

Seminar on Moduli Theory

Lecture 3

Neeraj Deshmukh

September 11, 2020

Last Week

- ① \mathbb{P}^n and its standard covering.
- ② The Proj construction.
- ③ Degree 2 generators of \mathbb{P}^1 .

More Examples

$V_+(x^2 + y^2 + z^2)$ over \mathbb{R} and \mathbb{C} .

More Examples

Blow-up of \mathbb{A}^2 at the origin.

More Examples

An example of a scheme without a closed point.

Morphisms

Definition

Let \mathcal{P} be a property of morphisms of schemes. Let $f : X \rightarrow Y$ be a morphism which satisfies \mathcal{P} . Then,

- 1 We say that \mathcal{P} is *affine-local on the target* if given any affine open cover $\{V_i\}$ of Y , $f : X \rightarrow Y$ has \mathcal{P} if and only if the restriction $f : f^{-1}(V_i) \rightarrow V_i$ has \mathcal{P} for each i .
- 2 We say that \mathcal{P} is *affine-local on the source* if given any affine open cover $\{U_i\}$ of X , $X \rightarrow Y$ has \mathcal{P} if and only if the composite $U_i \rightarrow Y$ has \mathcal{P} for each i .

Using *affine communication lemma* one can then show that it suffices to check the above statements on single affine open cover.

Something flat, something finitely presented/finite type, something finite.

Not all properties are like this. For example, separatedness, properness, quasi-compactness, etc.

Something ramified, something smooth, something singular.

What are morphisms to \mathbb{P}^n ?