Seminar on Moduli Theory

Lecture 10

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Last Week

1. Serre vanishing
2. Flat base change

Theorem

Let $X \to Y$ be a projective morphism with Y locally Noetherian. If $\mathcal{F}$ is a coherent sheaf on X which is flat over Y, then the Hilbert polynomial $\chi(\mathcal{F}_{X_y}, d)$ is locally constant for $y \in Y$.

Lemma

Let $S = \operatorname{Spec} A$ be a Noetherian local ring. Let $\mathcal{F}$ be a coherent sheaf on $X = \mathbb{P}^n_S$. Consider the following statements:

1. *$\mathcal{F}$ is flat over S;*
2. *$H^0(X, \mathcal{F}(m))$ is a free A-module of finite rank, for all $m \gg 0$;*
3. *for any $t \in S$, the Hilbert polynomial $\chi(\mathcal{F}_t, m)$ of $\mathcal{F}_t$ on X_t is independent of t.*

Then we have the implications, (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3). Moreover, if S is a domain then they are all equivalent.

$(1) \Leftrightarrow (2)$

$(2) \Rightarrow (3)$

The Hilbert functor

The Hilbert admits a stratification by Hilbert polynomials