UDACITY

6 — Normal Distribution

6.1 Probability Distribution Function

Definition 6.1 — Probability Distribution Function. The probability distribution function is a normal curve with an area of 1 beneath it, to represent the cumulative frequency of values.
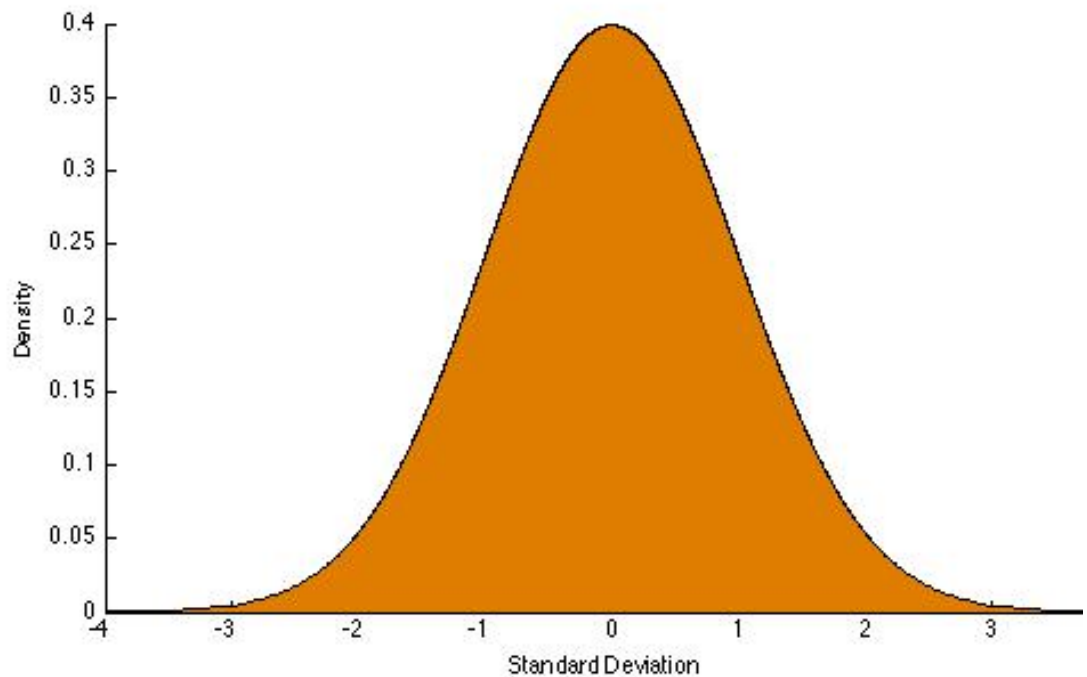

Figure 6.1: The area beneath the curve is 1

6.1.1 Finding the probability

We can use the PDF to find the probability of specific measurements occurring. The following examples illustrate how to find the area below, above, and between particular observations.

■ **Example 6.1**   The average height of students at a private university is 1.85 meters with a standard deviation of 0.15 meters. What percentage of students are shorter or as tall as Margie who stands at 2.00 meters.

To solve this problem the first thing we need to find is our z-score:

$$z = \frac{x - \mu}{\sigma} = \frac{2.05 - 1.85}{0.15} = 1.\bar{3}$$

Now we need to use the z-score table to find the proportion below a z-score of 1.33.

Ⓡ   The z-table only shows the proportion below. In this instance we are trying to find the orange area.
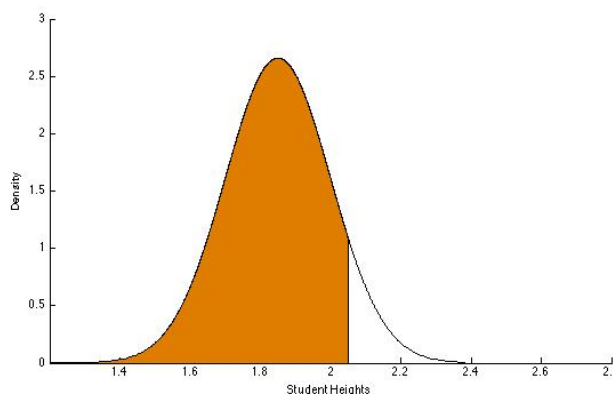

Figure 6.2: 85% is the shaded area

To use the z-table we start in the left most column and find the first two digits of our z-score (in this case 1.3) then we find the third digit along the top of the table. Where this row and column intersect is our proportion below that z-score.

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■ **Example 6.2**   Margie also wants to know what percent of students are taller than her. Since the area under the normal curve is 1 we can find that proportion:

$$1 - 0.9082 = 0.0918 = 9.18\%$$

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■ **Example 6.3**   Anne only measures 1.87 meters. What proportion of classmates are between Anne and Margies heights.

We already know that 90.82% of students are shorter that Margie. So lets first find the percent of students that are shorter than Anne.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633

Figure 6.3: using the z-table for 1.33
This means that Margie is taller than 90.82% of her classmates.

$$\frac{1.87 - 1.85}{0.15} = 0.1\bar{3}$$

If we use the z-table we see that this z-score corresponds with a proportion of 0.5517 or 55.17%. So to get the proportion in between the two we subtract the two proportions from each other. That is the proportion of people who's height's are between Anne and Margies height is $90.82 - 55.17 = 35.65\%$.

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6.2 Practice Problems

Problem 6.1   In 2007-2008 the average height of a professional basketball player was 2.00 meters with a standard deviation of 0.02 meters. Harrison Barnes is a basketball player who measures 2.03 meters. What percent of players are taller than Barnes?

Problem 6.2   Chris Paul is 1.83 meters tall. What proportion of Basketball players are between Paul and Barne's heights?

Problem 6.3   92% of candidates scored as good or worse on a test than Steve. If the average score was a 55 with a standard deviation of 6 points what was Steve's score?