Seminar on Moduli Theory

Lecture 15

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Last Week

1. Grassmannian of a coherent sheaf
2. Outline of proof of representability

Theorem (Grothendieck)

Let $\pi : X \to S$ be a projective morphism with S Noetherian. Then for any coherent sheaf E on X and any polynomial $\phi \in \mathbb{Q}[t]$, the functor $\mathfrak{Quot}^{\phi(t)}_{E/X/S}$ is representable by a projective S-scheme.

What if X is proper?

Hironaka's example: A proper threefold over $\mathbb{C}$ which is not projective

Hironaka's example

Hironaka's Example with a $\mathbb{Z}/2$-action

X_G parametrises a closed subgroup of $\mathfrak{Hilb}^n_X$

Castelnuovo-Mumford Regularity

Definition

Let $\mathcal{F}$ be a coherent sheaf on $\mathbb{P}^n_k$. Let m be an integer. $\mathcal{F}$ is said to be *m-regular* if we have

$$H^i(\mathbb{P}^n_k, \mathcal{F}(m-i)) = 0 \text{ for each } i \geq 1.$$

Theorem (Mumford)

Given any non-negative integers p and n, there exists a polynomial $F_{p,n}$ in $n+1$-variables with the following property:
If $\mathcal{F} \subset \bigoplus^p \mathcal{O}_{\mathbb{P}^n_k}$ is any coherent subsheaf with Hilbert polynomial

$$\chi(\mathcal{F}, r) = \sum_{i=0}^{n} a_i \binom{r}{i},$$

then $\mathcal{F}$ is $F_{p,n}(a_0, \ldots, a_n)$-regular.