

SEMINAR ON MODULI THEORY

August 24, 2020

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1. SUMMARY

One way to go through these sessions is go recall theory of schemes (at level of things in Hartshorne chapter two), by doing a beeline through all the definitions, properties, etc. However, I feel since the point of doing this exercise is to become more comfortable in working with scheme, we will just do lots of examples instead. By this, I mean we will just to prove some things in very concrete situations. This will help you build a concrete picture of the generalities.

List of some things to discuss (just do lots of examples):

- (1) Definition of a scheme.
- (2) Say affine communication lemma *stress this!*
- (3) examples
 - (a) \mathbb{P}^n and it sheaf theory! This already clarifies the $\mathcal{O}(n)$'s

Definition 1. A *scheme* is a locally ringed space with the property that every point has an open neighbourhood which is an affine scheme. A *morphism of schemes* is a morphism of locally ringed spaces. The category of schemes will be denoted Sch .

Things to say: finite-generation, valuative criteria for \mathbb{P}^1 (using DVR's - how giving a point and it's specialisation is the same as giving a map $\text{Spec } R \rightarrow \mathbb{P}^1$).

List of examples:

- (1) \mathbb{A}^1 with a double point (what are quasi-coherent sheaves on this?)
- (2) $\mathbb{A}^2 \setminus \{0, 0\}$ (what is the structure sheaf?).
- (3) \mathbb{P}^1 (its structure sheaf!).
- (4) $V_+(x^2 + y^2 + z^2)$
- (5) Blow-up of \mathbb{A}^2 at a point. (because everyone should know about blow-ups!)
- (6) $\text{Spec } R[x_1, x_2, \dots]$