CSCI 567 Homework 5

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Problem 1. HMM

- **Question 1.1** Following the procedure described in slide 35 lecture 18:

$$something_1 = p(x_1) = 0.7 \times 0.4 + 0.3 \times 0.2 = 0.34 \tag{1}$$

$$\alpha_1(1) = \frac{0.4 \times 0.7}{something_1} = 0.82 \tag{2}$$

$$\alpha_1(2) = \frac{0.3 \times 0.2}{something_1} = 0.18 \tag{3}$$

$$something_2 = \sum_j P(x_2|z_2 = s_j) \sum_i a_{ij}\alpha_1(i) \tag{4}$$

$$something_2 = 0.4 \times (0.8 \times 0.82 + 0.4 \times 0.18) + 0.2 \times (0.2 \times 0.82 + 0.6 \times 0.18) = 0.35 \tag{5}$$

$$\alpha_2(1) = \frac{0.4 \times (0.8 \times 0.82 + 0.4 \times 0.18)}{something_2} = 0.83 \tag{6}$$

$$\alpha_2(2) = \frac{0.2 \times (0.2 \times 0.82 + 0.6 \times 0.18)}{something_2} = 0.15 \tag{7}$$

$$something_3 = \sum_j P(x_3|z_3 = s_j) \sum_i a_{ij}\alpha_2(i) \tag{8}$$

$$something_3 = 0.1 \times (0.8 \times 0.83 + 0.4 \times 0.15) + 0.3 \times (0.2 \times 0.83 + 0.6 \times 0.15) = 0.15 \tag{9}$$

$$\alpha_3(1) = \frac{0.1 \times (0.8 \times 0.83 + 0.4 \times 0.15)}{something_3} = 0.48 \tag{10}$$

$$\alpha_3(2) = \frac{0.3 \times (0.2 \times 0.83 + 0.6 \times 0.15)}{something_3} = 0.51 \tag{11}$$

similarly:

$$something_4 = \sum_j P(x_4|z_4 = s_j) \sum_i a_{ij}\alpha_3(i) \tag{12}$$

$$something_4 = 0.4\times(0.8\times0.48+0.4\times0.51)+0.2\times(0.2\times0.48+0.6\times0.51) = 0.32 \tag{13}$$

$$\alpha_4(1) = 0.74, \alpha_4(2) = 0.25 \tag{14}$$

$$something_5 = \sum_j P(x_5|z_5 = s_j) \sum_i a_{ij}\alpha_4(i) \tag{15}$$

$$something_5 = 0.1\times(0.8\times0.74+0.4\times0.25)+0.3\times(0.2\times0.74+0.6\times0.25) = 0.16 \tag{16}$$

$$\alpha_5(1) = 0.44, \alpha_5(2) = 0.56 \tag{17}$$

$$something_6 = \sum_j P(x_6|z_6 = s_j) \sum_i a_{ij}\alpha_5(i) \tag{18}$$

$$something_6 = 0.4\times(0.8\times0.44+0.4\times0.56)+0.2\times(0.2\times0.44+0.6\times0.56) = 0.31 \tag{19}$$

$$\alpha_6(1) = 0.73, \alpha_6(2) = 0.27 \tag{20}$$

Thus

$$P(X_{1:6}) = something_1 \times ... \times something_6 = 0.34 \times 0.35 \times ... \times 0.31 = 0.000283 \tag{21}$$

- **Question 1.2** using Viterbi Algorithm:
  $\delta_0(1) = \delta_0(2) = 1$

$$\delta_1(1) = \delta_0(1) \times \pi_1 \times b_{1A} = 0.28 \tag{22}$$

$$\delta_1(2) = \delta_0(2) \times \pi_2 \times b_{2A} = 0.06 \tag{23}$$

$$\Psi_1(1) = \Psi_1(2) = *S* \tag{24}$$

$$\delta_2(1) = Max\delta_1(i)a_{i1}P(x_2|z_2 = s_1) = max(0.28\times0.8\times0.4, 0.06\times0.4\times0.4) = 0.09 \tag{25}$$

$$\psi_2(1) = 1 \tag{26}$$

$$\delta_2(2) = Max\delta_1(i)a_{i2}P(x_2|z_2 = s_2) = max(0.28 \times 0.2 \times 0.2, 0.06 \times 0.6 \times 0.2) = 0.0112 \tag{27}$$

$$\psi_2(2) = 1 \tag{28}$$

similarly we get:

$$\delta_3(1) = Max\delta_2(i)a_{i1}P(x_3|z_3 = s_1) = 0.09 \times 0.8 \times 0.1 = 0.0072 \tag{29}$$

$$\psi_3(1) = 1 \tag{30}$$

$$\delta_3(2) = Max\delta_2(i)a_{i2}P(x_3|z_3 = s_2) = 0.09 \times 0.2 \times 0.3 = 0.0054 \tag{31}$$

$$\psi_3(2) = 1 \tag{32}$$

$$\delta_4(1) = 0.0072 \times 0.8 \times 0.4 = 0.0023 \tag{33}$$

$$\psi_4(1) = 1 \tag{34}$$

$$\delta_4(2) = 0.0054 \times 0.6 \times 0.2 = 0.00065 \tag{35}$$

$$\psi_4(2) = 2 \tag{36}$$

$$\delta_5(1) = 0.0023 \times 0.8 \times 0.1 = 0.000184 \tag{37}$$

$$\psi_5(1) = 1 \tag{38}$$

$$\delta_5(2) = 0.0023 \times 0.2 \times 0.3 = 0.000138 \tag{39}$$

$$\psi_5(2) = 1 \tag{40}$$

$$\delta_6(1) = 0.000184 \times 0.8 \times 0.4 = 0.000059 \tag{41}$$

$$\psi_6(1) = 1 \tag{42}$$

$$\delta_6(2) = 0.000138 \times 0.6 \times 0.2 = 0.0000165 \tag{43}$$

$$\psi_6(2) = 2 \tag{44}$$

By backtracking we get the following the likely state path as:
$z^* = \{111111\}$