5/9/19
Thursday

Continuous Random Variable

p.d.f : (i) $f(x) \geq 0 \quad \forall x$.
(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Cummulative Distribution Function.

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = P(X \leq x).$$

$$P(a \leq X \leq b) = F(b) - F(a).$$

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx.$$

$$\sigma^2 = E(X^2) - \mu^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$E(X-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx.$$

Q. A college professor never finishes his lecture before the end of an hour and always finishes his lectures within 2 minute after the hour. Let X be the time between end of the hour of the lecture. The p.d.f. is given by,

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find -.

(i) k

(ii) what is the probability that the lecture ends within 1 minute of the end of the hour.

(iii) what is the Prob. that the lecture continues beyond the hour for between 60 sec to 90 sec.

(iv) what is Prob. that the lecture continues atleast 90 sec beyond the end of the hour.

(i) $$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

$$\cancel{\int_{-\infty}^{0} f(x)\,dx} + k\int_{0}^{2} f(x)\,dx + \cancel{\int_{2}^{\infty} f(x)\,dx} = 1$$

$$\cancel{\int_{-\infty}^{0} x^2}\; k\left[\cancel{\int_{-\infty}^{0} x^2 dx} + \int_{0}^{2} x^2\,dx + \cancel{\int_{2}^{\infty} x^2}\right] = 1.$$

$$k\left[\left[\frac{x^3}{3}\right]_0^2 + \cancel{\left[\frac{x^3}{3}\right]_2^{\infty}}\right] = 1.$$

$$k\left[\frac{8}{3}\right] = 1$$

$$k = \frac{3}{8}$$

5)

(ii) $\int_0^1 f(x)\,dx$

$= \int_0^1 \frac{3}{8} x^2\,dx$

$= \left[\frac{3x^3}{3\times 8}\right]_0^1$

$= \left[\frac{x^3}{8}\right]^1$

$= \frac{1}{8}$

(iii) $\int_1^{1.5} f(x)\,dx$

$\int_1^{1.5} \frac{3x^3}{8}\,dx$

$= \left[\frac{x^3}{8}\right]_1^{1.5}$

$= \frac{2.37}{8}$

(iv) ~~$\int_{1.5}^{2} f(x)\,dx = \int_{1.5}^{2}$~~

(iv) $P(X \geq 1.5) = 1 - P(X < 1.5)$

$= 0.578$

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Q. The current in a certain circuit is measured by an ammeter is a continuous random variable X with p.d.f

$$f(x) = \begin{cases} 0.075x + 0.2 , & 3 \le x \le 5 \\ 0 , & \text{otherwise.} \end{cases}$$

Find : (i) $P(X \le 4)$

(ii) $P(3.5 \le X \le 4.5)$

(iii) $P(4.5 < X)$.

(i) $$\int_{3}^{4} (0.075x + 0.2)\, dx$$

$$\left[\frac{0.075x^2}{2} + 0.2x \right]_3^4$$

$= 0.4625$

(ii) $$\int_{3.5}^{4.5} (0.075x + 0.2)\, dx = 0.5.$$

(iii) $$\int_{4.5}^{5} (0.075x + 0.2)\, dx = 0.2781.$$

5)

Q. $F(x) = \begin{cases} 0 , & x < 0. \\ \frac{x^2}{4} , & 0 \leq x < 2. \\ 1 , & 2 \leq x \end{cases}$

Compute : (i) $P(X \leq 1)$.
(ii) $P(0.5 \leq X \leq 1)$
(iii) $P(X > 1.5)$
(iv) $f(x)$ [p.d.f].
(v) $E(X)$
(vi) $V(X)$
(vii) σ_x.

(i) $P(X \leq 1) = F(1) = \frac{1}{4}$.

(ii) $P(0.5 \leq X \leq 1) = F(1) - F(0.5)$

$= \frac{1}{4} - \frac{(0.5)^2}{4} = \frac{3}{16}$.

(iii) $P(X > 1.5) = 1 - P(X \leq 1.5)$

$= 1 - F(1.5)$

$= 1 - \frac{(1.5)^2}{4}$

$= \frac{7}{16}$.

(iv) $f(x) = \begin{cases} \frac{x}{2} , & 0 \leq x < 2. \\ 0 , & \text{else}. \end{cases}$

(v) $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$.

$= \int_{0}^{2} \frac{x^2}{2}\, dx + \int_{0}$

$= \left[\frac{x^3}{6}\right]_0^2$

$= \frac{\cancel{8}\,4}{\cancel{6}\,3} = \frac{4}{3}$.

(vi) $\text{var}(x) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$.

$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$

$= \int_{0}^{2} x^2 \cdot \frac{x}{2} - \left(\frac{4}{3}\right)^2$

$= \frac{1}{2}\left[\frac{x^4}{4}\right]_0^2 - \frac{16}{9} = \frac{2}{9}$.

(vii) $\sigma_x = \sqrt{\frac{2}{9}}$

Q.No.7

$$F(x) = \begin{cases} 0, & x \le 0. \\ \frac{x}{4}\left[1 + \ln\left(\frac{4}{x}\right)\right], & 0 < x \le 4 \\ 1, & x > 4. \end{cases}$$

Compute : (i) $P(X \le 1)$

(ii) $P(1 \le X \le 3)$

(iii) $f(x)$.

1/9/19

NORMAL DISTRIBUTION. (mean and median are same)

Probability Distribution funcⁿ

A Continuous RV 'x' is said to have a normal distribution with parameter μ or σ (or σ^2). where $-\infty < \mu < \infty$ and $\sigma > 0$. If the p.d.f. of X is:

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty$$

where μ = mean.
σ = standard deviation.

$$X \sim N(\mu, \sigma)$$

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

STANDARD NORMAL VARIABLE.

The normal distribution with parameter $\mu = 0$ and $\sigma = 1$ is called standard normal distribution.

A random variable having a standard normal distribution is called a standard normal random variable and it is denoted by. (Z).

$$f(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-z^2/2}, \quad -\infty < Z < \infty$$

Properties of Standard Normal Distribution

Distribution function of z (standard normal variable).

The distribuⁿ funcⁿ of $\phi(z)$ of a standard normal variate is defined by

$$\phi(z) = P(Z \leq z)$$

$$= \int_{-\infty}^{z} f(t)\,dt$$

$$= \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-t^2/2}\,dt.$$

(1) Properties of Distribution funcⁿ.

(1) $\phi(-z) = 1 - \phi(z)$.

(2) $P(a \leq x \leq b) = \int_a^b = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$

$\|$

$P\left(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right).$

$$P(X \le a) = \phi\left(\frac{a-\mu}{\sigma}\right).$$

$$P(X \ge b) = 1 - P(X < b)$$
$$= 1 - \phi\left(\frac{b-\mu}{\sigma}\right).$$

Q. Let Z be a standard normal random variable then evaluate the following.

(i) $P(Z \le 1.25)$ $= \cancel{0.8944}$

(ii) $P(Z > 1.25)$ $=$

(iii) $P(Z \le -1.25)$

(iv) $P(-0.38 \le Z \le 1.25)$.

(i) 0.8944

1st property in both

(ii) $1 - P(Z \le 1.25)$
$= 1 - 0.8944$
$= 0.1056$.

(iii) 0.1056.

(iv) $\phi(1.25) - \phi(-0.38)$
$\phi(1.25) - 1 + \phi(0.38)$.
$= \phi(1.25) - 1 + 0.6480$.
$= 0.8944 - 1 + 0.6480$
$= 0.5424$

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11/9/19
Wednesday

NON-STANDARD Normal Distribution.

If X has a normal distribution with mean μ and standard deviation σ.

then $Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution. and the Probability of.

$$P(a \le X \le b) = \left(\frac{a-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right)$$

$$= P\left(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$$

Q. The break down voltage of a randomly chosen diode of a particular type is known to be normally distributed (i) what is the probability that a diode's breakdown voltage is within 1 S.D of its mean value.

$$P(\mu-\sigma \le X \le \mu+\sigma)$$

$$= P\left(\frac{\mu-\sigma-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{\mu+\sigma-\mu}{\sigma}\right)$$

$$= P(-1 \le z \le 1)$$
$$= \phi(1) - \phi(-1)$$
$$= 0.8413 - 0.1587.$$
$$= 0.6826$$

(ii) what is the Probability a diode breakdown voltage is with 2 S.D of its mean value.

$$P(\cancel{\alpha}\mu - 2\sigma \leq x \leq \mu + 2\sigma).$$

$$= P\left(\frac{\mu - 2\sigma - \mu}{\sigma} \leq \frac{x-\mu}{\sigma} \leq \frac{\mu + 2\sigma - \mu}{\sigma}\right).$$

$$= P(-2 \leq Z \leq 2)$$

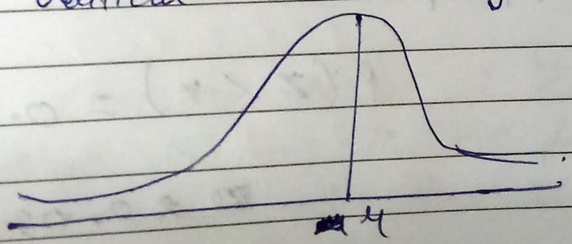
$$= \phi(2) - \phi(-2)$$

$$= 0.9772 - 0.0228$$

$$= \cancel{0.7492}\ 0.9544.$$

AREA UNDER NORMAL CURVE.

The curve of a normal distribution is uni-model and bell shape, with the highest point over the mean (μ). It is symmetric about a vertical line through μ.



The normal curve has following features—

1. The values of mean, median and mode are equal.

2. The curve is symmetrical about the coordinate at its means which locate the peak of the bell.

(3) The area covered by $\mu - \sigma$ and $\mu + \sigma$ is 0.6826 (68.26%) of the total area.

(4) The area covered b/w the limits $\mu - \sigma$ and $\mu + \sigma$ is 0.9544 (95.44%) of the total area.

12/9/19 Thursday

Percentile of Standard Normal distribution

For any 'p' b/w 0 to 1. The normal distribution table can be used to obtain $(100p)^{th}$ percentile of the standard normal distribution.

For eg: The 99^{th} percentile of the standard normal distribution is that value on the horizontal axis such that the area under the z curve to the left of the value is 0.99.

$$P(Z < z) = 0.99$$

~~Z < 0.09~~

$$2.33 \rightarrow 0.9901.$$

$$P(Z < 2.33) = 0.99$$

Find Z such that:

(i) $P(Z \le z) = 0.95$

$0.95 \rightarrow 0.9505 \rightarrow 1.65$

$0.95 \rightarrow 0.9495 \rightarrow 1.64$

$P(Z \le 1.645)$.

(ii) $P(Z \le z) = 0.97$

$0.97 \rightarrow 0.9699 \rightarrow 1.88$.

$P(Z \le 1.88)$

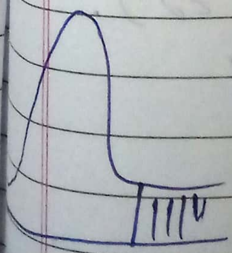
(iii) $P(Z \le z) = 0.92$

$0.92 \rightarrow 0.9207 \rightarrow 1.41$.

$P(Z \le 1.41)$

Z_α : CRITICAL VALUE : Z_α denote the value on the Z axis for which α of the area under the Z curve lies to the right of Z_α.

$P(Z \ge Z_\alpha) = \alpha$.



Z_α

$$P(Z \ge 1) = 1 - P(Z < 1) = 1 - \phi(1) = 1 - 0.8413 = 0.1587.$$

Since α under the Z curve lies in the right of Z_α, $(1-\alpha)$ of the area lies to its left.

Hence Z_α is the 100 times $(1-\alpha)$ times percentile of the standard normal distribution.

Q. Determine Z_α if (i) $\alpha = 0.0055$.

$$P(Z \geq Z_\alpha) = 0.0055.$$

$$1 - P(Z < Z_\alpha) = 0.0055.$$

$$1 - 0.0055 = P(Z < Z_\alpha)$$

$$0.9945 = P(Z < Z_\alpha)$$

$$\boxed{Z_\alpha = 2.54}$$

(ii) $\alpha = 0.09$

$$P(Z \geq Z_\alpha) = 0.09$$

$$1 - P(Z < Z_\alpha) = 0.09.$$

$$1 - 0.09 = P(Z < Z_\alpha)$$

$$0.91 = P(Z < Z_\alpha).$$

$$Z_\alpha = 1.34$$

(iii) $\alpha = 0.663.$

$$P(Z \geq Z_\alpha) = 0.663$$

$$1 - P(Z < Z_\alpha) = 0.663$$

$$1 - 0.663 = P(Z < Z_\alpha)$$

$$0.337 = P(Z < Z_\alpha)$$

$$Z_\alpha = -0.42$$

3. The mean and standard deviation of a normal variable X are 50 and 4 respectively. Find the value of the corresponding standard normal ~~distributi~~ variable. when.

$X = 42, 54, 84$. Hence find the probabilities.

$$\mu = 50$$

$$\sigma = 4$$

$$Z = \frac{X_1 - 50}{4}$$

$$Z_1 = \frac{42 - 50}{4}$$

$$= \frac{-8}{4}$$

$$= -2$$

$$Z_2 = 1$$

$$Z_3 = 8.5$$

13/9/19
Friday.

Q. The A, B, C Company uses a machine to fill boxes with soap powder. Assume that the net weight of the boxes of soap is normally distributed with mean 15 and standard deviation 0.8. What proportion of boxes will have net weight of more than 14 pound.

$\mu = 15$, $\sigma = 0.8$, $X = 14$

$P(X > 14) = Z = \frac{X - \mu}{\sigma}$

$= \frac{14-15}{0.8}$

$= \frac{-10}{8} = \frac{-5}{4}$

$= -1.25$

$= P(Z > -1.25)$

$= 1 - P(Z < -1.25)$

$= 1 - 0.1056$

$= 0.8944$

$= 89.44\%$

Hence 89.44% of the boxes will have there net weight more then 14 pound.

Q. Let X denote the no. of success in a test. If X is normally distributed with μ is 100 and σ is 15 then find Prob. that X doesnot exceed 130?

$$X = 130, \quad \mu = 100, \quad \sigma = 15.$$

$$Z = \frac{X - \mu}{\sigma} = \frac{130 - 100}{15}$$

$$= 2.$$

$$P(Z < 2) = \cancel{0.9772}.$$

$$= \phi(2) = 0.9772.$$

Q. Assume that the mean height of a soldier to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall.

Soln

$$\sigma^2 = 10.8$$

$$\sigma = \sqrt{10.8}$$

$$\sigma = 3.29$$

$$\mu = 68.22.$$

$$X = 72.$$

$$Z = \frac{72 - 68.22}{3.29} = \frac{3.78}{3.29} = 1.148.$$

$$P(Z > 1.148) = 1 - P(Z \leq 1.148)$$

$$= 1 - 0.8749 = 0.1251 = 12.51\%$$

Q. what is the Prob. that a standard normal variate Z will be lying between 1.25 and 2.75.

$$P(1.25 \le Z \le 2.75)$$

$$= \phi(2.75) - \phi(1.25)$$

$$= 0.1026.$$

Q. Suppose that blood chloride concn has a normal distribution with $\mu = 104$ and $\sigma = 5$. what is Prob. that

(i) chloride concn is less than 105.

(ii) Atmost 105.

(iii) chloride concn differs from the mean by more than 1 standard deviation

(iv) Does the Prob. depends on the value of μ and σ.

S. $\mu = 104, \sigma = 5$.

(i) $X = 105$,

$$Z = \frac{105-104}{5} = \frac{1}{5}.$$

$$P(Z < 0.2) = \phi(0.2) = 0.5793.$$

(ii) $P(Z \le 0.2) = \cancel{1 - \phi(0.2)}$

$= \cancel{0.4207}$.

$= 0.5793$

(iii) $P(\mu - \sigma \le X \le \mu + \sigma)$.

$$P\left(\frac{\mu - \sigma - \mu}{\sigma} \le \frac{X - \mu}{\sigma} \le \frac{\mu + \sigma - \mu}{\sigma}\right)$$

$P(-1 \le Z \le 1)$

$\varphi(1) - \varphi(-1)$

$= 0.8413 - 0.1587$

$= 0.6826 \quad = 1 - 0.6826 = 0.3174$

(iv) ~~Yes~~ NO It doesn't depends upon the values of μ and σ.

Suppose that 25% of all students at a large public university receive scholarship. Let X be the number of students in a random sample of size 50 who receive scholarship. Find the Prob. that

(i) atmost 10 student receives scholarship.

(ii) b/w 5 and 15 (inclusive) receives scholarship.

~~$\lambda = 50, \mu = 0.25.$~~

$$n = 50, \quad p = 0.25.$$

By poisson's.

$$\lambda = np$$

$$\lambda =$$

APPROXIMATION OF BINOMIAL BY NORMAL DISTRIBUTION

Q.. $P(X \leq 10) = \varphi\left(Z \leq \frac{10-\mu}{\sigma}\right)$

Let X be a ~~rv~~ Binomial random variable based on 'n' trials with success Probability 'p' then if the binomial Probability histogram is not too skewed, X has approximately a normal distribution with mean $\mu = np$ and standard deviation $\sigma = \sqrt{npq}$.

area under the normal curve to the left of $x+0.5$.

Then the Probability ① $P(X \leq x) = \varphi\left(\frac{x+0.5-np}{\sqrt{npq}}\right)$

② $P(a \leq X \leq b) = \varphi\left(\frac{a-0.5-np}{\sqrt{npq}}\right) - \varphi\left(\frac{b+0.5-np}{\sqrt{npq}}\right)$

$n = 50$

$p = 0.25$

$\mu = np = 50 \times 0.25 = 12.5$.

$\sigma = \sqrt{npq} = \sqrt{50 \times 0.25 \times 0.75} = 3.06$

(i). $P.(X \leq 10) = \varphi\left(\frac{10+0.5-12.5}{3.06}\right)$

$= \varphi(-0.653) = 0.2578.$

(ii) $P(5 \leq X \leq 15) = \varphi\left(\frac{15+0.5-12.5}{3.06}\right) -$

$\varphi\left(\frac{5-0.5-12.5}{3.06}\right)$

$$= \phi(0.98) - \phi(-2.61)$$

$$= 0.8320.$$

Q. Let X have a binomial distribution with parameters $n = 25$ and p. Calculate each of the following probabilities using normal distribution approximation for the cases- $p = 0.5, 0.6, 0.8$ and compare to the exact probabilities. Find -

(i) $P(X \leq 15)$
(ii) $P(X \geq 20)$
(iii) $P(15 \leq X \leq 20)$.

$p = 0.5, n = 25,$ $\therefore \mu = 25 \times 0.5$

$\mu = 12.5$

$$\sigma = \sqrt{25 \times 0.5 \times 0.5} = 2.5.$$

(i) $P(X \leq 15) = \phi\left(\frac{15 + 0.5 - 12.5}{2.5}\right).$

$$= \phi(1.2) = 0.8849.$$

Suppose only 75% of all ~~tribals~~ Drivers in a certain state regularly wear a seat belt. A random sample of 500 driver is selected what is Prob. that (i) b/w 360 and 400 (inclusive) of the drivers in the sample regularly wears seat belt (ii) Fewer than 400 of those wear seat belt.

(i) $P(360 \leq X \leq 400)$.

$n = 500, \; p = 0.75, \; q = 0.25$.

$\mu = np$

$\mu = 500 \times 0.75$

$\mu = 375$

$\sigma = \sqrt{npq} = 9.68$.

(i) $P(360 \leq X \leq 400) = \phi\left(\frac{400 + 0.5 - 355}{9.68}\right) - \phi\left(\frac{360 - 0.5 - 375}{9.68}\right)$.

$= \phi(\quad) - \phi(\quad)$.

(ii) $P(X < 400) = 1 - (X \geq 399)$

25/9/19
Wednesday

GAMMA DISTRIBUTION

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\, dx$$

$$\Gamma(n+1) = \int_0^\infty e^{-x} x^{n}\, dx$$

$$f(\alpha) = \frac{e^{-x} x^{\alpha-1}}{\Gamma(\alpha)};\ \alpha > 0$$

Is $f(\alpha)$ a p.d.f ?

$$\int_0^\infty f(x, \alpha)\, dx = \frac{1}{\Gamma(\alpha)} \int_0^\infty e^{-x} x^{\alpha-1}\, dx$$

$$= 1.$$

A gamma funcⁿ is defined by

$$\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1}\, dx,\ \alpha > 0$$

Properties :-

(i) $\Gamma(1/2) = \sqrt{\pi}$

(ii) $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$

(iii) $\Gamma(\alpha+1) = \alpha!\ ;\ \alpha \in Z^+$

$$\Gamma(3/2) = \Gamma(1/2+1) = 1/2\,\Gamma(1/2) = \frac{1}{2}\sqrt{\pi}$$

$$\Gamma(5/2) = 3/2 \times \frac{1}{2} \times \sqrt{\pi}$$

Gamma Distribution.

A continuous random variable 'X' is said to have a gamma distribution if the p.d.f of 'X' is given by :

$$f(x,\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} e^{-x/\beta} x^{\alpha-1} ; & x \geq 0 \\ 0 ; & x < 0 \end{cases} \quad \text{——} (1)$$

where α and β are parameters and both are positive.

> when $\beta = 1$, $\cancel{f(x}$ eq^n (1) reduces to

$$f(x,\alpha,1) = \begin{cases} \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad \text{——} (2)$$

Eq^n (2) is called standard gamma distribution.

> when $\alpha = 1$, $\beta = \frac{1}{\lambda}$

$$f(x,\lambda) = \underbrace{\begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}}_{\text{Exponential distribution.}}$$

The Parameters α and β in gamma distribution are known as : Shape Parameter and scale parameter respectively.

MEAN AND VARIANCE

The mean and variance of a random variable X having gamma distribuⁿ are given by –

$$\mu = E(X) = \alpha\beta.$$

$$\sigma^2 = E(X^2) - (E(X))^2$$
$$= \alpha\beta^2$$

when X is a standard gamma random variable the c.d.f. is given by

$$F(x, \alpha) = \int_0^x \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)} dy \; ; \alpha > 0 \quad (3)$$

Eqⁿ (3) is called Incomplete gamma function.

Let 'X' have a gamma distribution with Parameters α and β. Then for any $x > 0$ the c.d.f is given by -

$$P(X \leq x) = F(x, \alpha, \beta) = F\left(\frac{x}{\beta}, \alpha\right)$$

where $F(\cdot, \alpha)$ is incomplete gamma function.

Suppose the survival time X of a randomly selected mouse exposed to 240 ~~rats~~ rads of gamma radiation has a gamma distribution.

$\alpha = 8$, $\beta = 15$.

(i) Find the expected survival time of the mouse.

$$\mu = E(x) = \alpha\beta$$
$$= 8 \times 15$$
$$= 120.$$

(ii) $$\sigma^2 = \alpha\beta^2$$
$$= \overset{8}{\alpha} \times 225$$
$$= 1800.$$

(iii) $$P(60 \leq x \leq 120)$$

$$= F(b) - F(a).$$
$$= F(120) - F(60)$$
$$= P(X \leq 120) - P(X \leq 60).$$
$$= F\left(\frac{120}{15}, 8\right) - F\left(\frac{60}{15}, 8\right).$$
$$= F(8,8) - F(4,8).$$

(iv) $$P(X > 30) = 1 - P(X \leq 30).$$
$$= 1 - F\left(\frac{30}{15}, 8\right).$$
$$= 1 - F(2,8)$$

26/9/19
Thursday

The WEIBULL DISTRIBUTION.

A r.v X said to have weibull distribu^n with parameters α and β $(\alpha>0)$

So exponential distribuⁿ is a special case of both gamma and weibull distribuⁿ however there are gamma distribuⁿ which are not weibull distribuⁿ and vice versa.

The mean and variance of weibull distribuⁿ is given by –

$$\mu = \beta\,\Gamma\left(1+\frac{1}{\alpha}\right).$$

The cummulative distribuⁿ funcⁿ ~~of~~ of weibull distribuⁿ with parameters α and β is given by

$$F(x,\alpha,\beta) = \begin{cases} 0 & , x<0 \\ 1-e^{-(x/\beta)^{\alpha}} & , x \geq 0. \end{cases}$$

27/9/19
Friday

Q. Suppose the proportion 'X' of surface area in a randomly selected Quadrant, that is covered by a certain plant has standard β distribution with parameters $\alpha = 5$ and $\beta = 2$. Compute.

(i) $E(X)$
(ii) $V(X)$
(iii) $P(X \leq 0.2)$
(iv) $P(0.2 \leq X \leq 0.4)$.

$\alpha = 5 \quad , \beta = 2.$

(i) $\mu = A + (B - A) \dfrac{\alpha}{\alpha+\beta}$

(ii) $\sigma^2 = \dfrac{(B-A)^2 \, \alpha\beta}{(\alpha+\beta)^2 (\alpha+\beta+1)}$

$A = 0 \quad , B = 1$

(i) $\mu = 0 + (1-0) \dfrac{5}{7} , = 0.714$

$= \dfrac{5}{7}.$

(ii) $\sigma^2 = \dfrac{(1-0)^2 \, 10}{49 \times 8}$

$= \dfrac{\cancel{10}\, 5}{49 \times \cancel{8}\, 4}$

$= \dfrac{5}{196} = 0.02.$

(iii) $P(X < 0.2)$

$$P(X \le 0.2) = \int_0^{0.2} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \cdot x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$= \frac{\Gamma(7)}{\Gamma(5)\Gamma(2)} \int_0^{0.2} x^4 (1-x)\,dx.$$

$$= \frac{\Gamma(7)}{\Gamma(5)\cdot\Gamma(2)} \left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^{0.2}$$

$$= \frac{\Gamma(7)}{\Gamma(5)\cdot\Gamma(2)} \left[\frac{0.0003}{5} - \frac{0.00006}{6}\right]$$

$$= \frac{\Gamma(7)}{\Gamma(5)\cdot\Gamma(2)} \left[0.00006 - 0.00001\right]$$

$$= \frac{\Gamma(7)}{\Gamma(5)\cdot\Gamma(2)} (0.00005).$$

$$= \cancel{0.0012}\ 0.0015.$$

(iv) $P(0.2 \le X \le 0.4) =$

$$= \frac{\Gamma(7)}{\Gamma(5)\,\Gamma(2)} \int_{0.2}^{0.4} x^4(1-x)\,dx$$

$$= \frac{720}{24 \times 1} \left[\frac{x^5}{5} - \frac{x^6}{6}\right]_{0.2}^{0.4}$$

CHAPTER - 5 (upto 5.2) aage from another book

JOINT DISTRIBUTION.

Let X and Y be two discrete random variables defined on the sample space S of an experiment. The joint probability mass function $p(x, y)$ is defined for each pair of numbers (x, y) by

$$p(x,y) = P(X=x, Y=y).$$

Such that - (i) $p(x,y) \geq 0$
(ii) $\sum_x \sum_y p(x,y) = 1.$

MARGINAL PROBABILITY MASS FUNCTION

This of X is denoted by $P_X(x)$ and is defined by

$$P_X(x) = \sum_y p(x,y) \text{ for each}$$

possible value of x.
$p(x,y) > 0$.

Similarly the marginal probability mass function of Y is denoted by

$P_Y(y)$ and is defined by.

$$P_Y(y) = \sum_x p(x,y) \text{ for each}$$

possible value of y $p(x,y) > 0$

Q For the following Joint distribution (X, Y). Find
(i) $F(1,1)$ (iii) $p_X(x)$
(ii) $F(1,3)$ (iv) $p_Y(y)$

value of Y.

values of X

x/y	1	2	3	4
0	1/24	1/12	1/12	1/24
1	1/12	1/6	1/6	1/12
2	1/24	1/12	1/12	1/24

$p(0,1)$

(i) $F(1,1) = $

$$P(X=0, Y=1) = 1/24.$$

(iii) $p_X(x) \Rightarrow p_X(0) = \sum_y p(0,y)$

$$= p(0,1) + p(0,2) + p(0,3) + p(0,4)$$

$$= \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{24}$$

$$= \frac{1}{4} = 0.25$$

$p_X(1) = \sum_y p(1,y)$

$$= p(1,1) + p(1,2) + p(1,3) + p(1,4)$$

$$= \frac{1}{12} + \frac{1}{6} + \frac{1}{6} + \frac{1}{12}$$

$$= \frac{1+2+2+1}{12} = \frac{\cancel{6}^{1}}{\cancel{12}_{2}} = 0.5.$$

$p_X(2) = \sum_y p(2,y)$

$$= \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{24} = 0.25$$

(iv) $p_Y(y)$.

$p_Y(1) = \sum_x p(x, 1)$

$= p(0,1) + p(1,1) + p(2,1)$

$= \frac{1}{24} + \frac{1}{12} + \frac{1}{24}$

$= \frac{1+2+1}{24} = \frac{\cancel{4}1}{\cancel{24}6} = 0.17$.

$p_Y(2) = \frac{1}{12} + \frac{1}{6} + \frac{1}{12}$

$= \frac{1+2+1}{12} = \frac{\cancel{4}1}{\cancel{12}3} = 0.33$

$p_Y(3) = \frac{1}{12} + \frac{1}{6} + \frac{1}{12}$

$= 0.33$.

$p_Y(4) = \frac{1}{6}$.

(i) $F(1,1) = P(X \leq 1, Y \leq 1)$

$F(x) = P(X \leq x)$.

$F(1,1) = P(0,1) + P(1,1)$

$= \frac{1}{24} + \frac{1}{12}$

$= \frac{\cancel{3}1}{\cancel{24}8} = \frac{1}{8}$.

$6 \,|\, 12, 6, 24$
$2 \,|\, 2, 1, 4$
1

(ii) $F(1,3) = P(X \le 1, Y \le 3)$

$= P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)$

$= \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{6} + \frac{1}{6}$

$= \frac{1+2+2+2+4+4}{24}$

$= \frac{15}{24}$.

Q. From the following Table for Bi-variate distribution. Find –

(i) $P(X \le 1)$
(ii) $P(Y \le 3)$
(iii) $F(1,1)$
(iv) $F(2,3)$
(v) $F(3,6)$.

x/y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

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Joint Probability Mass Function.

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy.$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx.$$

Q. The Joint prob. density function of the random variable X and Y is given by

$$f(x,y) = \begin{cases} 8xy, & 0 \le x \le 1,\ 0 \le y \le x \\ 0, & \text{otherwise.} \end{cases}$$

Find the marginal ~~density~~ Prob. density func^n of X and Y.

$$f_X(x) = \int_0^x 8xy\,dy$$

$$= \frac{8xy^2}{2}$$

$$= \left[4xy^2\right]_0^x$$

$$= 4x^3. \ ; \ 0 \le x \le 1$$

$$f_Y(y) = \int_0^1 8xy\,dx.$$

classmate

Date

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$$= \left[4x^2y\right]_0^1$$

$$= 4y \quad , \quad 0 \le y \le x$$

$$f_X(x) = \begin{cases} 4x^3 & , 0 \le x \le 1 \\ 0 & , \text{else} \end{cases}$$

$$f_Y(y) = \begin{cases} 4y & , 0 \le y \le x \\ 0, & \text{else} \end{cases}$$

Each front tyre of a particular type of vehicle ~~value~~ is supposed to be filled to a pressure of 26 PSI suppose the actual Air pressure in each tyre is a random variable — X for the right tyre ; Y for the left tyre with Joint p.d.f.

$$f(x, y) = \begin{cases} k(x^2+y^2) & , 20 \le x \le 30, \ 20 \le y \le 30 \\ 0 & , \text{else} \end{cases}$$

(i) what is value of k.

(ii) Determine the distribuⁿ of air pressure in the right tyre alone. what is the prob.

(iii) Are X and Y independent.

what is prob that (iv) Find Marginal distribution for Y.

(v) The air pressure in ~~the~~ left tyre is b/w 24 to 30.

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$$\int_{20}^{30}\int_{20}^{30} f(x,y)\,dx\,dy = 1.$$

$$k\int_{20}^{30}\int_{20}^{30} (x^2+y^2)\,dx\,dy = 1$$

$$k\int_{20}^{30}\left[\frac{x^3}{3} + y^2 x\right]_{20}^{30} dy = 1$$

$$k\int_{20}^{30}\left[\frac{19000}{3} + 10y^2\right] dy = 1$$

$$k\left[\frac{19000}{3}y + \frac{10y^3}{3}\right]_{20}^{30} = 1$$

$$k\left[\frac{190000}{3} + \frac{190000}{3}\right] = 1$$

$$\cancel{k = \frac{380000}{3}} \qquad k = \frac{3}{380000}$$

(ii) $$f_X(x) = \int_{20}^{30} k(x^2+y^2)\,dy.$$

$$= \frac{3}{380000}\left(x^2 y + \frac{y^3}{3}\right)_{20}^{30}$$

$$= \frac{3}{380000}\left[10x^2 + \frac{19000}{3}\right]$$

$$= \frac{3\cancel{0}x^2}{38000\cancel{0}} + \frac{\cancel{57000}}{} \frac{\cancel{190}\,1}{\cancel{380}\,20}$$

$$= \frac{3x^2}{38000} + \frac{1}{20}$$

$$= \frac{3x^2}{38000} + 0.05 \; ; \; 20 \le x \le 30$$

$$f_Y(y) = \int_{20}^{30} k(x^2 + y^2)\,dx$$

$$= k\left[\frac{x^3}{3} + y^2 x\right]_{20}^{30}$$

$$= \frac{3}{380000}\left[\frac{19000}{3} + 10y^2\right]$$

$$= \frac{1}{20} + \frac{3y^2}{38000}$$

$$= \frac{3y^2}{38000} + 0.05 \; ; \; 20 \le y \le 30$$

(iii) In this case X and Y are not independent.

(v) $P(24 \le Y \le 30) = \int_{24}^{30} f_Y(y)\,dy.$

$$= \int_{24}^{30} \left(\frac{3y^2}{38000} + 0.05 \right) dy.$$

~~Indepan~~ Independent Random Variables

Two R.V. X and Y are said to be independent if for every pair x and y values

$p(x, y) = p_X(x) \cdot p_Y(y).$ when X and Y are Discrete.

$f(x,y) = f_X(x) \cdot f_Y(y)$ when X and Y continuous

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If X and Y are discrete random variables then the Joint probability distribution func^n is defined by.

$$F(x,y) = P(X \le x, Y \le y)$$

$$= \sum_{s \le x} \sum_{t \le y} p(s,t)$$

If X and Y are continuous random variable then the function defined by

$$F(x,y) = P(X \le x, Y \le y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(s,t)\,ds\,dt,$$

$-\infty < x < \infty$
$-\infty < y < \infty$

is called as Joint Prob. distri^n func^n.

Find the Marginal density function of X and Y and distribution function of X and Y.

(i) $f(x,y) = \begin{cases} x+y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{else.} \end{cases}$

In this ques. X and Y are dependent

(ii) $f(x,y) = \begin{cases} \frac{1}{4}(1+xy), & |x|<1,\ |y|<1 \\ 0, & \text{else.} \end{cases}$

(i) $f_X(x) = \int_0^1 (x+y)\,dy = \left(xy + \frac{y^2}{2}\right)_0^1$

$= x + \frac{1}{2}, \quad 0 \le x \le 1$

If $P(X \le 1)$ func. is asked we have to use marginal p.d.f of X

$f_Y(y) = y + \frac{1}{2}; \quad 0 \le y \le 1.$

$F(x,y) = \int_0^1 \int_0^1 (x+y)\,dx\,dy.$

$\Big(\quad 0 \quad , \text{ else}$

Find the Joint p.d.f of x and y.

$$f(x,y) = \frac{\partial^2}{\partial x \partial y}(F(x,y))$$

$$\cancel{= \frac{\partial^2 (1-e^{-x})(1-e^{-y})}{\partial x \partial y}}$$

$$= \frac{\partial^2}{\partial x \partial y}(1-e^{-x})(1-e^{-y}).$$

$$\cancel{f(x,y) = e^{-x}e^{-y}, \quad x>0, y>0}$$

$$\frac{\partial}{\partial x}\left[\frac{\partial}{\partial y}(1-e^{-x})(1-e^{-y})\right]$$

$$= \frac{\partial}{\partial x}(1-e^{-x}) \cdot (e^{-y})$$

$$= e^{-x}e^{-y} \quad , \quad x>0, y>0$$

Conditional Probability

Case : I

X, Y are discrete R.V

If ~~Prob~~ $P(X = x_i, Y = y_j)$ is the Joint probability mass function of R.V X and Y. then the conditional probability function of X given $Y = y_j$ is defined by

$$P(X = x_i \mid Y = y_j) = \frac{P(X = x_i, Y = y_j)}{P(Y = y_j)}$$

Similarly Probability of Y given $X = x_i$ is given by

$$P(Y = y_j \mid X = x_i) = \frac{P(X = x_i, Y = y_j)}{P(X = x_i)}$$

Case II : Let X and Y be two continuous R.V with Joint p.d.f $f(x, y)$ and marginal Prob. density funcⁿ of X as $f_X(x)$ then the conditional Probability density function of Y given $X = x$ is

X and Y are continuous R.V

$$f_{Y/X}(y/x) = \frac{f(x, y)}{f_X(x)}, \quad -\infty < y < \infty.$$

Similarly X given $Y = y$ is

$$f_{X/Y}(x/y) = \frac{f(x, y)}{f_Y(y)}, \quad -\infty < x < \infty$$

Q. The two Dimensional random variable X, Y have Joint P.M.F

$$f(x,y) = \frac{x+2y}{27}, \quad x = 0,1,2 \quad y = 0,1,2$$

Find the conditional distribution of Y for X = x.

$$\cancel{P(X = x / Y = y_j)}$$

$$P(Y = y_j \mid X = x_i) = \frac{P(X = x_i, Y = y_j)}{P(X = x_i)}$$

$$= \frac{P(X=0, Y=0)}{P(X=0)} = 0.$$

x/y	0	1	2
0	0	2/27	4/27
1	1/27	1/9	5/27
2	2/27	4/27	2/9

$$\frac{P(X=0, Y=1)}{P(X=0)} = \frac{2}{27}$$

$$\frac{P(X=0, Y=2)}{P(X=0)} = \frac{4}{27}.$$

$$\frac{P(X=1, Y=0)}{P(X=1)} = \frac{1}{27}.$$

$$\frac{P(X=1, Y=1)}{P(X=1)} = \frac{3}{27}.$$

$$\frac{P(X=1, Y=2)}{P(X=1)} = \frac{5}{27}$$

$P(Y = y_j \mid X = 0)$ $\quad$ $P(Y = y_j \mid X = 1)$ $\quad$ $P(Y = y_j \mid X = 2)$

$P(Y = 0 \mid X = 0) = 0$

$P(Y = 1 \mid X = 0) = \dfrac{\frac{2}{27}}{6/27} = \dfrac{1}{3}$

$P(Y = 2 \mid X = 0) = \dfrac{\frac{4}{27}}{6/27} = \dfrac{2}{3}$

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Q. An Instructor has given a short quiz consisting of two parts, For a randomly selected student, let X denotes the no. of points earned in the First part Y denotes no. of points earned in 2nd part.
suppose that the Joint PMF of X and Y is given as follows.

$p(x,y)$		y			
	0	0.02	0.06	0.02	0.10
x	5	0.04	0.15	0.20	0.10
	10	0.01	0.15	0.14	0.01

Find (i) expected no. of points scored in 1st part
(ii) Expected no. of points scored in 2nd part
(iii) If the score recorded in grade book is the total no. of points earned on the two parts what is the expected recorded score.

Yaha row sum tha.

(i) $E(X) = \sum x f_X(x)$
$= 0 \times f_X(0) + 5 \times f_X(5) + 10 \times f_X(10)$.
$= 5.55$

Yaha column sum hoga.

(ii) $E(Y) = \sum y P_Y(y) = 0 \times 0.07 + 5 \times 0.36 + 10 \times 0.36 + 15 \times 0.21$
$= 8.55$.

(iii) $E(X+Y) = \sum_x \sum_y (x+y) p(x,y)$

$= 0+0(0.02) + 0+5(0.04) + 0+10(0.01)$
$+ 5+0(0.06) + 5+5(0.15) + 5+10(0.15)$
$+ 10+0(0.02) + 10+5(0.20) + 10+10(0.14)$
$+ 15+0(0.10) + 15+5(0.10) + 15+10(0.01)$

$= 0.2 + 0.1 + 0.3 + 1.5 + 2.25$
$+ 0.2 + 3 + 2.8 + 1.5 + 2 + 0.25$

$= 14.10$

or.

$E(aX+b) = aE(X)+b$

$E(aX+bY) = aE(X)+bE(Y)$
$= 5.55 + 8.55$
$= 14.10$

Q. A large insurance agency services a no. of customers who have purchased both a home owner policy and an automobile policy from the agency. For each type of policy let X denote the deductable amount on the auto policy. Y denote the deductable amount on a homeowner policy and Joint PMF is given by

$p(x,y)$		y: 0	100	200
x	100	0.20	0.10	0.20
	250	0.05	0.15	0.30

Find (i) $E(X)$ (ii) $E(Y)$ (iii) $E(X+Y)$

the life of the ... justify its new business salvage value is 20 Lakh. ~~pro~~ project 10% with no profit Rs 3 Lakh 50 thousand. The annual net. Find the rate of return for new business.

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$$E(X) = \underset{\uparrow}{\overset{\text{row 1 sum}}{}}100 \times 0.5 + 250 \times 0.5 = 175$$

(row 1 sum, row 2 sum)

$$E(Y) = 0 \times 0.25 + 100 \times 0.25 + 200 \times 0.5 = 125$$

$$E(X+Y) = 175 + 125 = 300$$

Covariance

no. If covariance is +ve and large they r.v are strongly related. If -ve weakly related.

The covariance between two random variables X and Y is defined by

$$Cov(X, Y) = E\left[(X - \mu_X)(Y - \mu_Y)\right]$$

$$= \begin{cases} \sum_x \sum_y (x - \mu_x)(y - \mu_y)\, p(x, y), & X, Y \text{ discrete.} \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - \mu_x)(y - \mu_y)\, f(x, y)\, dx\, dy, & X, Y \text{ continuous} \end{cases}$$

Prev. Ques

$$\mu_X = 175, \quad \mu_Y = 125.$$

$$Cov(X, Y) = \sum_x \sum_y (x - 175)(y - 125)\, p(x, y)$$

$$= (100 - 175)(0 - 125) \times 0.2 + (100 - 175)(100 - 125) \times 0.1 + (100 - 175)(200 - 125) \times 0.2 +$$

$$(250 - 175) \times 0.05 + (250 - 175)(100 - 125) \times 0.15 + (250 - 175)(200 - 125) \times 0.30$$

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$$cov(x, x) = E\left[(x - \mu_x)^2\right] = var[x].$$

Mostly used.
$$cov(x, y) = E(xy) - \mu_x \mu_y.$$

NOTE : (i) For a strong +ve relationship co-variance of X, Y should be quite +ve.

(ii) For a strong -ve relationship the signs of $(x - \mu_x)$ and $(y - \mu_y)$ will tend to be opposite, Results in a negative product. Hence For a strong -ve relationship co-variance of X, Y should be quite -ve.

(iii) If X, Y are not strongly related, positive and negative products will tend to cancel one another, results in a covariance near zero.

limit $= 0$ to $1-x$

$$f(x,y) = \begin{cases} 24xy & ; 0 \le x \le 1, 0 \le y \le 1, x+y \le 1 \\ 0 & , \text{otherwise.} \end{cases}$$

Find $co(X, Y)$.

$$f_X(x) = \int_{-\infty}^{\infty} 24xy \, dy.$$

$$f_Y(y) = \int_{-\infty}^{\infty} 24xy \, dx.$$

$$\mu_X = E(X) = \int_{-\infty}^{\infty} x f_X(x) \, dx.$$

$$\mu_Y = E(Y) =$$