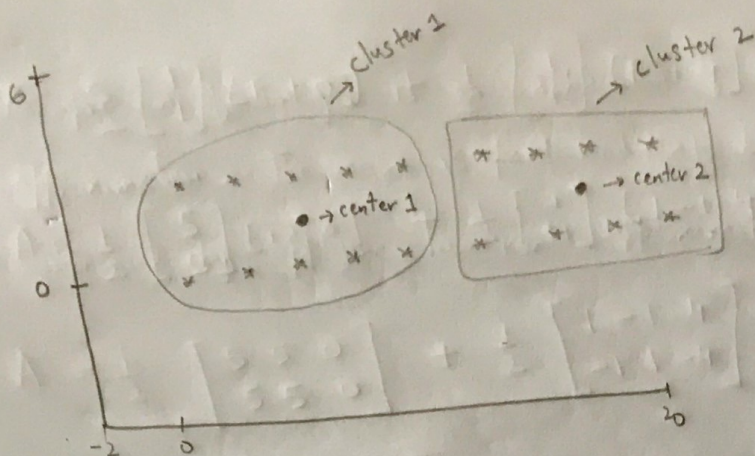①

② $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0 \qquad \Rightarrow (3-\lambda)^2 - 1 = 0 = 0$$

$$\Rightarrow (3-\lambda)^2 = 1$$

$$\Rightarrow 3 - \lambda = \pm 1$$

$$\boxed{\therefore \lambda_1 = 4 \quad \text{and} \quad \lambda_2 = 2}$$

$\Rightarrow$ When $\lambda = 4$, $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 0$

$\Rightarrow \quad u_{11} = u_{12}$

$$\boxed{\therefore \quad u_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$\Rightarrow$ When $\lambda = 2$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 0$

$\Rightarrow \quad u_{11} = -u_{12}$

$$\boxed{\therefore \quad u_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$

(3)

(a) $u_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $u_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $v_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $v_2 = \frac{1}{\sqrt{10}}\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$

(c) $\lambda_1 = 5$ $\lambda_2 = \sqrt{5}$

(4)
(a)

Model Selection is choosing the 'best' classifier of all available classifier based on the performance of Classifier on validation data.

Model Assessment is assessing how well the chosen classifier (in Model Selection) generalizes well on unseen data (based on the performance on test data).

(b) We can use clustering to reduce the number of distance calculations to be done for testing a datapoint. Thus, we can use clustering to simplify nearest neighbour classification

(4)

(c)

$$\therefore J(K) = \underset{\substack{z_1,\dots,z_n \\ \mu_1,\dots,\mu_k}}{\min} \sum_{i=1}^{N} \| x_i - \mu_{z_i} \|^2$$

Here, J is objective function

x_i's are training data

μ_{z_i}'s are cluster centers

z_i's are clusters

(d) Feature Selection for regression is used to improve prediction accuracy and interpretability (i.e., large weights correspond to strong response).

(e)

$$\hat{A} = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

where σ_i = Singular values

u_i = left Singular vectors

v_i = right singular vectors

(f) Compression of Images is an application of low rank approximation. The image is partioned into blocks and compressed by applying low-rank approximation.