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B.N.M. Institute of Technology

An Autonomous Institution under VTU

Semester End Assessment, October 2023

Fourth Semester BE, 2021-22 Scheme

Statistics, Probability and Graph Theory-21MAI141

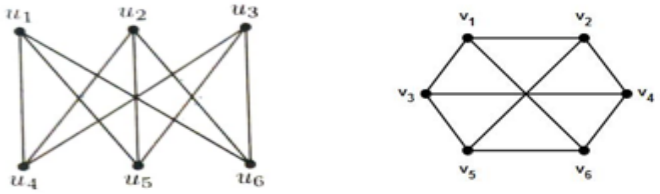
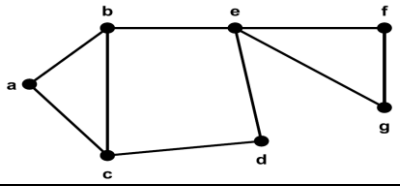
Duration: 3 Hours **Max. Marks: 100**

Note: 1. Answer one full question from each Module (5Q x 20M=100 Marks)

Q. No	Questions	Marks	CO	PO	Cognitive Level
	Module 1				
1 (a)	Fit a parabola of second degree $y = a + bx + cx^2$ for the following data. x: 0 1 2 3 4 5 y: 1 3 7 13 21 31	7	1	1,2	3
1 (b)	Calculate the coefficient of correlation and obtain the lines of regression for the following data. x: 1 2 3 4 5 6 7 8 9 y: 9 8 10 12 11 13 14 16 15 Also obtain an estimate for y which corresponds to $x = 6.2$.	7	1	1,2	3
1 (c)	Calculate the first four moments about the point $a = 4$ for the following data x: 0 1 2 3 4 5 6 7 8 y: 1 8 28 56 70 56 28 8 1	6	1	1,2	3
	OR				
2 (a)	Find a curve of best fit of the form $y = ax^b$ to the following data. x: 1 2 3 4 5 y: 0.5 2 4.5 8 12.5	7	1	1,2	3
2 (b)	In a partially destroyed laboratory record of correlation data, the following results only are available:Variance of x is 9 and the regression equations are $4x - 5y + 33 = 0,\ 20x - 9y = 107$. Find (i) the mean values of x and y, (ii) standard deviation of y, and (iii) the coefficient of correlation between x and y.	7	1	1,2	3
2 (c)	The first four moments about the point 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the four moments about mean. Also evaluate β_1 and β_2.	6	1	1,2	3
	Module 2				

3 (a)	The joint distribution of two random variables X and Y is as follows. Y / X: -2, -1, 4, 5 1: 0.1, 0.2, 0, 0.3 2: 0.2, 0.1, 0.1, 0 Determine the marginal distributions of X and Y. Also compute the following. (a) Expectations of X, Y and XY. (b) Standard deviations of X and Y. (c) Covariance of X and Y.	7	2	1,2	3
3 (b)	The marks of 1000 students in an examination follow a normal distribution with a mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75, (Given that $\phi(1) = 0.3413$)	7	2	1,2	3
3 (c)	The probability that an individual suffers a bad reaction from a certain injection is 0.002. Using Poisson distribution, determine the probability that out of 1000 individuals, (a) exactly 3, (b) more than 2 will suffer a bad reaction.	6	2	1,2	3
	OR				
4 (a)	The probability distribution of two discrete random variables X and $Y$is given by $f(x, y) = k(2x + y)$ where x and y are integers such that $0 \le x \le 2,\ 0 \le y \le 3$ (a) Find the value of the constant k. (b) Find the marginal distributions of X and Y. (c) Show that the random variables X and Y are independent.	7	2	1,2	3
4 (b)	A class of 100 contains 10 bright students. Five students from the class are picked at random. Find the following probabilities: (i) none of the picked is a bright student, and (ii) all the picked are bright students.	7	2	1,2	3
4 (c)	The sales per day in a shop is exponentially distributed with the average sale amounting to $Rs\ 100$ and net profit is 8%. Find the probability that the net profit exceeds Rs. 30 on two consecutive days.	6	2	1,2	3
	Module 3				
5 (a)	Prove that the Markov chain whose t.p.m $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. Find the corresponding stationary probability vector.	7	3	1,2	3
5 (b)	Each year a man trades his car for a new car in 3 brands of the popular company Maruti Udyog limited. If he has a 'Standard' he trades it for 'Zen'. If he has a 'Zen' he trades it for a'Esteem'. If he has a 'Esteem' he is just as likely to trade it for a new 'Esteem' or for 'Zen' or a 'Standard' one. In 1996 he bought his first car which was Esteem. (i) Find the probability that he has. (a) 1998 Esteem (b) 1998 Standard (c) 1999 Zen (d) 1999 Esteem (ii) In the long run how often will he have an Esteem?	7	3	1,2	3

<table>
<tr><td>5 (c)</td><td>A group of boys and girls were given an intelligence test. The mean score, S.D score and numbers in each group are as follows.
<table><tr><td></td><td>Boys</td><td>Girls</td></tr><tr><td>Mean</td><td>74</td><td>70</td></tr><tr><td>SD</td><td>8</td><td>10</td></tr><tr><td>n</td><td>12</td><td>10</td></tr></table>Is the difference between the means of the two groups significant at 5% level of significance $(t_{.05} = 2.086\ for\ 20\ d.f)$.</td><td>6</td><td>3</td><td>1,2</td><td>3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>6 (a)</td><td>There boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball</td><td>7</td><td>3</td><td>1,2</td><td>3</td></tr>
<tr><td>6 (b)</td><td>Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches.
$(t_{0.05} = 2.262\ for\ 9\ d.f)$.</td><td>7</td><td>3</td><td>1,2</td><td>3</td></tr>
<tr><td>6 (c)</td><td>Five dice were thrown 96 times and the numbers 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as below.
<table><tr><td>No. of dice showing 1,2 or 3</td><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td></tr><tr><td>Frequency</td><td>7</td><td>19</td><td>35</td><td>24</td><td>8</td><td>3</td></tr></table>Test the hypothesis that the data follows a binomial distribution.
$\left(x^2_{0.05} = 11.07\ for\ 5\ d.f\right)$</td><td>6</td><td>3</td><td>1,2</td><td>3</td></tr>
<tr><td colspan="6">Module 4</td></tr>
<tr><td>7 (a)</td><td>If people arrive to purchase cinema tickets at the average rate of 6 per minute. It takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2minutes before the picture starts and if it takes exactly 1.5minutes to reach the correct seat after purchasing the ticket.
a) Can he expect to be seated for the start of the picture?
b) What is the probability that he will be seated for the start of the picture?
c) How early must he arrive in order to be 99% sure of being seated forthe start of the picture?</td><td>7</td><td>4</td><td>1,2</td><td>3</td></tr>
<tr><td>7 (b)</td><td>Given an average arrival rate of 20 per hour, is it better for a customer to get service at a single channel with mean service rate of 22 customers per hour or at one of two channels in parallel with mean service rate of 11 customers per hour for each of the two channels. Assuming both the channels to be Poisson type.</td><td>7</td><td>4</td><td>1,2</td><td>3</td></tr>
<tr><td>7 (c)</td><td>Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour.
a) Find the effective arrival rate at the clinic.
b) What is the probability that an arriving patient will not wait?</td><td>6</td><td>4</td><td>1,2</td><td>3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>8 (a)</td><td>Customers arrive at a one-man barber shop according to a Poisson process with a mean inter arrival time of 12 minutes. Customers spend an average of 10 minutes in the barber's chair.
(a) What is the expected number of customers in the barber shop and in the</td><td>7</td><td>4</td><td>1,2</td><td>3</td></tr>
</table>

	queue? (b) Calculate the percentage of time an arrival can walk straight into barber's chair without having to wait. (c) How much time can a customer expect to spend in the barber's shop?				
8 (b)	There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. a) What is the probability that all the typists will be busy? b) What is the average number of letters waiting to be typed?	**7**	**4**	**1,2**	**3**
8 (c)	A supermarket has two girls attending sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour, (a) what is the probability that a customer has to wait for service? (b) what is the expected percentage of idle time for each girl? (c) if the customer has to wait in the queue, what is the expected length of his waiting time?	**6**	**4**	**1,2**	**3**
	Module 5				
9 (a)	Verify the following graphs for isomorphism. 	**7**	**5**	**1,2**	**3**
9 (b)	Write adjacency matrix and incidence matrix for the following graph 	**7**	**5**	**1,2**	**3**
9 (c)	Define the following. (i) In-degree and out-degree of a vertex (ii) Connected and disconnected graphs (iii) Sub-graph	**6**	**5**	**1,2**	**3**
	OR				
10(a)	How many vertices will the following graphs have if they contain: (i) 16 edges and all vertices of degree 4. (ii) 12 edges, 6 vertices of degree 3, and other vertices of degree less than 3.	**7**	**5**	**1,2**	**3**
10(b)	Using Euler's formula, show that Kuratowski's first and second graphs K5 and K3,3 are non-planar.	**7**	**5**	**1,2**	**3**
10(c)	Define the following. (i) Bipertite graph (ii) Eulerian graph (iii) Hamiltonian graph	**6**	**5**	**1,2**	**3**