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B.N.M. Institute of Technology

An Autonomous Institution under VTU

Model Question Paper

Fourth Semester BE, 2022-23 Scheme

Statistics, Probability and Graph theory -22MAI141

(Common to CSE, ISE & AIML)

Duration: 3 Hour   Max. Marks: 100

Note: 1. Answer one full question from each Module 1,2,3,4,5 (5Q x 20M = 100 Marks)

	Module 1				
Q. No	Questions	Marks	CO	PO	Cognitive Level
1 (a)	Fit a straight line $y = a + bx$ in the least square sense for the data 	x	1	3	4	6	8	9	11	14	 	y	1	2	4	4	5	7	8	9		7	1	1, 2	3
1 (b)	Calculate the first four moments about a=3.5(raw moments) and thereby deduce the first four central moments from the following distribution and also find the coefficients of skewness and kurtosis. 	x	2.0	2.5	3.0	3.5	4.0	4.5	5.0	 	f	5	38	65	92	70	40	10		7	1	3	1, 2
1 (c)	Find the coefficient of correlation and the equation of the lines of regression for the data 	x	1	2	3	4	5	 	y	2	5	3	8	7		6	1	3	1, 2
	OR				
2 (a)	Fit a curve of the form $y = ax^b$ from the following data 	x	1	2	3	4	5	6	 	y	2.98	4.26	5.21	6.1	6.8	7.5		7	1	3	1, 2
2 (b)	If the first four moments of the frequency distribution about a = 5 are moments of a distribution are 2, 20, 40 and 50. Compute the skewness and Kurtosis of the distribution.	7	1	3	1, 2
2 (c)	Given the regression lines $x = 19.13 - 0.87y \ and \ y = 11.64 - 0.5x$ Compute $\bar{x}$, $\bar{y}$ and the coefficient of correlation r.	6	1	3	1, 2
	Module 2				
3 (a)	The number of telephone lines busy at an instant of time is distributed binomially with probability 0.2. If at an instant , 10 lines are chosen randomly what is the probability that (i) 3 lines are busy (ii)At most 2 lines are busy (iii) All the lines are busy.	7	2	3	1, 2
3 (b)	In a test on electric bulbs, it was found that the life time of a particular brand was distributed normally with an average life of 2000 hours and Standard deviation of 60 hours. If a firm purchase 2500 bulbs, find the number of bulbs that are likely to last for (i) More than 2100hours (ii) Less than 1950hours (iii) Between 1900 to 2100hours	7	2	3	1, 2

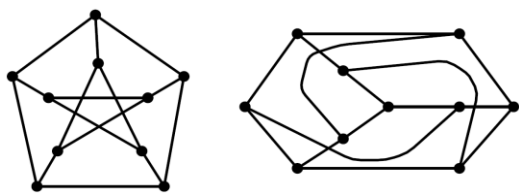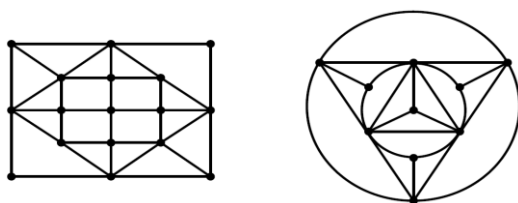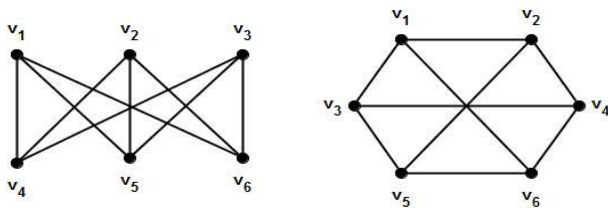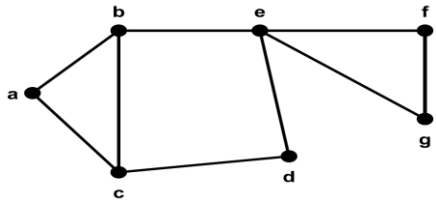|					
3 (c)	The joint probability distribution table for two random variables X and Y is as follows 	X \ Y	-4	2	7
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1	1/8	1/4	1/8
5	1/4	1/8	1/8 | Compute i. Expectation of X and Y and XY ii. Standard deviations of X and Y **iii.** Covariance of X and Y	6	2	3	1, 2
OR					
4 (a)	If the probability of a bad reaction from certain vaccination is 0.001, determine the chances that out of 2000 individuals i. More than 2 will get bad reaction. ii. None of them get bad reaction.	7	2	3	1,2
4 (b)	In an examination 7% of the students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed.	7	2	3	1,2
4 (c)	X & Y are independent random variables. X takes the values 2, 5, 7 with the probability ½, ¼, ¼ respectively. Y take the values 3, 4, 5 with the probability 1/3, 1/3, 1/3 . Find i. The joint probability distribution of X & Y ii. Covariance of X and Y iii. Probability distribution of Z= X +Y	6	2	3	1,2
Module 3					
5 (a)	A student's study habits are as follows he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60 % sure not to study the next night. In the long run, how often does he study?	7	3	3	1, 2
5 (b)	It has been found from the experience that the mean breaking strength of a particular brand of thread is 275.6grams with a standard deviation of 39.7grams. Recently, a sample of 36 pieces of thread showed a mean breaking strength of 253.2grams. Can one conclude at the significance level of (i) 0.05 % (ii) 0.01 % that the thread has become inferior. ($z_{0.05}$ = 1.645 and $z_{0.01}$ = 2.33)	7	3	3	1, 2
5 (c)	A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure. ($t_{0.05}$ for 11 d.f. =2.201)	6	3	3	1, 2
OR					
6 (a)	Find the unique fixed probability vector of the regular stochastic matrix $$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$	7	3	3	1, 2
6 (b)	In an elementary school examination, the mean grade of 32 boys	7	3	3	1, 2

	was 72 with a standard deviation of 8, while the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis that the performance of girls is better than boys.				
6 (C)	Five dice were thrown 96 times and the numbers 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as below	6	3	3	1, 2

No of dice showing 1,2 or 3	1	2	3	4	5
frequency	2	5	3	8	7

Module 4

7 (a)	Customers arrive at a one-man barber shop according to a Poisson process with a mean inter-arrival time of 12 min. Customers spend an average of 10 minutes in the barber's chair. i. What is the expected number of customers in the barber shop and in the queue? ii. Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait iii. How much time can a customer expect to spend in the barber's shop?	7	4	3	1, 2
7 (b)	A super market has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4min and if people arrive in Poisson fashion at the rate of 10 per hour. i. What is the Probability that a customer has to wait for service? ii. What is the expected percentage of idle time for each girl? iii. If the customer has to wait in the queue, what is the expected length of his waiting time	7	4	3	1, 2
7 (c)	At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20kms down the river. Tankers arrive according to a Poisson process with a mean of one every 2h. It takes for an unloading crew, on the average, 10hours to unload a tanker, the unloading time following an exponential distribution. Find a) How many tankers are at the port on the average? b) How long does a tanker spend at the port on the average? c) What is the average arrival rate at the overflow facility	6	4	3	1, 2

OR

8 (a)	At what average rate must a clerk in a supermarket work in order to ensure a probability of 0.90 that the customer will not wait longer than 12min? It is assumed that there is only one counter at which customers arrive in a Poisson fashion at an average rate of 15 per hour and that the length of the service by the clerk has an exponential distribution.	7	4	3	1, 2
8 (b)	A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour. a) What is the probability that an arrival would have to wait in line? b) Find the average waiting time, average time spent in the system and the average number of cars in the system c) For what percentage of time would a pump be idle on an average	7	4	3	1, 2

8 (c)	In a single server queueing system with Poisson input and Exponential service times, if the mean arrival rate is 3 calling per hour, the expected service time is 0.25hours and the maximum possible number of calling units in the system is 2, find P_n, $(n \geq 0)$, average number of calling units in the system and in the queue and average waiting time in the system and in the queue.	6	4	3	1, 2
Module 5					
9 (a)	How many vertices will the following graphs have if they contain: (i) 16 edges and all vertices of degree 4. (ii) 12 edges, 6 vertices of degree 3, and other vertices of degree less than 3.	7	5	3	1, 2
9 (b)	Verify the following graphs for isomorphism 	7	**5**	3	1, 2
9 (c)	Apply Euler's formula for the following planar graphs and find number of edges, vertices and regions. 	6	**5**	3	1, 2
OR					

10(a)	How many vertices will the following graph have, if they contain
	  i. 16 edges and all the vertices of degree 4
	  ii. 21 edges, 3 vertices of degree 4 and other vertices of degree 3
	  iii. 12 edges, 6 vertices of degree 3 and other vertices of degree less than 3.
		7	5	3	1, 2

10(b)	Verify the following graphs for isomorphism 	7	5	3	1, 2
10©	Write adjacency matrix and incidence matrix for the following graph 	6	5	3	1,2