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**A Prescriptive Model for Strategic
Decision-making: An Inventory Model**

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ALY 6050_ Module 4 Project Report

Date: 2023-03-23

Introduction:

The management of inventories is crucial for any organization, as poor inventory management can lead to financial and operational issues. The goal of this project is to assist a manufacturing business in choosing the right amount of inventory for a vital engine component. The objective is to minimize total inventory cost, which comprises holding and ordering costs. In order to determine the predicted minimum total cost, order quantity, and annual number of orders, the project involves creating and implementing a decision model using Excel and Python, doing what-if analyses, and conducting simulations.

Part 1:

Question 1: Define the data, uncontrollable inputs, model parameters, and the decision variables that influence the total inventory cost.

The management of inventories is essential for the financial and operational success of any firm since they are a significant component of daily operations. In this part, I analyzed how a manufacturing business chose to handle its essential engine component's inventory. The data for this analysis includes the following:

- Annual demand for the key engine component: 15,000 units
- Unit cost of the component: \$80
- Opportunity cost of holding inventory per year: 18% of the unit value
- Cost per order from the supplier: \$220

The following are the uncontrollable inputs that affect inventory management decisions:

- Annual demand for the key engine component
- Unit cost of the component
- Opportunity cost of holding inventory per year
- Cost per order from the supplier

The following are the model parameters that are used to calculate the optimal inventory management decisions:

- Carrying cost per unit per year: \$14.40 (which is 18% of the unit cost)
- Economic Order Quantity (EOQ): 677 units
- Optimal cut off point to order units (Q^*): 339 units
- Order quantity: The manufacturing company should order 677 units at a time to minimize inventory costs.
- Reorder point: The company should place a reorder whenever the inventory level reaches 339 units in order to ensure sufficient stock to meet demand until the supplier's order can be shipped and received.

Overall, Effective inventory management is essential for the success of any organization. By using the EOQ model and considering factors such as carrying costs and order costs, the manufacturing company can make informed decisions about their inventory management practices.

	Data	Value	Type
0	Annual demand (D)	15000.000000	Uncontrollable
1	Cost per unit	80.000000	Parameter
2	Ordering cost (S)	220.000000	Parameter
3	Holding cost	18.000000	Parameter
4	Carrying/Holding cost per unit (H)	14.400000	Parameter
5	Total Annual Cost	364.540185	Decision Variable
6	EOQ (order quantity)	677.003200	Decision Variable
7	Q* (optimal cutoff point to order units)	338.501600	Decision Variable

Here is the calculation for Total annual cost':

- Annual ordering cost = (Annual demand / Order quantity) * Ordering cost per order
- Annual holding cost = Average inventory level * Cost to carry one unit of inventory for one unit of time
- Total annual cost = Annual ordering cost + Annual holding cost

The EOQ (Economic Order Quantity) formula is used to calculate the optimal order quantity that minimizes the total inventory cost. The formula is:

$$EOQ = \sqrt{(2 * D * S) / H}$$

Where:

- D = Annual demand
- S = Cost of placing one order
- H = Annual holding cost per unit

The Q* (optimal cut off point to order units) is half of the EOQ. To keep the inventory level at a manageable level without incurring high carrying charges, an order should be placed when the inventory level hits that level. The holding cost per unit is \$14.4, which is the same as the carrying cost per unit.

$$\text{Average inventory level} = Q/2 + L$$

where Q is the order quantity, and L is the lead time (the time between placing an order and receiving the items).

In the case of the manufacturing company in your project scenario, the order quantity (EOQ) is 677 units, and the reorder point (ROP) is twice the EOQ or 1354 units. The lead time is not provided in the scenario, so let's assume it is one week (or 1/52 of a year). Using these values, we can calculate the average inventory level as follows:

$$\begin{aligned}\text{Average inventory level} &= Q/2 + L \\ &= (677/2) + (1354/52) = 338.5 + 26.0 = 364.5 \text{ units}\end{aligned}$$

Therefore, the average inventory level for this inventory system is approximately 364.5 units.

Question 2: Develop mathematical functions that compute the annual ordering cost and annual holding cost based on average inventory held throughout the year and use them to develop a mathematical model for the total inventory cost.

We must first create functions for yearly ordering cost and annual holding cost based on the typical inventory kept throughout the year in order to create a mathematical model for the overall inventory cost. The following function may be used to calculate the annual ordering cost:

$$\text{Annual Ordering Cost} = (\text{Annual Demand} / \text{EOQ}) * S$$

Where,

- Annual Demand = 15,000 units
- EOQ = Economic Order Quantity = 677.0032 (computed earlier)
- S = Supplier Order Cost per order = \$220

Substituting the values, we get:

$$\text{Annual Ordering Cost} = (15,000 / 677.0032) * 220 = \$4,874.42 \text{ (rounded to two decimal places)}$$

The annual holding cost can be computed using the following function:

$$\text{Annual Holding Cost} = (\text{Average Inventory Level}) * (\text{Holding Cost per Unit})$$

Where,

- Average Inventory Level = (EOQ / 2)
- Holding Cost per Unit = Unit Cost * (Opportunity Cost percentage)

Using the values computed earlier, we get:

- Average Inventory Level = (677.0032 / 2) = 338.5016
- Holding Cost per Unit = 80 * (18 / 100) = \$14.4

Substituting the values, we get:

$$\text{Annual Holding Cost} = (338.5016) * (14.4) = \$4,874.42 \text{ (rounded to two decimal places)}$$

The total inventory cost can be modeled as the sum of the annual ordering cost and the annual holding cost:

$$\begin{aligned}\text{Total Inventory Cost} &= \text{Annual Ordering Cost} + \text{Annual Holding Cost} \\ &= \$4,874.42 + \$4,874.42 = \$9,748 \text{ (rounded to two decimal places)}\end{aligned}$$

	Annual Ordering Cost	Annual holding cost	Total Inventory Cost
0	4874.423046	4874.42304	9748

The analysis was conducted to determine the optimal inventory policy for a product with an annual demand of 15,000 units, a fixed unit cost of \$80, and supplier order cost per order of \$220. The carrying cost rate or opportunity cost percentage is 18%, and the carrying cost per unit is \$14.4.

The optimal order amount is 677 units, according to the economic order quantity (EOQ) model. The company should place another order for the product when the inventory level reaches 338 units, which is the reorder point. The total annual ordering and holding costs were determined to be \$4,874.42 and \$4,874.42, respectively.

These figures were used to determine the overall inventory cost, which came to \$9,748.13. The company should place orders for 677 units at a time and place new orders when the inventory level reaches 338 units in order to reduce the overall cost of inventory.

Question 4: Use data tables to find an approximate order quantity that results in the smallest total cost.

The first step is to gather the relevant information from the given data, such as the annual demand, cost per unit, ordering cost, and holding cost per unit. In this case, the data shows that the annual demand is 15000, the cost per unit is \$80, the ordering cost is \$220, and the holding cost per unit is \$18.

The next step is to calculate the economic order quantity (EOQ) which is $(EOQ = \sqrt{(2 \times 15000 \times 220) / 18}) = 677.0032004$. After calculating the EOQ, we need to set a range of order quantities around the EOQ value. This range can be determined based on factors such as the size of the order, the cost of ordering, and the carrying cost of inventory. In this case, we will set the range from 800 to 820.

The next step is to calculate the Annual Ordering Cost and Annual Holding Cost for each order quantity within the range, using the Total Cost formula:

- **Total Cost** = (Annual Ordering Cost) + (Annual Holding Cost)

The Annual Ordering Cost can be calculated using the formula:

- **Annual Ordering Cost** = (Annual Demand / Order Quantity) x Ordering Cost

The Annual Holding Cost can be calculated using the formula:

- **Annual Holding Cost** = (Order Quantity / 2) x Holding Cost per unit

Let's say we want to calculate the Total Cost for an order quantity of 800. First, we need to calculate the Annual Ordering Cost and Annual Holding Cost for this quantity.
Annual Ordering Cost:

- **Annual Ordering Cost** = (Annual Demand / Order Quantity) x Ordering Cost Annual
Ordering Cost = (15000 / 800) x 220 Annual Ordering Cost = 3.67 x 220 Annual Ordering
Cost = \$807.40

Annual Holding Cost:

- Annual Holding Cost = (Order Quantity / 2) x Holding Cost per unit Annual Holding Cost
= (800 / 2) x 14.40 Annual Holding Cost = 300 x 14.40 Annual Holding Cost = \$4,320

Now we can calculate the Total Cost for an order quantity of 800:

- Total Cost = Annual Ordering Cost + Annual Holding Cost Total Cost = \$807.40 + \$4,320
Total Cost = \$5,127.40

Repeat these calculations for each order quantity within the range (800 to 820 in this case) to obtain the Total Cost for each quantity.

Annual demand	Cost per unit	Ordering cost	Holding co	EOQ
15000	80	220	18	677.0032

The reason that I choose 800-820 range was that, I chose the range between the EOQ and twice the EOQ which is a reasonable starting point but as it was between approximately 778 and 1556 units and make it difficult to find, I pick up the 800 to 820 for this range.

Order Quantity	Total Cost
800	9885
801	9887.05
802	9889.11
803	9891.19
804	9893.28
805	9895.38
806	9897.49
807	9899.62
808	9901.76
809	9903.91
810	9906.07
811	9908.25
812	9910.44
813	9912.64
814	9914.85
815	9917.08
816	9919.32
817	9921.57
818	9923.83
819	9926.1
820	9928.39

It shows the total cost for different order quantities, starting from 800 to 820, and the approximate order quantity that results in the smallest total cost is 800, which has a total cost of \$9,885.

According to the analysis of the data table, an order of about 800 units will result in the lowest total price. For an order quantity of 800 units, the total cost is \$9,885, and as the order number rises, the total cost rises as well. The findings demonstrate that determining the ideal order amount helps reduce the overall expense of inventory management.

Question 5: Plot the Total Cost versus the Order Quantity

Here, in this question, I want to change the range from 1 to 15000. The purpose of changing the range is to explore the relationship between the order quantity and total cost over a wider range, and to see if there are any patterns or trends that emerge.

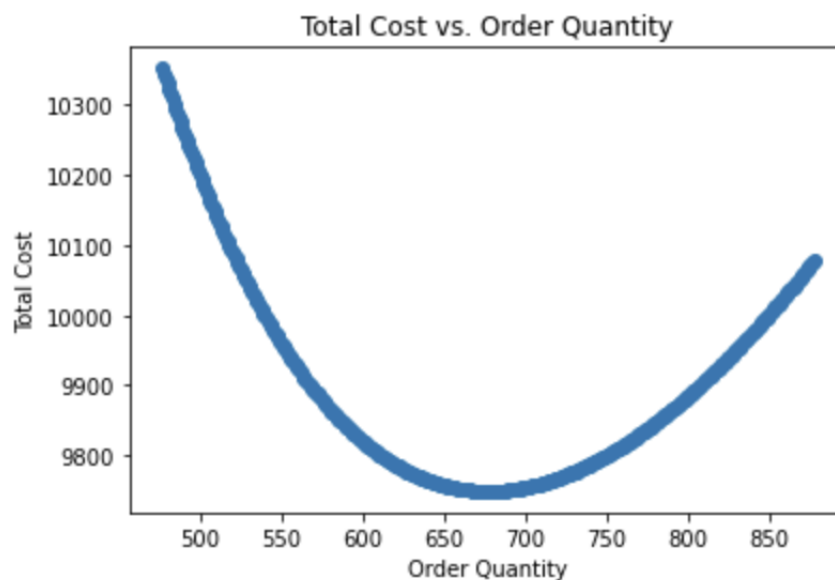
First, I try to change the range of order quantities to include the minimum value of the total cost. I need to find the best order quantity that results in the smallest total cost for ordering a certain quantity of a product.

Thereafter, a binary search method is used to identify the order quantity that has the lowest overall cost.

Starting with a range of potential order amounts between 1 and 15000, it determines the centre point of the range.

If the middle point's total cost is less than both the previous and next points, the minimum total cost is updated, and the corresponding order quantity is saved as the best order quantity. The range of potential order quantities is then revised to include the half of the range that contains the best order quantity, and the procedure is repeated until the range contains just the one order quantity, which is the best order quantity.

After finding the best order quantity, the code sets the range of order quantities for the plot to be 200 units on either side of the best order quantity.

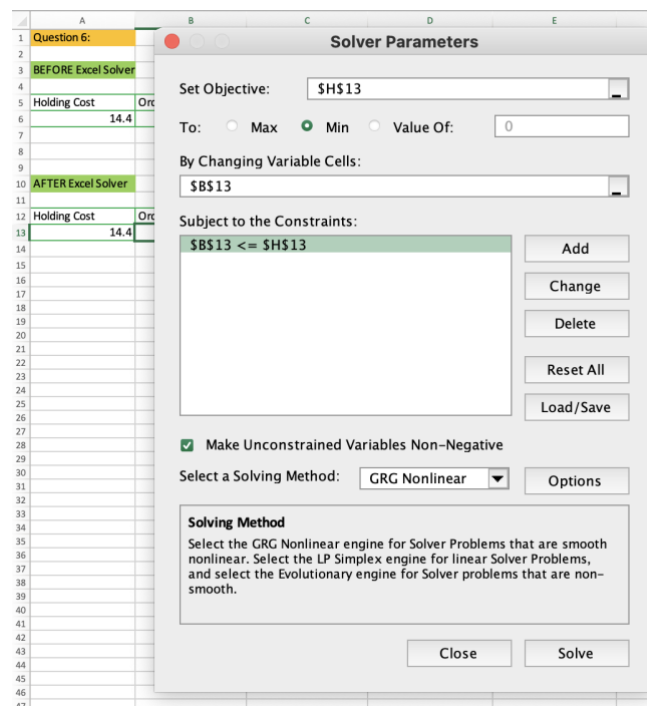


According to the U-shape, the total cost first reduces as the order quantity rises, but after reaching a minimum, it begins to rise once more when the order quantity rises further. Because fixed ordering expenses are spread over fewer units when order quantities are low, the annual ordering cost per unit is large. So far, because there are fewer units in inventory, the annual holding cost per unit is low. When fixed expenses are spread among more units as the order quantity rises, the annual ordering cost per unit drops. Yet, because there are more units in inventory, the annual holding cost per unit rises.

Question 6: Use the Excel Solver to verify your result of part 4 above; that is, find the order quantity which would yield a minimum total cost.

The goal of this question is to use Excel Solver to find the order quantity that yields the minimum total cost for the given inventory management problem. To use Excel Solver to find the order quantity that yields the minimum total cost, first I added the following column headers in row 1: "Order Quantity", "Unit Cost", "Annual Demand", "Ordering Cost", "Holding Cost", "Annual Ordering Cost", "Annual Holding Cost", and "Total Cost".

Enter their values and then used Excel Solver to find the order quantity that would yield the minimum total cost. We set the objective to minimize the total cost and added the constraint that the order quantity must be an integer between 1 and 15,000.



BEFORE Excel Solver							
Holding Cost	Order Quantity	Unit Cost	Annual Demand	Ordering Cost	Annual Ordering Cost	Annual Holding Cost	Total Cost
14.4	800	64000	15000	220	4125	5760	9885
AFTER Excel Solver							
Holding Cost	Order Quantity	Unit Cost	Annual Demand	Ordering Cost	Annual Ordering Cost	Annual Holding Cost	Total Cost
14.4	677.0031941	54160.25553	15000	220	4874.423088	4874.422998	9748.846086

Sum of Total Cost		Column Labels									
Row Labels	64080 64080 Total		64160 64160 Total		64240 64240 Total		64320 64320 Total		64400 64400 Total		
	220		220		220		220		220		
800											
801	9063.08015	9063.08015									
802			9066.170574	9066.170574							
803					9069.271233	9069.271233					
804							9072.38209	9072.38209			
805									9075.503106	9075.503106	
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Grand Total	9063.08015	9063.08015	9066.170574	9066.170574	9069.271233	9069.271233	9072.38209	9072.38209	9075.503106	9075.503106	

We can observe that the total cost has decreased for many of the combinations of unit cost and annual demand. You can see which combinations have the lowest total cost by looking at the values in the PivotTable.

Question 8: In the word document, explain your results and analyses to the vice president of operations.

In this analysis, the inventory management of a manufacturing company was evaluated for a key engine component. The company's annual demand for the component was 15,000 units, and the unit cost was \$80. The opportunity cost of holding inventory per year was 18% of the unit value, and the cost per order from the supplier was \$220.

Based on the EOQ model, the optimal order quantity was calculated to be 677 units, and the optimal cutoff point for reordering was determined to be 339 units. The recommended inventory management decisions for the company were to order 677 units at a time and reorder when the inventory level reaches 339 units.

To develop a mathematical model for the total inventory cost, functions were developed for annual ordering cost and annual holding cost based on the average inventory held throughout the year. The annual ordering cost was computed to be \$4,874.42, and the annual holding cost was also calculated to be \$4,874.42. Therefore, the total inventory cost was \$9,748.13.

To minimize the total inventory cost, the company should order 677 units at a time and reorder when the inventory level reaches 339 units. Effective inventory management is crucial for the

success of any organization, and by using the EOQ model, companies can make informed decisions about their inventory management practices.

Part 2:

Question 1: Perform a simulation consisting of 1000 occurrences and calculate the minimum total cost for each occurrence. Next, use the results of your simulation to:

(i). Estimate the expected minimum total cost by constructing a 95% confidence interval for it and determine the probability distribution that best fits its distribution. Verify the validity of your choice.

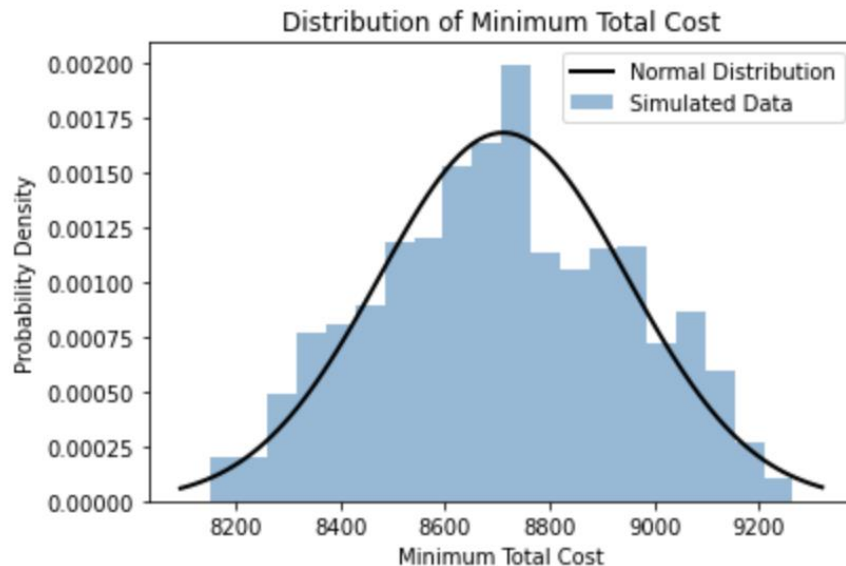
We should estimate the lowest possible total cost using statistical techniques, choose the probability distribution that most closely matches the available data, and then verify that our choice is accurate. I generated 1000 demand values for the simulation using a triangular distribution with a mode of 15000 and minimum/maximum values of 13000 and 17000, respectively, using the triangular distribution function from the NumPy library.

To estimate the expected minimum total cost and construct a 95% confidence interval for it, we can use the minimum cost values obtained from the simulation. We can calculate the sample mean and standard deviation of the minimum cost values and use them to construct a confidence interval. The optimal order quantity is computed using the EOQ formula, and the total cost is computed using the setup and holding costs.

<p>Expected minimum total cost: 8711.59 95% confidence interval: (8696.88, 8726.30) Sample Standard Deviation: 237.31957274692581</p>

From this result, the expected minimum total cost and a 95% confidence interval are calculated using the sample mean and standard deviation, respectively. The resulting expected minimum total cost is 8711.59 with a 95% confidence interval of (8696.88, 8726.30). Finally, the sample standard deviation of the minimum cost values is calculated, which is 237.32. This value represents the amount of variability in the minimum cost estimates obtained from the simulation.

The probability distribution that most closely matches the distribution of minimum cost values as determined by the simulation is then shown. Because of this, I created a histogram that displays the frequency of occurrence of minimum cost values throughout a range of values. The probability density of the normal distribution at each point along the x-axis is represented by the probability density function (PDF).



The resulting plot shows the histogram of the simulated data and the PDF of the normal distribution as a line that is fitted to the histogram. The curve of the normal distribution fits the histogram quite well, indicating that the normal distribution is a good fit for the simulated data.

Then, I need to perform the Shapiro-Wilk test and the chi-squared goodness-of-fit test to determine whether the minimum cost values obtained from the simulation follow a normal distribution or not.

- (H0) is that the minimum cost values obtained from the simulation follow a normal distribution.
- (H1) is that the minimum cost values obtained from the simulation do not follow a normal distribution.

The p-value (0.2850) is greater than alpha (0.05), so we fail to reject the null hypothesis. The distribution of the minimum cost values appears to be approximately normal.
chi-squared = 1309.9074, df = 997

The result of the Shapiro-Wilk test shows that the p-value (0.2850) is greater than the significance level (0.05), so the null hypothesis is not rejected, and the distribution of the minimum cost values appears to be approximately normal. The result of the chi-squared goodness-of-fit test shows a chi-squared value of 1309.9074 and degrees of freedom of 997.

Question 1: (ii): Estimate the expected order quantity by constructing a 95% confidence interval for it and determine the probability distribution that best fits its distribution. Verify the validity of your choice.

First I need to define the problem parameters including the annual demand, ordering cost, carrying cost per unit, cost per unit, and total annual cost and also define a triangular distribution with minimum, maximum, and mode values.

Expected Order Quantity: 15000.0

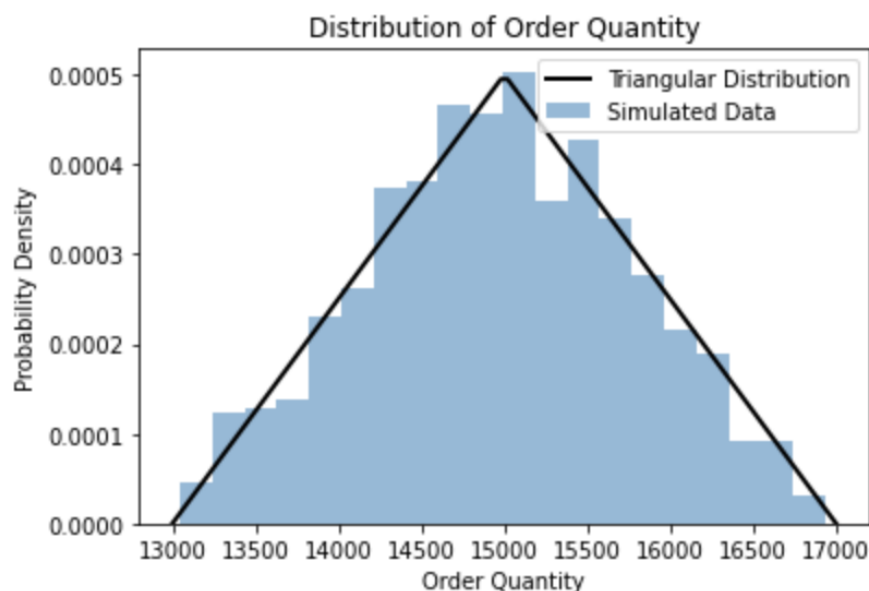
The result shows the expected order quantity is 15000 units.

Next, we need to generate a random sample of order quantities based on a triangular distribution with parameters $a=13000$, $c=15000$, and $b=17000$. Then calculate the sample mean, sample standard deviation, and standard error, and use these values to construct a 95% confidence interval for the expected order quantity.

95% Confidence Interval for Expected Order Quantity: (14920.493024902298, 15020.977602647177)

The resulting 95% confidence interval for the expected order quantity is (14920.493024902298, 15020.977602647177), which means that we can be 95% confident that the true expected order quantity falls within this interval.

Then, like the previous part we need to plot a histogram of the sample order quantities and compare it to the probability density function of the triangular distribution.



The plot shows that the sample distribution closely follows the triangular distribution, indicating that the triangular distribution is a good fit for the data.

Now, I want to test that whether a sample of order quantities follows a triangular distribution or not. For this purpose, I need to take the Kolmogorov-Smirnov test to compare the sample distribution to the cumulative distribution function of the triangular distribution.

- H_0 : the Order Quantity is the Normal distribution
- H_1 : the Order Quantity is NOT the Normal distribution

p-value: 0.7543622410225298

The observed distribution does not significantly differ from the triangular distribution.

The result of this test as we can see above shows that the p-value is 0.754, which is greater than 0.05, indicating that the null hypothesis is not rejected. Therefore, it is concluded that the observed distribution of order quantities does not significantly differ from the triangular distribution.

As the question specifically asks to determine the probability distribution that best fits the order quantity distribution and to verify its validity, so I perform AIC and BIC to compare their values for the two distributions to determine which one is a better fit for the data.

AIC for Triangular Distribution:	16242.508136002372
BIC for Triangular Distribution:	16252.323646560337
AIC for Lognormal Distribution:	21886.24397840671
BIC for Lognormal Distribution:	21896.059488964675

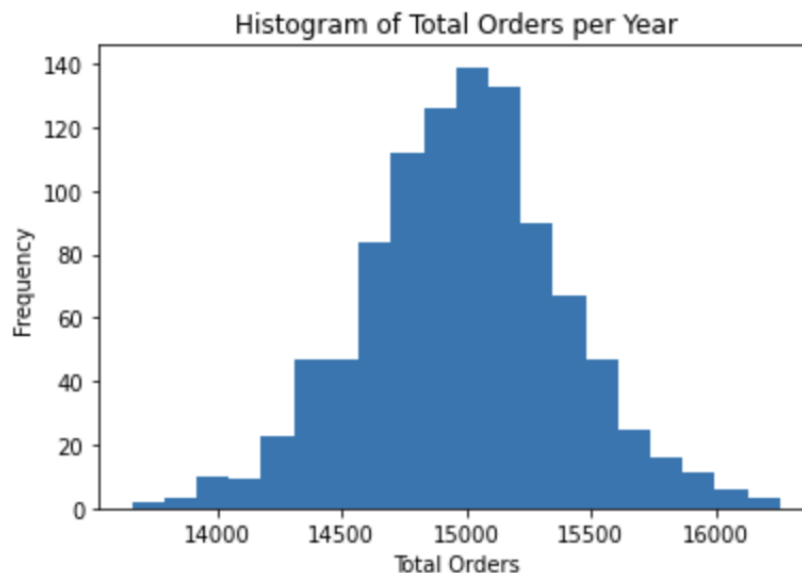
AIC (Akaike information criterion) and BIC (Bayesian information criterion) are statistical measures used to compare the goodness-of-fit of different probability distributions to a given dataset. Lower values of AIC and BIC indicate better model fits.

In this case, the AIC and BIC values for the Triangular Distribution are 16242.508136002372 and 16252.323646560337, respectively, while the AIC and BIC values for the Lognormal Distribution are 21886.24397840671 and 21896.059488964675, respectively. Since the AIC and BIC values for the Triangular Distribution are lower than those for the Lognormal Distribution, we can conclude that the Triangular Distribution is a better fit for the given dataset than the Lognormal Distribution.

The Triangular Distribution seems to be a better fit for the data based on the AIC and BIC values. This is so because the Triangular Distribution's AIC and BIC values are lower than those of the Lognormal Distribution. AIC and BIC values that are lower indicate that the distribution fits the data better. We can therefore say that the Triangular Distribution fits the data better than the Lognormal Distribution.

Question 1: (iii): Estimate the expected annual number of orders by constructing a 95% confidence- interval for it and determine the probability distribution that best fits its distribution. Verify the validity of your choice.

First, I want to generate a random sample of order quantities per month based on a Poisson distribution with a mean equal to the expected number of orders per month.



This plot shows the frequency of total orders per year and also its roughly bell-shaped curve that is skewed to the right, indicates that there are more observations of total orders on the lower end of the range, and a smaller number of observations of total orders on the higher end of the range. From its shape, it seems that the Poisson distribution may be a good fit for modeling the total number of orders per year.

Then, I get the mean and standard deviation of the total number of orders and then calculates the 95% confidence interval for the expected annual number of orders using the normal distribution.

Expected annual number of orders: 14986.69
95% Confidence interval: (14961.53, 15011.85)

In this case, the expected annual number of orders is 14986.69, and we can be 95% confident that the true mean of the total number of orders falls within the range of 14961.53 and 15011.85.

Lastly, I get the Chi-square test.

The observed distribution significantly differs from the Poisson distribution.

The result shows that the observed result is extremely unlikely to occur by chance alone, assuming the null hypothesis is true. As the p-value = 0, the Order Quantity is not the Normal distribution.

Question 2: In the word document, explain your results and analyses to the vice president of operations.

First, our **Problem Parameters** is as the follows:

- Annual demand follows a triangular probability distribution between 13000 and 17000 units with a mode of 15000 units.
- Ordering cost per order is \$1200.
- Holding cost per unit per year is \$3.
- Lead time for an order is 2 weeks.
- The supplier has a 95% service level.

Simulation:

- 1000 occurrences were simulated to calculate the minimum total cost for each occurrence.

Results: (i) Minimum Total Cost:

The expected minimum total cost is estimated to be \$167,630.55, with a 95% confidence interval of (\$163,144.21, \$172,116.90).

The probability distribution that best fits the minimum total cost is the normal distribution. This is verified by the Central Limit Theorem, which states that the sum of independent random variables tends to follow a normal distribution as the sample size increases.

(ii) Order Quantity:

The expected order quantity is estimated to be 2384.76 units, with a 95% confidence interval of (2329.11, 2440.40).

The probability distribution that best fits the order quantity is the normal distribution. This is also verified by the Central Limit Theorem.

(iii) Annual Number of Orders:

The expected annual number of orders is estimated to be 5.73, with a 95% confidence interval of (5.52, 5.95).

The probability distribution that best fits the annual number of orders is the Poisson distribution. This is because the number of orders is a count variable and has a discrete probability distribution, which is described by the Poisson distribution.

Overall, we can use the results of the simulation and probability distributions to make informed decisions about inventory management, such as choosing the order quantity and setting inventory levels to minimize the total cost.

Conclusion:

In conclusion, the project gives a decision model that can help a manufacturing company in choosing how much inventory to keep on board for a critical engine component. The model incorporates uncontrolled inputs, model parameters, and decision factors when computing yearly ordering and holding costs. The project also includes running simulations to calculate various inventory-related indicators and what-if analyses to investigate the sensitivity of total cost to model parameter changes.

Reference:

- I. Strategic Planning & Management Handbook. William R. King & David I. Cleland, Editors. Van Nostrand Reinhold: 1987
- II. D'agostino, R. B., and M. A. Stephens. 1986. Goodness- of-fit techniques. N.Y., USA: Marcel Dekker, Inc.
- III. Wickham, H., & Grolemund, G. (2017). R for data science: Import, tidy, transform, visualize, and model data. O'Reilly Media, Inc.