Ellipse Drawing

Properties of Ellipses

- Equation simplified if ellipse axis parallel to coordinate axis

$$\left(\frac{x-x_c}{r_x}\right)^2+\left(\frac{y-y_c}{r_y}\right)^2=1$$

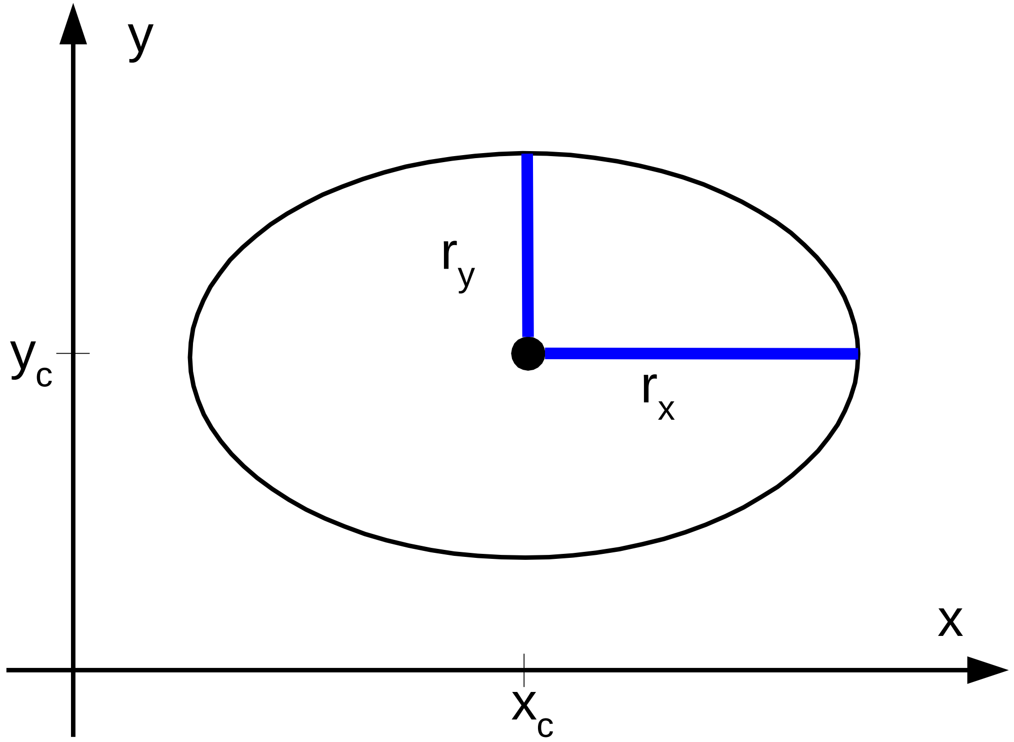
- If center of the ellipse is at origin,

$$f(x,y)=r_y^2x^2+r_x^2y^2-r_x^2r_y^{\,2}$$

- Parametric form

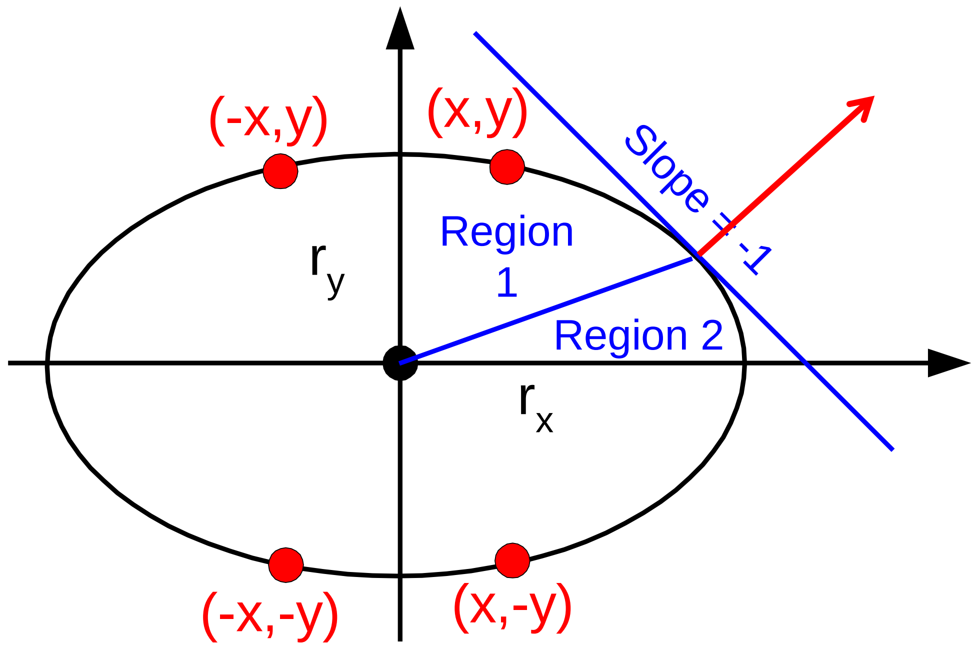
$$x=x_c+r_x\cos\theta$$

$$y=y_c+r_y\sin\theta$$



Symmetry Considerations

- 4-way symmetry
- Unit steps in *x* until reach region boundary
- Switch to unit steps in *y*



$$f(x,y) = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$$

$$\frac{dy}{dx} = -\frac{r_y^2 x}{r_x^2 y}$$

$$\frac{dy}{dx} = -1$$

$$r_y^2 x = r_x^2 y$$

- Step in *x* while

$$r_y^2 x < r_x^2 y$$

- Switch to steps in *y* when

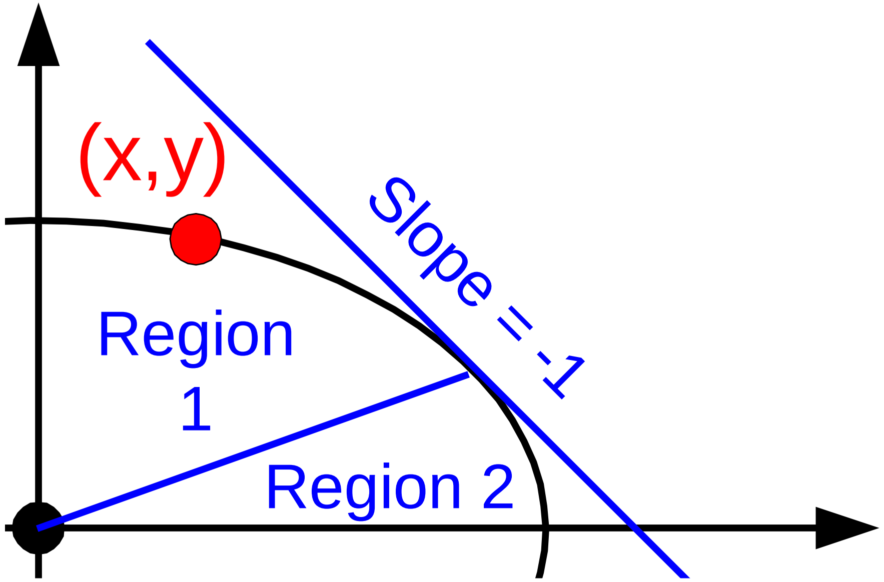
$$r_y^2 x \geq r_x^2 y$$

Midpoint Algorithm (initializing)

- Similar to circles
- The initial value for region 1

$$D_{init1} = f(1, r_y - \tfrac{1}{2}) = r_y^2 - r_x^2 r_y + \tfrac{1}{4} r_x^2$$

- The initial value for region 2



$$D_{init2} = f(x_p + \tfrac{1}{2}, y_p - 1) = r_y^2 (x_p + \tfrac{1}{2})^2 + r_x^2 (y_p - 1)^2 - r_x^2 r_y^2$$

- We have initial values, now we need the increments

Making a Decision

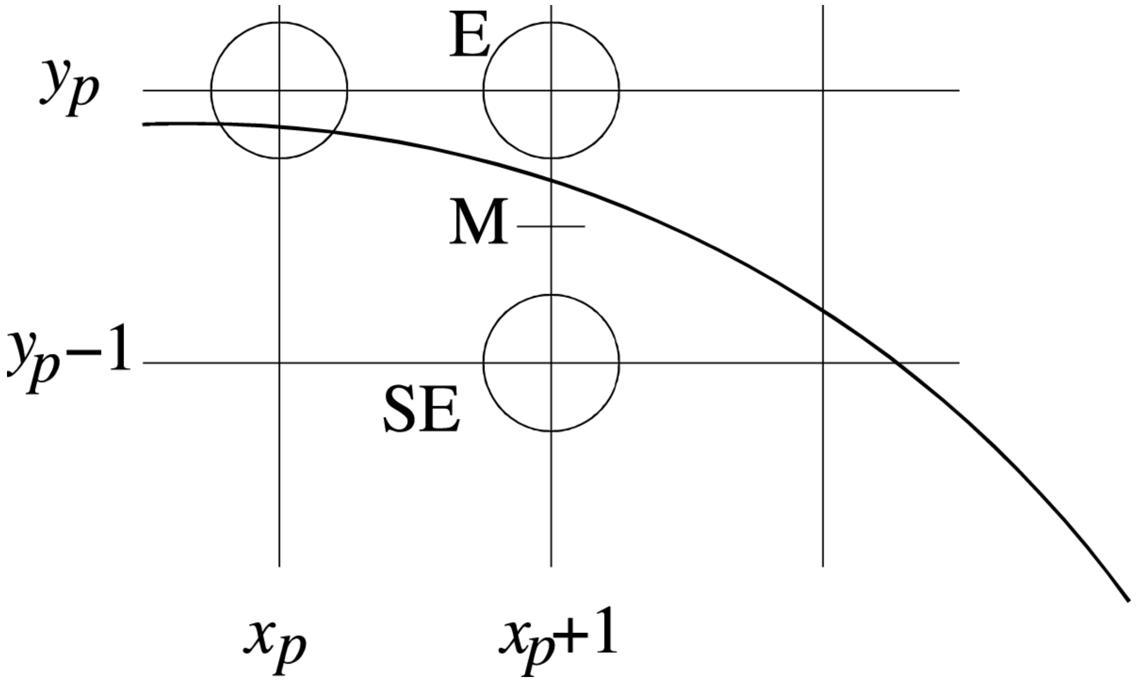
- Computing the decision variable/ discriminant

$$D = f(x_p + 1, y_p - \tfrac{1}{2})$$

$$= r_y^2(x_p + 1)^2 + r_x^2(y_p - \tfrac{1}{2})^2 - r_x^2 r_y^2$$

- If $D < 0$ then M is *below* the arc, hence the E pixel is closer to the line.

- If $D \geq 0$ then M is *above* the arc, hence the SE pixel is closer to the line.



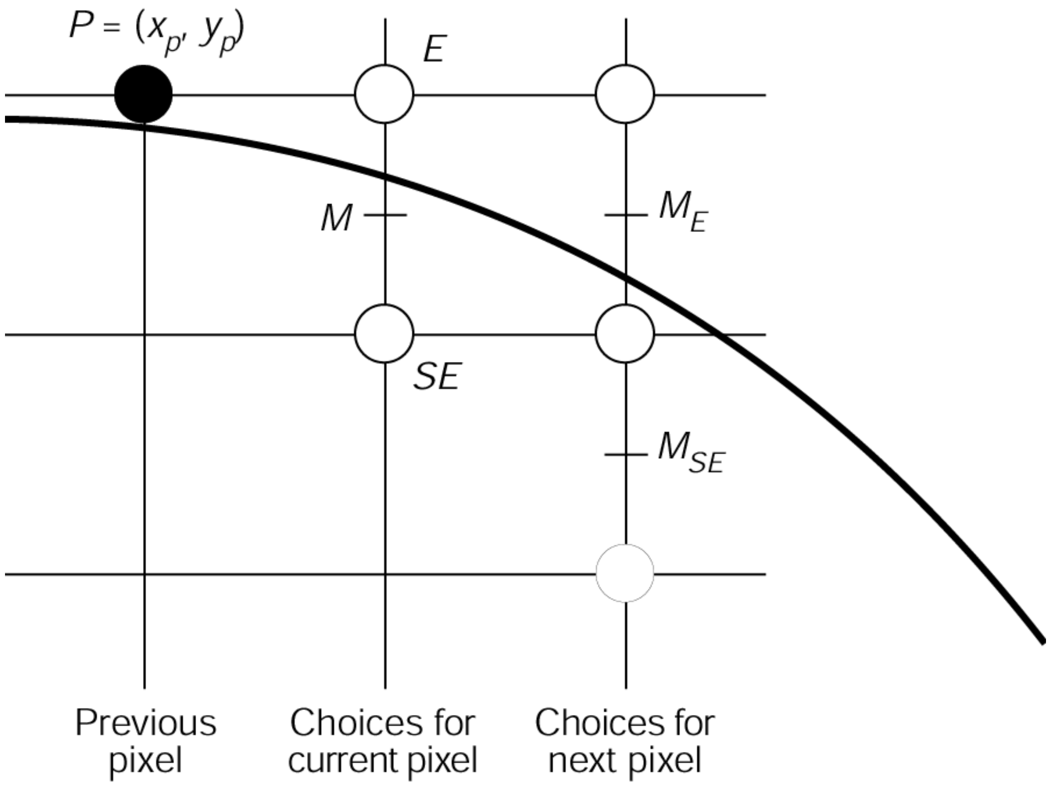
Computing the rate of change of Discriminant

E case

$$D_{new} = f(x_p + 2, y_p - \tfrac{1}{2})$$
$$= r_y^2 (x_p + 2)^2 + r_x^2 (y_p - \tfrac{1}{2})^2 - r_x^2 r_y^2$$
$$= D_{old} + r_y^2 (2x_p + 3)$$

SE case

$$D_{new} = f(x_p + 2, y_p - \tfrac{3}{2})$$
$$= r_y^2 (x_p + 2)^2 + r_x^2 (y_p - \tfrac{3}{2})^2 - r_x^2 r_y^2$$
$$= D_{old} + r_y^2 (2x_p + 3) + r_x^2 (-2y_p + 2)$$



$$increment = \begin{cases} r_y^2 (2x_p + 3) & D_{old} < 0 \\ r_y^2 (2x_p + 3) + r_x^2 (-2y_p + 2) & D_{old} \geq 0 \end{cases}$$

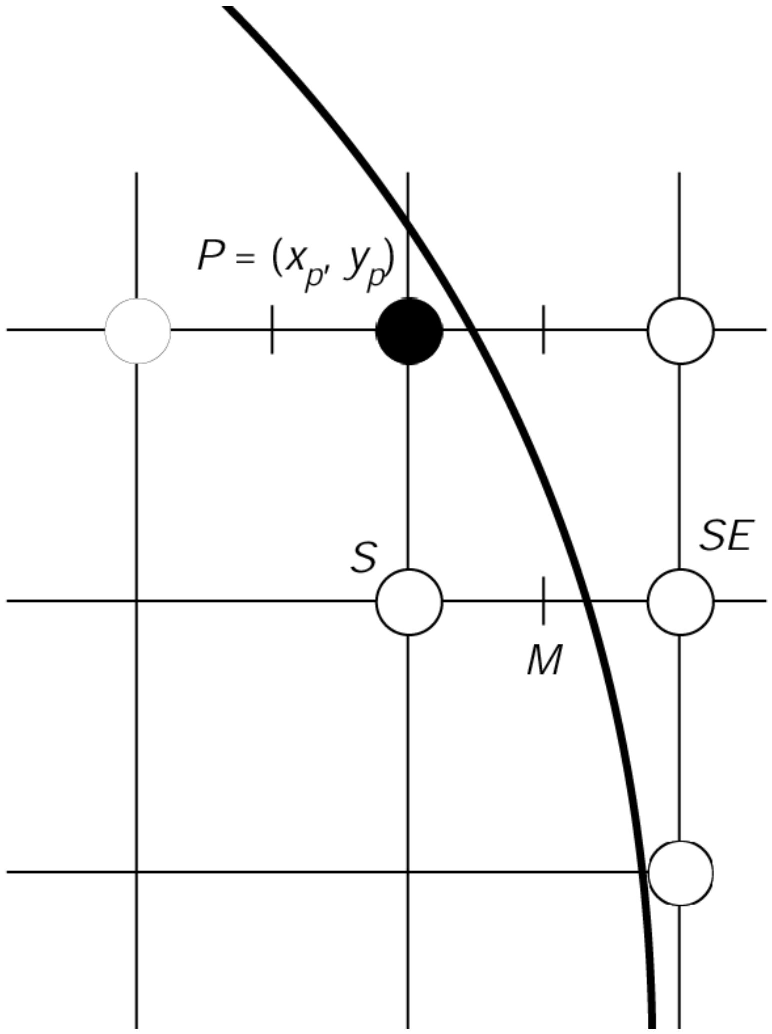
Computing the Increment in 2nd Region

- Decision variable in 2nd region

$$D = f(x_p + \tfrac{1}{2}, y_p - 1)$$
$$= r_y^2 (x_p + \tfrac{1}{2})^2 + r_x^2 (y_p - 1)^2 - r_x^2 r_y^2$$

- If $D < 0$ then M is *left of* the arc, hence the *SE* pixel is closer to the line.

- If $D \geq 0$ then M is *right of* the arc, hence the *S* pixel is closer to the line.



Computing the Increment in 2nd Region

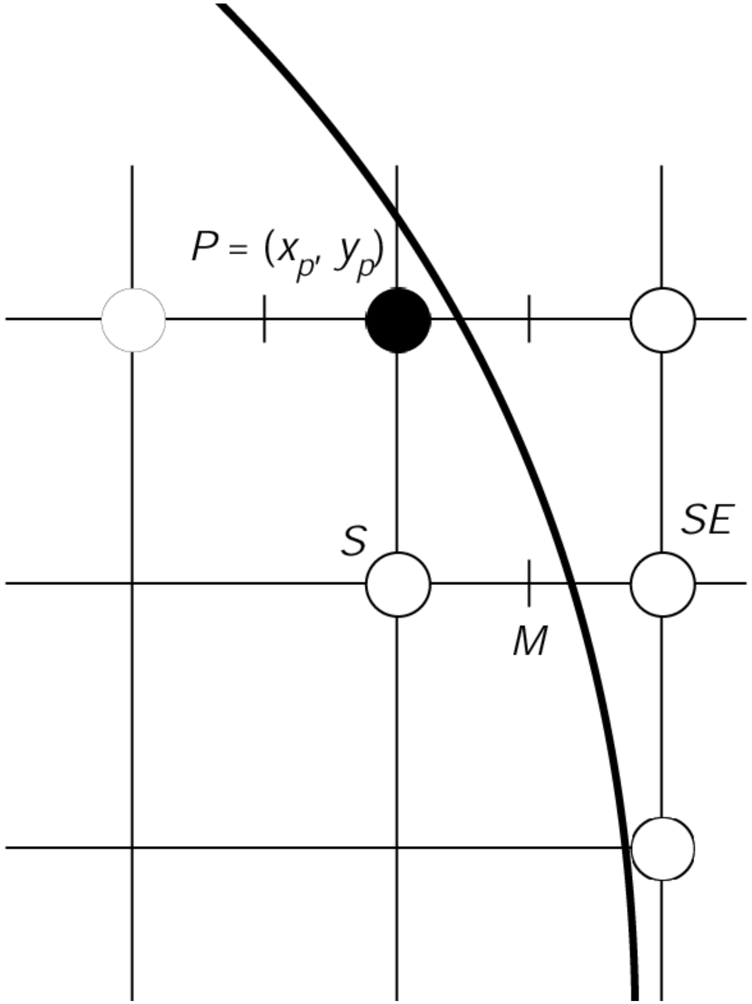
SE case

$$D_{new} = f(x_p + \tfrac{3}{2}, y_p - 2)$$
$$= r_y^2(x_p + \tfrac{3}{2})^2 + r_x^2(y_p - 2)^2 - r_x^2 r_y^2$$
$$= D_{old} + r_x^2(-2y_p + 3) + r_y^2(2x_p + 2)$$

S case

$$D_{new} = f(x_p + \tfrac{1}{2}, y_p - 2)$$
$$= r_y^2(x_p + \tfrac{1}{2})^2 + r_x^2(y_p - 2)^2 - r_x^2 r_y^2$$
$$= D_{old} + r_x^2(-2y_p + 3)$$

$$increment = \begin{cases} r_x^2(-2y_p + 3) + r_y^2(2x_p + 2) & D_{old} < 0 \\ r_x^2(-2y_p + 3) & D_{old} \geq 0 \end{cases}$$



Midpoint Algorithm for Ellipses

Region 1

Set first point to $(0, r_y)$

Set the Decision variable to

$$D_{init1} = r_y^2 - r_x^2 r_y + \tfrac{1}{4} r_x^2$$

Loop $(x = x+1)$

If $D < 0$ then pick E and

$$D \mathrel{+}= r_y^2(2x_p + 3)$$

If $D \geq 0$ then pick SE and

$$D \mathrel{+}= 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + 3r_y^2 + 2r_x^2$$

$y = y - 1;$

Use symmetry to complete the ellipse

Until $r_y^2(x_k + 1) > r_x^2(y_k - ½)$

Region 2

Set first point to the last computed

Set the Decision variable to from previous value

Or, $D_{init2} = r_y^2(x_p + \tfrac{1}{2})^2 + r_x^2(y_p - 1)^2 - r_x^2 r_y$

Loop $(y = y - 1)$

If $D < 0$ then pick SE and

$$D \mathrel{+}= r_y^2(2x_p + 2) + r_x^2(-2y_p + 3)$$

x = x + 1;

If $D \geq 0$ then pick S and

$$D \mathrel{+}= r_x^2(-2y_p + 3)$$

Use symmetry to complete the ellipse

Until $y < 0$