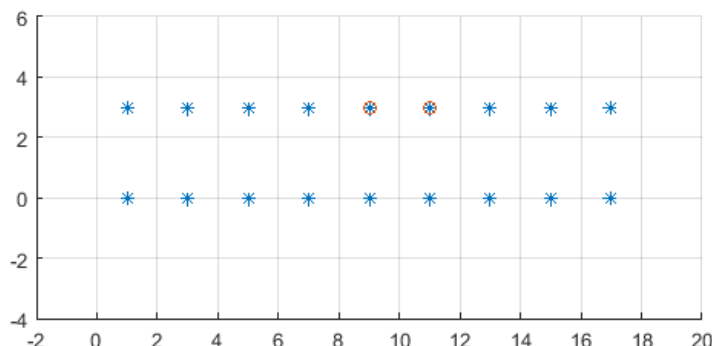


QUIZ 2: 7:00-7:40 pm (60 points total)

Note: This is a **closed-book** exam. Write your answers on clean sheets of paper using a pen or pencil, and upload the scanned pages on Gradescope, indicating the page numbers corresponding to your answers.

1. [K-Means Clustering (10 points)]

Apply K -means clustering to the points shown below. Use the circled points $(9, 3)$ and $(11, 3)$ as your initial cluster centers. Clearly mark what your final clusters are and their centers.



2. [Eigen-decomposition (10 points)]

Find the eigenvalues and orthonormal eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}.$$

Hint: First show that the eigenvalues are $\lambda_1 = 4$ and $\lambda_2 = 2$.

3. [SVD (10 points)]

Suppose the matrix A can be written as

$$A = \frac{5}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 4 \end{bmatrix}$$

- What are the left singular vectors of A ?
- What are the right singular vectors of A ?
- What are the singular values of A ?

Hint: Recall that the SVD of a $m \times n$ matrix can be written as

$$A = \sum_{i=1}^{\min\{m,n\}} \sigma_i \underline{u}_i \underline{v}_i^\top,$$

where the \underline{u}_i 's and \underline{v}_i 's are the orthonormal left and right eigenvectors, respectively.

4. [Concepts (30 points)]

Answer in one or two complete sentences.

- What is the difference between Model Selection and Model Assessment?
- Describe how clustering can be used to simplify nearest neighbor classification.
- Explain in words (or via an equation) the objective of K-means clustering.
- What is the purpose of feature selection in regression problems?
- How do you obtain a low rank approximation of a matrix using the SVD?
- Describe one application of low rank approximation.