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Probability And Statistics.

Experiment – Is an act or process that leads to a single outcome which cannot be predicted with certainty.

Random Experiment – An ~~random~~ experiment in which all the outcomes can be enumerated but it is not known which particular outcome will result. is called a random experiment.

Event – The result of an experiment or observation is called an event.

Mutually Exclusive – Events are said to be mutually ~~exp~~ exclusive if the occurrence of one is independent of occurrence of other.

$E_i \cap E_j = \phi \ (i \neq j)$ let $E_1, E_2 \ldots E_n$ be an mutually exclusive events.

Exhaustive Events – All possible events in any trial are known as exhaustive events.

$A \cup B = S$, Let $E_1, E_2 \ldots E_n$ are exhaustive events i.e. $E_1 \cup E_2 \cup \ldots \cup E_n = S$ sample space.

Mutually Exclusive and Exhaustive events –

~~$E_i \cap E_j$~~ $E_1, E_2 \ldots E_n$ are said to be mutually exclusive and exhaustive events if $E_i \cap E_j = \phi \ (i \neq j)$

$\bigcup_{i=1} E_i = S$.

Certain Events – Probability is one.
Impossible Events – Probability is zero.

Probability – If an experiment can result in 'n' exhaustive, mutually exclusive and equally likely cases and 'm' of them are favourable to the event 'E' then the probability of occurance of the event 'E' is denoted by P(E) and is defined by

$$\frac{\text{no. of fav. cases to the Event } E}{\text{Total no. of exhaustive, mutually exclusive and equally likely cases}} = \frac{m}{n}$$

Properties of Probability.

① If 'E' is an event of a sample space 'S' then
(i): $0 \leq P(E) \leq 1$.

② If $E_1, E_2 --- E_K$ are K mutually exclusive and exhaustive events in the sample space 'S' of an Experiment then

$$\sum_{i=1}^{k} P(E_i) = 1.$$

E is an Event : Probability : $P(E)$
non-occurrence : $P(E^c)$

$$P(E) + P(E^c) = 1$$
$$E \cup E^c = S.$$

Theorem 1 : If events 'A' and 'A^c' are complimentary events in a sample space 'S' then,

$$P(A) = 1 - P(A^c)$$
$$P(A^c) = 1 - P(A)$$

* Theorem 2 : If A and B are two arbitrary events in a sample space 'S' then,

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Case-1 : If A and B are mutually exclusive events of a sample space 'S' then,

$$P(A \cup B) = P(A) + P(B) + P(\phi)$$
$$\Rightarrow P(A) + P(B)$$

Q. The probability that atleast one of the event A and B occur is 0.7. and that they occurs simultaneously is 0.2. Find $P(\bar{A}) + P(\bar{B})$.

$$\cancel{P(A)=0.} \quad P(A \cup B) = 0.7$$
$$P(A \cap B) = 0.2.$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.7 = \cancel{P(A} \; 1 - P(\bar{A}) + 1 - P(\bar{B}) - 0.2$$
$$\cancel{0.9}. \quad P(\bar{A}) + P(\bar{B}) = 1.1.$$

Q. Two events A and B have probabilities 0.25 and 0.5 respectively. The probability that both A and B occur is 0.14. Find the probability that neither A nor B occurs

$$\cancel{0.14 = 0.5 + 0.25} - P(A \cap$$

$$P(A) = 0.25$$
$$P(B) = 0.5$$
$$P(A \cap B) = 0.14.$$
$$P(\bar{A} \cap \bar{B}) = ?$$

$$P(A \cup B) = 0.25 + 0.5 - 0.14$$
$$P(A \cup B) = 0.61$$
$$P(\overline{A \cup B}) = 1 - P(A \cup B)$$
$$= 1 - 0.61$$
$$= 0.39.$$

$X \sim N(x, n, M, N)$

SYMBOLIC NOTATIONS.

Let S denote the sample space. If A and B are any 2 events of the experiment then –

1. The non occurrence of A is $A^c / A' / \bar{A}$.
2. Either A or B is, $A \cup B$.
3. Both A and B is; $A \cap B$.
4. If $A \cap B$ is ϕ (A and B are mutually exclusive.

   then $A \cup B = A + B$.
5. Neither A nor B. $\bar{A} \cap \bar{B} = \overline{A \cup B} = 1 - (A \cup B)$ [Demorgans law]
6. A occurs and B doesnot occurs, $A \cap B^c = A - B$
7. If A, B, C are 3 events then,
   - (a) all 3 occurs, $A \cap B \cap C$.
   - (b) at least one of A, B, C occurs, $A \cup B \cup C$.
   - (c) Neither A, B, C occurs, $\bar{A} \cap \bar{B} \cap \bar{C} = \overline{A \cup B \cup C}$.

$2 \mid 2,4,8$
$2 \mid 1,2,4$
$2 \mid 1,1,2$

If $P(A) = \frac{3}{8}$

$P(B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$

find (a) $P(A \cup B)$ (b) $P(\bar{A})$ (c) $P(\bar{B})$ (d) $P(\bar{A} \cap \bar{B})$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{3}{8} + \frac{1}{2} - \frac{1}{4}$$
$$= \frac{3 + 4 - 2}{8} = \cancel{\frac{9}{8}}\ \frac{5}{8}$$

$$P(\bar{A}) = 1 - \frac{3}{8}$$
$$= \frac{5}{8}$$

$$P(\bar{B}) = 1 - \frac{1}{2}$$
$$= \frac{1}{2}$$

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) + P(\bar{B}) -$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$
$$= \cancel{\frac{9}{8}}\ 1 - \frac{5}{8} = \frac{3}{8}$$

Q I/ A and B are two mutually Exclusive events such that.

$A \cup B = S$ and $P(B) = 2P(A)$ Then $P(A) = 1/3$

Q. Two Dice are rolled find Probability of getting a sum of 10 or 11.

10	11
5+5	6+5
4+6	5+6
6+4	

$\frac{\cancel{3}\,1}{\cancel{36}\,12}$ $\frac{\cancel{2}\,1}{\cancel{36}\,18}$

$P(10) = \frac{1}{12}$, $P(11) = \frac{1}{18}$.

$P(A \cap B) = \phi$. (Mutually exclusive)

$P(A \cup B) = P(A) + P(B)$.

$= \frac{1}{12} + \frac{1}{18}$

$=$

• 3 students A, B, C are running in a race. A and B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins.

$$P(A) + P(B) + P(C) = 1$$
$$P(A) = P(B) = 2P(C)$$

$$P(A) + P(B) + P(C) = 1$$
$$5P(C) = 1$$
$$P(C) = \frac{1}{5}.$$

$$P(B) = \frac{2}{5}$$

$$P(B \cup C) = P(B) + P(C)$$
$$= \frac{2}{5} + \frac{1}{5}$$
$$= \frac{3}{5}.$$

INDEPENDENT EVENTS :

Two events A and B are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$.

CONDITIONAL PROBABILITY :

Let A and B denote two events associated with a random experiment. Then Probability of $P(B/A)$ represents the conditional probability of occurrence of B given that A has already occurred. (Also referred as the probability of occurrence of B relative to A).

It is defined by

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

MULTIPLICATION THEOREM :-

If A and B are two events then probability of $A \cap B$ will be

$$P(A \cap B) = P(A)\,P(B/A) = P(B) \cdot P(A/B).$$

Q. A can solve 90% of the Problems given in a book. and B can solve 70% of the Problems. what is the probability that atleast one of them will solve a problem selected at random from the book.

BAYES THEOREM - 'n'

Let $E_1, E_2, ---, E_n$ be mutually exclusive and exhaustive events with non-zero probability. If A is any arbitrary event of the sample space of the above experiment, then.

$$P(E_i/A) = \frac{P(E_i) P(A/E_i)}{\sum_{j=1}^{n} P(E_j) P(A/E_j)}.$$

Eg : Assume that a factory has 2 machine. Past records shows that machine 1 produces 20% of the items and machine 2 produces 80% of the items. Further it shows that 6% of the items produced by machine 1 are defective and only 1% are defective from machine 2. If a defective item is drawn at random what is the probability that it was produced by machine 1.

$$P(E_1) = 0.2$$

so:

$$P(E_2) = 0.8$$

$$P(E_1/A)$$

Q. Suppose 5 men out of 100 and 25 out of 10000 are colorblind. A person chosen at random. What is the probability that person is a male. (Assume that male ~~and~~ and female are in equal number.

$P(A) = \frac{5}{100}$ $P(B) = \frac{25}{10000}$

$P(M$.

$$P(M) = \frac{1}{2}$$

$$P(M \cup B) = P(M) + P(B) - P(M \cap B)$$

$$= \frac{1}{2} + \frac{5}{100} - P(M \cap B)$$

RANDOM VARIABLE

A random variable is a ~~numb~~ numerical valued variable defined on a sample space of an experiment. usually random variables are denoted by

X, Y, Z etc and the corresponding lower cases x, y, z are used to denote the numerical value taken by the random variable.

$$X : S \longrightarrow R$$

Let S be the sample space of a random experiment and R denote the set of real numbers, then the func^n is called a random variable.

① If X, Y are random variable ~~d~~ on the sample space of ~~a~~ an experiment then X+Y, X−Y, XY is also a random variable

② If a and b are any two real numbers then aX + bY is also a random variable one.

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Q. Let us consider that 3 coins are tossed

$$S = \{HHH, HHT, HTH, THH, THT, HTT, TTH, TTT\}.$$

$$P(\text{getting } 1 \text{ Head}) = \frac{3}{8}$$

$$P(\text{getting } 2 \text{ head}) = \frac{3}{8}$$

$$P(\text{getting } 3 \text{ head}) = \frac{1}{8}$$

$$P(\text{getting } 0 \text{ head}) = \frac{1}{8}$$

$X \rightarrow$ no. of heads

$$X = \underbrace{0, 1, 2, 3,}_{x}$$

$$P(X=0) = 1/8 \qquad P(X = x_i)$$

$$P(X=1) = 3/8.$$

$$P(X=2) = 3/8$$

$$P(X=3) = 1/8$$

- Random variables are classified as -
  1. Discrete R.V.
  2. Continuous R.V.

(3) Random variables are also known as 'stochastic' variable.

- Discrete R.V — A random variable which can take only finite no. of values or countably infinite no. of values is called a discrete R.V.

Probability Mass func^n — Let X be a discrete random variable which ~~test~~ takes the values $x_0 = x_1, x_2, \ldots, x_n$ corresponding to the various outcomes of a random experiment. If the probability of occurrence of

$$X = x_i \ (i = 1, 2, \ldots, n) \text{ is } P(X = x_i)$$

such that :-

(i) $P(X = x_i) = p_i \geq 0 \ \forall i$

(ii) $\sum_{i=1}^{n} P(X = x_i) = \sum_{i=1}^{n} p_i = 1$.

then the func^n $P(x)$ is called the probability func^n of the random variable 'X' and the set

$$\{(x_1, P(X = x_1)); (x_2, P(X = x_2)), \ldots, (x_n, P(X = x_n))\}$$

and this set is called the Probability distribu^n of the random Variable X.

D iscrete
cont..

$\int dx$

P.D.f → fun
F(x)

The probability funcⁿ $P(X = x)$ is also denoted by $f(x)$ and is called the probability mass funcⁿ of discrete random variable.

(i) $f(x_i) \geq 0 \; \forall i$.

(ii) $\sum_x f(x) = 1$.

Probability Distribution function.
Let X be a discrete random variable then the funcⁿ ~~F(x)~~ $F(x) = P(X \leq x)$. is called the distribution funcⁿ of X.

(4) If X is a discrete R.V and $F(x)$ is the distribution function of X.
Then; $P(a < X \leq b) = F(b) - F(a)$.

Discrete	Continuous $\frac{d}{dx}F(x)$
PDF → CDF	PDF → CDF $f(x)$ $F(x)$ $\int f(x)\,dx$

Q. Consider an Experiment in which 4 coins are tossed. If we define a random variable as the no. of heads obtained then find the probability distribution.

4 coins.

$X \to 0, 1, 2, 3, 4.$

no heads
all tails

$P(X=0) = \frac{1}{16}$

$P(X=1) = \frac{4}{16} = \frac{1}{4}.$

$P(X=2) = \frac{6}{16} = \frac{3}{8}.$

$P(X=3) = \frac{4}{16} = \frac{1}{4}$

$P(X=4) = \frac{1}{16}.$

$X = x_i$	0	1	2	3	4
$P(X=x_i)$	1/16	1/4	6/16	1/4	1/16

$F(1) = P(X \le 1)$
$= P(X=0) + P(X=1) = \frac{1}{16} + \frac{4}{16} = \frac{5}{16}.$

$F(2) = P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$
$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16} = \frac{11}{16}$

$F(3) = P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16} + \frac{4}{16} = \frac{15}{16}$.

$F(4) = P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = 1$.

$$F(x) = \begin{cases} 0 & ; \; x < 0 \\ 1/16 & ; \; 0 \leq x < 1 \\ 5/16 & ; \; 1 \leq x < 2 \\ 11/16 & ; \; 2 \leq x < 3 \\ 15/16 & ; \; 3 \leq x < 4 \\ 1 & ; \; x \geq 4 . \end{cases}$$

> If X is a discrete random variable and $F(x)$ is the distribution function of X then

(i) $P(a \leq X \leq b) = P(a < X \leq b) + P(X=a)$
$= F(b) - F(a) + P(X=a)$

(ii) $P(a < X < b) = F(b) - F(a) - P(X=b)$

(iii) $P(a \leq X < b) = F(b) - F(a) + P(X=a) - P(X=b)$

Q. A Random variable X has the following probability distribution

$X = x_i$	0	1	2	3	4
$P(X = x_i)$	$3k$	$3k$	k	$2k$	$6k$

Find (i) k (ii) $P(X>2)$ (iii) $P(0<X\leq 2)$
(iv) $P(2\leq X\leq 4)$ (v) $P(X\leq 1)$.

(1) Sum of all probability $= 1$

$$3k + 3k + k + 2k + 6k = 1$$

$$k = \frac{1}{15}$$

(2) $P(X>2) = P(X=3) + P(X=4)$

or

$1 - P(X\leq 2)$

$$= 2k + 6k$$

$$= \frac{2}{15} + \frac{6}{15} = \frac{8}{15}$$

(3) $P(0<X\leq 2) = F(b) - F(a)$

$$= F(2) - F(0)$$

$$= 7k - 3k$$

$$= 4k$$

$$= \frac{4}{15}$$

$P(x \le 4)$ $F(5)$ $F(3)$ $= P(0) + P(1) + P(2) + P(3)$
$F(4)$

(4) $P(2 \le x \le 4)$
$= F(b) - F(a) + P(x=a)$
$= F(4) - F(2) + P(x=2)$
$= 15k - 7k + k$
$= 15 \times \frac{1}{15} - 7 \times \frac{1}{15} + \frac{1}{15}$
$= \frac{9}{15}$

or (5) $P(x \le 1)$
$= P(x=0) + P(x=1)$
$= \frac{6}{15} = \frac{2}{5}$

Q. A random variable X has the following P.D.-

$X = x_i$	-2	-1	0	1	2	3
$P(x=x_i)$	0.1	k	0.2	2k	0.3	k

Find (1) k (2) $P(x \le 0)$
(3) The probability distribution function $F(x)$

(1) $0.1 + k + 0.2 + 2k + 0.3 + k = 1$
$0.6 + 4k = 1$
$4k = 0.4$
$k = \frac{0.4}{4} = \cancel{\frac{4}{10}}$
$k = \frac{1}{10}$

(2) $P(x \le 0) = 0.1 + k + 0.2$
$= 0.1 + 0.1 + 0.2 = 0.4$

Chiding go why you not going

19/7/19

Mean And Variance of Discrete Distribution.

Let X be a discrete random variable then the mean and variance of the discrete distribution is defined as follows -

$$\text{Mean } (\mu) = \sum_{i=1}^{n} x_i P(X = x_i) = \boxed{\sum_{i=1}^{n} x_i p_i}$$

$$\text{Variance } (\sigma^2) = \text{var}[X] = \boxed{\sum_{i=1}^{n} (x_i - \mu)^2 p_i}$$

$$= \sum_{i=1}^{n} (x_i^2 + \mu^2 - 2x_i\mu) p_i$$

$$= \sum_{i=1}^{n} x_i^2 p_i + \sum_{i=1}^{n} \mu^2 p_i - \sum_{i=1}^{n} 2x_i p_i \mu$$

$$= \sum_{i=1}^{n} x_i^2 p_i + \mu^2 \left(\sum_{i=1}^{n} p_i\right) - 2\mu \left(\sum_{i=1}^{n} x_i p_i\right)$$

$$= \sum_{i=1}^{n} x_i^2 p_i + \mu^2 - 2\mu^2$$

$$\boxed{\sigma^2 = \sum_{i=1}^{n} x_i^2 p_i - \mu^2}$$

back side prob (k wala).

(ii) Mean $= \sum_{i=1}^{n} x_i p_i$

$= 0 \times 3k + 1 \times 3k + 2 \times k + 3 \times \frac{3 \times 2}{15}$

$+ \frac{4 \times 6}{15}$

$= \frac{7}{3}$

Variance $\Rightarrow \sigma^2 = \sum_{i=1}^{n} x_i^2 p_i - \mu^2$

$= 0 + 1 \times \frac{3}{15} + 4 \times \frac{81}{15} + 9 \times \frac{2}{15} + 16 \times \frac{6}{15}$

$- \frac{49}{9}$

$= \frac{3}{15} + \frac{4}{15} + \frac{18}{15} + \frac{96}{15} - \frac{49}{9}$

$\sigma^2 = 2.62$

Q. 2 cards are drawn successively with replacement from a well shuffled pack of card. And the probability distribution of the no. of kings that can be drawn.

Let X be the random variable which denotes card is a king.

$X = 0, 1, 2.$

$P(X = x_i)$

$^nC_4 / ^{52}C_4$ $\quad$ $^nC_x = \frac{n!}{x!(n-x)!}$

$$P(\text{getting a king}) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52}$$

$$= \frac{\frac{4!}{1!\ 3!}}{\frac{52!}{1!\ 51!}} = \frac{\frac{4 \times 3!}{3!}}{\frac{52 \times 51!}{51!}} = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{not getting a king}) = 1 - \frac{1}{13} = \frac{12}{13}$$

$$P(X=0) = \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$$

$$P(X=1) = \frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13} = \frac{24}{169}$$

$$P(X=2) = \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$$

$X = x_i$	0	1	2
$P(X = x_i)$	144/169	24/169	1/169

$$F(x) = \begin{cases} 0 & x < 0 \\ 144/169 & 0 \le x < 1 \\ 168/169 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

Q.1 find the probability distribution of no. of white balls drawn when 3 balls are drawn with replacement from a bag containing 4 white and 6 red balls.

Q.2 without replacement same Question.

(1) Total white balls = 4.
-"- red -"- = 6.
Total balls = 10.

$X = 0, 1, 2, 3.$

$P(\text{getting } \cancel{\text{first}} \text{ ball white}) = \frac{^4C_1}{^{10}C_1} = \frac{\cancel{4}\,2}{\cancel{10}\,5}$

$P(\text{not getting ball white}) = 1 - \frac{2}{5} = \frac{3}{5}$

P(1st ball is white) =

(get 0 white balls 3 times.)

$P(X=0) = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$

get 1 white ball 3 time

$P(X=1) = \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5}$

$+ \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} = \frac{3 \times 3 \times 2}{125} \times 3 = \frac{54}{125}$.

getting 2 white balls.

$P(X=2) = \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} \times 3 = \frac{36}{125}$

getting 3 white balls.

$P(X=3) = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$.

$X = x_i$	0	1	2	3
$P(X=x_i)$	27/125	54/125	36/125	8/125

(2) Total white ball = 4.
— " — red — " — 6
Total balls = 10.

$X = 0, 1, 2, 3 \ldots$

$P(\text{getting white ball}) = \frac{^4C_1}{^{10}C_1} = \frac{2}{5}$.

$P(\text{not getting white ball}) = 1 - \frac{2}{5} = \frac{3}{5}$.

get 0 white ball 3 times =

0000
000000

0 white balls $= \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$.

RRW
RWR
WRR

1 white balls $= \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$

$+ \frac{6}{10} \times \frac{4}{9} \times \frac{5}{8}$

$+ \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8}$.

RWW
WRW
WWR

2 white balls $= \frac{6}{10} \times \frac{4}{9} \times \frac{3}{8}$

$+ \frac{4}{10} \times \frac{6}{9} \times \frac{3}{8}$

$+ \frac{4}{10} \times \frac{3}{9} \times \frac{6}{8}$

WWW

3 white balls $= \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}$.

Continuous Random Variable.

Let X be a random variable. If X takes uncountable infinite no. of values then X is called a continuous random variable.

If X is a continuous random variable then the range of X is an interval is an interval on the real line.

Eg: The life of an Electric bulb, the range of a RADAR, range of Bluetooth, WIFI, Hotspot etc.

Probability Density Function.

Let X be a continuous random variable if for every x in the range of X we assign a real number 'f(x)' satisfying.

(i) $f(x) \geq 0$, $-\infty < x < \infty$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

then the funcn $f(x)$ is called the probability density funcn of X.

Cummulative Distribution Function.

If X is a continuous random variable having $f(x)$ as its probability density funcn then the distribution function or the cummulative distribution funcn is defined by.

$$F(x) = P(X \le x)$$

$$= \int_{-\infty}^{\infty} f(x)\,dx \quad ; -\infty < x < \infty$$

* If $f(x)$ is a probability density function of a random variable X, then we have

$$\boxed{P(a \le X \le b) = \int_a^b f(x)\,dx} \;.$$

for any real constants a and b, $a \le b$.

$$P\left(\cancel{a < X < b}\right) = \int_a^b f(x)\,dx \;\cancel{- P(X=a) - P(X=b)}$$

$P(\cancel{a \le X}$

Theorem : Let $f(x)$ be a continuous real valued function, Let $F(x)$ be another func^n such that $F'(x) = f(x)$. then,

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

$\int f(x)$

dP

<u>P.d.f</u>   <u>C.d.f.</u>

$$\begin{tikzcd} f \arrow[r] & F \arrow[l, bend left, "F'"] \end{tikzcd}$$

P.M.F or
C.D.F

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Mean And Variance of continuous random variable.

Let X be a continuous random variable with $f(x)$ as the Probability density function. then.

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x) dx.$$

$$\text{Variance } (\sigma^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2.$$

Q. Let the P.D.F of a continuous random variable X be given by :-

$$f(x) = \begin{cases} \frac{x}{6} + k & ; 0 < x \leq 1. \\ 0 & ; \text{else where.} \end{cases}$$

Find (i) k (ii) μ (iii) $F(x)$

(i) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\Rightarrow \int_{-\infty}^{0} 0\, dx + \int_{0}^{1} \left(\frac{x}{6} + k\right) dx + \int_{1}^{\infty} 0\, dx = 1.$$

$F(x)$
$f(x)$

$$\int_0^1 \left(\frac{x}{6} + k\right) dx = 1$$

$$\left[\frac{x^2}{12} + kx\right]_0^1 = 1.$$

$$\frac{1}{12} + k = 1.$$

$$k = 1 - \frac{1}{12}$$

$$k = \frac{11}{12}$$

(ii) $$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_0^1 x\left(\frac{x}{6} + k\right) dx.$$

$$= \int_0^1 \left[\frac{x^2}{6} + kx\right] dx$$

$$= \int_0^1 \left[\frac{x^2}{6} + \frac{11}{12}x\right] dx$$

$$= \left(\frac{x^3}{18} + \frac{11}{24}x^2\right)_0^1$$

$$= \frac{1}{18} + \frac{11}{24} = \frac{111}{216} = 0.514$$

(iv) Note:

(iii) $F(x) = \int f(x)\,dx$

$$= \begin{cases} \frac{x^2}{12} + \frac{11}{12}x, & 0 < x \le 1 \\ 0, & \text{else.} \end{cases}$$

Q. $f(x) = \begin{cases} Cx^2, & |x| \le 1 \\ 0, & \text{else where.} \end{cases}$

Find (i) c (ii) μ (iii) σ^2 (iv) $P(x \ge \frac{1}{2})$.

A (i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-1}^{1} Cx^2\,dx = 1$$

$$\left[C\frac{x^3}{3}\right]_{-1}^{1} = 1$$

$$\frac{C}{3} + \frac{C}{3} = 1$$

$$\frac{2C}{3} = 1$$

$$C = \frac{3}{2}.$$

(ii) $\mu = \int_{-\infty}^{\infty} x f x \, dx$

$\mu = c\int_{-1}^{1} x^3 \, dx$

$= c\left[\frac{x^4}{4}\right]_{-1}^{1} = 0$

$= \frac{c}{4} + \frac{c}{4}$

$\mu c = \frac{c}{2}$

$c = \frac{3}{2}$. $\mu = \frac{3}{4}$.

(iii) $= 0 \cdot \frac{3}{8} \cdot 0$

(iii) $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) \, dx - \mu^2$

$= c\int_{-1}^{1} x^4 \, dx - (0)^2$ 0.

$= c\left[\frac{x^5}{5}\right]_{-1}^{1} - 0$

$= c\left[\frac{1}{5} + \frac{1}{5}\right]$

$= c \times \frac{2}{5}$

$= \frac{3}{2} \times \frac{2}{5} = \frac{3}{5}$.

$cx^3, |x| \le 1$

(b) $P(x \ge 1/2)$

$$\int_{1/2}^{1} f(x)\,dx$$

$$\int_{1/2}^{1} cx^2\,dx$$

$= 7/16.$

Q.1 $f(x) = \begin{cases} cx^3 & , |x| \le 1 \\ 0 & , \text{elsewhere.} \end{cases}$ $|x| = \begin{cases} -x, & x<0 \\ x, & x \ge 0 \end{cases}$

Find (i) c (ii) μ (iii) σ^2 (iv) $P(x \ge 1/2)$

(v) $P(0.25 < X < 0.75)$.

Q.2 The C.D.F of a random variable X is given by:

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1/2\left(x^2 - \frac{3}{2}x\right) & , 0 < x \le 1 \\ 1 & , x > 1 \end{cases}$$

Find (i) $f(x)$

(ii) $P(0.3 < X < 0.5)$.

(i) $f(x) = F'(x)$.

$f(x) = \begin{cases} 0 & , x < 0 \\ x - 3/4 & , 0 < x < 1 \\ 0 & , x > 1 \end{cases}$

$= \begin{cases} x - \frac{3}{4} & ; \ 0 < x \leq 1 \\ 0 & , \ \text{elsewhere.} \end{cases}$

$$P(0.3 < X < 0.5) = \int_{0.3}^{0.5} f(x)\,dx$$

$$= F(0.5) - F(0.3)$$

Q. Given :

$$F(x) = \begin{cases} 0 & , \ x < 0 \\ x^2 & , \ 0 \leq x \leq 1 \\ 1 & ; \ x > 1 \end{cases}$$

Find (i) P.D.F

(ii) $P(0.5 < X < 0.75)$

(i) $P.D.F = F'(x)$

$P.D.F = 2x$

(ii) $P(0.5 < X < 0.75)$

$\int_{0.5}^{0.75} f(x)\,dx$ instead of this we can use

$$\int_{0.5}^{0.75} = F(0.75) - F(0.5)$$

$$= 0.3125$$

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~~K C~~ ~~X~~ P

Q. Find k such that ~~f(x)~~ $f(x)$ is a p.d.f. of a continuous random variable X. where

$$f(x) = \begin{cases} kxe^{-x}, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

also find (i) μ (ii) σ^2.

we know. ~~p.d.f.~~ $= \int_{-\infty}^{\infty} kxe^{-x} = 1$.

$$= k \int_{0}^{\infty} xe^{-x} = 1.$$

$$\cancel{k \int x} \Rightarrow \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} k\,xe^{-x}dx + \int_{1}^{\infty} 0\,dx = 1.$$

$$k\left[-xe^{-x} - e^{-x}\right]_0^1 = 1.$$

$$k\left[\left(-e^{-1} - e^{-1}\right) - (-1)\right] = 1$$

$$k\left(\frac{-2}{e} + 1\right) = 1$$

$$k = \frac{e}{e-2}.$$

$$\int u v \, dx = u \int v \, dx - \int \left(\frac{d}{dx} u \int v \, dx \right) dx$$

(i) $$\mu = \int_{-\infty}^{\infty} x f(x) \, dx$$

$$= k \int_0^1 x^2 e^{-x} \, dx$$

$$= \cancel{k \left[-x^2 e^{-x} \left[x \right. \right.}$$

$$= k \left[-x^2 e^{-x} + \int (2x)(e^{-x}) \right]$$

$$= \cancel{k \left[-x^2 e^{-x} - 2e^{-x} \right]}$$

$$= k \left[-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} \right]_0^1$$

$$= k \left[\left(-e^{-1} - 2e^{-1} - 2e^{-1} \right) - (-2) \right]$$

$$= k \left(-5e^{-1} \right) + 2 \cancel{= 1}$$

$$\cancel{k \left(-5e^{-1} \right) = -1}$$

$$k \left(\frac{-5}{e} + 2 \right)$$

$$\left(\frac{e}{e-2} \right) \times \left(\frac{2e - 5}{e} \right)$$

$$= \frac{2e - 5}{e - 2}$$

$37 \times 2 = 74$, $74 / 2 = 37$

$e^2 - 4e + 4$

$e^2 - 2e - 2e +$

$e(e-2) - 2(e-$

$(e-2)(e-$

classmate
Date
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(ii) $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$

$= \int_0^1 k x^3 e^{-x}\, dx - \mu^2$.

$= k \int x^3 e^{-x}\, dx - \mu^2$

$= k\left[-x^3 e^{-x} + \int (3x^2)(e^{-x}) \right] - \mu^2$

$= k\left[-x^3 e^{-x} - 3x^2 e^{-x} - 2xe^{-x} - 2e^{-x} \right]_0^1$

$= k\left[-e^{-1} - e^{-1} - 2e^{-1} - 2e^{-1} \right] - (-2) - \mu^2$

$= k(-6e^{-1}) + 2 - \left(\frac{2e-5}{e-2} \right)^2$

$= k\left(\frac{-6}{e} + 2 \right) - \frac{4e^2 + 25 - 20e}{e^2 + 4 - 4e}$.

$= \frac{\cancel{e}}{e-2} \times \left(\frac{2e-6}{\cancel{e}} \right) - \frac{4e^2 + 25 - 20e}{e^2 + 4 - 4e}$.

$= \frac{2e-6}{e-2} - \frac{4e^2 + 25 - 20e}{(e-2)^2}$

$= \frac{(2e-6)(e-2) - 4e^2 + 25 - 20e}{(e-2)^2}$.

$= \frac{2e^2 - 4e - 6e + 12 - 4e^2 + 25 - 20e}{(e-2)^2}$

$= \frac{-2e^2 - 30e + 37}{(e-2)^2}$

$\begin{array}{r} 37 \\ \times 2 \\ \hline 74 \\ \boxed{37} \end{array}$

$e^2 - 4e + 4$
$e^2 - 2e - 2e + 4$
$e(e-2) - 2(e-2)$
$(e-2)(e-2)$

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Q. For the following density function of the (Probability) ~~Probabil~~ random variable X. Find.

(a) $f(x) = \begin{cases} ae^{-|x|} & ; -\infty < x < \infty \\ 0 & , \text{elsewhere} \end{cases}$

find (i) a (ii) μ (iii) σ^2

$$|x| = \begin{cases} -x & , x < 0 \\ x & , x > 0 \end{cases}$$

$$f(x) = \begin{cases} ae^{x} & , -\infty < x < 0 \\ ae^{-x} & , 0 < x < \infty \end{cases}$$

$$\int_{-\infty}^{0} ae^{x}dx + \int_{0}^{\infty} ae^{-x}dx = 1$$

$$a\int_{-\infty}^{0} e^{x}dx + a\int_{0}^{\infty} e^{-x}dx = 1.$$

$$\cancel{a\left[xe^{x} - \int (1)e^{x}\right]} = a\left[e^{x}\right]_{-\infty}^{0} + a\left[-e^{-x}\right]_{0}^{\infty} = 1$$

$$= a\left[e^{0} - e^{-\infty}\right] + a\left[-e^{-\infty} + e^{0}\right] = 1$$

$$= a(1) + a(1) = 1$$

$$\Rightarrow 2a = 1$$

$$a = 1/2.$$

(ii) $\mu = \int_{-\infty}^{\infty} x f(x)\, dx.$

$$\mu = \int_{-\infty}^{0} -x a e^{x} + \int_{0}^{\infty} x a e^{-x}.$$

$$= -a\int_{-\infty}^{0} x e^{x} + a\int_{0}^{\infty} x e^{-x}$$

$$= -a\left[x e^{x} - \int (1)(e^{x})\right]_{-\infty}^{0} + a\left[-x e^{-x} + \int (1)(e^{-x})\right]_{0}^{\infty}$$

$$= -a\left[x e^{x} - e^{x}\right]_{-\infty}^{0} + a\left[-x e^{-x} + e^{-x}\right]_{0}^{\infty}$$

$$= -a\left[0 - 1\right] + a\left[-1\right]$$

$$= +a - a$$

$$= 0.$$

(iii) $\sigma^2 = 2.$

Q.1 A random variable X has the p.d.f

$$f(x) = \begin{cases} 2x, & 0<x<1 \\ 0, & \text{else.} \end{cases}$$

Find (i) $F(x)$
(ii) $P(X<1/2)$
(iii) $P\left(1/4<X<1/2\right)$.

Q.2 A continuous random variable X has the following p.d.f.

$$f(x) = \begin{cases} 1/2, & -1<x<0. \\ 1/4(2-x), & 0<x<2. \\ 0, & \text{elsewhere.} \end{cases}$$

Find : (i) $F(x)$ (ii) μ (iii) σ^2.

Q.3

$$f(x) = \begin{cases} 12x^3-21x^2+10x, & 0\le x\le 1. \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) $F(x)$
(ii) μ
(iii) $P(X\le 1/2)$
(iv) $P(X>1/2)$
(v) $P(0.3<X<0.7)$.

31/7/19
Wednesday

Q. Consider a group of 5 potential blood donors a, b, c, d, e, of whom a, b only have O+ blood. 5 blood samples, one from each individual will be typed on random order until an O+ individual is identified. Let Y be the random variable that denote the number of typing necessary to identify an O+ individual. Find the probability mass func" and hence represent in the form of graph.

Y = the no. of typing necessary to identify an O+ individual.

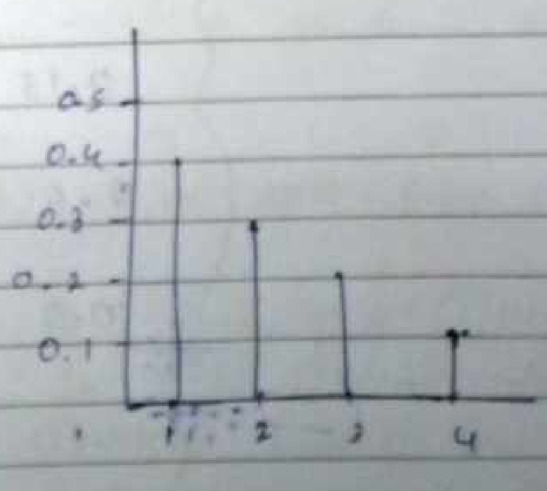
$Y = 1, 2, 3, 4$

$P(Y=1) = \frac{2}{5}$ not getting $= 0.4$

$P(Y=2) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10} = 0.3$

$P(Y=3) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = 0.2$

$P(Y=4) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = 0.1$



Q. An electronic store carries pen-drives with either 1, 2, 4, 8, 16 GB. The following table gives the following distribution of Y.

Y = the amount of memory in a pen drive

y	1	2	4	8	16
p(y)	0.05	0.10	0.35	0.40	0.10

Find F(y).

$F(1) = P(Y \le 1) = 0.05$

$F(2) = P(Y \le 2) = P(Y=1) + P(Y=2) = 0.15$

$\cancel{F(y) = P(0) = 0}$

$\cancel{F(1) = 0.05}$

$\cancel{P(2) =}$

⋮

$F(16)$.

$$F(y) = \begin{cases} 0 & y < 1 \\ 0.05 & 1 \le y < 2 \\ 0.15 & 2 \le y < 4 \\ 0.5 & 4 \le y < 8 \\ 0.9 & 8 \le y < 16 \\ 1 & 16 \le y \end{cases}$$

Secti...

Q. Starti... gender ... born ... Succ... varia... Find

1/8/19 Thursday

Q. A...

Section - 3.2 (Q.

Q. Starting at a fix time a hospital observe the gender of each new born child until a boy is born. Let $p = P(B)$ assume that the successive births are independent define the random variable X as the number of birth observed. Find Probability mass function. Find distribution, mean, variance also.

X = no. of Birth observed.

$P(X=1) = p.$ (1st is boy)

$P(X=2) = (1-p) \times p$
$P(GB)$

$P(X=3) = (1-p)^2 \times p.$

$$P(X=x) = \begin{cases} (1-p)^{x-1} p, & x=1,2,-x. \\ 0, & \text{else} \end{cases}$$

$P(X=x) = (1-p)^{x-1} \cdot p, \quad x = 1, 2, ----, x$

1/8/19 Thursday.

Q. A computer bussiness has 6 telephone lines. Let x denote the no. of lines in use at a specific time. Suppose the probability mass func^n of a random variable x is given by -

x	0	1	2	3	4	5	6
$P(x)$	0.10	0.15	0.20	0.25	0.20	0.06	0.04

Calculate the prob. of - 1 atmost 3 lines are in use. 2 fewer than 3 lines are in use 3 atleast 3 lines are in use. 4 b/w 2 and 5 lines, inclusive are in use. 5 b/w 2 and 4 lines, inclusive, are not in use 6 atleast 4 lines are not in use.

(1) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.10 + 0.15 + 0.20 + 0.25$

$= 0.70$

(2) $P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

(3) $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$.

(4) $P(2 \leq X \leq 5) = F(5) - F(2) + P(X=2)$

$= P(X \leq 5) - P(X \leq 2) + P(X=2)$

$= P(X=3) + P(X=4) + P(X=5) + P(X=2)$.

$= 0.71$

(5) $1 - P(2 \leq X \leq 4)$.

(6) $1 - P(X \geq 4)$.

Q A consumer organisation that evaluates new automobiles customers reports the no. of major defect in each car examined. Let X denote the no. of major defect for a randomly selected car of a certain type. The cumulative distribution funcⁿ of X is given by :-

$$F(x) = \begin{cases} 0, & x<0 \\ 0.06, & 0 \le x < 1 \\ 0.19, & 1 \le x < 2 \\ 0.39 & 2 \le x < 3 \\ 0.67 & 3 \le x < 4 \\ 0.92 & 4 \le x < 5 \\ 0.97 & 5 \le x < 6 \\ 1 & 6 \le x \end{cases}$$

① calculate $P(X=2)$ ② $P(X>3)$.
③ $P(2 \le X \le 5)$ ④ $P(2<X<5)$.

① $P(X=2) =$

P(x) 0.06 0.13 0.20 0.28 0.25 0.05 0.03

① $P(x=2) = 0.20.$

② $P(x>3) = P(x=4) + P(x=5) + P(x=6)$

$= 0.25 + 0.05 + 0.03$

$= 0.33.$

$1-P(x\leq 3)$

③ $P(2 \leq x \leq 5) = F(b) - F(a) + P(x=a)$

$= F(5) - F(2) + P(x=2)$

$= \cancel{0.05 - 0.20 + 0.20}$

$= P(x=5) + P(x=4) + P(x=3) + P(x=2)$

$= 0.05 + 0.25 + 0.28 + 0.20$

$= 0.78.$

④ $P(2 < x < 5) = F(b) - F(a) - P(x=b)$

$= F(5) - F(2) - P(x=5)$

$= P(x=3) + P(x=4) + P(x=5) - P(x=5)$

$= 0.28 + 0.25$

$= 0.53.$

$P(X>3)$
$1-P(X\le 3)$

Q. An insurance company offers its policy holders a no. of different premium payment options. For a random selected policy holder let X define the no. of months between successive payments. The cumulative distribution of X is as follows -

$$F(x) = \begin{cases} 0, & x<1 \\ 0.30, & 1\le x<3 \\ 0.40, & 3\le x<4 \\ 0.45, & 4\le x<6 \\ 0.60, & 6\le x<12 \\ 1, & 12\le x \end{cases}$$

Compute ① PMF of X.
② using just the CDF, find $P(3\le X\le 6)$ and $P(4\le X)$.

x	0	1	3	4	6	12
$P(x)$		0.30	0.1	0.05	0.1	

①

x	1	3	4	6	12
$P(x)$	0.30	0.10	0.05	0.15	0.4.

$F(3)$

② $P(3\le X\le 6)$
$= F(6) - F(a) + P(X=a)$
$= F(6) - F(3) + P(X=3)$
$= 0.60 - 0.40 + 0.10$
$= 0.3$

$P(X = x) = p(x)$

$P(X \le 3)$

or

$$P(4 \le X) = 1 - P(4 > X)$$
$$= 1 - 0.40$$
$$= 0.6.$$

EXPECTATIONS :-

Let X be a random variable that takes the values $x_1, x_2 \ldots x_k$ with corresponding probabilities $p_1, p_2 \ldots p_k$ respectively. Then the expectation of X is denoted by $E(X)$ and is defined as

$$E(X) = \sum_{i=1}^{k} x_i p_i. \text{ (discrete)}$$

In probability theory the expected value of random variable is intuitively the long run average value of repetitions of the same experiment it represents.

2/8/19 Friday

Properties of Expectation.

1. If the random variable X has a set of possible values 'D' and p.m.f $p(x)$. then the expected value of $h(x)$ is given by.

$$\mu = E[h(x)] = \sum_{D} h(x) \cdot p(x)$$

value of x.

2. Expectation of $E(ax+b) = aE(X) + b$ for any constant a and b.

$\mu = \sum x_i p_i$

$\sum (x) p(x)$

$E(ax+b) = aE(x)+b$.

$$E(ax+b) = \sum_x (ax+b) \cdot p(x).$$

$$= \sum_x ax\, p(x) + \sum_x b\, p(x).$$

$$= a\left(\sum_x x\, p(x)\right) + b\left(\sum_x p(x)\right).$$

$$\boxed{E(ax+b) = aE(x)+b}$$

$a=1$, $E(x+b) = E(x)+b$.

$E(k) = k$

$b=0$, $E(ax) = aE(x)$

Variance :

$$Var[X] = \sigma^2 = \sum_{i=1}^{n} x_i^2 p_i - \mu^2 = E(x^2) - (E(x))^2$$

or $\sum_i (x_i - \mu)^2 p_i = E(x-\mu)^2$.

Properties of Variance.

(1) If X is a random variable and k is a real constant then,

(i) $Var[kX] = k^2 Var[X]$.

(ii) $Var[X+k] = Var[X]$

$$var[kx] = E(k^2x^2) - \cancel{E(xk)}\,(E(kx))^2$$

$$var[kx] = k^2E(x^2) - k^2(E(x))^2$$

$$\boxed{var[kx] = k^2[E(x^2) - (E(x))^2] = k^2 var[x]}$$

$$\boxed{var[x+k] = var[x].}$$

$$var[x+k] = var[x]$$

$$= \cancel{E(x^2) - (E(x))^2}$$

$$= E(x^2+k^2) - (E(x+k))^2$$

$$= E(x^2+k^2+2xk) - [E(x)+k]^2$$

$$= E(x^2) + 2k\cancel{E(x)} + \cancel{k^2} - (E(x))^2 - \cancel{k^2} - 2k\cancel{E(x)}$$

$$= E(x^2) - (E(x))^2$$

$$= var[x]$$

(2) The positive square root of variance is called standard deviation.

(3) $var[x] = 0$ if and only if x takes only one value with probability 1.

(4) variance of a constant is 0.

The p.m.f of a discrete random variable is as follows.

$X = x$	1	2	4	8	16
$P(X=x)$	0.05	0.10	0.25	0.40	0.10

Compute : (i) $E(x)$
(ii) $var[x]$ (using both formula)
(iii) standard deviation of X.

(i) $E(x) = \sum_{i=1}^{k} x_i p_i$

$= 0.05 + 0.20 + 1.4 + 3.2 + 1.6$

$= 6.45 = \mu$

~~(ii) $var[x] = E(x^2) - (E(x))^2$~~

$=$

(ii) $var[x] = \sigma^2 = \sum x_i^2 p_i - \mu^2$

$= 0.05 + 0.40 + 5.6 + 25.6 + 25.6 - (6.45)^2$

$= 57.25 - 41.60$

$= 15.65$

$var[x] = \sigma^2 = \sum (x_i - \mu)^2 p_i$

$= 1.48 + 1.98 + 2.1 + 0.96 + 9.12$

$= 15.64$.

scc - 3.3

(iii) $S.D = \sqrt{\sigma^2} = \sqrt{16.65} = 3.955$

7/8/19 Wednesday

BINOMIAL DISTRIBUTION.

The Binomial distribution is a discrete distribution. The following condition should be satisfied for the application of Binomial distribution.

1. The exp. consist of a sequence of 'n' smaller experiments called 'trials' where n is fixed in advance.

2. Each trial can result in one of the same two possible outcomes generally denoted by success (s) and failure (f).

3. The probability of S remains the same from trial to trial. The probability of S is denoted by 'p' and probability of failure is denoted by 'q' and '$p + q = 1$'.

4. All the trials are independent so that the outcome of any particular trial doesnot influence the outcome of any other trial.

If X denotes the no. of success in n trials under the conditions stated above then, X is said to follow binomial distribution with parameter n and p.

Binomial, no. of trials, prob. of success.

$$X \sim B(n, p).$$

Probability mass function of Binomial Distribution.

A discrete random variable taking the values 0, 1, 2, --- n is said to follow binomial distribution with parameters n and p if its probability mass funcⁿ is given by.

$$P(X=x) = p(x) = \begin{cases} {}^nC_x p^x q^{n-x} & ; x = 0,1,2 \ldots n \\ & 0<p<1,\ p+q=1. \\ 0 & , \text{otherwise}. \end{cases}$$

Mean and variance of Binomial Distribution.

$$\text{Mean } (\mu) = E(X). \qquad \{\mu = \Sigma x_i p_i\}$$

$$= \sum_{x=0}^{n} x\, {}^nC_x\, p^x q^{n-x}.$$

because for zero value is 0 issue 1 to n

$$= \sum_{x=1}^{n} x \frac{n!}{x!(n-x)!} p^x \cdot q^{n-x}.$$

$$= \sum_{x=1}^{n} \frac{\cancel{x} n!}{\cancel{x}(x-1)!(n-x)!} p^x q^{n-x}.$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\{(n-1)-(x-1)\}!} p^{x-1} q^{(n-1)-(x-1)}$$

$$np\left(\sum_{x=1}^{n} {}^{n-1}C_{x-1}\, p^{x-1} q^{n-1-(x-1)}\right)$$

$$= np \cdot (p+q)^{n-1}$$

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots$$

$$\boxed{\mu = np}$$

$$\text{Variance} = \text{var}[x] = E(x^2) - (E(x))^2.$$

Now $E(x^2) = \cancel{\sum x^2 p_i}\ E(x^2 - x + x)$

$$= E(x^2 - x) + E(x)$$

$$= E(x(x-1)) + E(x).$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^nC_x\, p^x q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!} p^x q^{n-x} + np.$$

$$= \sum_{x=2}^{n} \frac{n!}{(x-2)!\,(n-x)!} p^x q^{n-x} + np$$

$$= n(n-1)p^2 \left(\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,((n-2)-(x-2))!} p^{x-2} q^{n-2-(x-2)} \right) + np$$

$$= n(n-1)p^2 (p+q)^{n-2} + np$$

$$= n^2p^2 - np^2 + np.$$

$$\text{Var}[x] = E(x^2) - (E(x))^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np(1-p)$$

$$\boxed{\text{var}[x] = npq}$$

$\frac{4}{6}$ 'p' x or

Q. A die is thrown 4 times getting a number greater than 2 is a success. Find the probability of getting —

① Exactly one success.

② Less than 3 success.

$n = 4.$

$p = \frac{4}{6}.$

$q = 1 - p = 1 - \frac{4}{6} = \frac{2}{6}$

① $P(X=1) = {}^4C_1 \left(\frac{4}{6}\right)^1 \left(\frac{2}{6}\right)^3 = \frac{8}{81}$

② $P(X<3) = {}^4C_1 \left(\frac{4}{6}\right)^1 \left(\frac{2}{6}\right)^3 + {}^4C_2 \left(\frac{4}{6}\right)^2 \left(\frac{2}{6}\right)^2$

$+ {}^4C_0 \left(\frac{2}{6}\right)^4$

$= \frac{33}{81}$

Q. If the chance that any one of 5 telephone lines is busy at any instant is 0.01, what is the probability that —

① all the lines are busy.

② More than 3 lines are busy.

③ atleast 2 lines are busy.

④ atmost 2 lines are busy.

$X = 5$

$n = 5$, $p = 0.01$, $q = 1 - 0.01$
$= 0.99$

① $P(X=5) = 10^{-10}$

② $P(X>3) = 4.96 \times 10^{-8}.$

③ $P(X \geq 2)$

④ $P(X \leq 2).$

8/8/19

Q A die is thrown 3 times, getting a 3 or 6 is considered to be success. Find the probability of getting (1) atleast 2 success
(2) exactly 3 success
(3) atmost 2 success.

A. $n = 3$.

$p = \frac{1}{3}$.

$q = \frac{2}{3}$.

(1) $P(X \geq 2) = P(X=2) + P(X=3)$

$= {}^3C_2 \cdot \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1 + {}^3C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^0$

$= 3 \times \frac{1}{9} \times \frac{2}{3} + \frac{1}{27} = \frac{7}{27}$.

(2) $P(X=3) = \frac{1}{27}$.

(3) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= {}^3C_0 \cdot \left(\frac{2}{3}\right)^3 + {}^3C_1 \left(\frac{1}{3}\right) \times \left(\frac{2}{3}\right)^2 + \frac{2}{9}$

$= \frac{8}{27} + \frac{4}{9} + \frac{2}{9} = \frac{26}{27}$.

$p = 0.20 \quad n = 4$

Q. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolt chosen at random.

1. 1 defective.
2. no defective.
3. 2 are defective.

$n = 4$

$p = \frac{20}{100} = \frac{1}{5}$

$q = \frac{4}{5}$

① $P(x=1) = {}^4C_1 \left(\frac{1}{5}\right)^1 \times \left(\frac{4}{5}\right)^3$

$= 4 \times \frac{1}{5} \times \frac{64}{125}$

$= \frac{256}{625}$

② $P(x=0) = {}^4C_0 \left(\frac{4}{5}\right)^4$

$= \frac{256}{625}$

③ $P(x=2) = {}^4C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^2$

$\frac{24 \times 3 \times 2}{2 \times 2} \times \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^2 \Rightarrow \quad = \frac{96}{625}$

Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let X denote the number amount 15 randomly selected copies that fail the test. Find probability that –

1. atmost 8 fail the test.
2. exactly 8 fail the test
3. atleast 8 fail the test.
4. The probability that b/w 4 and 7, inclusive, fail the test.

$n = 15$; $p = \frac{1}{5}$, $q = \frac{4}{5}$.

1. $(P X \leq 8) = 1 - P(X > 8)$.

2. $P(X = 8) =$

3. $P(X \geq 8) = 1 - P(X < 8)$.

4. $P(4 \leq X \leq 7)$

Q. The average % of failure in a certain examination is 40. What is the probability that out of a group of 6 candidates atleast 4 pass in the examination.

$n = 6 \quad , p = 0.6 \quad , q = 0.4 .$

$P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$=$

Q. $X \sim B(n, p)$

such that $\boxed{4P(X=4) = P(X=2)}$

If $n=6$, $p = ?$

$4 \cdot P(X=4) = P(X=2)$

$4 \, {}^6C_4 \, p^{\cancel{4}2} \cancel{q^2} = {}^6C_2 \, p^2 q^{\cancel{4}} q^2$

$\Rightarrow 4 \times {}^6C_4 \, p^4 q^2 = {}^6C_2 \, p^2 q^4$

$4 \times \cancel{15} \, p^2 = q^2 .$

$= 4 \times \frac{6!}{4!2!} p^4 q^2 = \frac{6!}{2!4!} p^2 q^4$

$4p^2 - 1 - p^2 + 2p = 0 .$

$= 4 \frac{p^2}{q^2} = 1$

$3p^2 + 2p - 1 = 0$

$p = -1, \; 1/3 .$

$4 \, {}^6C_4 \, p^{\cancel{4}2} q^{\cancel{2}} = {}^6C_2 \, p^2 q^{4 \, 2}$

$4 \frac{\cancel{6!}}{\cancel{2!}4!} (p)^2 = \frac{\cancel{6!}}{2!\cancel{4!}} q^2$

$4p^2 = q^2$

$4p^2 = (1-p)^2$

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If the sum of μ and σ^2 of a binomial distribution of 5 trials is $\frac{9}{5}$. Find binomial distribution.

$$np + npq = \frac{9}{5}$$

$$5p(1+q) = \frac{9}{5}$$

Binomial distribution is

$$B\left(x, 5, \frac{1}{5}\right) = P(X=x) = 5C_x \, p^x q^{n-x}$$

$$= 5C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{5-x} \quad ; \; x = 0, 1, 2 \ldots 5$$

Find the maximum n (no. of trial) such that the probability of getting no head and tossing a coin n time is greater than 0.1.

2/8/19
Tuesday

$\alpha P(X=x)$

$N \rightarrow M$

n $x \rightarrow n$

$N = 25$
$M = 5$

$n = 20$
$M = 5$

HYPERGEOMETRIC DISTRIBUTION.

Assumptions / Conditions.

The assumptions leading to hypergeometric distribution are as follows –

(1) The population or ~~set~~ set to be sampled consist of N individuals or objects or elements.

(2) Each individual can be categorized as a success (S) or a failure (F) and there are M no. of successes in the population.

(3) A sample of 'n' individuals is selected without replacement in such a way that each subset of size 'n' is equally likely to be chosen.

The random variable X denote the no. of success in the sample. The probability distribution of X depends on the parameters 'n' 'M', 'N'.
i.e. $X \sim h(x, n, M, N)$.

Probability Mass function.

If x is the no. of success in a completely random sample of size 'n', drawn from a population consisting of 'M' successes and '(N − M)' failures, then the probability distribution of X called the hypergeometric distribution is given by $P(X=x) = h(x, n, M, N)$

$$P(X=x) = h(x, n, M, N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

For x, an integer satisfying
$\max(0, n-N+M) \le x \le \min(n, M)$

Q. 5 Individuals from an animal population thought to be near extinction in a certain region have been caught, taged and released to mix into the population. After they have had an opportunity to mix, a random sample of 10 of these animals is selected. Let X denote the no. of taged animals in the second sample. If there are actually 25 animals of this type in the region, what is the prob. that -

(a) $X = 2$.
(b) $X \le 2$.

$N = 25$
$M = 5$
$n = 10$. $\quad P(X=2) = h(2, 10, 5, 25)$

① $P(X=2) = {}^{M}C_{x} \cdot {}^{N-M}C_{n-x} / {}^{N}C_{n}$

$$P(X=2) = \frac{{}^{5}C_{2} \times {}^{20}C_{8}}{{}^{25}C_{10}}$$

$= 0.3853$.

$\left(\frac{N-n}{N-1}\right) n\frac{M}{N}$ $1-\frac{M}{N}$

20

25

(2) $P(X\leq 2) = P(X=0)+P(X=1)+P(X=2)$.

$$= \frac{{}^5C_0 \times {}^{20}C_{10}}{{}^{25}C_{10}} + \frac{{}^5C_1 \times {}^{20}C_9}{{}^{25}C_{10}} + \frac{{}^5C_2 \times {}^{20}C_8}{{}^{25}C_{10}}$$

$= 0.699$.

Q. An Electronic store has received a shipment of 20 radios that have connections ~~for~~ an ipod or iphone. 12 of these have 2 slots, and the other 8 have single slot. Suppose that 6 of the 20 radios are selected at random to be stored in a shelf and the remaining are placed in the store room. Let X denotes the no. amount the radios stored for display in the shelf that have 2 slots.

(1) what kind of distribution does X have.

(2) compute $P(X=2)$; $P(X\leq 2)$, $P(X\geq 2)$

(3) calculate the mean and S.D of X.

Mean and Variance of Hypergeometric distribution

The mean and variance is given by -

$$\mu = E(x) = n.\frac{M}{N}$$

$$\text{var}[x] = (\sigma^2) = \left(\frac{N-n}{N-1}\right).n.\frac{M}{N}\left(1-\frac{M}{N}\right).$$

$n\frac{M}{N}$

$\left(\frac{N-n}{N-1}\right) n\frac{M}{N}\left(1-\frac{M}{N}\right)$

$N=50$
$n=15$
$x=10$
$M=30$ $1 \to 20$
$2 \to 30$

$M=12$
$N=20$
$n=6.$

② $P(X=2) = \dfrac{{}^{12}C_2 \times {}^{8}C_4}{{}^{20}C_6}$

$= 0.1192.$

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.1373.$

$P(X \ge 2) = 1 - P(X<2)$
$= 1 - \{P(X=0) + P(X=1)\}$
$= 0.9819.$

③ $\text{Mean} = 4 = E(X) = n \cdot \dfrac{M}{N}$

$= 3.6$

$\text{var}[X] = \sigma^2 = 1.06$
$S.D = \sqrt{1.06}$
$= 1.03.$

$\left(1-\frac{M}{R}\right)$

R

p defective
$\to$ less scratches
$N=12.$
$M=8$
$n=6$

14/8/19
Wednesday

POISSON DISTRIBUTION.

CONDITIONS:

The condition for the applicibility of poisson distribution are same as those of the binomial distribution.

The additional requirements are as follows

(1) The number of trials is indefinitely very large.

(2) The probability of success in a trial is very very small.

(3) The product of ~~Dand P~~ n and p must be a constant.

eg: The number of car accidents in a year at some particular place.

The no. of earthquakes in a particular place

Similarly the no. of breakdown of an electronic gadget.

Probability Mass Function.

A discrete random variable X taking the ~~and~~ values 0, 1, 2, ... is said to follow Poisson distribution with parameters "λ" if the probability mass funcⁿ is given by.

$$P(X=x) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} & ; x = 0, 1, 2, 3, \ldots \\ 0 & , \text{otherwise} \end{cases}$$

$\lambda : np.$

① $P(X=x) \geq 0$.

② $\sum_x P(X=x) = 1$

$$\sum_{x=0}^{\infty} \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots\right]$$

$$= e^{-\lambda} e^{\lambda} = 1.$$

Note : If X is a discrete random variable which follow poisson distribution then show that

$$E(x) = V[x] = \lambda.$$

Q. There are 50 telephone lines in a exchange. The prob. of them being busy is 0.1. what is the prob. that all the lines are busy.

$$n = 50 \ , \ p = 0.1$$

$$\lambda = np$$
$$\lambda = 50 \times 0.1$$
$$\lambda = 5$$

$$P(X=50) = \frac{5^{50} e^{-5}}{50!} = 1.9676 \times 10^{-32}.$$

Q. The prob. that a bomb dropped from an aeroplane ~~des~~ will strike a certain target is 1/5. If 6 bombs are dropped find the Prob. that

(1) Exactly two will strike the target.
(2) atleast 2 will strike the target.

$$n = 6 \ , \ p = \frac{1}{5}$$

$$\lambda = 6 \times \frac{1}{5} = \frac{6}{5} = 1.2.$$

$$P(X=2) = \frac{(1.2)^2 e^{-1.2}}{2!} =$$

$$P(X \geq 2) = 1 - P(X<2)$$
$$= 1 - P(X=0) - P(X=1).$$
$$=$$

16/8/19
Friday

Q. A car hire firm has 2 cars. The no. of demands for a car on each day is distributed as a poisson's distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demands are refused.

$$\lambda = 1.5$$
$$n = 2$$
$$p = 0.75 = \frac{3}{4}$$

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P(X=0) = e^{-1.5} = 0.223$$

$$P(X>2) = 1 - P(X \leq 2)$$
$$= 1 - P(X=0) - P(X=1) - P(X=2)$$
$$= 1 - 0.223 - 0.334 - 0.251$$
$$= 0.191$$

Q. Let $P(X=2) = \frac{2}{3} P(X=1)$

then find $P(X=0)$

$$\frac{\cancel{e^{-\lambda}}\lambda^{\cancel{2}}}{2} = \frac{2}{3} \cancel{e^{-\lambda}} \cdot \cancel{\lambda}$$
$$\lambda = \frac{4}{3}$$
$$P(X=0) = e^{-4/3} = 0.263$$

Q) If $P(X=x)$ for $x=0$ is 0.1 find μ

$$e^{-\lambda} = 0.1$$
$$-\lambda = \ln(0.1)$$
$$\lambda = -\ln(0.1)$$
$$\lambda = 2.303 = \mu.$$

Q) P is getting no misprint in a page of a book is e^{-4}. What is the P that a page contains more than 2 misprint.

$$1 - P(X=0) - P(X=1) - P(X=2)$$

$$P(X=0) = e^{-\lambda} = e^{-4}$$
$$\lambda = 4.$$

$$P(X>2) = 1 - e^{-4} - 4.e^{-4} - 8.e^{-4}$$
$$= 1 - 13e^{-4}$$
$$= 0.761.$$

Q. At a busy traffic intersection, The P of an individual car having an accident is 0.0001. However during a certain part of the day, a large no of cars, say, 1000 pass through the intersection. Under these condi'n what is the P that 2 or more accidents occurs during that period?

$p = 0.0001 \qquad n = 1000$

$\lambda = 0.1$

$P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - e^{-0.1} - 0.1e^{-0.1}$

$= 4.67 \times 10^{-3}$

Suppose 2% of the people on an avg are left handed. Find.

① P of finding 3 or more left handed.
② P of at least 1 left handed.

$\lambda = 0.02$

① $P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - e^{-0.02} - (0.02 e^{-0.02} - \frac{(0.02)^2 e^{-0.02}}{2}$

$= 1.31 \times 10^{-6}$

② $P(X \geq 1) = 1 - P(X=0)$

$= 0.019$

NEGATIVE BINOMIAL DISTRIBUTION

(used when no. of trials are not fixed)

Conditions :-

The -ve binomial random variable and based on the four conditions

1. The experiment consists of a sequence of independent trials.

2. Each trial results in either S or F.

3. The Probability of S(P) is constant from trial to trial.

4. The experiment continues until a total no. of 'r' success have been observed where 'r' is a +ve no.

The random variable of interest is X
X = no. of failures that preceeds the r^{th} success.

X is called a negative binomial random variable.

PMF of Negative B.D. is given by

$$p(X=x) = nb(x, r; p) = \binom{x+r-1}{r-1} p^r (1-p)^x, \quad x = 0, 1, 2, \ldots$$

r : no of success.
p : probability of success.

Alternate Form.

$$P(X=x) = \binom{n-1}{r-1} p^r (1-p)^{n-r}, \quad n = r, r+1, r+2, \ldots$$

where, n = no. of trials.
r = no. of success.

Mean and variance

$$\mu = E(X) = \frac{r(1-p)}{p}$$

$$\sigma^2 = Var[X] = \frac{r(1-p)}{p^2}$$

$x = n - r$

$\sigma^2 = \frac{r(1-p)}{p^2}$

$\binom{x+r-1}{r-1} p^r (1-p)^x$

$\mu = \frac{r(1-p)}{p}$

$\sigma^2 = \frac{r(1-p)}{p^2}$

11/10/23 Wednesday

Q. An NGO surveying people exiting ~~peopl~~ from a polling booth and asking them if they voted independent. The probability p that a person voted independent is 25%. What is the probability that 15 people must be asked before the NGO will find 5 people who voted independent.

$$p = 0.25$$
$$r = 5$$
$$x = 10$$

$$nb(10, 5, 0.25) = \binom{14}{4}(0.25)^5(0.75)^{10}$$
$$= 0.055$$

Q. A reality show wishes to recruit 5 couples.

① If p (randomly selected couples agree to participate) = 0.2. What is the probability that 12 couples must be asked before 5 are found who agree to participate.

$$p = 0.2$$
$$r = 5$$
$$x = 7$$

$$nb(7, 5, 0.2) = \binom{11}{4}(0.2)^5(0.8)^7$$
$$= 0.02$$

② What is the probability that atmost 15 couples are asked before 5 are found who agree to participate.

4 $P(X=4)$ 2^{a}

Q. A couple wishes to have exactly 2 girls in their family. They will have children until this condition is fulfilled. What is the probability that the family has x (no. of male children).

1. 4 childrens
2. atmost 4 children.
3. How many male childrens could you expect to have.
4. ~~How many children you expect this family to have.~~

(1) $p = P(\text{male birth}) = 0.5$

$r = 2.$

(i) $nb(x, 2, 0.5) = \binom{x+r-1}{r-1} p^r (1-p)^x.$

MFF
FMF
~~FFMM~~
FFMM
FMMF
~~MMFF~~

(ii) $P(X=2) = {}^3C_1 (0.5)^2 (0.5)^2 = \binom{x+1}{1}(0.5)^2(0.5)^x$

$= (x+1)(0.5)^{x+2}$

$= 3 \times (0.5)^4 = 0.1875$

(iii) $P(\text{atmost 4 children}) = nb(0, 2, 0.5) + nb(1, 2, 0.5) + nb(2, 2, 0.5).$

$P(X \leq 4) = P(X=2) + P(X=3) + P(X=4).$

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