Seminar on Moduli Theory

Lecture 8

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Last Week

1. Characterisation of fpqc sheaf property.
2. representable functors have fpqc sheaf property.
3. Examples of representable morphisms.

The Hilbert function: $\chi(M, n) : n \mapsto dim_k(M_n)$

A function $f : \mathbb{Z}_{\geq n_0} \to Z$ is said to be *polynomial-like* if there exists a polynomial $P_f(X)$ such that $f(n) = P_f(n)$ for $n \gg 0$. We will show that the Hilbert function is polynomial-like.

Theorem

$\chi(M, n)$ is a polynomial-like function of n, of degree $\leq r$.

Examples of Hilbert function

Examples of Hilbert function

Let $X \overset{i}{\hookrightarrow} Y \hookrightarrow \mathbb{P}^n_k$ be a sequence of closed embeddings.

Euler characteristic and Hilbert polynomial

Theorem (Serre Vanishing)

Let $\mathcal{F}$ be a coherent sheaf on a projective k-scheme X, then for $m \gg 0$, $H^i(X, \mathcal{F}(m)) = 0$ for all $i > 0$.