}\right)$$

$$= 14.47^\circ$$

$$\cancel{\sin\theta} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = \underline{30^\circ}$$

$e \sin\theta_{max} = n_{max}\lambda$

$\Rightarrow \theta_{max} = \pi/2$

$\Rightarrow e = 4\lambda$.

$\Rightarrow 4\lambda = n_{max}\lambda$.

$\Rightarrow \underline{\underline{n_{max} = 4}}$

) In above ques findout angle for 1st order secondary maxima.

If $\lambda = 5893 \mathring{A}$

$\alpha = \frac{3\pi}{2}$

$\frac{\pi e \sin\theta}{\lambda} = \frac{3\pi}{2}$

$\Rightarrow e \sin\theta = \frac{3}{2}\lambda$

$\Rightarrow$ ~~$4\lambda \sin\theta$~~.

~~$e \sin\theta = \frac{3}{2} \times 5893 \times 10^{-8}$~~

$\Rightarrow \sin\theta = \frac{3}{8}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{3}{8}\right) = \underline{\underline{22.02^{0}}}$