5/9/19 Thursday

Continuous Random Variable

P.d.f : (i) $f(x) \geq 0 \quad \forall x$.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Cumulative Distribution Function.

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = P(X \leq x).$$

$$P(a \leq x \leq b) = F(b) - F(a).$$

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx.$$

$$\sigma^2 = E(X^2) - \mu^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$E(X-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx.$$

Q. A college professor never finishes his lecture before the end of an hour and always finishes his lectures within 1 minute after the hour. Let X be the time between end of the hour of the lecture. The P.d.f is given by.

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find -.

(i) k

(ii) what is the probability that the lecture ends within 1 minute of the end of the hour.

(iii) what is the Prob. that the lecture continues beyond the hour for between 60 sec to 90 sec.

(iv) what is Prob. that the lecture continues atleast 30 sec beyond the end of the hour.

(i) $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

$$\int_{-\infty}^{0} f(x)\,dx + k\int_{0}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx = 1$$

$$k\left[\int_{0}^{2} x^2\,dx\right] = 1$$

$$k\left[\left[\frac{x^3}{3}\right]_0^2\right] = 1$$

$$k\left[\frac{8}{3}\right] = 1$$

$$k = \frac{3}{8}$$

5)

(ii) $\int_0^1 f(x)\,dx$

$= \int_0^1 \frac{3}{8} x^2\,dx$

$= \left[\frac{3x^3}{8 \times 3}\right]_0^1$

$= \left[\frac{x^3}{8}\right]_0^1$

$= \frac{1}{8}$

(iii) $\int_1^{1.5} f(x)\,dx$

$\int_1^{1.5} \frac{3x^3}{8}\,dx$

$= \left[\frac{x^3}{8}\right]_1^{1.5}$

$= \frac{2.37}{8}$

(iv) $\int_{1.5}^{2} f(x)\,dx = \int_{1.5}^{12}$

(v) $P(X \geq 1.5) = 1 - P(X < 1.5)$

$= 0.578$

Q. The current in a certain circuit is measured by an ammeter is a continuous random variable X with pdf

$$f(x) = \begin{cases} 0.075x + 0.2, & 3 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$$

Find : (i) $P(X \le 4)$

(ii) $P(3.5 \le X \le 4.5)$

(iii) $P(4.5 < X)$.

(i) $$\int_3^4 (0.075x + 0.2)\,dx$$

$$\left[\frac{0.075x^2}{2} + 0.2x\right]_3^4$$

$$= 0.4625$$

(ii) $$\int_{3.5}^{4.5} (0.075x + 0.2)\,dx = 0.5$$

(iii) $$\int_{4.5}^{5} (0.075x + 0.2)\,dx = 0.2781$$

51

0. $$F(x) = \begin{cases} 0, & x < 0. \\ \frac{x^2}{4}, & 0 \le x < 2. \\ 1, & 2 \le x \end{cases}$$

Compute: (i) $P(X \le 1)$.
(ii) $P(0.5 \le X \le 1)$
(iii) $P(X > 1.5)$
(iv) $f(x)$ [p.d.f].
(v) $E(X)$
(vi) $V(X)$
(vii) σ_X.

(i) $P(X \le 1) = F(1) = \frac{1}{4}$

(ii) $P(0.5 \le X \le 1) = F(1) - F(0.5)$

$= \frac{1}{4} - \frac{(0.5)^2}{4} = \frac{3}{16}$

(iii) $P(X > 1.5) = 1 - P(X \le 1.5)$

$= 1 - F(1.5)$

$= 1 - \frac{(1.5)^2}{4}$

$= \frac{7}{16}$

(iv) $$f(x) = \begin{cases} \frac{x}{2}, & 0 \le x < 2 \\ 0, & \text{else.} \end{cases}$$

(v) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$.

$= \int_{0}^{2} \frac{x^2}{2}\,dx + \int_{0}^{\infty}$

$= \left[\frac{x^3}{6}\right]_0^2$

$= \frac{8}{6} = \frac{4}{3}$.

(vi) $\text{Var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$.

$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$= \int_{0}^{2} x^2 \cdot \frac{x}{2} - \left(\frac{4}{3}\right)^2$

$= \frac{1}{2}\left[\frac{x^4}{4}\right]_0^2 - \frac{16}{9} = \frac{2}{9}$.

(vii) $\sigma_x = \sqrt{\frac{2}{9}}$.

Q.No. 8

$$f(x) = \begin{cases} 0, & x \le 0 \\ \frac{x}{4}\left[1 + \ln\left(\frac{4}{x}\right)\right], & 0 < x \le 4 \\ 1, & x > 4 \end{cases}$$

Compute : (i) $P(X \le 1)$
(ii) $P(1 \le X \le 3)$
(iii) $f(x)$.

NORMAL DISTRIBUTION (mean and median are same)

Probability Distribution func^n

A Continuous RV 'X' is said to have a normal distribution with parameters μ or σ (or σ^2). where $-\infty < \mu < \infty$ and $\sigma > 0$. If the P.d.f of X is.

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

where μ = mean.

σ = standard deviation.

$$X \sim N(\mu, \sigma)$$

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

STANDARD NORMAL VARIABLE:

The normal distribution with parameters $\mu = 0$ and $\sigma = 1$ is called standard normal distribution.

A random variable having a standard normal distribution is called a standard normal random variable and it is denoted by.

(Z)

$$f(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-z^2/2}, \quad -\infty < z < \infty$$

Properties of Standard Normal Distribution

Distribution function of z (standard normal variable).

The distribution function $\phi(z)$ of a standard normal variate is defined by

$$\phi(z) = P(Z \le z)$$

$$= \int_{-\infty}^{z} f(t)\,dt$$

$$= \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-t^2/2}\,dt.$$

(2) Properties of Distribution func^n.

(1) $\phi(-z) = 1 - \phi(z)$

(2) $P(a \le x \le b) = \int_a^b \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$

$P\left(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right)$.

$$P(X \le a) = \Phi\left(\frac{a-\mu}{\sigma}\right)$$

$$P(X \ge b) = 1 - P(X \le b)$$
$$= 1 - \Phi\left(\frac{b-\mu}{\sigma}\right)$$

2. Let Z be a standard normal random variable then evaluate the following.

(i) $P(Z \le 1.25)$ = 0.8944

(ii) $P(Z > 1.25)$

(iii) $P(Z \le -1.25)$

(iv) $P(-0.38 \le Z \le 1.25)$

(i) 0.8944

(ii) $1 - P(Z \le 1.25)$
$= 1 - 0.8944$
$= 0.1056$

1st property in both

(iii) 0.1056.

(iv) $\Phi(1.25) - \Phi(-0.38)$
$= \Phi(1.25) - 1 + \Phi(0.38)$
$= 0.8944 - 1 + 0.6480$
$= 0.5424$

11/9/19 Wednesday

NON-STANDARD Normal Distribution.

If X has a normal distribution with mean μ and standard deviation σ.

then $Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution. and the Probability of

$$P(a \leq X \leq b) = \left(\frac{a-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{b-\mu}{\sigma}\right)$$

$$= P\left(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$$

Q. The break down voltage of a randomly chosen diode of a particular type is known to be normally distributed (i) what is the Probability that a diode's breakdown voltage is within 1 S.D of its mean value.

$$P(\mu - \sigma \leq X \leq \mu + \sigma)$$

$$= P\left(\frac{\mu - \sigma - \mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{\mu + \sigma - \mu}{\sigma}\right)$$

$$= P(-1 \leq z \leq 1)$$

$$= \phi(1) - \phi(-1)$$

$$= 0.8413 - 0.1587.$$

$$= 0.6826$$

(ii) what is the probability a diode breakdown voltage is with 2 S.D of its mean value.

$$P(\mu - 2\sigma \leq x \leq \mu + 2\sigma)$$

$$= P\left(\frac{\mu - 2\sigma - \mu}{\sigma} \leq \frac{x - \mu}{\sigma} \leq \frac{\mu + 2\sigma - \mu}{\sigma}\right)$$

$$= P(-2 \leq Z \leq 2)$$

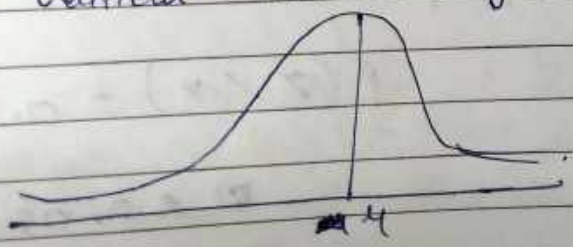
$$= \Phi(2) - \Phi(-2)$$

$$= 0.9772 - 0.0228$$

$$= 0.9544.$$

AREA UNDER NORMAL CURVE.

The curve of a normal distribution is uni-model and a bell shape, with the highest point over the mean (μ). It is symmetric about a vertical line through μ.



The Normal curve has following features—

1. The values of mean, median and mode are equal.
2. The curve is symmetrical about the coordinate at its means which locate the peak of the bell.

(3) The area covered by $\mu - \sigma$ and $\mu + \sigma$ is 0.6826 (68.26%) of the total area.

(4) The area covered blw the limits $\mu - \sigma$ and $\mu + \sigma$ is 0.9544 (95.44%) of the total area.

12/9/19 Thursday

Percentile of Standard normal distribution

For any 'p' blw 0 to 1. The normal distribution table can be used to obtain $(100p)^{th}$ percentile of the standard normal distribution.

For eg: The 99^{th} percentile of the standard normal distribution is that value on the horizontal axis such that the area under the z curve to the left of the value is 0.99.

$$P(Z \leq z) = 0.99$$

~~z = 0.09~~

$2.33 \rightarrow 0.9901$

$$P(Z \leq 2.33) = 0.99$$

Find z such that.

(i) $P(z \leq z) = 0.95$

$0.95 \rightarrow 0.9505 \rightarrow 1.65$

$0.9495 \rightarrow 1.64$

$P(z \leq 1.645)$.

(ii) $P(z \leq z) = 0.97$

$0.97 \rightarrow 0.9699 \rightarrow 1.88$.

$P(z \leq 1.88)$

(iii) $P(z \leq z) = 0.92$

$0.92 \rightarrow 0.9207 \rightarrow 1.41$.

$P(z \leq 1.41)$

Z_α : CRITICAL VALUE : Z_α denote the value on the z axis for which α of the area under the z curve lies to the right of Z_α.

$P(z \geq z_\alpha) = \alpha$.

$P(Z \geq 1) = 1 - P(z < 1)$

$= 1 - \phi(1) = 1 - 0.8413$

$= 0.1587$.

Since α under the Z curve lies in the right of Z_α, $(1-\alpha)$ of the area lies to its left.

Hence Z_α is the 100 times $(1-\alpha)$ times percentile of the standard normal distribution.

Q. Determine Z_α if (i) $\alpha = 0.0055$.

$$P(Z \geq Z_\alpha) = 0.0055.$$

$$1 - P(Z < Z_\alpha) = 0.0055.$$

$$1 - 0.0055 = P(Z < Z_\alpha)$$

$$0.9945 = P(Z < Z_\alpha)$$

$$\boxed{Z_\alpha = 2.54}$$

(ii) $\alpha = 0.09$

$$P(Z \geq Z_\alpha) = 0.09$$

$$1 - P(Z < Z_\alpha) = 0.09.$$

$$1 - 0.09 = P(Z < Z_\alpha)$$

$$0.91 = P(Z < Z_\alpha).$$

$$Z_\alpha = 1.34$$

(iii) $\alpha = 0.663$

$P(Z \geq z_\alpha) = 0.663$

$1 - P(Z \geq z_\alpha) = 0.663$

$1 - 0.663 = P(Z < z_\alpha)$

$0.337 = P(Z < z_\alpha)$

$z_\alpha = -0.42$.

3. The mean and standard deviation of a normal variable X are 50 and 4 respectively. Find the value of the corresponding standard normal ~~distributi~~ variable. ~~X~~ when.

$X = 42, 54, 84$. Hence find the probabilities.

$\mu = 50$

$\sigma = 4$.

$Z = \frac{X_1 - 50}{4}$

$z_1 = \frac{42 - 50}{4}$

$= \frac{-8}{4}$

$= -2$.

$z_2 = 1$

$z_3 = 8.5$.

13/9/19
Friday

Q. The A, B, C company uses a machine to fill boxes with soap powder. Assume that the net weight of the boxes of soap is normally distributed with mean 15 and standard deviation 0.8. What proportion of boxes will have net weight of more than 14 pound.

$\mu = 15$, $\sigma = 0.8$, $X = 14$

$P(X > 14) = Z = \frac{X - \mu}{\sigma}$

$= \frac{14 - 15}{0.8}$

$= \frac{-10}{8} \frac{5}{4} = \frac{-5}{4}$

$= -1.25$.

$= P(Z > -1.25)$

$= 1 - P(Z < -1.25)$

$= 1 - 0.1056$

$= 0.8944$.

$= 89.44\%$

Hence 89.44% of the boxes will have their net weight more than 14 pound.

9. Let X denote the no. of success in a test. If X is normally distributed with $\mu = 100$ and σ is 15 then find Prob. that X does not exceed 130?

$$X = 130, \quad \mu = 100, \quad \sigma = 15.$$

$$Z = \frac{X - \mu}{\sigma} = \frac{130 - 100}{15}$$

$$= 2.$$

$$P(Z < 2) = 0.9772.$$

$$= \phi(2) = 0.9772.$$

1. Assume that the mean height of a soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall.

Soln

$$\sigma^2 = 10.8$$

$$\sigma = \sqrt{10.8}$$

$$\sigma = 3.29$$

$$\mu = 68.22.$$

$$X = 72.$$

$$Z = \frac{72 - 68.22}{3.29} = \frac{3.78}{3.29} = 1.148.$$

$$P(Z > 1.148) = 1 - P(Z \leq 1.148)$$

$$= 1 - 0.8749 = 0.1251 = 12.51\%$$

Q. What is the Prob. that a standard normal variable Z will be lying between 1.25 and 2.75.

$$P(1.25 \leq Z \leq 2.75)$$

$$= \phi(2.75) - \phi(1.25)$$

$$= 0.1026.$$

Q. Suppose that blood chloride conc^n has a normal distribu^n with $\mu = 104$ and $\sigma = 5$. What is Prob. that.

(i) Chloride conc^n is less than 105.

(ii) Atmost 105.

(iii) Chloride conc^n differs from the mean by more than 1 standard deviation

(iv) Does the Prob. depends on the value of μ and σ.

(8) $\mu = 104, \sigma = 5$.

(i) $X = 105$.

$$Z = \frac{105 - 104}{5} = \frac{1}{5}.$$

$$P(Z < 0.2) = \phi(0.2) = 0.5793.$$

(ii) $P(Z \leq 0.2) = 1 - \phi(0.2)$

$= 0.4207$.

$= 0.5793$

(iii) $P(\mu - \sigma \leq X \leq \mu + \sigma)$.

$$P\left(\frac{\mu - \sigma - \mu}{\sigma} \leq \frac{X - \mu}{\sigma} \leq \frac{\mu + \sigma - \mu}{\sigma}\right)$$

$$P(-1 \leq Z \leq 1)$$

$$\Phi(1) - \Phi(-1)$$

$$= 0.8413 - 0.1587$$

$$= 0.6826 \quad = 1 - 0.6826 = 0.3174$$

(iv) No document. It depends upon the values of μ and σ.

Suppose that 25% of all students at a large public university receive scholarship. Let X be the number of students in a random sample of size 50 who receive scholarship. Find the prob. that

(i) atmost 10 student receives scholarship.

(ii) b/w 5 and 15 (inclusive) receives scholarship.

$X = 50$, $p = 0.25$.

$n = 50$, $p = 0.25$.

By poisson's.

$\lambda = np$

$\lambda =$

APPROXIMATION OF BINOMIAL BY NORMAL DISTRIBUTION

$$P(X \leq 10) = \Phi\left(z \leq \frac{10-\mu}{\sigma}\right)$$

Let X be a Binomial random variable based on 'n' trials with success probability 'p' then if the binomial probability histogram is not too skewed, X has approximately a normal distribution with mean $\mu = np$ and standard deviation $\sigma = \sqrt{npq}$.

Then the Probability ① $P(X \leq x) = \Phi\left(\frac{x + 0.5 - np}{\sqrt{npq}}\right)$

② $P(a \leq X \leq b) = \Phi\left(\frac{a - 0.5 - np}{\sqrt{npq}}\right) - \Phi\left(\frac{b + 0.5 - np}{\sqrt{npq}}\right)$

$n = 50$

$p = 0.25$

$\mu = np = 50 \times 0.25 = 12.5$

$\sigma = \sqrt{npq} = \sqrt{50 \times 0.25 \times 0.75} = 3.06$

(i) $P(X \leq 10) = \Phi\left(\frac{10 + 0.5 - 12.5}{3.06}\right)$

$= \Phi(-0.653) = 0.2578.$

(ii) $P(5 \leq X \leq 15) = \Phi\left(\frac{15 + 0.5 - 12.5}{3.06}\right) - \Phi\left(\frac{5 - 0.5 - 12.5}{3.06}\right)$

$$= \Phi(0.98) - \Phi(-2.61)$$

$$= 0.8320.$$

Q. Let X have a binomial distribution with parameters $n = 25$ and p. Calculate each of the following probabilities using normal distribution approximation for the cases- $p = 0.5, 0.6, 0.8$ and compare to the exact probabilities.. Find -

(i) $P(X \leq 15)$

(ii) $P(X \geq 20)$

(iii) $P(15 \leq X \leq 20)$.

$p = 0.5, n = 25,$ $\therefore \mu = 25 \times 0.5$

$\mu = 12.5$

$$\sigma = \sqrt{25 \times 0.5 \times 0.5} = 2.5$$

(i) $$P(X \leq 15) = \Phi\left(\frac{15 + 0.5 - 12.5}{2.5}\right)$$

$$= \Phi(1.2) = 0.8849.$$

Suppose only 75% of all ~~Tribals~~ Drivers in a certain state regularly wears a seat belt. A random sample of 500 driver is selected. what is Prob. that (i) b/w 360 and 400 (inclusive) of the drivers in the sample regularly wears seat belt (ii) Fewer than 400 of those wear seat belt.

(i) $P(360 \leq X \leq 400)$

$n = 500$, $p = 0.75$, $q = 0.25$.

$\mu = np$

$\mu = 500 \times 0.75$

$\mu = 375$

$\sigma = \sqrt{npq} = 9.68$.

(i) $P(360 \leq x \leq 400) = \Phi\left(\frac{400 + 0.5 - 375}{9.68}\right) - \Phi\left(\frac{360 - 0.5 - 375}{9.68}\right)$.

$= \Phi(\quad) - \Phi(\quad)$.

(ii) $P(X < 400) = 1 - (X \geq 399)$

25/9/19 Wednesday

GAMMA DISTRIBUTION

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\, dx.$$

$$\Gamma(n+1) = \int_0^\infty e^{-x} x^n dx$$

$$f(\alpha) = \frac{e^{-x} x^{\alpha-1}}{\Gamma(\alpha)}; \quad \alpha > 0.$$

Is $f(\alpha)$ a p.d.f?

$$\int_0^\infty f(x, \alpha)\, dx = \frac{1}{\Gamma(\alpha)} \int_0^\infty e^{-x} x^{\alpha-1} dx$$

$$= 1.$$

A gamma func^n is defined by

$$\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1}\, dx, \quad \alpha > 0$$

Properties :-

(i) $\Gamma(1/2) = \sqrt{\pi}$.

(ii) $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$

(iii) $\Gamma(\alpha+1) = \alpha!$; $\alpha \in Z^+$

$\Gamma(3/2) = \Gamma(1/2+1) = 1/2\,\Gamma(1/2) = \frac{1}{2}\sqrt{\pi}$.

$\Gamma(5/2) = 3/2 \times \frac{1}{2} \times \sqrt{\pi}$

Gamma Distribution.

A continuous random variable 'X' is said to have a gamma distribution if the p.d.f of 'X' is given by :

$$f(x, \alpha, \beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} e^{-x/\beta} x^{\alpha-1}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad \text{---(1)}$$

where α and β are parameters and both are positive.

> when $\beta = 1$, eq^{n} (1) reduces to

$$f(x, \alpha, 1) = \begin{cases} \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad \text{---(2)}$$

eq^{n} (2) is called standard gamma distribution.

> when $\alpha = 1$, $\beta = \frac{1}{\lambda}$.

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Exponential distribution.

The Parameters α and β in gamma distribution are known as : Shape Parameter and Scale parameter respectively.

MEAN AND VARIANCE

The mean and variance of a random variable X having gamma distribution are given by -

$$\mu = E(X) = \alpha\beta,$$

$$\sigma^2 = E(x^2) - (E(x))^2$$
$$= \alpha\beta^2$$

When X is a standard gamma random variable, the C.d.f. is given by

$$F(x, \alpha) = \int_0^x \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)} dy ; \quad \alpha > 0 \quad (3)$$

Eq^n (3) is called Incomplete gamma function.

Let 'X' have a gamma distribution with Parameters α and β. Then for any $x > 0$ the C.d.f is given by -

$$P(X \leq x) = F(x, \alpha, \beta) = F\left(\frac{x}{\beta}, \alpha\right)$$

where $F(\cdot, \alpha)$ is incomplete gamma function.

Suppose the survival time X of a randomly selected mouse exposed to 240 rads of gamma radiation has a gamma distribution.

$\alpha = 8, \beta = 15$.

(i) Find the expected survival time of the mouse.

$$\mu = E(X) = \alpha\beta$$
$$= 8 \times 15$$
$$= 120.$$

(ii) $$\sigma^2 = \alpha\beta^2$$
$$= 8 \times 225$$
$$= 1800.$$

(iii) $$P(60 \leq X \leq 120)$$
$$= F(b) - F(a).$$
$$= F(120) - F(60)$$
$$= P(X \leq 120) - P(X \leq 60)$$
$$= F\left(\frac{120}{15}, 8\right) - F\left(\frac{60}{15}, 8\right).$$
$$= F(8,8) - F(4,8).$$

(iv) $$P(X > 30) = 1 - P(X \leq 30)$$
$$= 1 - F\left(\frac{30}{15}, 8\right)$$
$$= 1 - F(2,8)$$

26/9/13
Thursday

The WEIBULL DISTRIBUTION

A r.v X said to have weibull distribution with parameters α and β ($\alpha, \beta > 0$)

So Exponential distribu" is a special case of both gamma and weibull distribu" however there are gamma distribu" which are not weibull distribut" and vice versa.

The mean and variance of weibull distribu" is given by

$$\mu = \beta\, \Gamma\left(1 + \frac{1}{\alpha}\right)$$

The cumulative distribu" func" with parameters α and β of weibull distribu" is given by

$$F(x, \alpha, \beta) = \begin{cases} 0, & x < 0 \\ 1 - e^{-(x/\beta)^{\alpha}}, & x \geq 0. \end{cases}$$

27/9/19
Friday

0. Suppose the proportion 'X' of surface area in a randomly selected Quadrant. that is covered by a certain plant has standard β distribution. with parameters.
$\alpha = 5$ and $\beta = 2$. Compute.

(i) $E(X)$
(ii) $V(X)$
(iii) $P(X \leq 0.2)$
(iv) $P(0.2 \leq X \leq 0.4)$.

$\alpha = 5$, $\beta = 2$.

(i) $\mu = A + (B - A) \frac{\alpha}{\alpha + \beta}$

(ii) $\sigma^2 = \frac{(B-A)^2 \alpha\beta}{(\alpha+\beta)^2 (\alpha+\beta+1)}$

$A = 0$, $B = 1$

(i) $\mu = 0 + (1-0) \frac{5}{7}$, $= 0.714$

$= \frac{5}{7}$.

(ii) $\sigma^2 = \frac{(1-0)^2 10}{49 \times 8}$

$= \frac{10 5}{49 \times 84}$

$= \frac{5}{196} = 0.02$.

(iii) $P(X \leq 0.2)$

$$P(X \leq 0.2) = \int_0^{0.2} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \cdot x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$= \frac{\Gamma(7)}{\Gamma(5)\Gamma(2)} \int_0^{0.2} x^4 (1-x)\,dx$$

$$= \frac{\Gamma(7)}{\Gamma(5)\cdot\Gamma(2)} \left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^{0.2}$$

$$= \frac{\Gamma(7)}{\Gamma(5)\cdot\Gamma(2)} \left[\frac{0.0003}{5} - \frac{0.00006}{6}\right]$$

$$= \frac{\Gamma(7)}{\Gamma(5)\cdot\Gamma(2)} \left[0.00006 - 0.00001\right]$$

$$= \frac{\Gamma(7)}{\Gamma(5)\cdot\Gamma(2)} (0.00005)$$

$$= 0.00112 \quad 0.0015.$$

(iv) $P(0.2 < X \leq 0.4) =$

$$= \frac{\Gamma(7)}{\Gamma(5)\,\Gamma(2)} \int_{0.2}^{0.4} x^4 (1-x)\,dx$$

$$= \frac{720}{24 \times 1} \left[\frac{x^5}{5} - \frac{x^6}{6}\right]_{0.2}^{0.4}$$

CHAPTER – 5 (upto 5.2) copy from online by

JOINT DISTRIBUTION.

Let X and Y be two discrete random variables defined on the sample space S of an experiment. The joint probability mass function $p(x,y)$ is defined for each pair of numbers (x,y) by

$$p(x,y) = P(X=x, Y=y).$$

such that – (i) $p(x,y) \geq 0$

(ii) $\sum_x \sum_y p(x,y) = 1.$

MARGINAL PROBABILITY MASS FUNCTION

This of X is denoted by $P_X(x)$ and is defined by

$$P_X(x) = \sum_y p(x,y) \text{ for each}$$

possible value of x.

$p(x,y) > 0$

Similarly the Marginal probability mass function of Y is denoted by

$P_Y(y)$ and is defined by.

$$P_Y(y) = \sum_x p(x,y) \text{ for each}$$

possible value of y $p(x,y) > 0$

6. For the following Joint distribution (X, Y). Find
(i) $F(1,1)$ (iii) $p_X(x)$
(ii) $F(1,3)$ (iv) $p_Y(y)$

x / y	1	2	3	4
0	1/24	1/12	1/12	1/24
1	1/12	1/6	1/6	1/12
2	1/24	1/12	1/12	1/24

values of X

$p(0,1)$

value of Y.

(i) $F(1,1) = P(X=0, Y=1) = 1/24.$

(iii) $p_X(x) \Rightarrow p_X(0) = \sum_y p(0,y)$

$= p(0,1) + p(0,2) + p(0,3) + p(0,4)$

$= \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{24}$

$= \frac{1}{4} = 0.25$

$p_X(1) = \sum_y p(1,y)$

$= p(1,1) + p(1,2) + p(1,3) + p(1,4)$

$= \frac{1}{12} + \frac{1}{6} + \frac{1}{6} + \frac{1}{12}$

$= \frac{1+2+2+1}{12} = \frac{6}{12} = \frac{1}{2} = 0.5$

$p_X(2) = \sum_y p(2,y)$

$= \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{24} = 0.25$

(iv) $p_Y(y)$.

$p_Y(y) = \sum_x p(x,1)$

$= p(0,1) + p(1,1) + p(2,1)$

$= \frac{1}{24} + \frac{1}{12} + \frac{1}{24}$

$= \frac{1+2+1}{24} = \frac{4}{24} = \frac{1}{6} = 0.17$

$p_Y(2) = \frac{1}{12} + \frac{1}{6} + \frac{1}{12}$

$= \frac{1+2+1}{12} = \frac{4}{12} = \frac{1}{3} = 0.33$

$p_Y(3) = \frac{1}{12} + \frac{1}{6} + \frac{1}{12}$

$= 0.33$

$p_Y(4) = \frac{1}{6}$

(i) $F(1,1) = P(X \le 1, Y \le 1)$

$F(x) = P(X \le x)$

$F(1,1) = P(0,1) + P(1,1)$

$= \frac{1}{24} + \frac{1}{12}$

$= \frac{3}{24} = \frac{1}{8}$

(ii) $F(1,3) = P(X \le 1, Y \le 3)$

$= P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)$

$= \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{6} + \frac{1}{6}$

$= \frac{1+2+2+2+4+4}{24}$

$= \frac{15}{24}$.

From the following Table for Bi-variate distribution. Find —

(i) $P(X \le 1)$ (v) $F(3,6)$.

(ii) $P(Y \le 3)$.

(iii) $F(1,1)$

(iv) $F(2,3)$

x \ y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32.
1	1/16	1/16	1/8	1/8	1/8	1/8.
2	1/32	1/32	1/64	1/64	0	2/64.

16/10/19
Wednesday.

Joint Probability Mass Function.

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy.$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx.$$

Q. The Joint prob. density function of the random variable x and y is given by

$$f(x,y) = \begin{cases} 8xy, & 0 \le x \le 1,\ 0 \le y \le x \\ 0, & \text{otherwise.} \end{cases}$$

Find the marginal density Prob. density func of x and y.

$$f_X(x) = \int_0^x 8xy\,dy$$

$$= \frac{8xy^2}{2}$$

$$= \left[4xy^2\right]_0^x$$

$$= 4x^3; \quad 0 \le x \le 1$$

$$f_Y(y) = \int_0^1 8xy\,dx.$$

$$= [4x^2y]_0^1$$

$$= 4y \;, \quad 0 \le y \le x$$

$$f_X(x) = \begin{cases} 4x^3 \;, & 0 \le x \le 1 \\ 0 \;, & \text{else} \end{cases}$$

$$f_Y(y) = \begin{cases} 4y \;, & 0 \le y \le x \\ 0 \;, & \text{else} \end{cases}$$

Each front tyre of a particular type of vehicle is supposed to be filled to a pressure of 26 PSI. Suppose the actual Air pressure in each tyre is a random variable. —
X for the right tyre, Y for the left tyre with Joint p.d.f.

$$f(x,y) = \begin{cases} k(x^2+y^2) \;, & 20 \le x \le 30, \; 20 \le y \le 30 \\ 0 \;, & \text{else.} \end{cases}$$

(i) what is value of k.
(ii) Determine the distribⁿ of air pressure in the right tyre alone. What is its Prob.
(iii) Are X and Y independent.
(iv) Find Marginal distribution for Y.
(v) The air pressure in left tyre is b/w 24 to 30.

$$\int_{20}^{30}\int_{20}^{30} f(x,y)\,dx\,dy = 1$$

$$k\int_{20}^{30}\int_{20}^{30} (x^2+y^2)\,dx\,dy = 1$$

$$k\int_{20}^{30}\left[\frac{x^3}{3}+y^2x\right]_{20}^{30} dy = 1$$

$$k\int_{20}^{30}\left[\frac{19000}{3}+10y^2\right] dy = 1$$

$$k\left[\frac{19000}{3}y+\frac{10y^3}{3}\right]_{20}^{30} = 1$$

$$k\left[\frac{190000}{3}+\frac{190000}{3}\right] = 1$$

$$k = \frac{3}{380000}$$

(ii) $f_x(x) = \int_{20}^{30} k(x^2+y^2)\,dy$

$$= \frac{3}{380000}\left(x^2y+\frac{y^3}{3}\right)_{20}^{30}$$

$$= \frac{3}{380000}\left[10x^2 + \frac{19000}{3}\right]$$

$$= \frac{30x^2}{380000} + \frac{5700}{380}\cdot\frac{190}{20}$$

$$= \frac{3x^2}{38000} + \frac{1}{20}$$

$$= \frac{3x^2}{38000} + 0.05 \; ; \; 20 \le x \le 30$$

$$f_Y(y) = \int_{20}^{30} k(x^2 + y^2)\,dx$$

$$= k\left[\frac{x^3}{3} + y^2 x\right]_{20}^{30}$$

$$= \frac{3}{380000}\left[\frac{19000}{3} + 10y^2\right]$$

$$= \frac{1}{20} + \frac{3y^2}{38000}$$

$$= \frac{3y^2}{38000} + 0.05 \; ; \; 20 \le y \le 30$$

(iii) In this case X and Y are not independent.

(v) $P(24 \le Y \le 30) = \int_{24}^{30} f_Y(y)\,dy$.

$= \int_{24}^{30} \left(\frac{3y^2}{38000} + 0.05\right) dy$.

Independent Random Variables

Two R.V. X and Y are said to be independent if for every pair x and y values

$p(x,y) = p_X(x) \cdot p_Y(y)$ — when X and Y are Discrete.

$f(x,y) = f_X(x) \cdot f_Y(y)$ — when x and y Continuous

17/10/19 Thursday

If X and Y are discrete random variables then the Joint probability distribution func^n is defined by.

$F(x,y) = P(X \le x, Y \le y)$

$= \sum_{s \le x} \sum_{t \le y} p(s,t)$

If X and Y are continuous random variable then the function defined by

$F(x,y) = P(X \le x, Y \le y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(s,t)\,ds\,dt$,

$-\infty < x < \infty$, $-\infty < y < \infty$

is called as Joint Prob. distrin func^n.

Find the Marginal density function of x and y and distribu^n func^n of x and y.

(i) $f(x,y) = \begin{cases} x+y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{else.} \end{cases}$

In this ques. x and y are dependent

(ii) $f(x,y) = \begin{cases} \frac{1}{4}(1+xy), & |x|<1,\ |y|<1 \\ 0, & \text{else.} \end{cases}$

(i) $f_x(x) = \int_0^1 (x+y)\,dy = \left(xy + \frac{y^2}{2}\right)_0^1$

$= x + \frac{1}{2}, \quad 0 \le x \le 1$

$f_y(y) = y + \frac{1}{2}; \quad 0 \le y \le 1$

$F(x,y) = \int_0^1 \int_0^1 (x+y)\,dx\,dy$

$\begin{cases} 0 & , \text{else} \end{cases}$

Find the Joint P.d.f of x and y.

$$f(x,y) = \frac{\partial^2}{\partial x \partial y}(F(x,y))$$

$$= \frac{\partial^2}{\partial x \partial y}(1-e^{-x})(1-e^{-y})$$

$$= \frac{\partial^2}{\partial x \partial y}(1-e^{-x})(1-e^{-y})$$

$$f(x,y) = e^{-x}e^{-y}, \quad x>0, y>0$$

$$\frac{\partial}{\partial x}\left[\frac{\partial}{\partial y}(1-e^{-x})(1-e^{-y})\right]$$

$$= \frac{\partial}{\partial x}(1-e^{-x})\cdot(e^{-y})$$

$$= e^{-x}e^{-y}, \quad x>0, y>0$$

Conditional Probability

If $p(x=x_i, y=y_j)$ is the joint probability mass function of R.V x and Y then the conditional probability function of X given $Y=y_j$ is defined by

$$P(x=x_i \mid Y=y_j) = \frac{P(x=x_i, Y=y_j)}{P(Y=y_j)}$$

Similarly Probability of Y given $x=x_i$ is given by

$$P(Y=y_j \mid x=x_i) = \frac{P(x=x_i, Y=y_j)}{P(x=x_i)}$$

Case II: Let X and Y be two continuous R.V with joint P.d.f $f(x,y)$ and marginal Prob. density func of X as $f_X(x)$ then the conditional Probability density function of Y given $X=x$ is

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}, \quad -\infty < y < \infty.$$

Similarly X given $Y=y$ is

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)}, \quad -\infty < x < \infty$$

Q. The two dimensional random variable X, Y have joint P.M.F

$$f(x,y) = \frac{x+2y}{27}, \quad \begin{matrix} x=0,1,2 \\ y=0,1,2 \end{matrix}$$

Find the conditional distribution of Y for $X=x$.

$$P(Y=y_j \mid X=x_i) = \frac{P(X=x_i, Y=y_j)}{P(X=x_i)}$$

$$= \frac{P(X=0, Y=0)}{P(X=0)} = 0.$$

Y	0	1	2	Row Sum
0	0	2/27	4/27	
1	1/27	1/9	5/27	
2	2/27	4/27	2/9	

$$\frac{P(X=0, Y=1)}{P(X=0)} = \frac{2}{27}$$

$$\frac{P(X=0, Y=2)}{P(X=0)} = \frac{4}{27}.$$

$$\frac{P(X=1, Y=0)}{P(X=1)} = \frac{1}{27}.$$

$$\frac{P(X=1, Y=1)}{P(X=1)} = \frac{3}{27}.$$

$$\frac{P(X=1, Y=2)}{P(X=1)} = \frac{5}{27}$$

$P(Y=y_j \mid X=0)$ $P(Y=y_j \mid X=1)$ $P(Y=y_j \mid X=2)$

$P(Y=0 \mid X=0) = 0$

$$P(Y=1 \mid X=0) = \frac{\frac{2}{27}}{\frac{6}{27}} = \frac{1}{3}$$

$$P(Y=2 \mid X=0) = \frac{\frac{4}{27}}{\frac{6}{27}} = \frac{2}{3}$$

23/10/19
Wednesday

Q. An Instructor has given a short quiz consisting of two parts. For a randomly selected student, let X denotes the no. of points earned in the first part Y denotes no. of points earned in 2nd part. Suppose that the joint PMF of X and Y is given as follows.

p(x,y)		y			
	0	0.02	0.06	0.02	0.10
x	5	0.04	0.15	0.20	0.10
	10	0.01	0.15	0.14	0.01

Find (i) expected no. of points scored in 1st part.

(ii) Expected no. of points scored in 2nd part.

(iii) If the score recorded in grade book is the total no. of points earned on the two parts what is the expected recorded score.

Value row sum pro.

(i) $E(X) = \sum x f_X(x)$

$= 0 \times f_X(0) + 5 \times f_X(5) + 10 \times f_X(10)$.

$= 5.55$

Value column sum prop.

(ii) $E(Y) = \sum y f_Y(y) = 0 \times 0.07 + 5 \times 0.36 + 10 \times 0.36 + 15 \times 0.21$

$= 8.55$

(iii) $E(x+y) = \sum_x \sum_y (x+y) f(x,y)$

$= 0+0(0.02) + 0+5(0.04) + 0+10(0.01)$
$+ 5+0(0.06) + 5+5(0.15) + 5+10(0.15)$
$+ 10+0(0.02) + 10+5(0.20) + 10+10(0.14)$
$+ 15+0(0.10) + 15+5(0.10) + 15+10(0.01)$

$= 0.2 + 0.1 + 0.3 + 1.5 + 2.25$
$+ 0.2 + 3 + 2.8 + 1.5 + 2 + 0.25$

$= 14.10$

or.

$E(ax+b) = aE(x)+b$

$E(ax+bY) = aE(x)+bE(Y)$
$= 5.55 + 8.55$
$= 14.10$

A large insurance agency services a no. of customers who have purchased both a home on a policy and an automobile policy from the agency. For each type of policy let X denote the deductable amount on the auto policy. Y denote the deductable amount on a homeowner policy and Joint PMF is given by

$p(x,y)$		y		
		0	100	200
x	100	0.20	0.10	0.20
	250	0.05	0.15	0.30

Find (i) $E(x)$ (ii) $E(Y)$ (iii) $E(x+Y)$

the life of the [illegible] project is 10% with no salvage value. Rs 3 lakh. The annual net profit is Rs 3 lakh 50 thousand. Find the rate of returns for new business.

23/10/19 Wednesday

$$E(X) = 100 \times 0.5 + 250 \times 0.5 = 175$$

$$E(Y) = 0 \times 0.25 + 100 \times 0.25 + 200 \times 0.5 = 125$$

$$E(X+Y) = 175 + 125 = 300$$

Covariance If covariance is +ve and large no. they are strongly related.
If -ve weakly related.

The covariance between two random variables X and Y is defined by

$$\text{Cov}(X,Y) = E\left[(X-\mu_X)(Y-\mu_Y)\right]$$

$$= \begin{cases} \sum_x \sum_y (x-\mu_x)(y-\mu_y) p(x,y), & x,y \text{ discrete.} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x-\mu_x)(y-\mu_y) f(x,y)\,dx\,dy, & x,y \text{ continuous.} \end{cases}$$

Prev. Ques

$$\mu_X = 175, \quad \mu_Y = 125.$$

$$\text{cov}(X,Y) = \sum_x \sum_y (x-175)(y-125) p(x,y)$$

$$= (100-175)(0-125) \times 0.2 + (100-175)(200-125) \times 0.2 + (250-175)(100-125) \times 0.15 + (250-175)(200-125) \times 0.30$$
$$+ (100-125) \times 0.1 + (250-175) \times 0.05$$

23/10/19 Wednesday

$$\text{cov}(x,x) = E\left[(x-\mu_x)^2\right] = \text{var}[x].$$

Mostly used. $$\text{cov}(x,y) = E(xy) - \mu_x \mu_y.$$

NOTE : (i) For a strong +ve relationship co-variance of x, y should be quite +ve.

(ii) For a strong -ve relationship the signs of $(x-\mu_x)$ and $(y-\mu_y)$ will tend to be opposite, Results in a negative product. Hence For a strong -ve relationship co-variance of x, y should be quite -ve.

(iii) If x, y are not strongly related, positive and negative products will tend to cancel one another, results in a covariance near zero.

limit = 0 to 1-x

$$f(x,y) = \begin{cases} 24xy & ; 0 \le x \le 1,\ 0 \le y \le 1,\ x+y \le 1 \\ 0 & ; \text{otherwise.} \end{cases}$$

Find $\text{cov}(x,y)$.

$$f_X(x) = \int_{-\infty}^{\infty} 24xy\, dy.$$

$$f_Y(y) = \int_{-\infty}^{\infty} 24xy\, dx.$$

$$\mu_X = E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx.$$

$$\mu_Y = E(Y) =$$