

Seminar on Moduli Theory

Lecture 13

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Last Week

- ① \mathbb{P}^n as a Quot functor
- ② Moduli of hypersurfaces and lines in the plane $\mathbb{A}_{\mathbb{C}}^2$
- ③ Hilbert functor of points

Theorem (Grothendieck)

Let $\pi : X \rightarrow S$ be a projective morphism with S Noetherian. Then for any coherent sheaf E on X and any polynomial $\phi \in \mathbb{Q}[t]$, the functor $\mathrm{Quot}_{E/X/S}^{\phi(t)}$ is representable by a projective S -scheme.

Notions of projectivity

Schemes of morphisms

Schemes of automorphisms

Grassmannians, classically

Grassmannians, classically (Plücker coordinates)

Grassmannian scheme

Grassmannian scheme (universal quotient and Plücker embedding)

$$\mathrm{Quot}_{\oplus^r \mathcal{O}_{\mathbb{Z}}/\mathbb{Z}/\mathbb{Z}}^d \text{ for } 1 \leq d \leq r$$

Grassmannian of a coherent sheaf