

Seminar on Moduli Theory

Lecture 11

Neeraj Deshmukh

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Last Week

- ① Hilbert polynomial is constant in flat families
- ② The Hilbert and Quot functors
- ③ Their stratification by Hilbert polynomials

Definition (Moduli problem)

A *moduli problem* is any functor $F : \text{Sch} \rightarrow \text{Sets}$. When F is representable we say that the moduli problem admits a *fine moduli space*.

When a fine moduli space M exists, then the identity map id_M corresponds to an element $\xi \in F(M)$. We say that ξ is the *universal family* over M .

Moduli of elliptic curves

Coarse moduli spaces

The Hilbert functor

The Quot functor

Theorem (Grothendieck)

Let $\pi : X \rightarrow S$ be a projective morphism with S Noetherian. Then for any coherent sheaf E on X and any polynomial $\phi \in \mathbb{Q}[t]$, the functor $\mathrm{Quot}_{E/X/S}^{\phi(t)}$ is representable by a projective S -scheme.

Moduli of hypersurfaces

Lines in the plane

Moduli of finite locally free covers

Finite group actions and the Hilbert functor of points