Data Visualization

Density estimation

June 12th, 2023

New dataset - Stephen Curry's shots

Created dataset of shot attempts by the Stephen Curry in 2021-2022 season using `nbastatR`

```
library(tidyverse)
curry_shots <-
  read_csv("https://shorturl.at/xFI18")
head(curry_shots)
```

```
## # A tibble: 6 × 8
##   shot_x shot_y shot_distance is_shot_made period fg_type        shot_…¹ shot_…²
##    <dbl>  <dbl>         <dbl> <lgl>         <dbl> <chr>          <chr>   <chr>
## 1   -109    260            28 FALSE             1 3PT Field Goal Above … Pullup…
## 2     48    257            26 FALSE             1 3PT Field Goal Above … Runnin…
## 3   -165    189            25 TRUE              1 3PT Field Goal Above … Jump S…
## 4    -13     12             1 FALSE             1 2PT Field Goal Restri… Drivin…
## 5    -15     22             2 FALSE             1 2PT Field Goal Restri… Layup …
## 6     18     16             2 FALSE             1 2PT Field Goal Restri… Drivin…
## # … with abbreviated variable names ¹shot_zone, ²shot_type
```

- each row / observation is a shot attempt by Curry in the 2021 season
- **Categorical** / qualitative variables: `is_shot_made`, `fg_type`, `shot_zone`, `shot_type`
- **Continuous** / quantitative variables: `shot_x`, `shot_y`, `shot_distance`

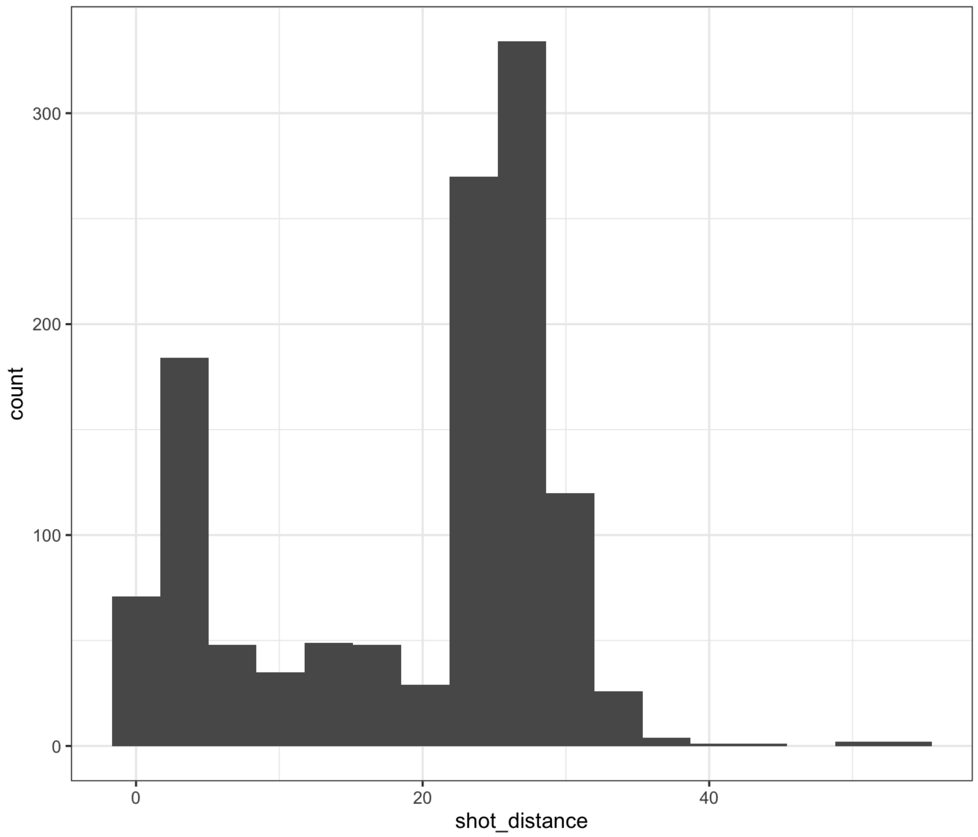
Back to histograms...

```
fd_bw <- 2 * IQR(curry_shots$shot_distance) /
curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_histogram(binwidth = fd_bw) +
  theme_bw()
```

- Split observed data into **bins**
- **Count** number of observations in each bin

Need to choose the number of bins, adjust with:

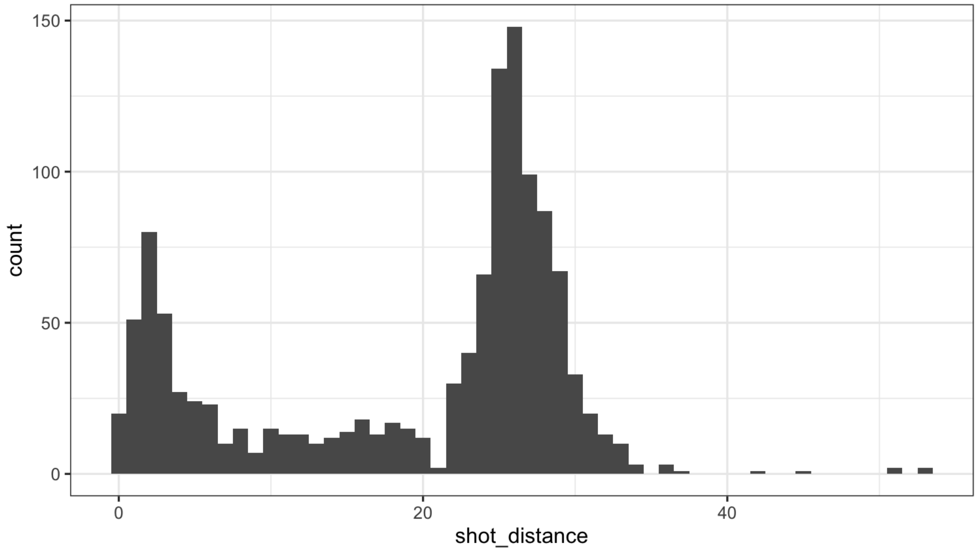
- `bins` - number of bins (default is 30)
- `binwidth` - literally the width of bins (overrides `bins`), various rules of thumb
  - e.g., see `fd_bw` for Freedman–Diaconis rule
- `breaks` - vector of bin boundaries (overrides both `bins` and `binwidth`)



Adjusting the bin width

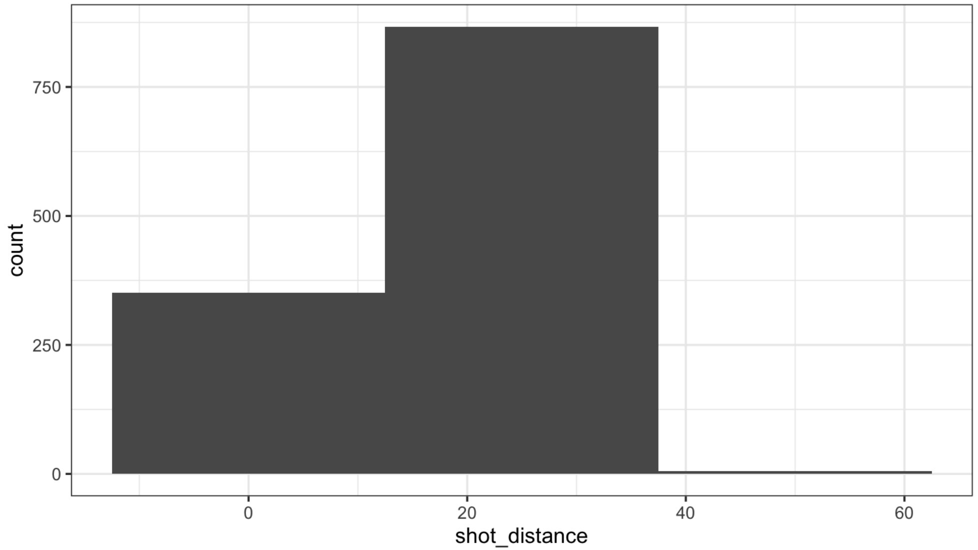
Small `binwidth` → *"undersmooth"* / spiky

```
curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_histogram(binwidth = 1) +
  theme_bw()
```



Large `binwidth` → *"oversmooth"* / flat

```
curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_histogram(binwidth = 25) +
  theme_bw()
```



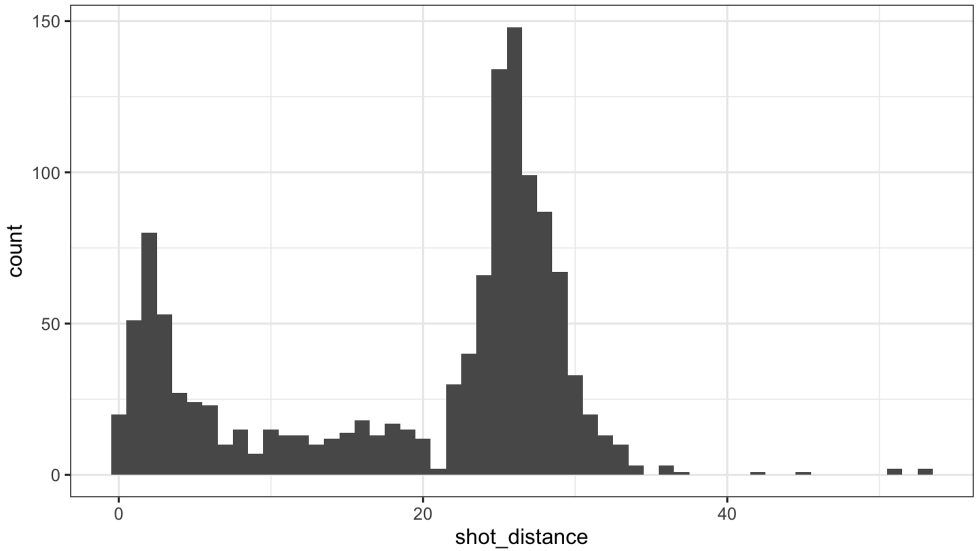
Try several approaches, the `R` / `ggplot2` default is NOT guaranteed to be an optimal choice

A subtle point about the histogram code...

By default the bins are centered on the integers...

- left-closed, right-open intervals
- starting at -0.5 to 0.5, 0.5 to 1.5, ...

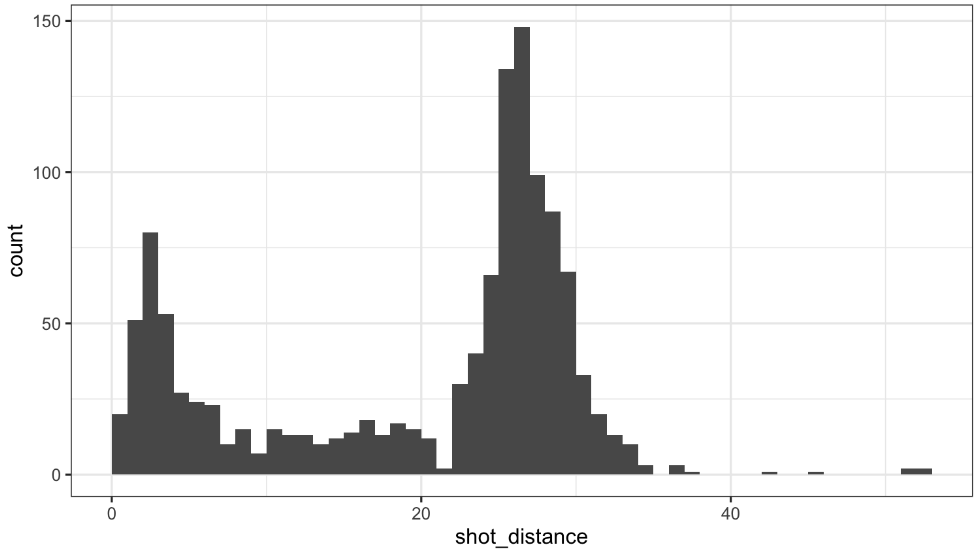
```
curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_histogram(binwidth = 1) +
  theme_bw()
```



Specify center of one bin (e.g. 0.5)

- Reminder to use `closed = "left"`...

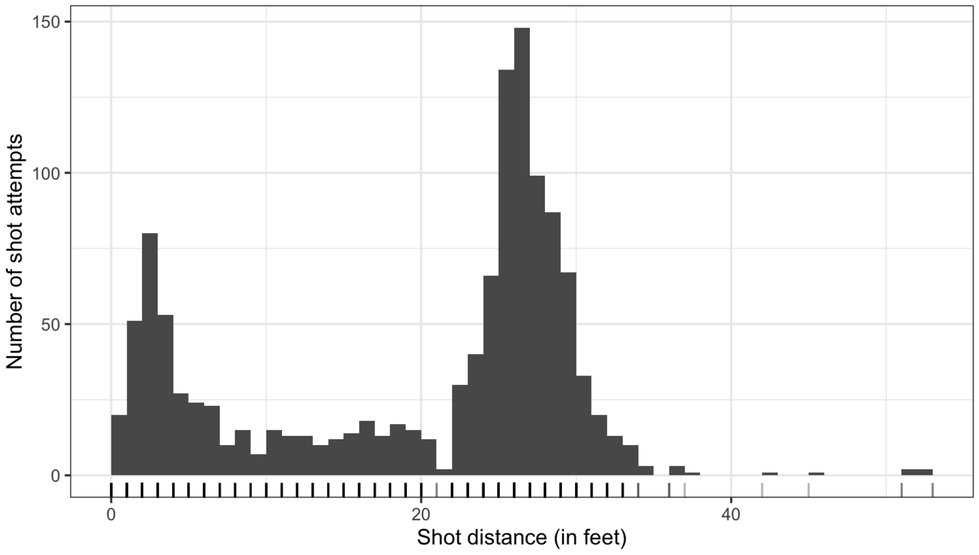
```
curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_histogram(binwidth = 1, center = 0.5,
                 closed = "left") +
  theme_bw()
```

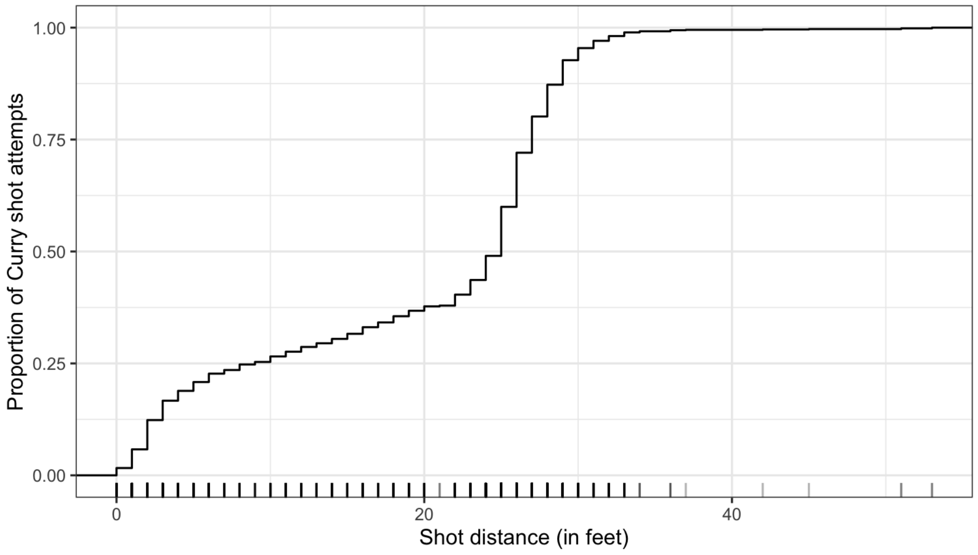


How do histograms relate to the PDF and CDF?

Remember: we use the **probability density function (PDF)** to provide a **relative likelihood**

- PDF is the **derivative** of the cumulative distribution function (CDF)
- Histograms approximate the PDF with bins, and **points are equally likely within a bin**





What can say about the relative likelihood of data we have not observed?

- we want **non-zero density** between our observations, e.g., just beyond 20 feet

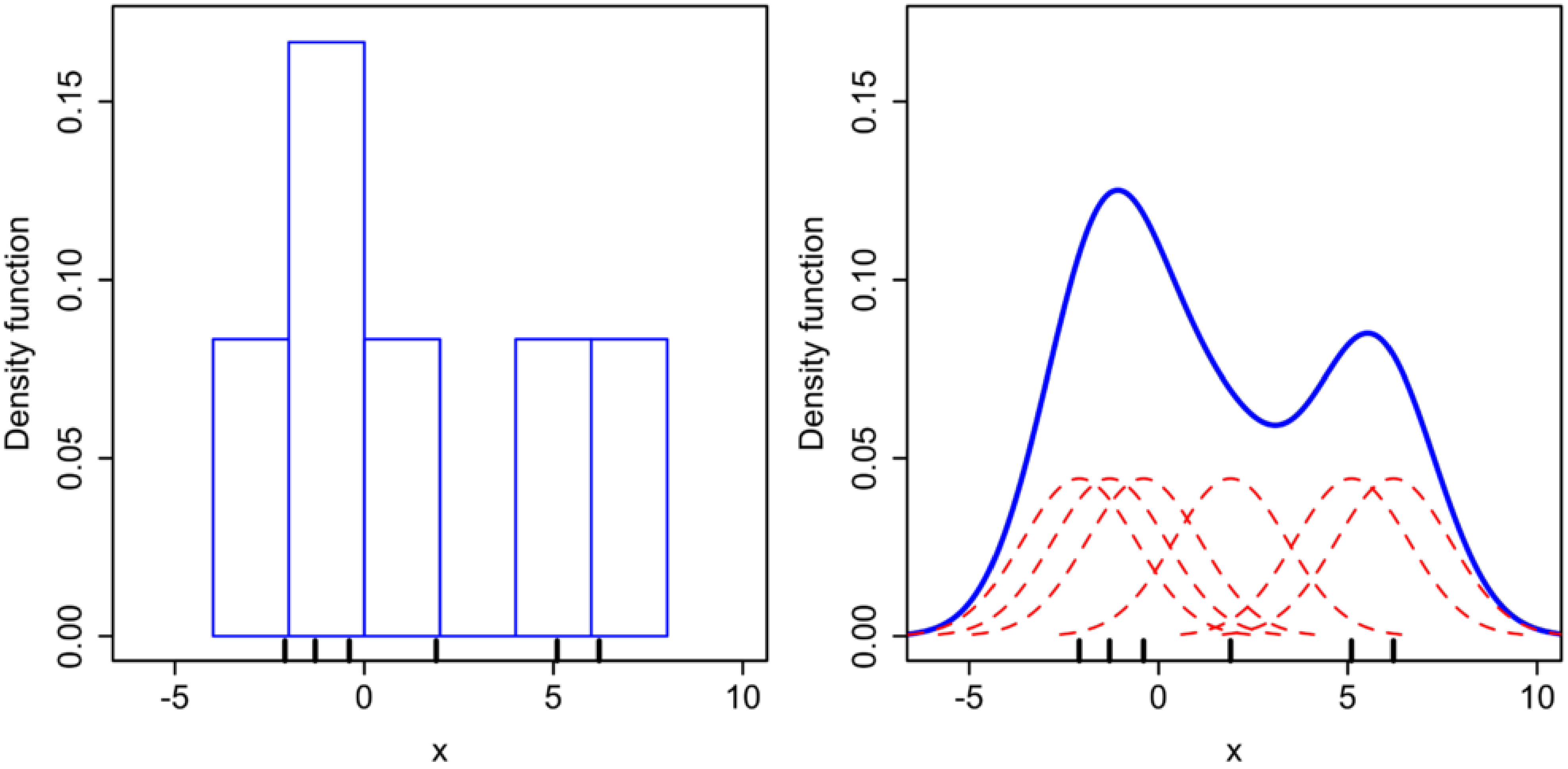
Kernel density estimation

Goal: estimate the PDF $f(x)$ for all possible values (assuming it is continuous / smooth)

$$\text{Kernel density estimate: } \hat{f}(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h}K_h(x - x_i)$$

- $n =$ sample size, $x =$ new point to estimate $f(x)$ (does NOT have to be in dataset!)
- $h =$ **bandwidth**, analogous to histogram bin width, ensures $\hat{f}(x)$ integrates to 1
- $x_i = i$th observation in dataset
- $K_h(x - x_i)$ is the **Kernel** function, creates **weight** given distance of ith observation from new point
    - as $|x - x_i| \to \infty$ then $K_h(x - x_i) \to 0$, i.e. further apart ith row is from x, smaller the weight
    - as **bandwidth** $h \uparrow$ weights are more evenly spread out (as $h \downarrow$ more concentrated around x)
    - typically use **Gaussian** / Normal kernel: $\propto e^{-(x-x_i)^2/2h^2}$
    - $K_h(x - x_i)$ is large when x_i is close to x

Wikipedia example

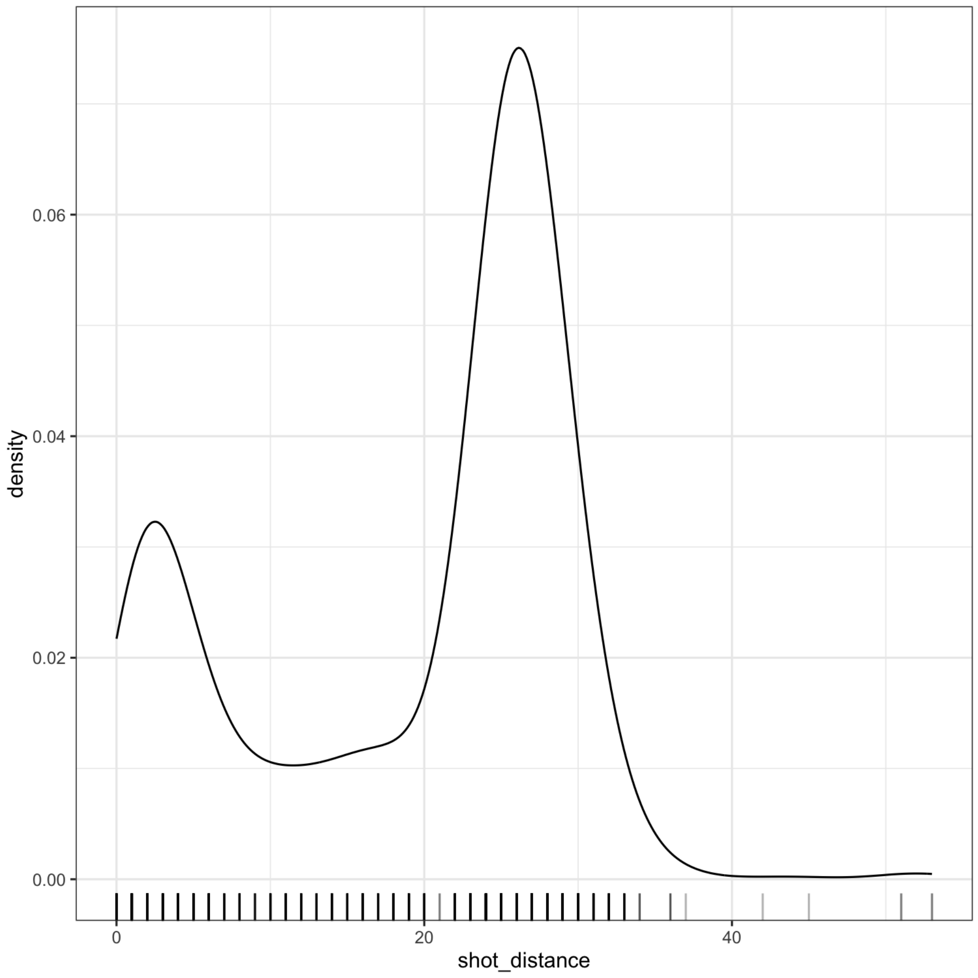


How do we compute and display the density estimate?

- We make **kernel density estimates** with `geom_density()`

```
curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_density() +
  geom_rug(alpha = 0.3) +
  theme_bw()
```

- **Pros**:
  - Displays full shape of distribution
  - Can easily layer
  - Add categorical variable with color
- **Cons**:
  - Need to pick bandwidth and kernel...

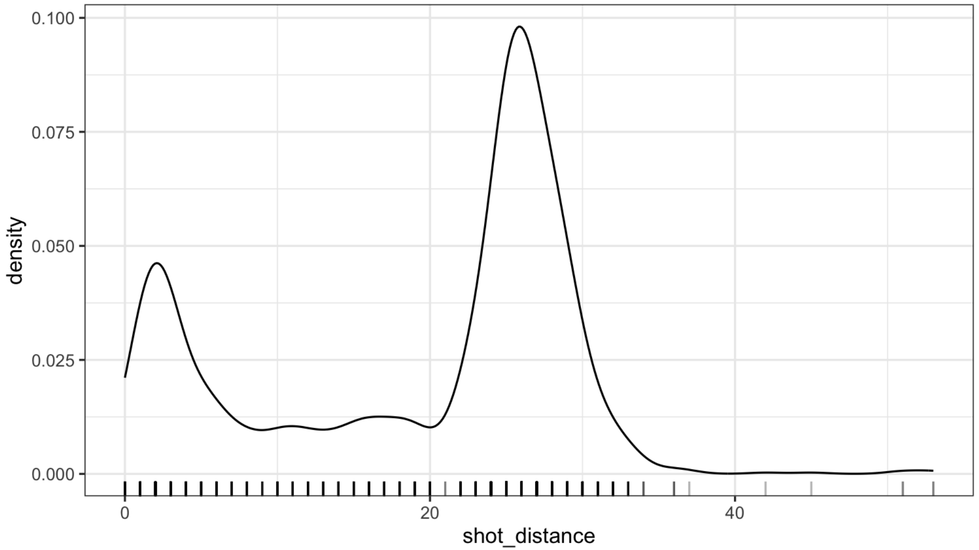


What about the bandwidth? See Chapter 14 for more...

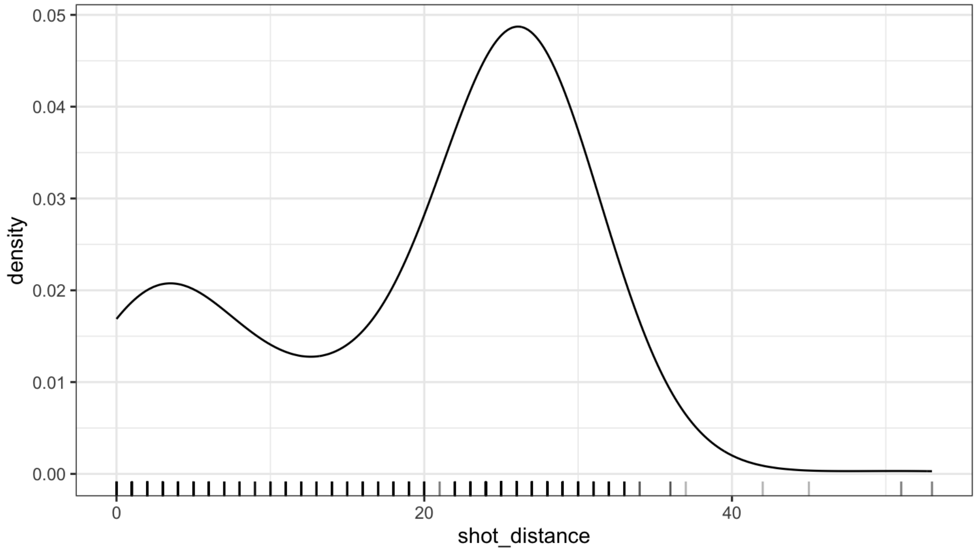
Use **Gaussian reference rule** (*rule-of-thumb*) $\approx 1.06 \cdot \sigma \cdot n^{-1/5}$, where σ is the observed standard deviation

Modify the bandwidth using the `adjust` argument - **value to multiply default bandwidth by**

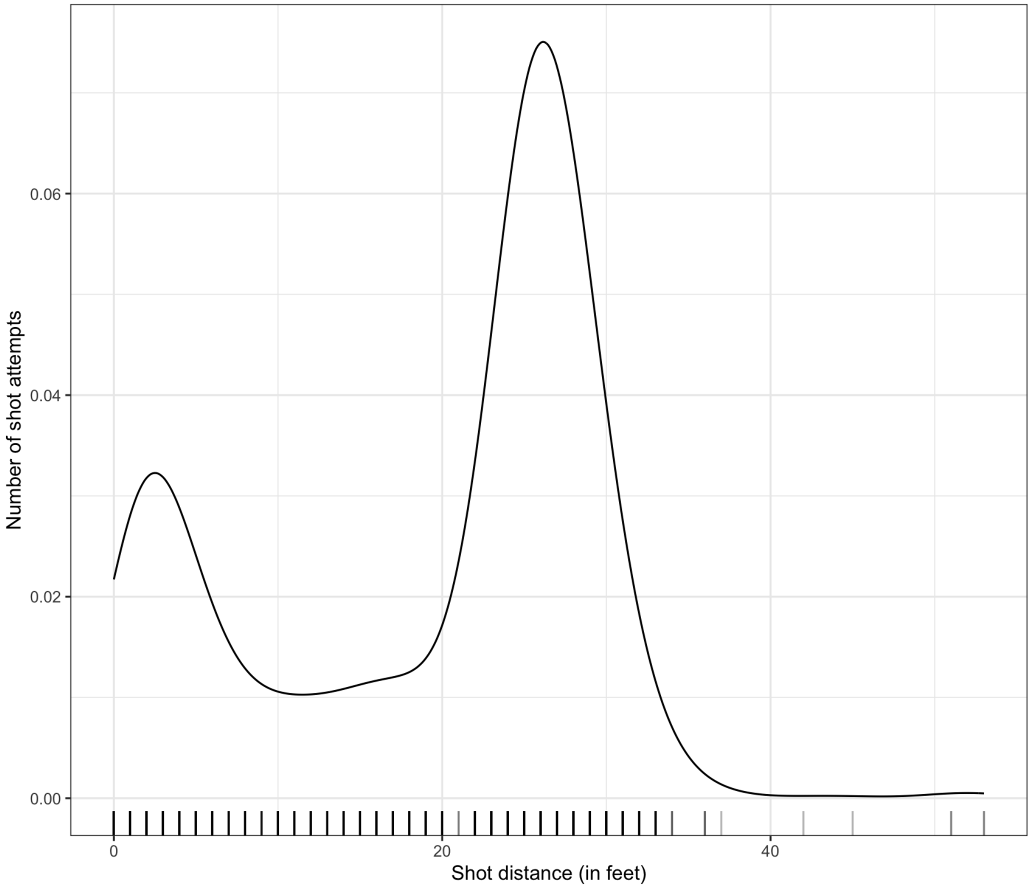
```r
curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_density(adjust = 0.5) +
  geom_rug(alpha = 0.3) + theme_bw()
```

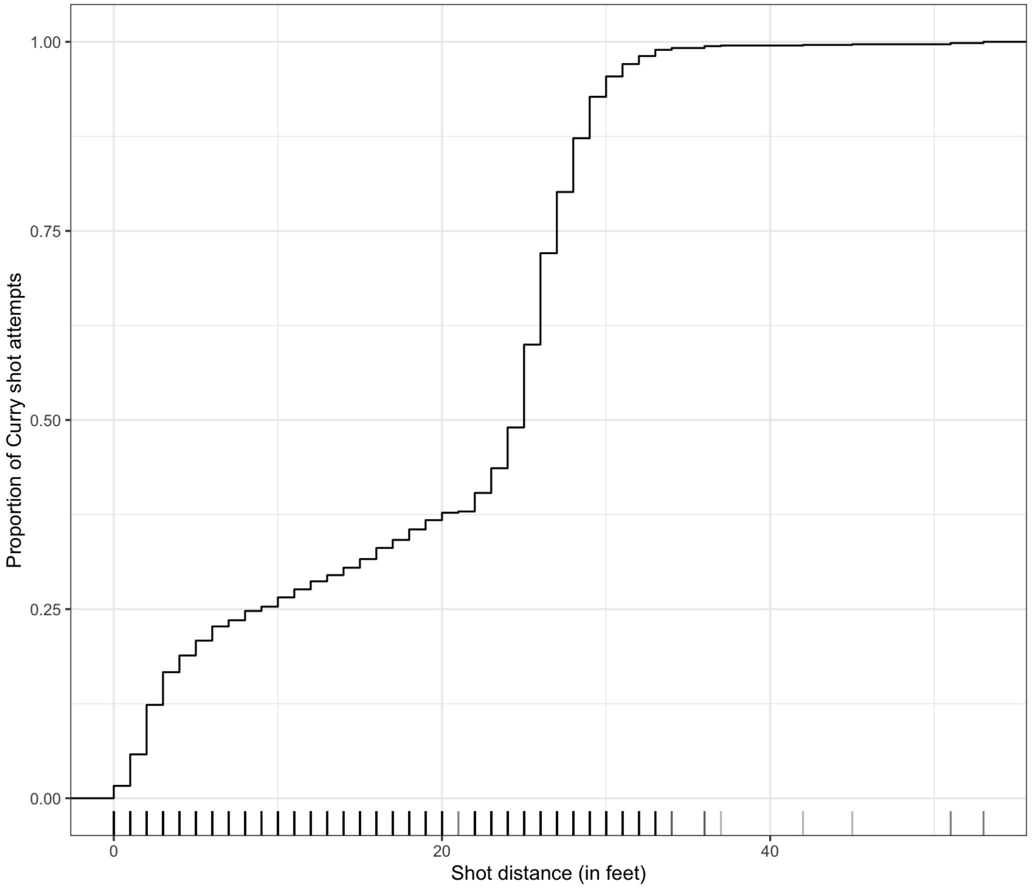



```r
curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_density(adjust = 2) +
  geom_rug(alpha = 0.3) + theme_bw()
```



Use density curves and ECDFs together



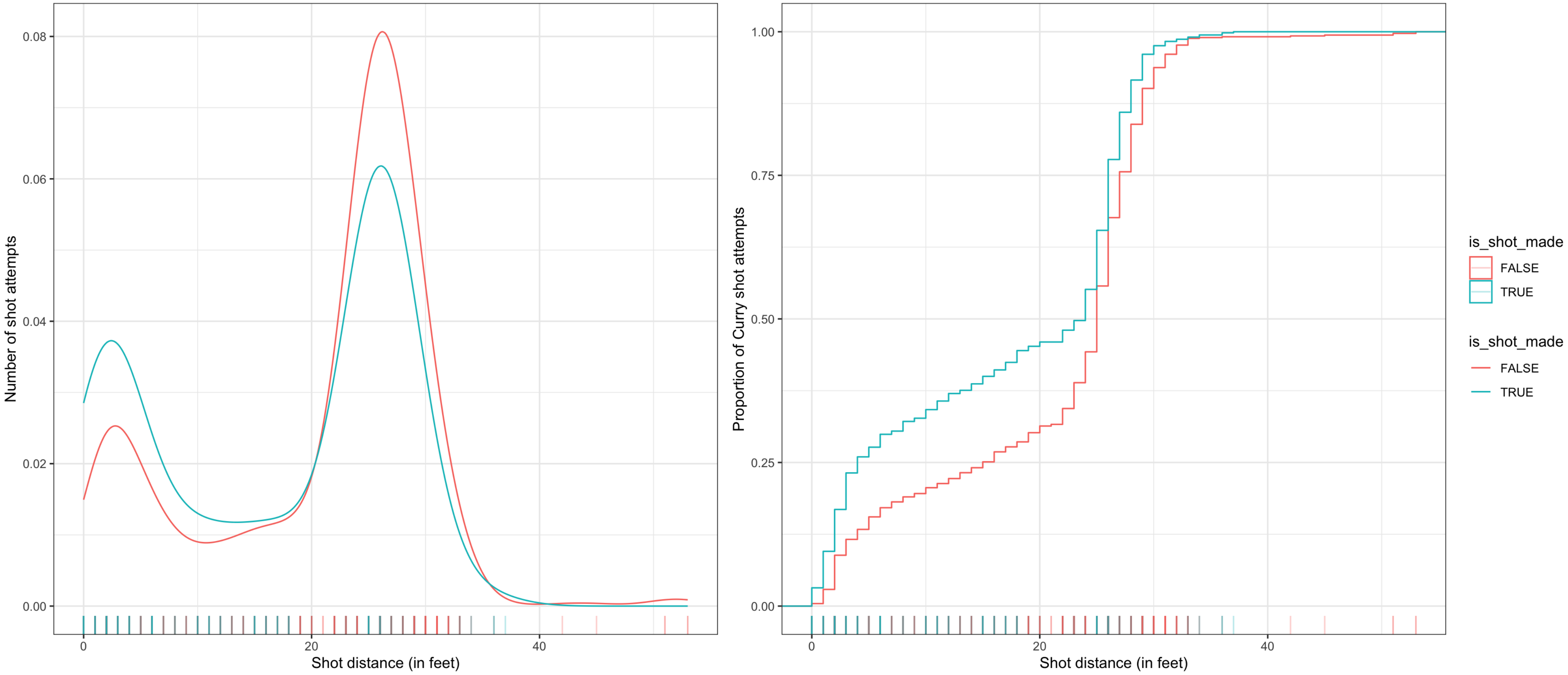


Code interlude: easy way to arrange multiple figures

Use the new `patchwork` package to easily arrange your plots (see also `cowplot`)

```r
library(patchwork)
curry_shot_dens <- curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  geom_density() +
  geom_rug(alpha = 0.3) +
  theme_bw() +
  labs(x = "Shot distance (in feet)",
       y = "Number of shot attempts")
curry_shot_ecdf <- curry_shots %>%
  ggplot(aes(x = shot_distance)) +
  stat_ecdf() +
  geom_rug(alpha = 0.3) +
  theme_bw() +
  labs(x = "Shot distance (in feet)",
       y = "Proportion of Curry shot attempts")
curry_shot_dens + curry_shot_ecdf
```

Use density curves and ECDFs together



Another code interlude: collect the legends

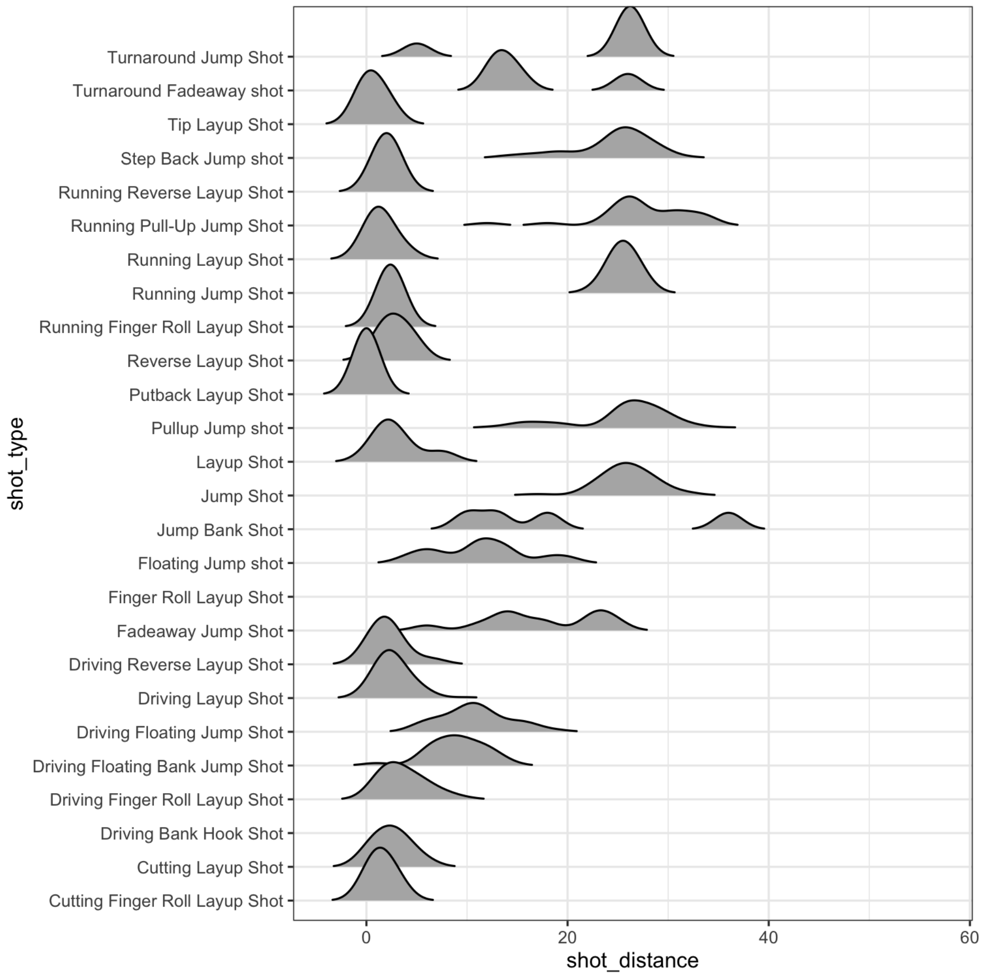
```
curry_shot_dens_made <- curry_shots %>%
  ggplot(aes(x = shot_distance,
             color = is_shot_made)) +
  geom_density() +
  geom_rug(alpha = 0.3) +
  theme_bw() +
  labs(x = "Shot distance (in feet)",
       y = "Number of shot attempts")
curry_shot_ecdf_made <- curry_shots %>%
  ggplot(aes(x = shot_distance,
             color = is_shot_made)) +
  stat_ecdf() +
  geom_rug(alpha = 0.3) +
  theme_bw() +
  labs(x = "Shot distance (in feet)",
       y = "Proportion of Curry shot attempts")
curry_shot_dens_made + curry_shot_ecdf_made + plot_layout(guides = 'collect')
```

Alternative to violins - ridge plots

- Check out the `ggridges` package for a variety of customization options

```
library(ggridges)
curry_shots %>%
  ggplot(aes(x = shot_distance,
             y = shot_type)) +
  geom_density_ridges(rel_min_height = 0.01)
  theme_bw()
```

- Useful to display conditional distributions across many levels

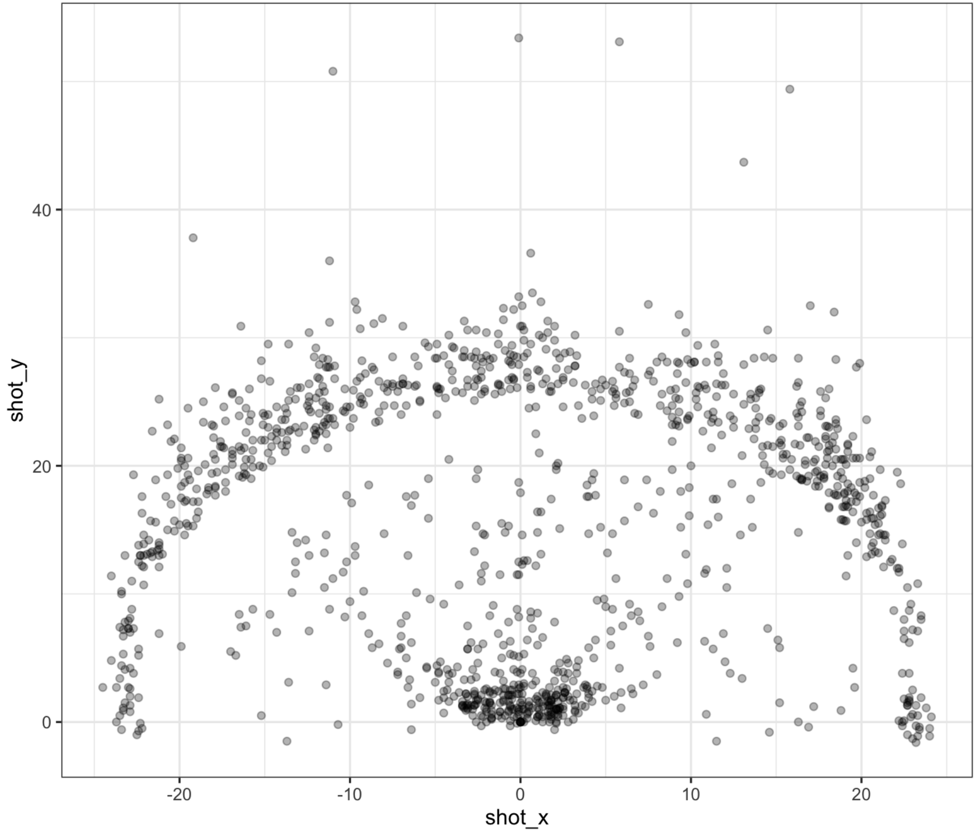


What about for 2D? (two continuous variables)

We can visualize all of the shot locations: (`shot_x`, `shot_y`)

```
curry_shots %>%
  # Modify the shot coordinates
  mutate(shot_x = -shot_x / 10,
         shot_y = shot_y / 10) %>%
  ggplot(aes(x = shot_x, y = shot_y)) +
  geom_point(alpha = 0.3) +
  theme_bw()
```

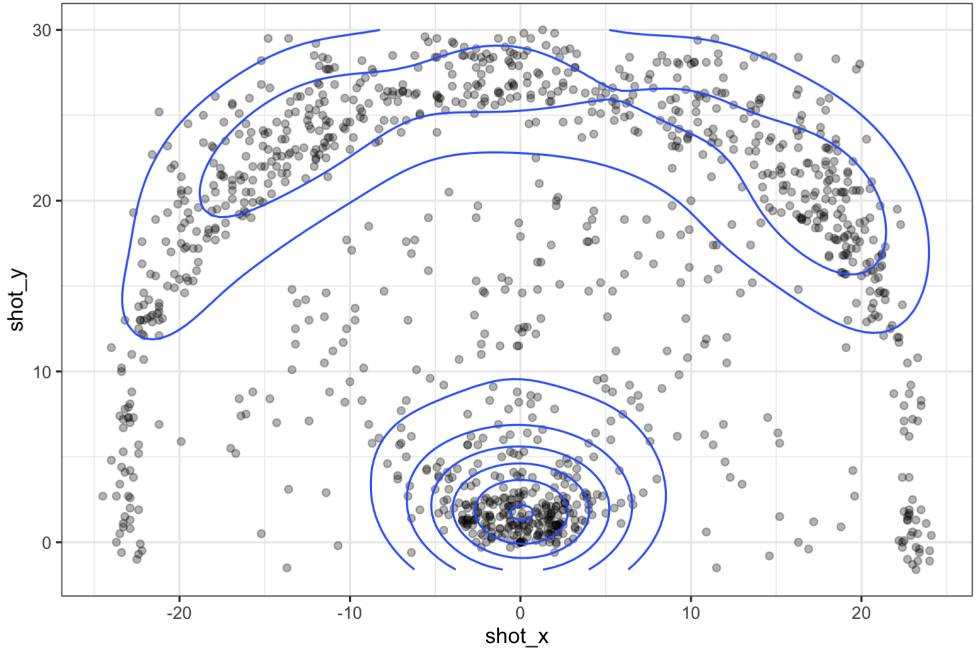
- Adjust transparency with `alpha` for overlapping points



Create contours of 2D kernel density estimate (KDE)

- We make 2D KDE **contour** plots using `geom_density2d()`

```
curry_shots %>%
  # Modify the shot coordinates
  mutate(shot_x = -shot_x / 10,
         shot_y = shot_y / 10) %>%
  filter(shot_y <= 30) %>%
  ggplot(aes(x = shot_x, y = shot_y)) +
  geom_point(alpha = 0.3) +
  geom_density2d() +
  theme_bw() + theme(legend.position = "botto
  coord_fixed()
```



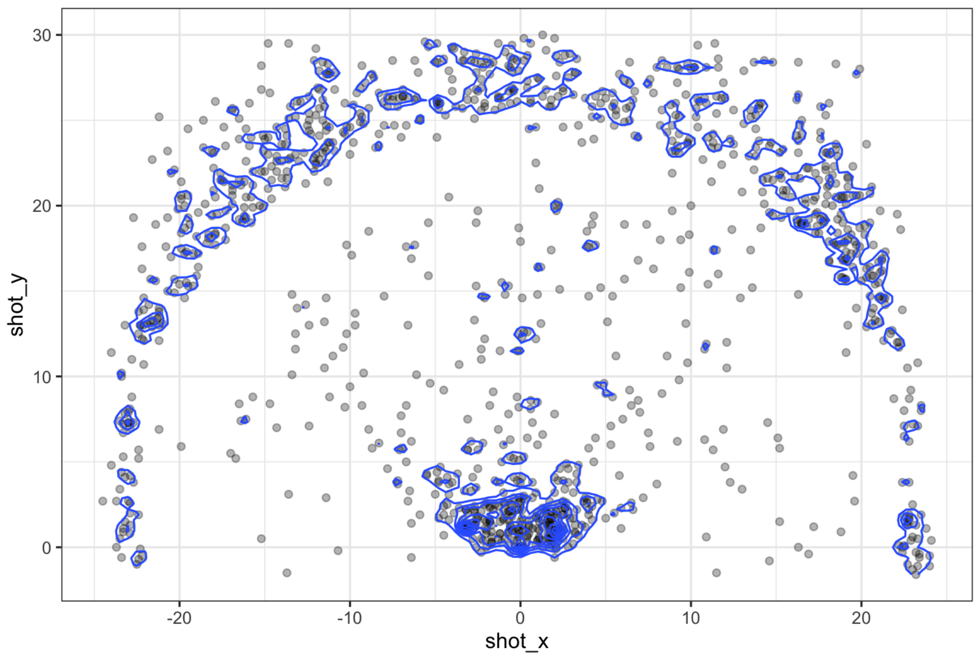
- Extend KDE for joint density estimates in 2D (see section 14.4.2 for details)

- `coord_fixed()` forced a fixed ratio

Create contours of 2D kernel density estimate (KDE)

- We make 2D KDE **contour** plots using `geom_density2d()`

```
curry_shots %>%
  # Modify the shot coordinates
  mutate(shot_x = -shot_x / 10,
         shot_y = shot_y / 10) %>%
  # Remove the outlier shots:
  filter(shot_y <= 30) %>%
  ggplot(aes(x = shot_x, y = shot_y)) +
  geom_point(alpha = 0.3) +
  geom_density2d(adjust = 0.1) +
  theme_bw() +
  theme(legend.position = "bottom") +
  coord_fixed()
```

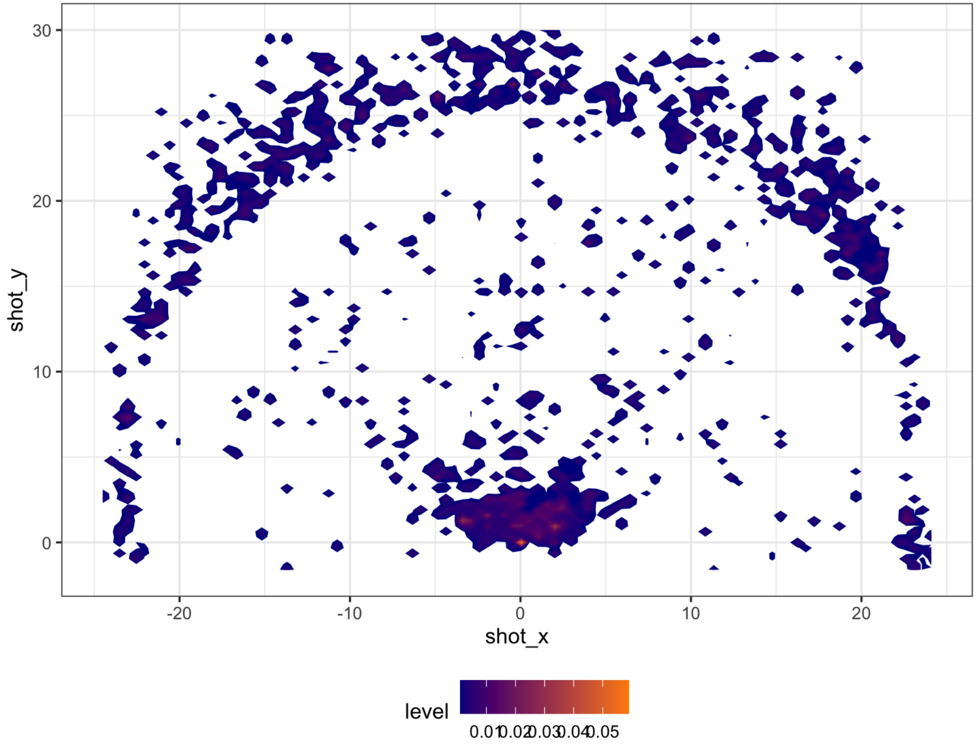


- Can use `adjust` to modify the multivariate bandwidth

Contours are difficult... let's make a heatmap instead

- We make 2D KDE **heatmap** plots using `stat_density_2d()` and the `..` or `after_stat()` function

```
curry_shots %>%
  mutate(shot_x = -shot_x / 10,
         shot_y = shot_y / 10) %>%
  filter(shot_y <= 30) %>%
  ggplot(aes(x = shot_x, y = shot_y)) +
  stat_density2d(h = 0.5, bins = 60,
                 aes(fill = after_stat(level)
                 geom = "polygon") +
  scale_fill_gradient(low = "darkblue",
                      high = "darkorange") +
  theme_bw() + theme(legend.position = "botto
  coord_fixed()
```

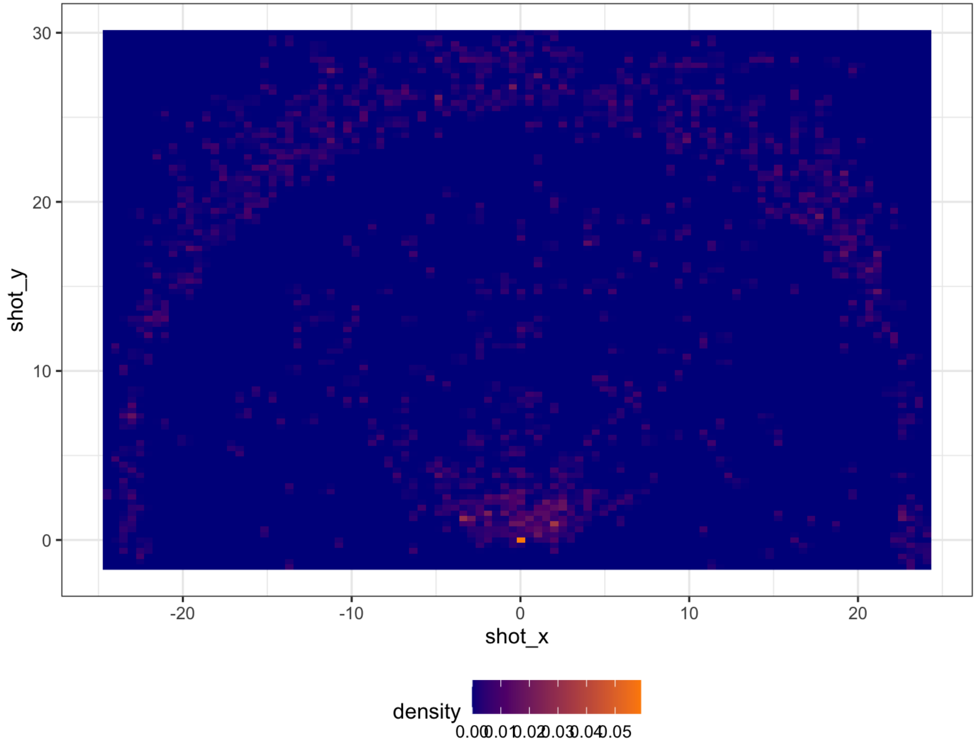


Multivariate density estimation can be difficult

Turn off contours and use tiles instead

- We make 2D KDE **heatmap** plots using `stat_density_2d()` and the .. or `after_stat()` function

```
curry_shots %>%
  mutate(shot_x = -shot_x / 10,
         shot_y = shot_y / 10) %>%
  filter(shot_y <= 30) %>%
  ggplot(aes(x = shot_x, y = shot_y)) +
  stat_density2d(h = 0.5, bins = 60,
                 contour = FALSE,
                 aes(fill = after_stat(densit
                 geom = "raster") +
  scale_fill_gradient(low = "darkblue",
                      high = "darkorange") +
  theme_bw() + theme(legend.position = "botto
  coord_fixed()
```

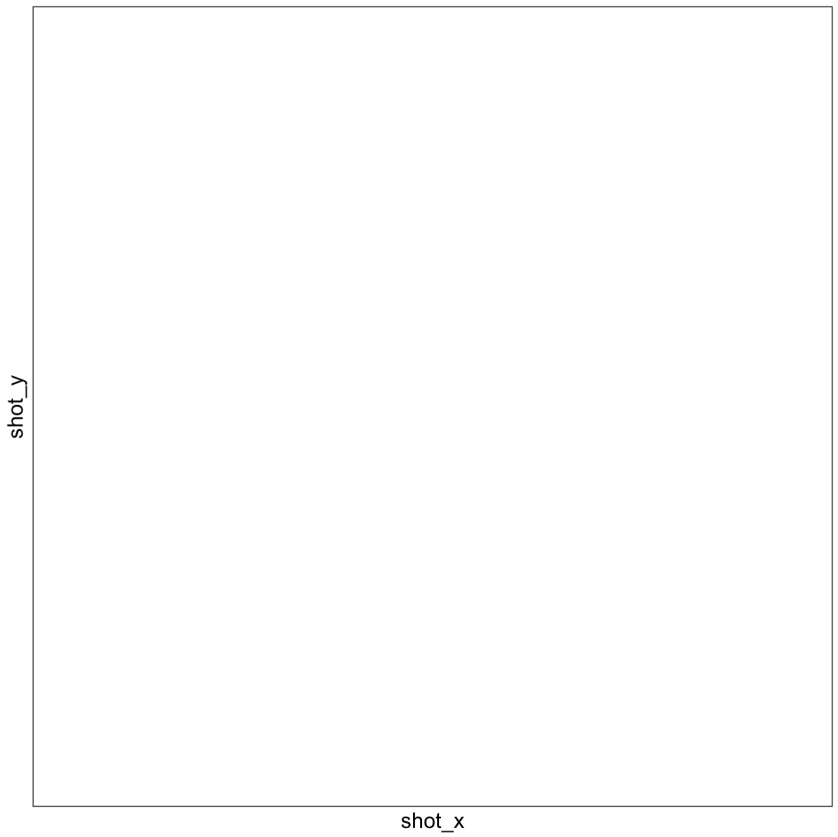


Best alternative? Hexagonal binning

- We make **hexagonal heatmap** plots using `geom_hex()`
- Need to have the `hexbin` package installed

```
curry_shots %>%
  mutate(shot_x = -shot_x / 10,
         shot_y = shot_y / 10) %>%
  filter(shot_y <= 30) %>%
  ggplot(aes(x = shot_x, y = shot_y)) +
  geom_hex(binwidth = c(1, 1)) +
  scale_fill_gradient(low = "darkblue",
                      high = "darkorange") +
  theme_bw() + theme(legend.position = "botto
  coord_fixed()
```

- Can specify `binwidth` in both directions
- Avoids limitations from smoothing



What about his shooting efficiency?

- Can compute a function of another variable inside hexagons with `stat_summary_hex()`
- Check out BallR for code examples to make shot charts and drawing courts

```
curry_shots %>%
  mutate(shot_x = -shot_x / 10,
         shot_y = shot_y / 10) %>%
  filter(shot_y <= 30) %>%
  ggplot(aes(x = shot_x, y = shot_y,
             z = is_shot_made,
             group = -1)) +
  stat_summary_hex(binwidth = c(2, 2),
                   color = "black",
                   fun = mean) +
  scale_fill_gradient(low = "darkblue",
                      high = "darkorange") +
  theme_bw() + theme(legend.position = "botto
  coord_fixed()
```