SEMINAR ON MODULI THEORY

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1. Summary

One way to go to through these sessions is go recall theory of schemes (at level of things in Hartshorne chapter two), by doing a beeline through all the definitions, properties, etc. However, I feel since the point of doing this exercise is to become more comfortable in working with scheme, we will just do lots of examples instead. By this, I mean we will just to prove some things in very concrete situations. This will help you build a concrete picture of the generalities.

List of some things to discuss (just do lots of examples):

(1) Definition of a scheme.
(2) Say affine communication lemma *stress this!*
(3) examples
    (a) $\mathbb{P}^n$ and it sheaf theory! This already clarifies the $\mathcal{O}(n)$'s

Definition 1. A *scheme* is a locally ringed space with the property that every point has an open neighbourhood which is an affine scheme. A *morphism of schemes* is a morphism of locally ringed spaces. The category of schemes will be denoted Sch.

Things to say: finite-generation, valuative criteria for $\mathbb{P}^1$ (using DVR's - how giving a point and it's specialisation is the same as giving a map $\operatorname{Spec} R \to \mathbb{P}^1$).

List of examples:

(1) $\mathbb{A}^1$ with a double point (what are quasi-coherent sheaves on this?)
(2) $\mathbb{A}^2 \setminus \{0, 0\}$ (what is the structure sheaf?).
(3) $\mathbb{P}^1$ (its structure sheaf!).
(4) $V_+(x^2 + y^2 + z^2)$
(5) Blow-up of $\mathbb{A}^2$ at a point. (because everyone should know about blow-ups!)
(6) $\operatorname{Spec} R[x_1, x_2, \ldots]$ as an example of something non-noetherian.
(7) An example of a scheme without a closed point.

List of morphisms:

(1) $x \mapsto x^2$ (more, generally x^n). This covers ramified, finitely presented, flat.
(2) a non-quasicompact open-immersion. Polynomial ring in infinitely many variables and knock off the origin. Also, the origin of is an example of something