

Understanding and Correcting Your Loan Payoff Calculations

This document clarifies and corrects the financial formulas for calculating loan payoff amounts, based on the image you provided.

Key Concepts and Provided Variables

You've correctly identified the core components:

- **Initial Balance (IB):** The total amount (principal + interest) originally expected to be paid over the loan's life.
- **Monthly Payment (EMI):** The fixed amount paid each month.
- **Balance Amount (BA):** The total remaining amount (principal + interest) expected to be paid over the remaining life of the loan.

The Financial Formulas (Correct)

The two main formulas you outlined are standard financial equations used in loan amortization. These are correct:

1. **Original Principal (PV - Present Value):** This calculates the actual initial loan amount borrowed, before any interest accrues.

$$PV = EMI \cdot \left[\frac{1 - (1 + r)^{-n}}{r} \right]$$

2. **Payoff Amount (Remaining Principal):** This calculates the outstanding principal balance after a certain number of payments have been made. This is the amount you'd need to pay to clear the loan, *excluding* any interest accrued since the last payment and *before* any new interest accrues.

$$Payoff = PV \cdot \left[\frac{(1 + r)^n - (1 + r)^m}{(1 + r)^n - 1} \right]$$

Critical Correction: Calculating 'n' and 'm'

The most important correction relates to how n (total tenure) and m (payments made) are derived. Your formulas $n = IB / EMI$ and $m = n - BA / EMI$ are **incorrect** because they perform a simple division of total/remaining *balance with interest* by the monthly payment. This does not accurately give the number of months due to the compounding nature of interest.

Instead, n and m must be determined based on the **loan's original terms** and the **number of payments already successfully applied**.

Correct Variable Definitions for Calculation:

- **PV:** The **Original Principal** loan amount (the actual money borrowed).
- **EMI:** The fixed **Equated Monthly Installment** (your monthly payment).
- **r:** The **monthly interest rate** (annual rate / 12).
 - *Example:* For a 12% Annual Interest Rate: $r = 0.12 / 12 = 0.01$ (or 1%).
- **n:** The **total number of payments** for the entire loan term (the full tenure in months, as agreed upon initially). This value should be a fixed parameter of the loan.
- **m:** The **number of payments already successfully made**. This value changes over time and needs to be accurately tracked.

Step-by-Step Correct Calculation Example

Let's use the data you provided and assume a 12% annual interest rate.

Given Data from ESS/Loan Offer:

- **Monthly Payment (EMI):** TSH 177,697
- **Annual Interest Rate:** 12% (so, $r = 0.01$ monthly)
- **Initial Balance (IB):** TSH 6,397,076.07 (This is the sum of all payments over the full term, including interest)
- **Remaining Balance (BA):** TSH 4,264,712.07 (This is the sum of remaining payments, including future interest)

Critical Missing Information (Must be Known or Derived):

- **Total Tenure (n):** This is the total number of months the loan was originally set for. We can derive this from IB, EMI, and r .

Step 1: Determine the Total Tenure (n) and Original Principal (PV)

Since **IB** (Total Balance with Interest) is given as TSH 6,397,076.07 and **EMI** is TSH 177,697, we can infer **n** from these.

- If we assume **IB** is simply $EMI \cdot n$, then $n = 6,397,076.07 / 177,697 = 36 \text{ months}$.
- Now, we use this **n** and your **EMI** to find the true **PV** (Original Principal).

$$PV = 177,697 \cdot \left[\frac{1 - (1 + 0.01)^{-36}}{0.01} \right]$$

$$PV = 177,697 \cdot \left[\frac{1 - 0.698924}{0.01} \right]$$

$$PV = 177,697 \cdot \left[\frac{0.301076}{0.01} \right]$$

$$PV = 177,697 \cdot 30.1076$$

Original Principal (PV) \approx TSH 5,350,000.00

Step 2: Determine the Number of Payments Made (m)

Now, we need to find how many payments have already been made. We cannot simply divide $(IB - BA) / EMI$ because IB and BA include interest. Instead, we use the Remaining Balance (BA) to infer m .

The BA (TSH 4,264,712.07) represents the sum of the *remaining EMI payments*.

- **Remaining Payments:** $4,264,712.07 / 177,697 = 24$ months (approx).
- If $n = 36$ and 24 payments are remaining, then $m = n - \text{remaining_payments}$. $m = 36 - 24 = 12$ payments made.

Step 3: Calculate the Payoff Amount (Outstanding Principal)

Now we have all the correct variables to use your second formula:

- **PV:** TSH 5,350,000.00
- **r:** 0.01
- **n:** 36
- **m:** 12

$$Payoff = 5,350,000 \cdot \left[\frac{(1 + 0.01)^{36} - (1 + 0.01)^{12}}{(1 + 0.01)^{36} - 1} \right]$$

$$Payoff = 5,350,000 \cdot \left[\frac{1.43076876 - 1.12682503}{1.43076876 - 1} \right]$$

$$Payoff = 5,350,000 \cdot \left[\frac{0.30394373}{0.43076876} \right]$$

$$Payoff = 5,350,000 \cdot 0.705574$$

Payoff Amount (Outstanding Principal) \approx TSH 3,775,000.00

Final Payoff Amount to ESS

The TSH 3,775,000.00 calculated above is the *outstanding principal*. To get the **Total Payoff Amount** that you send to the ESS, you must add any interest that has accrued since the last payment date.

Final Payoff Amount = Outstanding Principal + Accrued Interest

- **Accrued Interest:** Calculate this based on the Outstanding Principal, the monthly interest rate (r), and the number of days since the last payment.

This detailed breakdown, using the correct financial principles, should help you accurately implement the payoff calculation in your system.