Seminar on Moduli Theory

Lecture 7

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Last Week

1. fppf $\Rightarrow$ fpqc, and a non-subcanonical site.
2. Representable morphisms of functors.
3. Functors that are schemes.

Characterising fpqc sheaf property

Lemma

Let $F : Sch \to Sets$ be a presheaf. Then F satisfies the sheaf property for the fpqc topology if and only if it satisfies

1. *the sheaf property for every Zariski covering, and*
2. *the sheaf property for $\{V \to U\}$ with V, U affine and $V \to U$ faithfully flat.*

Characterising fpqc sheaf property

Theorem (Grothendieck)

Every representable functor satisfies the sheaf property in the fpqc topology.

Amitsur's Lemma

Let $f : A \to B$ be a faithfully flat ring map. Then, the following sequence of A-modules is exact:

$$0 \to A \xrightarrow{f} B \overset{e_1 - e_2}{\to} B \otimes_A B$$

What happens at $B \otimes_A B$?

Two examples of representable morphisms of functors

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