

Seminar on Moduli Theory

Lecture 7

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Last Week

- ① $\text{fppf} \Rightarrow \text{fpqc}$, and a non-subcanonical site.
- ② Representable morphisms of functors.
- ③ Functors that are schemes.

Characterising fpqc sheaf property

Lemma

Let $F : \text{Sch} \rightarrow \text{Sets}$ be a presheaf. Then F satisfies the sheaf property for the fpqc topology if and only if it satisfies

- ① *the sheaf property for every Zariski covering, and*
- ② *the sheaf property for $\{V \rightarrow U\}$ with V, U affine and $V \rightarrow U$ faithfully flat.*

Characterising fpqc sheaf property

Theorem (Grothendieck)

Every representable functor satisfies the sheaf property in the fpqc topology.

Amitsur's Lemma

Let $f : A \rightarrow B$ be a faithfully flat ring map. Then, the following sequence of A -modules is exact:

$$0 \rightarrow A \xrightarrow{f} B \xrightarrow{e_1 - e_2} B \otimes_A B$$

What happens at $B \otimes_A B$?

Two examples of representable morphisms of functors

Two examples of representable morphisms of functors

What's next?