

Imaging and Polarimetric Signatures of Konoplya–Zhidenko Black Holes with Various Thick Disk

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Abstract

We investigate the imaging properties of spherically symmetric Konoplya–Zhidenko (KZ) black holes surrounded by geometrically thick accretion flows, adopting a phenomenological radiatively inefficient accretion flow (RIAF) model and an analytical ballistic approximation accretion flow (BAAF) model. General relativistic radiative transfer is employed to compute synchrotron emission from thermal electrons and generate horizon-scale images. For the RIAF model, we analyze the dependence of image morphology on the deformation parameter, observing frequency, and flow dynamics. The photon ring and central dark region expand with increasing deformation parameter, with brightness asymmetries arising at high inclinations and depending on flow dynamics and emission anisotropy. The BAAF disk produces narrower rings and darker centers, while polarization patterns trace the brightness distribution and vary with viewing angle and deformation, revealing spacetime structure. These results demonstrate that intensity and polarization in thick-disk models provide probes of KZ black holes and near-horizon accretion physics.

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1 Introduction

Accumulating astronomical evidence strongly supports the existence of supermassive black holes (SMBHs) at galactic centers. Their presence has been confirmed through multiple independent observations, from the detection of gravitational waves by LIGO/Virgo [1] to the direct horizon-scale images captured by the Event Horizon Telescope (EHT) [2, 3]. Recent EHT polarization data [4, 5] further reveal detailed information about the magnetized plasma and radiation near event horizons, providing valuable probes of accretion physics and spacetime geometry. Motivated by these developments, many studies have focused on modeling black hole accretion flow images within both general relativity (GR) [6, 7] and various modified gravity theories [8–11].

Einstein’s general relativity remains one of the most elegant and experimentally verified theories of nature [12]. Nonetheless, current observations do not completely exclude the possibility of deviations from GR, leaving room for alternative theories. Konoplya and Zhidenko [13] proposed a rotating non-Kerr black hole metric incorporating a static deformation, which can be viewed as an axisymmetric vacuum solution of an as-yet-unknown modified gravity theory [14]. The motivation for this metric lies in exploring whether gravitational-wave observations might hint at departures from GR [13]. Several studies have shown that the Konoplya–Zhidenko (KZ) spacetime may describe a realistic astrophysical black hole [15, 16], and that such non-Kerr geometries could modify strong-field signatures and observational phenomena. Recent works have examined particle dynamics [17], gravitational lensing [18], and black hole shadows in the KZ background [19, 20], as well as the superradiant behavior of scalar and electromagnetic fields [21]. Further investigations include tests against the hot-spot data of Sgr A* [22], analyses of plasma effects [20] and relativistic viscous accretion flows [23], and studies of energy extraction via magnetic reconnection [24]. More recently, the observational appearance and imaging features of the KZ black hole surrounded by a thin accretion disk have also been explored [25].

SMBHs are generally believed to accrete hot, magnetized plasma, forming a luminous accretion disk whose thermal synchrotron emission likely dominates the observed black hole image frequencies [26, 27]. The observed disk features reflect both gravitational effects and the underlying plasma properties, including the electron distribution, temperature, and magnetic field geometry [27]. Previous imaging studies in KZ spacetimes have mainly focused on geometric lensing effects, often assuming simple, geometrically thin emission models [11, 25, 28–36]. However, observations by the EHT and other studies [2, 27, 37, 38] have suggested that, under the dominant influence of gravity, accretion flows near SMBHs may become geometrically thick and optically thin, owing to the suppression of vertical cooling and compression of matter [39]. In such situations, it becomes essential to consider the detailed distributions of streamlines, particle number density, electron temperature, and magnetic field configuration.

Theoretical studies of geometrically thick accretion disks remain an active area of research [40]. Many works have focused on *radiatively inefficient accretion flow* (RIAF) models [41–49], in which the vertically averaged electron density and temperature typically follow nearly power-law profiles with radius [41]. This phenomenological framework, physically motivated and broadly consistent with general relativistic magnetohydrodynamic (GRMHD) simulations, has been successful in reproducing the submillimeter spectra and overall image morphology of sources such as M87* [27]. However, RIAF models neglect certain physical effects—such as outflows, non-thermal particles, and full GRMHD dynamics—which limit their predictive power, especially for polarization and variability studies [50–52].

Traditionally, RIAF models have been employed to describe the large-scale behavior of accretion flows. However, the advent of horizon-resolving observations has enabled direct probes of plasma dynamics at event-horizon scales, making the study of near-horizon magnetofluids increasingly essential for understanding black hole environments. Recently, Hou *et al.* [6, 7] developed a self-consistent analytical model for horizon-scale accretion flows within the GRMHD framework, which we refer to as the *ballistic approximation accretion flow* (BAAF) model. Assuming that gravity dominates the fluid acceleration close to the event horizon, this model provides explicit expressions for thermodynamic variables and magnetic field configurations, offering a physically motivated description of the morphology and dynamics of geometrically thick accretion flows in the near-horizon region of black holes.

In this work, we investigate the imaging properties of the non-rotating KZ black hole using two representative accretion models: the phenomenological RIAF and the analytical BAAF. The images are computed using a general relativistic radiative transfer (GRRT) method. We analyze not only the intensity morphology under different parameters, viewing angles, and frequencies, but also the impact of emission anisotropy and distinct flow dynamics. Furthermore, we explore the polarization signatures predicted by the BAAF model.

The remaining sections of this paper are structured as follows. In Sec. 2, we briefly review the properties of non-rotating black holes in KZ gravity and derive the corresponding photon sphere. Sec. 3 introduces the synchrotron radiation mechanism and the general relativistic radiative transfer (GRRT) methodology. In Sec. 4, we analyze the imaging characteristics under the RIAF model, including the effects of emission anisotropy and distinct flow dynamics. Sec. 5 focuses on the BAAF model and presents the resulting intensity and polarization patterns. Finally, Sec. 6 presents a summary and outlook. In this work, we have set the fundamental constants c, G to unity, unless otherwise specified, and we will work in the convention $(-, +, +, +)$.

2 The Konoplya-Zhidenko Spacetime and Photon Sphere

We consider a spherically symmetric black hole metric in Konoplya–Zhidenko gravity [13, 53]. Its line element can be expressed as

$$ds^2 = -(1 - \frac{2M}{r} - \frac{\eta}{r^3})dt^2 + \frac{1}{1 - \frac{2M}{r} - \frac{\eta}{r^3}}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \quad (2.1)$$

where M is the black hole mass. It is deformed on the Schwarzschild spacetime by adding a static deformation which changes the relation between the black hole mass and position of the event horizon, but preserves asymptotic properties of the Schwarzschild spacetime. Deviations from the Schwarzschild metric are measured by the parameter η . Eq. (2.1) becomes the Schwarzschild metric when η is set to zero. The location of the event horizons is determined by the roots of the equation $g_{tt} = 1 - \frac{2M}{r} - \frac{\eta}{r^3} = 0$. This equation can have more than one root. Since in this paper we are only concerned with the exterior spacetime, we will refer hereafter to the outer horizon simply as the event horizon. The event horizon exists for $\eta \geq -32/27$ at $r_h/M = 4/3$, and its radius increases with increasing η . More details can be found in [53]. Furthermore, for simplicity and without loss of generality, we shall set $M = 1$ in the next discussion.

Since the KZ black hole is spherically symmetric, we consider photons moving in the equatorial plane $\theta = \pi/2$. Along a geodesic, a photon possesses two conserved quantities, namely the energy $E = -p_t$ and the angular momentum $L = p_\phi$. Combining these with the null normalization condition $u^\mu u_\mu = 0$, one can obtain the radial equation of motion:

$$\left(\frac{dr}{d\lambda}\right)^2 = -V_{\text{eff}}(r, E, L). \quad (2.2)$$

where

$$V_{\text{eff}}(r, E, L) = E^2 \left(1 - \frac{1 - \frac{2}{r} - \frac{\eta}{r^3}}{r^2} b^2\right). \quad (2.3)$$

Here, we define an impact parameter $b \equiv L/E$. The circular orbits can then be determined by solving the equations $V_{\text{eff}} = 0$ and $\partial_r V_{\text{eff}} = 0$ with respect to r and b . Assuming a small parameter η , the radius of the photon sphere r_{ph} and the critical value of the impact parameter b_c can be expanded perturbatively in η as

$$\begin{aligned} r_{\text{ph}} &= 3 + \frac{5}{18}\eta - \frac{25}{486}\eta^2 + \mathcal{O}(\eta^3), \\ b_c &= 3\sqrt{3} + \frac{\sqrt{3}}{6}\eta - \frac{2\sqrt{3}}{81}\eta^2 + \mathcal{O}(\eta^3). \end{aligned} \quad (2.4)$$

To illustrate the effect of the deformation parameter η , we present in Fig. 1 the numerical results for the dependence of several characteristic quantities of the KZ black hole on η . As shown in panel (a), the horizon radius r_h increases monotonically with η , deviating from the Schwarzschild value at

$\eta = 0$. A similar trend is observed for the photon sphere radius r_{ph} in panel (b) and for the critical impact parameter b_c in panel (c). In all cases, larger η leads to a more extended spacetime structure, while the Schwarzschild case serves as the reference baseline (black dashed line).

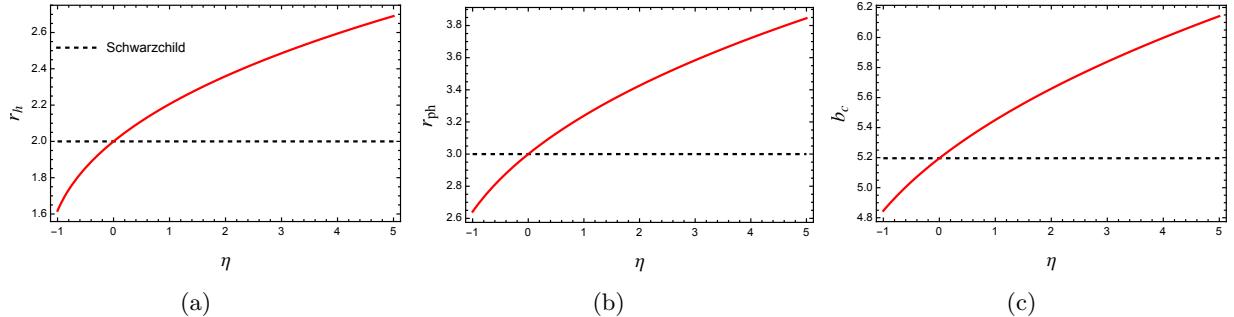


Figure 1: Dependence of the characteristic quantities of the KZ black hole on the deformation parameter η : (a) horizon radius r_h , (b) photon sphere radius r_{ph} , and (c) critical impact parameter b_c . The red solid curves correspond to the KZ black hole, while the black dashed lines denote the Schwarzschild case ($\eta = 0$).

3 Synchrotron Radiation and Radiative Transfer

We consider the presence of a magnetic field in the vicinity of the black hole, whose explicit configuration will be specified in later sections. In this field, thermal or nonthermal electrons within the accreting plasma undergo synchrotron emission as they are accelerated by the Lorentz force. Our analysis focuses on synchrotron radiation produced by highly relativistic electrons. In this section, we outline the general mechanism of synchrotron emission in the surrounding fluid and describe the subsequent propagation of the emitted radiation from the source to the observer's image plane. All quantities are expressed in CGS units.

In the plasma, synchrotron radiation is predominantly produced by electrons. To correctly describe the emission, absorption, and rotation of polarized radiation in curved spacetime, it is essential to construct an appropriate frame of reference. Following the formulation of [54], we define the fluid coordinate system by the four-velocity u^μ , the photon wave four-vector k^μ , and the local magnetic field b^μ . The corresponding orthonormal basis vectors are introduced as follows:

$$e_{(0)}^\mu = u^\mu, \quad e_{(3)}^\mu = \frac{k^\mu}{\omega} - u^\mu, \quad e_{(2)}^\mu = \frac{1}{\mathcal{N}} \left(b^\mu + \beta u^\mu - C e_{(3)}^\mu \right), \quad e_{(1)}^\mu = \frac{\epsilon^{\mu\nu\rho\sigma} u_\nu k_\sigma b_\rho}{\omega \mathcal{N}}, \quad (3.1)$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor, with

$$b^2 = b_\mu b^\mu, \quad \beta = u_\mu b^\mu, \quad \omega = -k_\mu u^\mu, \quad C = \frac{k_\mu b^\mu}{\omega} - \beta, \quad \mathcal{N} = \sqrt{b^2 + \beta^2 - C^2}. \quad (3.2)$$

With this choice of frame, all emission, absorption, and Faraday rotation coefficients associated with the Stokes parameter U vanish. The remaining nonzero emissivities correspond to the Stokes parameters I , Q , and V , which are given by [55–57]:

$$\begin{aligned} j_I &= \frac{\sqrt{3}e^3 B \sin \theta_B}{4\pi m_e c^2} \int_0^\infty d\gamma N(\gamma) F\left(\frac{\nu}{\nu_s}\right), \\ j_Q &= \frac{\sqrt{3}e^3 B \sin \theta_B}{4\pi m_e c^2} \int_0^\infty d\gamma N(\gamma) G\left(\frac{\nu}{\nu_s}\right), \\ j_V &= \frac{\sqrt{3}e^3 B \sin \theta_B}{4\pi m_e c^2} \int_0^\infty d\gamma N(\gamma) \frac{4 \cot \theta_B}{3\gamma} H\left(\frac{\nu}{\nu_s}\right), \end{aligned} \quad (3.3)$$

where $N(\gamma)$ denotes the electron energy distribution function, whose form determines the detailed synchrotron emissivity. Here and in what follows, B is the magnitude of the local magnetic field, ν is the emitted frequency, and $\nu_s = \frac{3eB \sin \theta_B \gamma^2}{4\pi m_e c}$ is the characteristic synchrotron frequency. The electron Lorentz factor is $\gamma = 1/\sqrt{1 - \beta^2}$, where e , m_e , and c are the elementary charge, electron mass, and speed of light, respectively. The pitch angle θ_B is the angle between the wave vector and the magnetic field in the fluid rest frame.

The synchrotron functions corresponding to total, linearly, and circularly polarized emission are defined as

$$\begin{aligned} F(x) &= x \int_x^\infty dy K_{5/3}(y), \\ G(x) &= x K_{2/3}(x), \\ H(x) &= \int_x^\infty dy K_{1/3}(y) + x K_{1/3}(x), \end{aligned} \quad (3.4)$$

where $K_n(z)$ denotes the modified Bessel function of the second kind of order n .

We adopt a relativistic thermal (Maxwellian) electron distribution, which is commonly used for astrophysical synchrotron sources:

$$N(\gamma) = \frac{n_e \gamma^2 \beta}{\theta_e K_2(1/\theta_e) \exp(-\frac{\gamma}{\theta_e})}, \quad (3.5)$$

where n_e is the electron number density and $\theta_e = \frac{k_B T_e}{m_e c^2}$ is the dimensionless electron temperature. Here k_B is the Boltzmann constant and T_e the thermodynamic temperature. In the ultra-relativistic regime ($\beta \approx 1$, $\theta_e \gg 1$), the modified Bessel function can be approximated by $K_2(1/\theta_e) \simeq 2\theta_e^2$. Introducing $z \equiv \gamma/\theta_e$, the emissivities become

$$\begin{aligned} j_I &= \frac{\sqrt{3}n_e e^3 B \sin \theta_B}{8\pi m_e c^2} \int_0^\infty dz z^2 \exp(-z) F\left(\frac{\nu}{\nu_s}\right), \\ j_Q &= \frac{\sqrt{3}n_e e^3 B \sin \theta_B}{8\pi m_e c^2} \int_0^\infty dz z^2 \exp(-z) G\left(\frac{\nu}{\nu_s}\right), \\ j_V &= \frac{\sqrt{3}n_e e^3 B \sin \theta_B \cot \theta_B}{6\pi m_e \theta_e c^2} \int_0^\infty dz z \exp(-z) H\left(\frac{\nu}{\nu_s}\right). \end{aligned} \quad (3.6)$$

Defining $x = (\nu/\nu_s)z^2$, the emissivities can be written compactly as

$$\begin{aligned} j_I &= \frac{n_e e^2 \nu}{2\sqrt{3}c\theta_e^2} I_I(x), \\ j_Q &= \frac{n_e e^2 \nu}{2\sqrt{3}c\theta_e^2} I_Q(x), \\ j_V &= \frac{2n_e e^2 \nu \cot \theta_B}{3\sqrt{3}c\theta_e^3} I_V(x), \end{aligned} \quad (3.7)$$

where $x \equiv \nu/\nu_c$ is the ratio of the emitted photon frequency to the characteristic frequency of the system $\nu_c = \frac{3eB \sin \theta_B \theta_e^2}{4\pi m_e c}$. The dimensionless thermal synchrotron integrals are

$$\begin{aligned} I_I(x) &= \frac{1}{x} \int_0^\infty dz z^2 \exp(-z) F\left(\frac{x}{z^2}\right), \\ I_Q(x) &= \frac{1}{x} \int_0^\infty dz z^2 \exp(-z) G\left(\frac{x}{z^2}\right), \\ I_V(x) &= \frac{1}{x} \int_0^\infty dz z \exp(-z) H\left(\frac{x}{z^2}\right). \end{aligned} \quad (3.8)$$

For a hot electron plasma, the absorption coefficients satisfy Kirchhoff's law,

$$a_\nu = \frac{j_\nu}{B_\nu}, \quad (3.9)$$

where B_ν represents the Planck black body radiation function.

Finally, the Faraday rotation coefficients are given by the following expressions:

$$\begin{aligned} r_Q &= \frac{n_e e^4 B^2 \sin^2 \theta_B}{4\pi^2 m_e^3 c^3 \nu^3} f_m(X) + \left(\frac{K_1(\theta_e^{-1})}{K_2(\theta_e^{-1})} + 6\theta_e \right), \\ r_V &= \frac{n_e e^3 B \cos \theta_B}{\pi m_e^2 c^2 \nu^2} \frac{K_0(\theta_e^{-1}) - \Delta J_5(X)}{K_2(\theta_e^{-1})}, \end{aligned} \quad (3.10)$$

with

$$X = \left(\frac{3}{2\sqrt{2}} \times 10^{-3} \frac{\nu}{\nu_c} \right)^{-1/2}. \quad (3.11)$$

Hence, the emissivity, absorption, and Faraday coefficients primarily depend on the electron number density, magnetic-field strength, field-wave vector angle, and electron temperature.

In order to obtain the image at the observer's location, we propagate the radiation from the light source to the observer's screen. To describe the interaction between light rays and matter in radiative transfer, we employ the tensor form of the covariant radiative transfer equation, following [58]:

$$k^\mu \nabla_\mu \mathcal{S}^{\alpha\beta} = \mathcal{J}^{\alpha\beta} + H^{\alpha\beta\mu\nu} \mathcal{S}_{\mu\nu}, \quad (3.12)$$

where $\mathcal{S}^{\alpha\beta}$ is the polarization tensor, k^μ is the photon wave vector, $\mathcal{J}^{\alpha\beta}$ characterizes emission from the source, and $H^{\alpha\beta\mu\nu}$ encodes absorption and Faraday rotation effects. Numerical solutions of the above equation can be obtained using the public code COPORT 1.0 [55].

Based on COPORT 1.0, we exploit the gauge invariance of $\mathcal{S}^{\alpha\beta}$ to simplify computations in a suitably chosen parallel-transported frame. In this framework, Eq. (3.12) can be decomposed into two parts. The first part encodes gravitational effects:

$$k^\mu \Delta_\mu f^a = 0, \quad f^a k_a = 0, \quad (3.13)$$

where f^μ is a normalized spacelike vector orthogonal to k^μ . The second part accounts for the plasma effects:

$$\frac{d}{d\lambda} S = R(\chi) J - R(\chi) M R(-\chi) S, \quad (3.14)$$

where

$$S = \begin{pmatrix} \mathcal{I} \\ Q \\ \mathcal{U} \\ \mathcal{V} \end{pmatrix}, \quad J = \frac{1}{\nu^2} \begin{pmatrix} j_I \\ j_Q \\ j_U \\ j_V \end{pmatrix}, \quad M = \nu \begin{pmatrix} a_I & a_Q & a_U & a_V \\ a_Q & a_I & r_V & -r_U \\ a_U & -r_V & a_I & r_Q \\ a_V & r_U & -r_Q & a_I \end{pmatrix}, \quad (3.15)$$

Here $\mathcal{I} = I/\nu^3$, and similarly for \mathcal{Q} , \mathcal{U} , and \mathcal{V} . The matrix $R(\chi)$ represents the rotation between the synchrotron emission frame and the parallel-transported reference frame:

$$R(\chi) = \begin{pmatrix} 1 & & & \\ & \cos(2\chi) & -\sin(2\chi) & \\ & \sin(2\chi) & \cos(2\chi) & \\ & & & 1 \end{pmatrix}, \quad (3.16)$$

where the rotation angle χ is defined as the angle between the reference vector f^μ and the magnetic field B^μ :

$$\chi = \text{sign}(\epsilon_{\mu\nu\alpha\beta} u^\mu f^\nu B^\rho k^\sigma) \arccos \left(\frac{P^{\mu\nu} f_\mu B_\nu}{\sqrt{(P^{\mu\nu} f_\mu f_\nu)(P^{\alpha\beta} B_\alpha B_\beta)}} \right), \quad (3.17)$$

and $P^{\mu\nu} = g^{\mu\nu} - \frac{k^\mu k^\nu}{\omega^2} + \frac{u^\mu k^\nu + k^\mu u^\nu}{\omega}$ is the induced metric on the transverse subspace.

At the observer's location, the Stokes parameters are projected onto the image plane using the same rotation matrix (3.16), with the corresponding rotation angle

$$\chi_o = \text{sign}(\epsilon_{\mu\nu\rho\sigma} u_o^\mu f^\nu d^\rho k^\sigma) \arccos \left(\frac{P^{\mu\nu} f_\mu d_\nu}{\sqrt{(P^{\mu\nu} f_\mu f_\nu)(P^{\alpha\beta} d_\alpha d_\beta)}} \right), \quad (3.18)$$

where u_o^μ is the observer's four-velocity and d^μ denotes the y -axis direction of the observer's screen. In this work, we adopt $d^\mu = -\partial_\theta^\mu$. The projected Stokes parameters are then

$$\mathcal{I}_o = \mathcal{I}, \quad \mathcal{Q}_o = \mathcal{Q} \cos 2\chi_o - \mathcal{U} \sin 2\chi_o, \quad \mathcal{U}_o = \mathcal{Q} \sin 2\chi_o + \mathcal{U} \cos 2\chi_o, \quad \mathcal{V}_o = \mathcal{V}. \quad (3.19)$$

The observed Stokes parameters encode the polarization state of the radiation. The total intensity is given by \mathcal{I}_o , while \mathcal{V}_o describes circular polarization — positive (negative) values correspond to left-(right-) handed circular polarization. The linear polarization intensity and electric-vector position angle (EVPA) are given by

$$P_o = \sqrt{\mathcal{Q}_o^2 + \mathcal{U}_o^2}, \quad \Phi_{\text{EVPA}} = \frac{1}{2} \arctan \frac{\mathcal{U}_o}{\mathcal{Q}_o}. \quad (3.20)$$

In the following, we investigate the image features of the KZ black hole using two representative models of geometrically thick accretion flows. The first is a phenomenological RIAF model, while the second is an analytic thick-disk model proposed by Hou *et al.* [6]. For convenience, we shall refer to the latter as the ballistic approximation accretion flow model (hereafter BAAF).

4 RIAF Model

We work in cylindrical coordinates, where $R = r \sin \theta$ denotes the cylindrical radius and $z = r \cos \theta$ measures the vertical height from the equatorial plane at $\theta = \pi/2$. Following the analytic construction of radiatively inefficient accretion flow (RIAF) models [59], we prescribe the radial and vertical profiles of the number density and temperature distributions as

$$n_e = n_h \left(\frac{r}{r_h} \right)^{-2} \exp \left(-\frac{z^2}{2(\alpha R)^2} \right), \quad T_e = T_h \left(\frac{r}{r_h} \right), \quad (4.1)$$

where r_h is the outer horizon radius, and n_h and T_h denote the electron number density and temperature at the horizon, respectively. The parameter α is a dimensionless constant controlling the disk thickness.

The local magnetic field strength is characterized by the cold magnetization parameter σ , defined as

$$B = \sqrt{\sigma \rho}, \quad (4.2)$$

where $\rho = n_e m_p c^2$ is the fluid mass density. In general, σ can be modeled as a spatially varying distribution, e.g., $\sigma = \sigma_h r_h / r$ [45], where σ_h is its value at the event horizon. For simplicity, we assume σ to be constant and fix it to $\sigma = 0.1$ in all models considered below.

The fluid dynamics in this model is relatively unconstrained. We therefore consider three representative kinematic configurations:

(a) Orbiting motion For circular motion around the black hole [60], the fluid four-velocity has only u^t and u^ϕ components, given by

$$u^\mu = u^t (1, 0, 0, \Omega), \quad (4.3)$$

where

$$u^t = \sqrt{-\frac{1}{g_{tt} + g_{\phi\phi}\Omega^2}}, \quad \Omega = \frac{u^\phi}{u^t} = \frac{g_{tt}l}{g_{\phi\phi}}, \quad l = -\frac{u_\phi}{u_t} = l_0 \frac{R^{3/2}}{C + R}, \quad R = r \sin \theta, \quad (4.4)$$

Here, l denotes the specific angular momentum, and l_0 and C are free parameters. Following Ref. [60], we set $l_0 = C = 1$ throughout this work.

(b) Infalling motion Assuming that the fluid is at rest at spatial infinity, i.e., $u_t = -1$, the four-velocity is given by

$$u^\mu = (-g^{tt}, \sqrt{-(1 + g^{tt})g^{rr}}, 0, 0), \quad (4.5)$$

which corresponds to a special case of the conical solution.

(c) Combined motion A more general flow configuration can be constructed as a combination of orbiting and infalling motion [45], with

$$u^\mu = (u^t, u^r, 0, u^\phi), \quad (4.6)$$

where

$$\Omega = \frac{u^\phi}{u^t} = \Omega_o + \beta_1(\Omega_i - \Omega_o), \quad u^r = u_o^r + \beta_2(u_i^r - u_o^r), \quad (4.7)$$

and

$$\Omega_o = -\frac{g_{tt}l}{g_{\phi\phi}}, \quad \Omega_i = 0, \quad u_o^r = 0, \quad u_i^r = \sqrt{-(1 + g^{tt})g^{rr}}. \quad (4.8)$$

Here, the subscripts “ i ” and “ o ” refer to the infalling and orbiting components, respectively. The parameters β_1 and β_2 are adjustable coefficients in the range $(0, 1)$, controlling the relative weights of the two components: smaller values correspond to predominantly orbiting motion, while larger values indicate a more infalling flow. Throughout this work, we adopt $\beta_1 = \beta_2 = 0.2$. The temporal component u^t is then determined by $u^2 = -1$.

$$u^t = \sqrt{-\frac{1 + g_{rr}(u^r)^2}{g_{tt} + \Omega^2 g_{\phi\phi}}}. \quad (4.9)$$

Next, let us consider the magnetic field. Under the assumption of ideal MHD,

$$u_\mu F^{\mu\nu} = 0. \quad (4.10)$$

the electric field in the rest frame of the fluid, $e^\nu = u_\mu F^{\mu\nu} = 0$, and the magnetic field $b^\nu = u_\mu {}^*F^{\mu\nu}$ is orthogonal to the fluid four-velocity, $b_\mu u^\mu = 0$. For all three flow prescriptions above, we assume a purely toroidal magnetic field,

$$b^\mu \sim (l, 0, 0, 1), \quad (4.11)$$

Depending on whether the emissivity depends on the angle between the magnetic field and the emitted photons, we distinguish two cases: isotropic and anisotropic emission. In the isotropic case, the emissivity is independent of this angle, and only the magnetic field strength needs to be specified. We employ an angle-averaged emissivity defined as

$$\bar{j}_\nu = \frac{1}{2} \int_0^\pi j_\nu \sin \theta_B d\theta_B, \quad (4.12)$$

with the fitting formula from [61]:

$$\bar{j}_\nu = \frac{n_e e^2 \nu}{2\sqrt{3}c\theta_e^2} I(x), \quad x = \frac{\nu}{\nu_c}, \quad \nu_c = \frac{3eB\theta_e^2}{4\pi m_e c}, \quad (4.13)$$

where the dimensionless function $I(x)$ is approximated as [61]

$$I(x) = \frac{4.0505}{x^{1/6}} \left(1 + \frac{0.4}{x^{1/4}} + \frac{0.5316}{x^{1/2}} \right) \exp(-1.8899x^{1/3}). \quad (4.14)$$

For the anisotropic case, the direction of the magnetic field must be accounted for. Adopting the toroidal field configuration in Eq. (4.11), the corresponding fitting function is

$$I(x) = 2.5651 \left(1 + 1.92x^{-1/3} + 0.9977x^{-2/3} \right) \exp(-1.8899x^{1/3}). \quad (4.15)$$

In the next section, we present the black hole images obtained from the phenomenological RIAF model, based on the methodology described above.

4.1 Isotropic Radiation Case

Fig. 2 shows the intensity maps of the KZ black hole in the RIAF model with isotropic emission. The accretion flow follows an infalling motion, and the observing frequency is fixed at 230GHz. Each panel corresponds to a different choice of the deformation parameter η and the observer's inclination angle θ_o : from top to bottom, the inclination varies as $\theta_o = 0.001^\circ, 17^\circ, 60^\circ, 85^\circ$, while from left to right, the deformation parameter takes the values $\eta = -1, 0.1, 2$. For a more quantitative comparison, we also analyze the intensity distributions along the screen's x - and y -axes in Fig. 3 and Fig. 4, where $\eta = 2$, $\eta = 0.1$, and $\eta = -1$ are shown in red, green, and blue, respectively.

The panels in Fig. 2 consistently exhibit a pronounced bright ring, corresponding to the peaks seen in Figs. 3 and 4. This feature originates from higher-order images, namely photons that orbit the black hole one or more times before reaching the observer, and is a direct consequence of strong gravitational lensing. Outside the ring lies the primary image, formed by photons reaching the observer directly from the disk. The dark regions in the images correspond to the event horizon. For geometrically thin disks, the horizon produces a distinct inner shadow that could be detected by the EHT [32].

In contrast, for geometrically thick disks, emission from off-equatorial regions obscures the horizon boundary, making the inner shadow harder to observe.

By comparing different subplots, we can examine the effects of the deformation parameter η and the inclination angle θ_o on the black hole image. For fixed θ_o , increasing η enlarges both the bright ring and the central dark region without altering their shapes, since the horizon radius r_h and the critical impact parameter b_c of the photon sphere both grow with η , as discussed in Sec. 2. For fixed η , varying θ_o produces the following trends: for polar viewing, the bright ring (higher-order images) and the dark region remain centered and isotropic; at $\theta_o = 17^\circ$, a clear up-down asymmetry in the bright ring emerges; at $\theta_o = 60^\circ$, two distinct dark regions appear inside the bright ring, with the upper one slightly dimmer; and at $\theta_o = 85^\circ$, the image becomes nearly symmetric, though the left-right brightness still exceeds the up-down one. The persistent left-right brightness symmetry arises from the spherical symmetry of the spacetime and the choice of infalling accretion flow. In contrast, the up-down brightness dependence reflects the equatorial symmetry of the thick disk: for observers near the equatorial plane, high-latitude emission partially fills the dark region, while near the poles, insufficient photons reach the observer. Figs. 3 and 4 further illustrate these effects through horizontal and vertical intensity cuts.

In Fig. 5, the dependence of the KZ black hole image on observing frequency is illustrated for the RIAF model with isotropic emission. For fixed parameters $\theta_o = 85^\circ$ and $\eta = -1$, the image transitions from a diffuse morphology at 85 GHz to a well-defined ring at 230 GHz and 345 GHz. At the lowest frequency, the dark region is almost entirely hidden, and the distinction between the primary and higher-order features becomes unclear. With increasing frequency, the contribution from regions far outside the photon ring is strongly suppressed, resulting in a more pronounced visibility of both the higher-order structures and the dark region boundary. The corresponding horizontal and vertical intensity cuts confirm this trend: higher observing frequencies enhance the contrast of the photon ring while diminishing the diffuse background contribution.

In Fig. 6, we further compare the effects of different accretion-flow dynamics on the image morphology in the same model. The observing frequency is fixed at $\nu_o = 230$ GHz, with parameters $\theta_o = 85^\circ$ and $\eta = -1$. Panels (a)–(c) correspond to the orbiting, infalling, and combined motion, respectively, illustrating how the image morphology depends on the flow pattern. Panels (d) and (e) show the corresponding horizontal (x -axis) and vertical (y -axis) intensity cuts through the image center, where red, green, and blue denote the orbiting, infalling, and combined cases, respectively. Since we set $\beta_1 = \beta_2 = 0.2$, the orbiting component dominates the combined flow, explaining the close similarity between the orbiting and combined images. The orbiting motion tends to obscure the dark region by enhancing off-equatorial emission and substantially increasing the total brightness, whereas the radial inflow produces more distinct direct and higher-order features. This difference originates from their

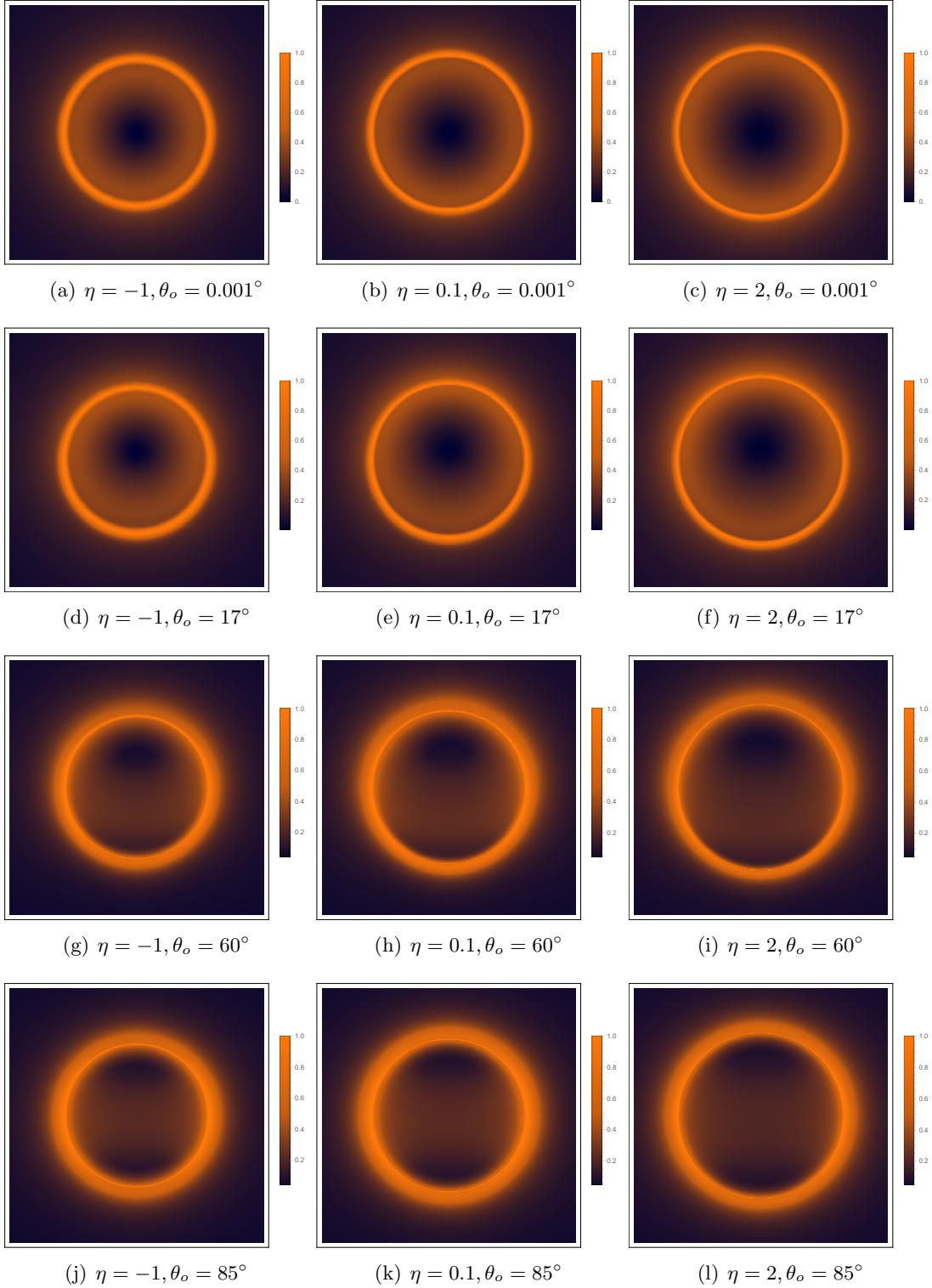


Figure 2: Intensity maps of the KZ black hole in the RIAF model with isotropic emission. The accretion flow follows the infalling motion, the observing frequency is 230 GHz.

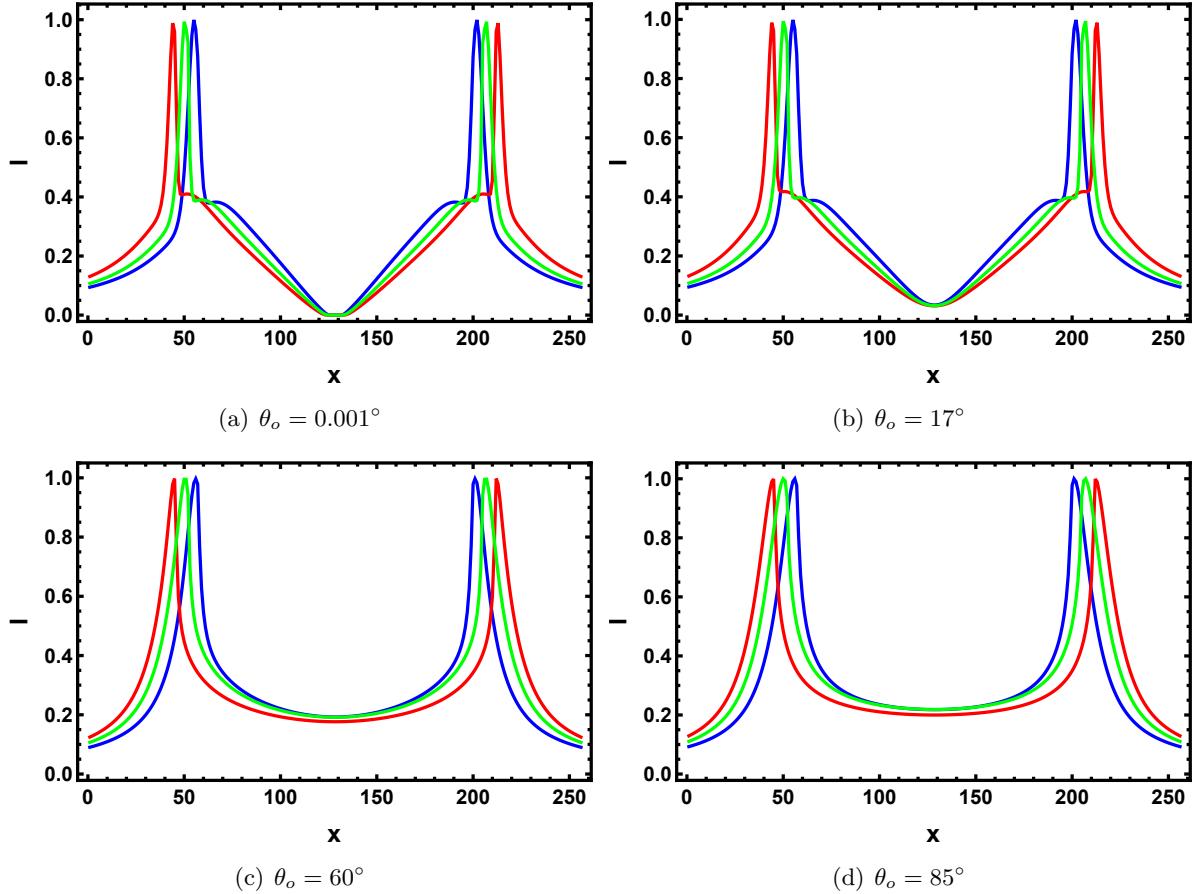


Figure 3: Intensity distribution along the screen x -axis for the KZ black hole in the RIAF model with isotropic emission. The accretion flow follows the infalling motion, and the observing frequency is fixed at 230 GHz. The curves correspond to different values of η : red for $\eta = 2$, green for $\eta = 0.1$, and blue for $\eta = -1$.

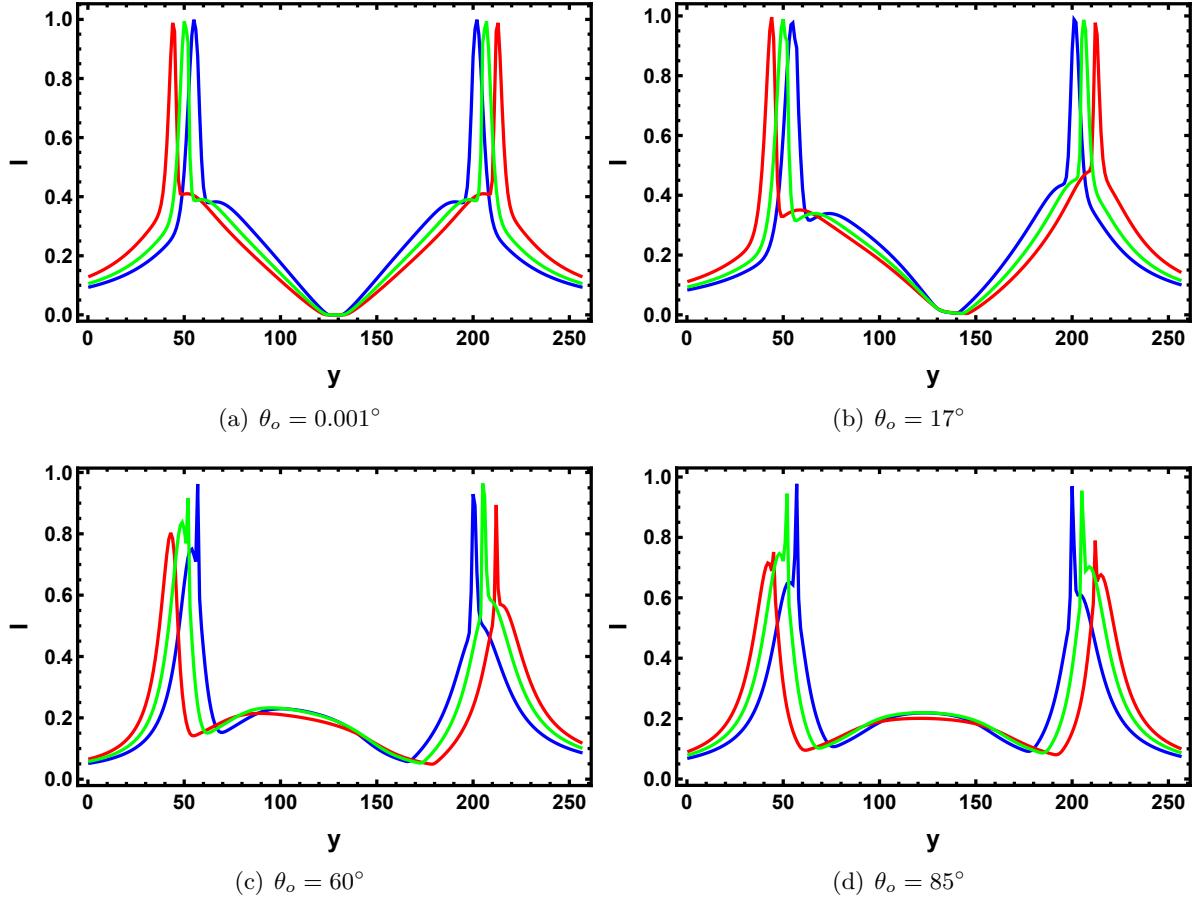


Figure 4: Intensity distribution along the screen y -axis for the KZ black hole in the RIAF model with isotropic emission. The accretion flow follows the infalling motion, and the observing frequency is fixed at 230 GHz. The curves correspond to different values of η : red for $\eta = 2$, green for $\eta = 0.1$, and blue for $\eta = -1$.

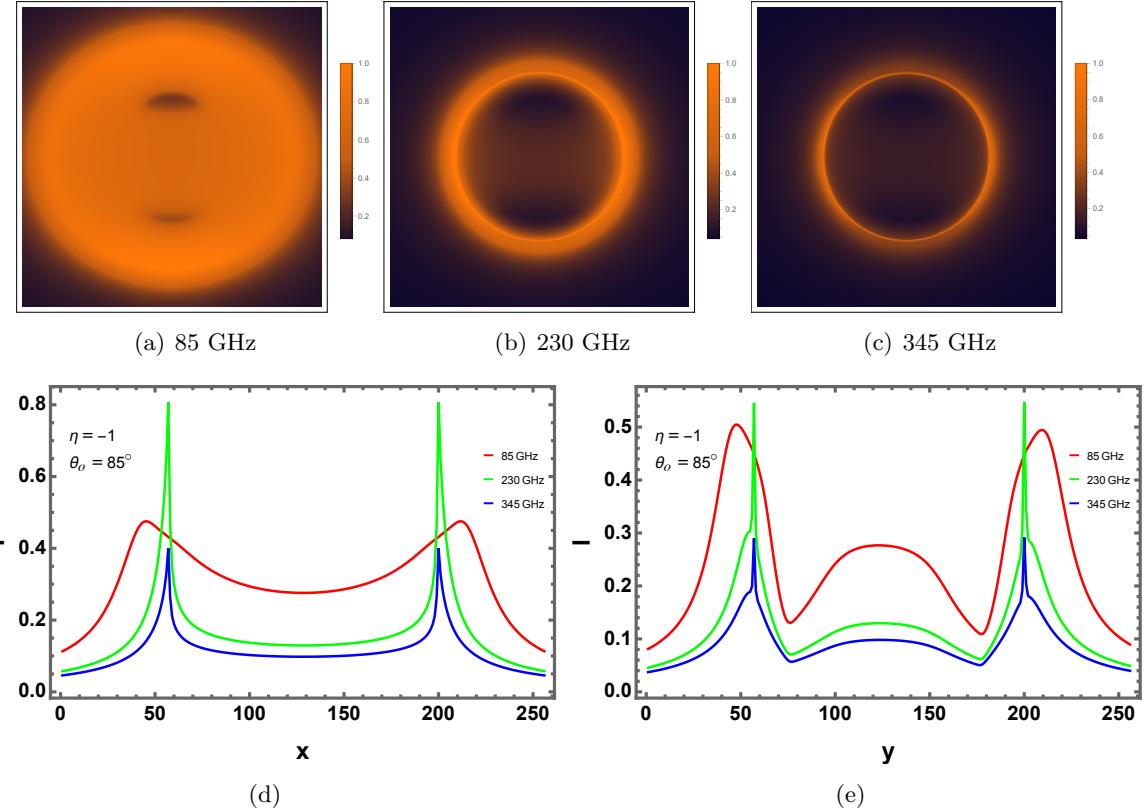


Figure 5: Intensity maps of the KZ black hole in the RIAF model with isotropic emission at different observing frequencies. The accretion flow follows the infalling motion, with fixed parameters $\theta_o = 85^\circ$ and $\eta = -1$. Panels (a)–(c) show the images at observing frequencies of 85 GHz, 230 GHz, and 345 GHz, respectively. Panels (d) and (e) display the corresponding intensity profiles along the horizontal (x -axis) and vertical (y -axis) cuts through the image center, where the curves for different frequencies are color-coded: red for 85 GHz, green for 230 GHz, and blue for 345 GHz.

respective Doppler patterns: for a purely radial inflow, the Doppler shifts remain nearly uniform in all directions, yielding a relatively sharp image; in contrast, a purely azimuthal flow produces strong redshifts and blueshifts in different azimuthal sectors. Gravitational lensing subsequently maps these sectors onto overlapping regions of the observer’s sky, such that photons with very different frequency shifts contribute to the same pixel, effectively smearing the image.

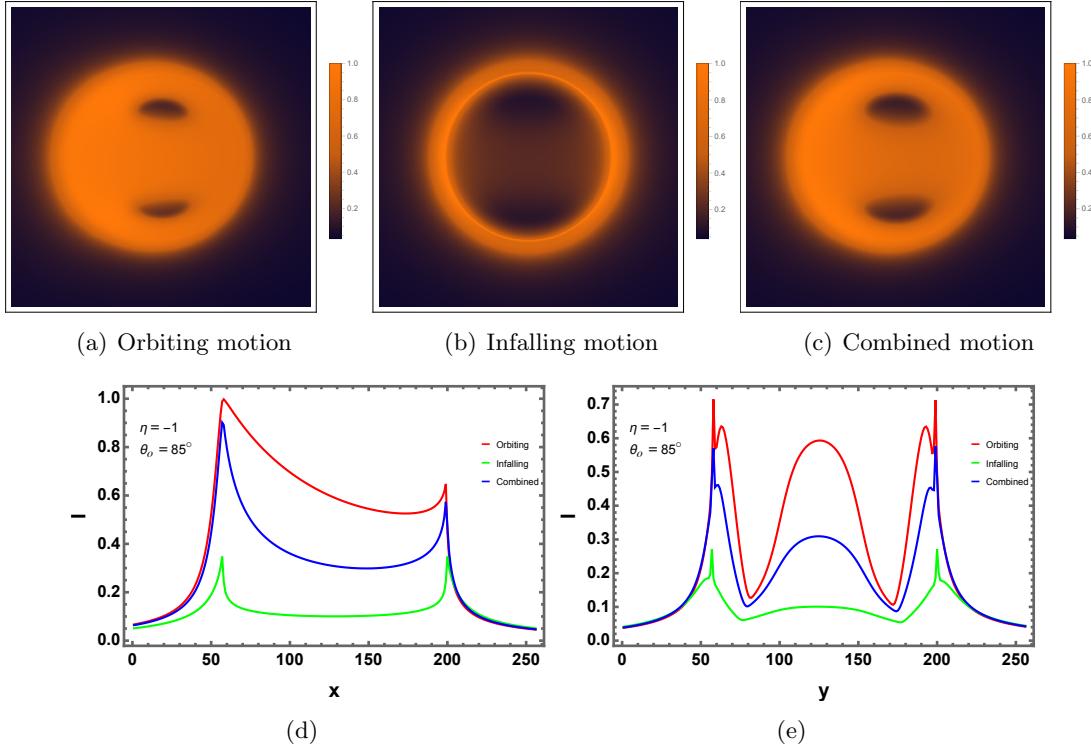


Figure 6: Intensity maps of the KZ black hole in the RIAF model with isotropic emission at different flow dynamics. Other parameters are set to $\nu_o = 230\text{ GHz}$, $\theta = 85^\circ$ and $\eta = -1$. Panels (a)–(c) show the images with flow of orbiting, infalling, and combined motion, respectively. Panels (d) and (e) display the corresponding intensity profiles along the horizontal (x -axis) and vertical (y -axis) cuts through the image center, where the curves for different frequencies are color-coded: red for orbiting motion, green for infalling motion, and blue for combined motion.

4.2 Anisotropic Radiation Case

In the previous subsection, we examined the imaging features of the KZ black hole in the RIAF thick-disk model under isotropic synchrotron emission. It is important to note, however, that synchrotron radiation is intrinsically anisotropic—its emissivity depends sensitively on the emission direction. This dependence is governed by the pitch angle θ_B , defined as the angle between the photon wave vector and the magnetic field in the fluid rest frame. In this subsection, we extend our analysis to account for this anisotropy, assuming a toroidal magnetic field as specified in Eq. (4.11).

Fig. 7 shows the intensity maps of the KZ black hole in the RIAF model with anisotropic synchrotron emission. The accretion flow follows the infalling motion, and the observing frequency is fixed at 230 GHz. Rows correspond to different inclination inclinations, $\theta_o = 0.001^\circ, 17^\circ, 60^\circ$, and 85° , while columns vary the deformation parameter as $\eta = -1, 0.1$, and 2 . To facilitate quantitative comparisons, the corresponding horizontal and vertical intensity profiles are displayed in Fig. 8 and Fig. 9, respectively. The overall morphology closely resembles that of the isotropic case (Fig. 2), featuring a pronounced bright ring encircling a dark central region—both of which expand with increasing η . At higher inclination angles, the brightness distribution becomes strongly nonuniform, and two dark zones emerge within the ring. A key distinction of the anisotropic case is the development of a vertically elongated and elliptical ring structure at large inclinations. This asymmetry originates from the angular dependence of synchrotron emissivity: for photons emitted from the upper and lower regions of the disk, the propagation direction is nearly perpendicular to the magnetic field, enhancing the emission and stretching the ring vertically.

5 BAAF Model

In the previous section, we examined the imaging characteristics of the KZ black hole surrounded by a phenomenological RIAF thick disk, considering both isotropic and anisotropic synchrotron emission. We now turn to a more analytically tractable model—the *BAAF*-to further explore the geometric and radiative properties of thick accretion disk.

The BAAF disk [6] describes a steady, axisymmetric accretion configuration in which the flow satisfies $u^\theta \equiv 0$, indicating that the streamlines are confined to constant- θ surfaces. Under this assumption, the mass conservation equation reduces to

$$\frac{d}{dr}(\sqrt{-g}\rho u^r) = 0, \quad \Rightarrow \rho = \rho_0 \frac{\sqrt{-g}u^r|_{r=r_0}}{\sqrt{-g}u^r}, \quad (5.1)$$

where $\rho_0 = \rho(r_0)$ is the rest mass density at a reference radius r_0 , typically chosen to be the event horizon, $r_0 = r_h$. Moreover, the projection of the energy-momentum conservation equation $\nabla_\mu T^{\mu\nu} = 0$ along the four-velocity yields

$$d\Xi = \frac{\Xi + p}{\rho} d\rho, \quad (5.2)$$

where Ξ denotes the internal energy density of the fluid. Defining the proton-to-electron temperature ratio $z = T_p/T_e$, which quantifies the relative thermal states of the two species in the plasma, the internal energy density satisfies

$$\Xi = \rho + \rho \frac{3}{2}(z+2) \frac{m_e}{m_p} \theta_e, \quad (5.3)$$

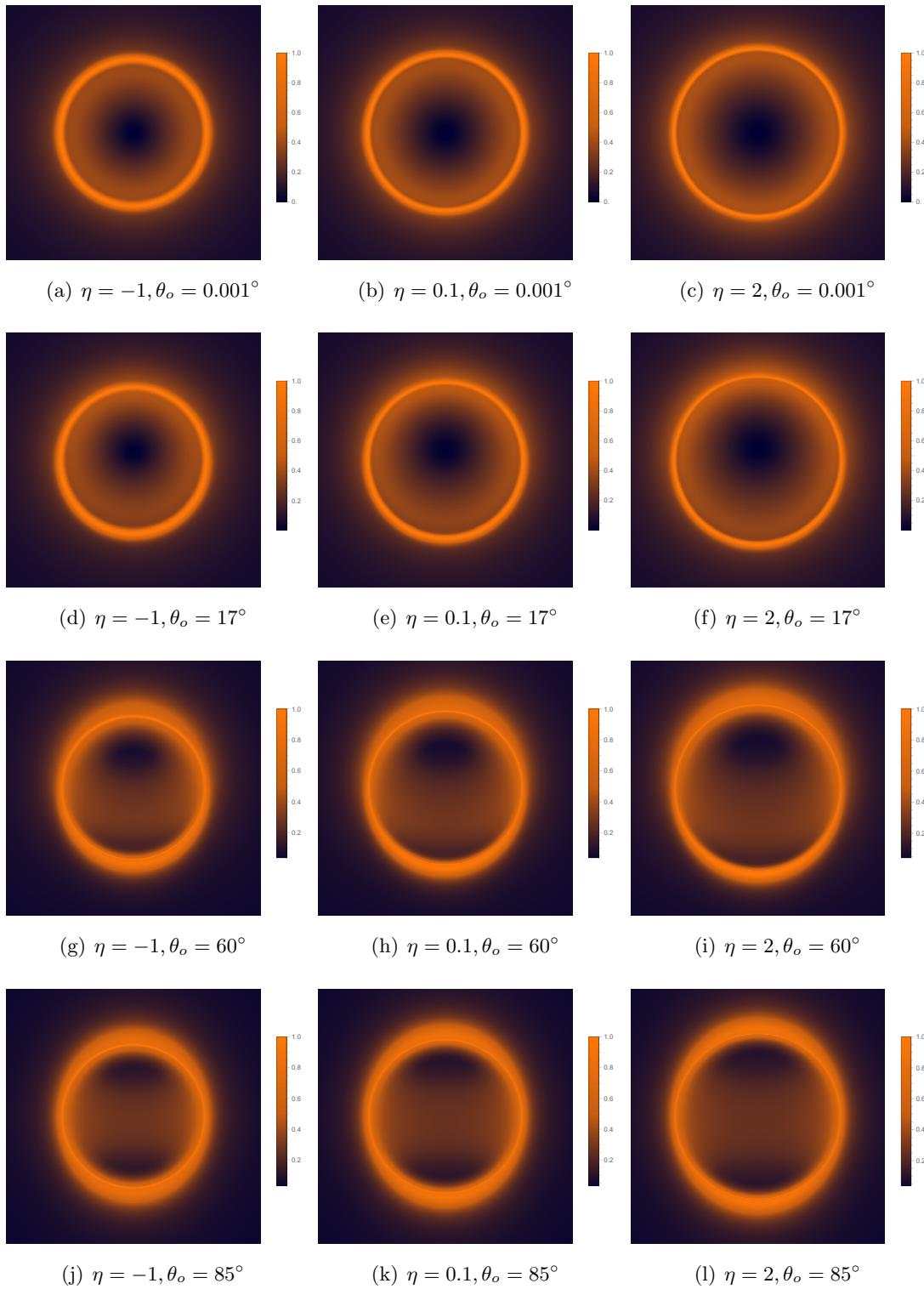


Figure 7: Intensity maps of the KZ black hole in the RIAF model with anisotropic emission. The accretion flow follows the infalling motion, the observing frequency is 230 GHz.

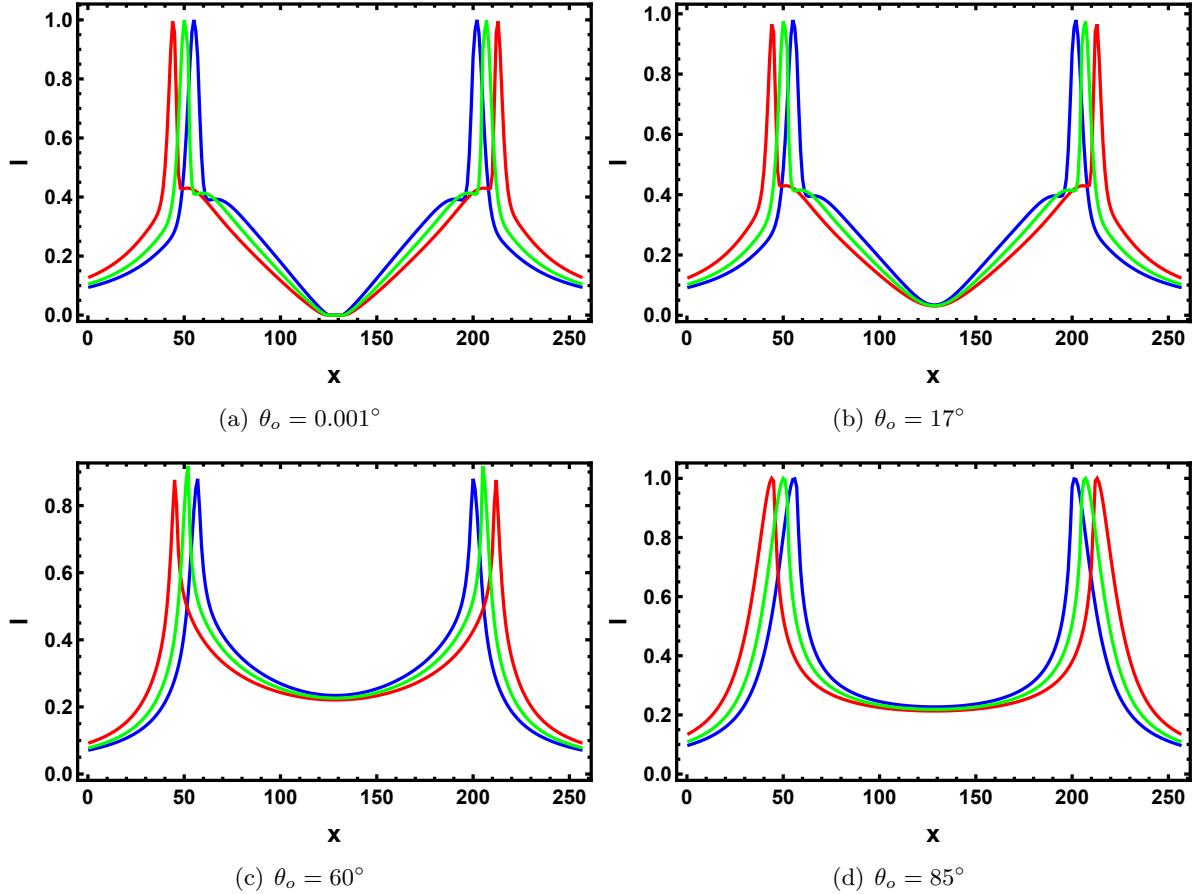


Figure 8: Intensity distribution along the screen x -axis for the KZ black hole in the RIAF model with anisotropic emission. The accretion flow follows the infalling motion, and the observing frequency is fixed at 230 GHz. The curves correspond to different values of η : red for $\eta = 2$, green for $\eta = 0.1$, and blue for $\eta = -1$.

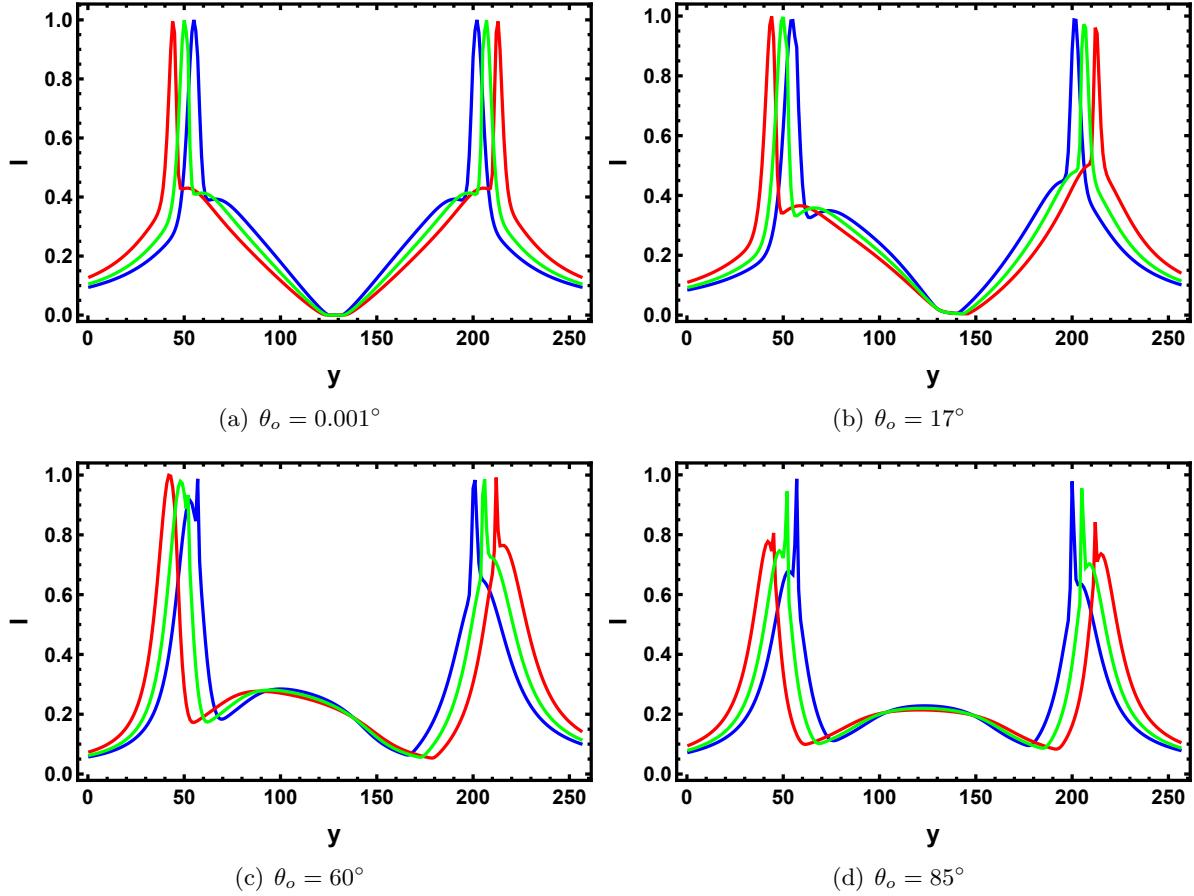


Figure 9: Intensity distribution along the screen y -axis for the KZ black hole in the RIAF model with anisotropic emission. The accretion flow follows the infalling motion, and the observing frequency is fixed at 230 GHz. The curves correspond to different values of η : red for $\eta = 2$, green for $\eta = 0.1$, and blue for $\eta = -1$.

where, as before, θ_e denotes the dimensionless electron temperature. Using the ideal gas equation of state, one obtains

$$p = nk_B(T_p + T_e) = \rho(1+z)\frac{m_e}{m_p}\theta_e, \quad (5.4)$$

and substituting Eqs. (5.3) and (5.4) into Eq. (5.2), one obtains after integration

$$\theta_e = (\theta_e)_0 \left(\frac{\rho}{\rho_0}\right)^{\frac{2(1+z)}{3(2+z)}}, \quad (5.5)$$

where $(\theta_e)_0 = \theta_e(r_0)$.

Assuming an infalling flow that satisfies Eq. (4.5), the analytical expressions for the rest-mass density and the electron temperature become

$$\begin{aligned} \rho(r, \theta) &= \rho(r_h, \theta) \left(\frac{r_h}{r}\right)^2 \sqrt{\frac{1}{1 - g^{rr}}} = \rho(r_h, \theta) \sqrt{\frac{r_h^4}{2r^3 + \eta r}}, \\ \theta_e(r, \theta) &= \theta_e(r_h, \theta) \left(\frac{r_h}{r}\right)^{\frac{4(1+z)}{3(2+z)}} \left(\frac{1}{1 - g^{rr}}\right)^{\frac{1+z}{3(2+z)}} = \theta_e(r_h, \theta) \left(\frac{r_h^4}{2r^3 + \eta r}\right)^{\frac{1+z}{3(2+z)}}. \end{aligned} \quad (5.6)$$

For convenience in subsequent analysis, the angular dependence of $\rho(r_h, \theta)$ is modeled by a Gaussian profile, while in the conical solution we take $\theta_e(r_h, \theta)$ to be constant:

$$\rho(r_h, \theta) = \rho_h \exp\left[-\left(\frac{\sin \theta - \sin \theta_J}{\sigma}\right)^2\right], \quad \theta(r_h, \theta) = \theta_h. \quad (5.7)$$

Here, θ_J specifies the central latitude of the distribution and σ its angular width. We consider an equatorially symmetric thick disk with $\theta_J = \pi/2$ and $\sigma = 1/5$. For the M87* black hole, observational estimates suggest $\rho_h \simeq 1.5 \times 10^3 \text{ g cm}^{-3} \text{s}^{-2}$, and $\theta_h \simeq 16.86$, corresponding to $n_h \simeq 10^6 \text{ cm}^{-3}$, $T_h \simeq 10^{11} \text{ K}$.

Assuming stationarity, axisymmetry, and the ideal MHD condition [62], the magnetic field takes the general form [6, 62]

$$B^\mu = \frac{\Psi}{\sqrt{-g} u^r} \left((u_t + \Omega_B u_\phi) u^\mu + \delta_t^\mu + \Omega_B \delta_\phi^\mu \right), \quad (5.8)$$

where $\Psi = F_{\theta\phi}$ is a component of the electromagnetic tensor and Ω_B denotes the field line angular velocity, which characterizes the rotation of magnetic field lines. The spatial part B^i is parallel to u^i , indicating that the magnetic field is frozen into the streamlines. For simplicity and without loss of generality, we set $\Omega_B = 0$ in what follows. In this work, we adopt a separable magnetic monopole configuration [63]

$$\Psi = \Psi_0 \operatorname{sign}(\cos \theta) \sin \theta. \quad (5.9)$$

Finally, based on the methodology introduced earlier, we present the black hole images illuminated by the BAAF disk and analyze their corresponding intensity and polarization distributions.

Fig. 10 illustrates the total intensity maps of the KZ black hole obtained from the BAAF disk model with anisotropic synchrotron emission. The accretion flow is assumed to follow a purely infalling motion, and the observing frequency is fixed at 230 GHz. Each panel corresponds to a specific combination of the deformation parameter η and the observer's inclination angle θ_o , as indicated. The corresponding horizontal and vertical intensity profiles are presented in Figs. 11 and 12, respectively, providing a quantitative representation of the brightness distribution. The bright ring visible in all images originates from higher-order images, while the central dark region corresponds to the event horizon. Overall, the dependence of image morphology on the deformation parameter η and inclination angle θ_o closely resembles that observed in the RIAF thick disk model: increasing η enlarges both the bright ring and the inner dark region. The dependence on the inclination angle likewise follows the symmetry trends discussed previously. However, a few notable distinctions emerge. Compared with the RIAF model, the bright ring in the BAAF disk case appears generally thinner, and the separation between the primary and higher-order images becomes more pronounced. Furthermore, at large inclination angles, the higher-order image does not exhibit the two distinct dark zones observed in the RIAF case. This suggests that, in the RIAF model, radiation from off-equatorial regions more effectively obscures the event-horizon silhouette. Such differences may stem from the fact that, for certain parameter choices, the BAAF disk in the conical approximation is geometrically thinner than the RIAF disk in some regions.

We now turn to the analysis of the polarization properties. Fig. 13 presents the spatial distributions of the observed Stokes parameters, \mathcal{I}_o , \mathcal{Q}_o , \mathcal{U}_o , and \mathcal{V}_o , for the KZ black hole in the BAAF model. The accretion flow is assumed to be purely infalling, with fixed parameters $\eta = 2$, $\theta_o = 85^\circ$, and an observing frequency of 230 GHz. The \mathcal{I}_o panel shows the total intensity distribution, where the arrows indicate the electric vector position angle (EVPA), Φ_{EVPA} , and the color encodes the linear polarization degree, P_o . Since the EVPA is predominantly perpendicular to the global magnetic field, B^μ , the polarization pattern suggests that the magnetic field is roughly radial. The combined distributions of \mathcal{Q}_o and \mathcal{U}_o qualitatively determine the EVPA orientation, while $\mathcal{V}_o < 0$ corresponds to right-handed circular polarization.

Fig. 14 presents the corresponding polarized images of the KZ black hole in the BAAF model with anisotropic emission, for various values of η and θ_o . The accretion flow is assumed to be purely infalling, and the images are computed at an observing frequency of 230 GHz. The polarization patterns exhibit a clear dependence on both parameters, with significant variations in the structure and intensity of the polarized flux across the different panels. Overall, the EVPA pattern is nearly perpendicular to the radial direction, reflecting the assumed alignment of the magnetic field with the radially infalling flow. In the vicinity of the higher-order images, the polarization exhibits rapid spatial variations. Along each row of panels, as η increases, both the size of the higher-order images and the

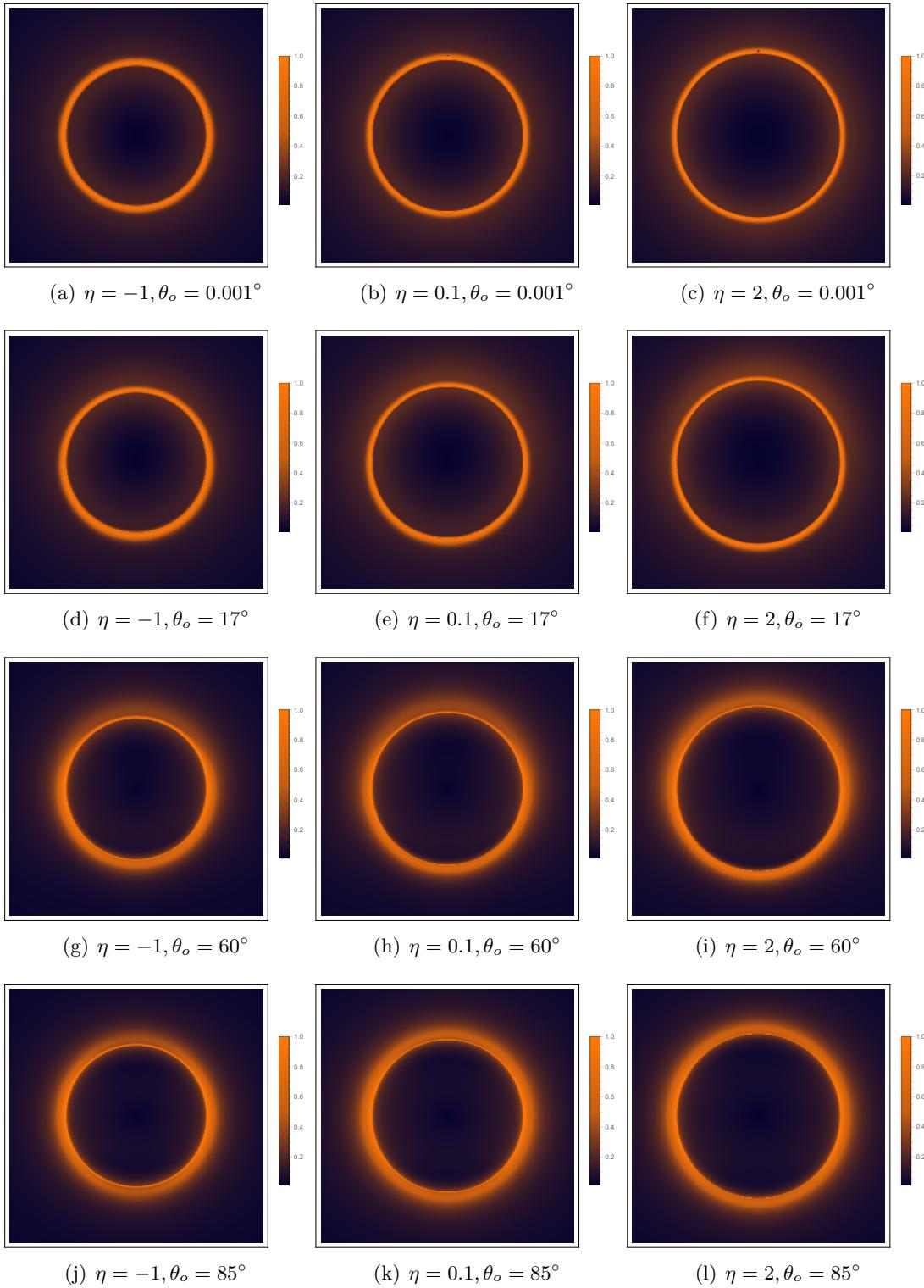


Figure 10: Intensity maps of the KZ black hole in the BAAF model with anisotropic emission. The accretion flow follows the infalling motion, the observing frequency is 230 GHz.

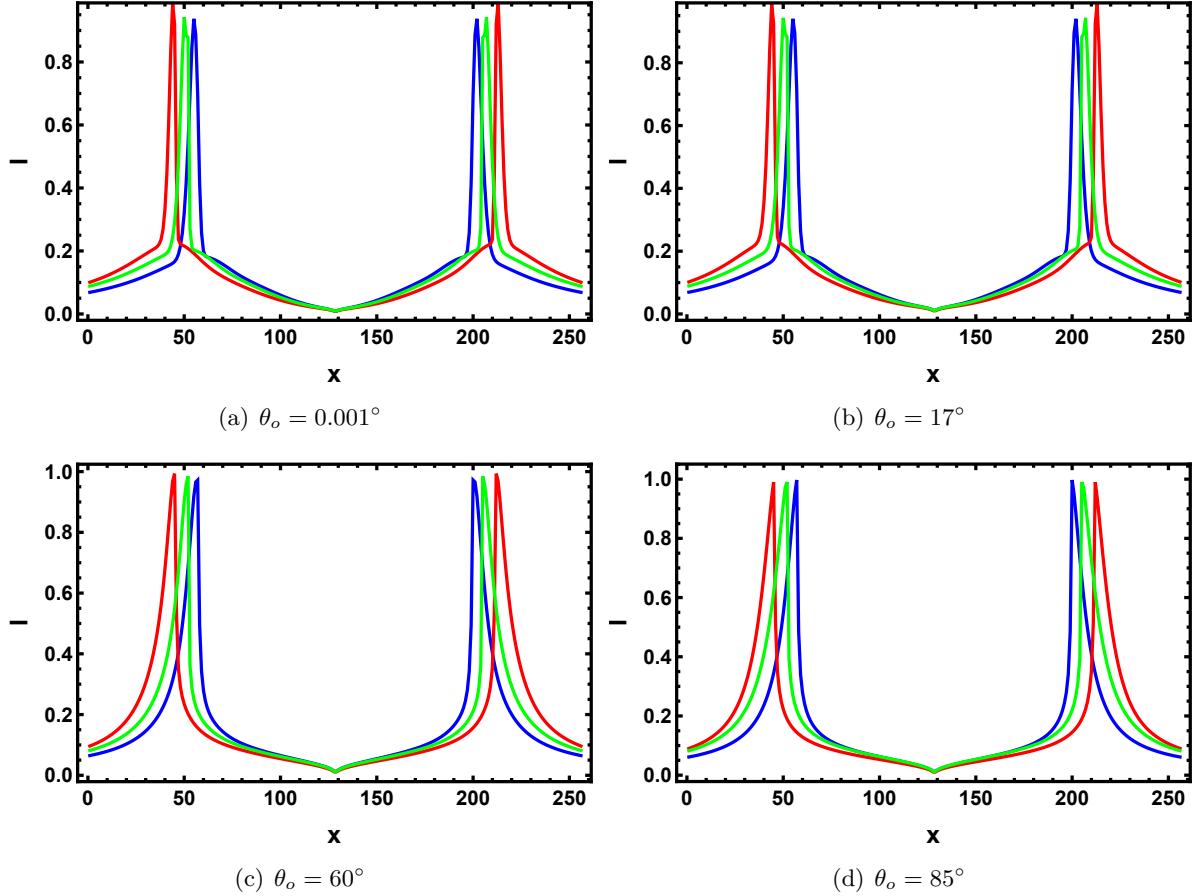


Figure 11: Intensity distribution along the screen x -axis for the KZ black hole in the BAAF model with anisotropic emission. The accretion flow follows the infalling motion, and the observing frequency is fixed at 230 GHz. The curves correspond to different values of η : red for $\eta = 2$, green for $\eta = 0.1$, and blue for $\eta = -1$.

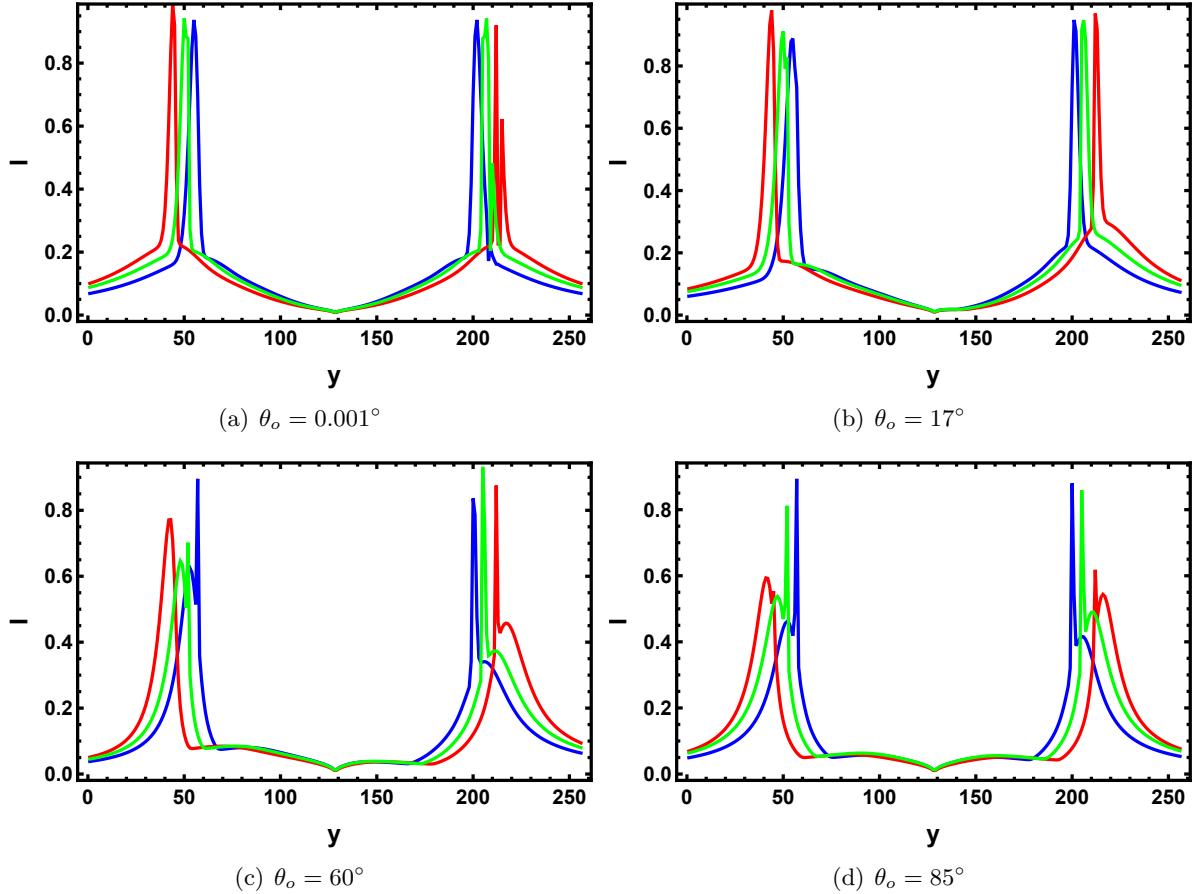


Figure 12: Intensity distribution along the screen y -axis for the KZ black hole in the BAAF model with anisotropic emission. The accretion flow follows the infalling motion, and the observing frequency is fixed at 230 GHz. The curves correspond to different values of η : red for $\eta = 2$, green for $\eta = 0.1$, and blue for $\eta = -1$.

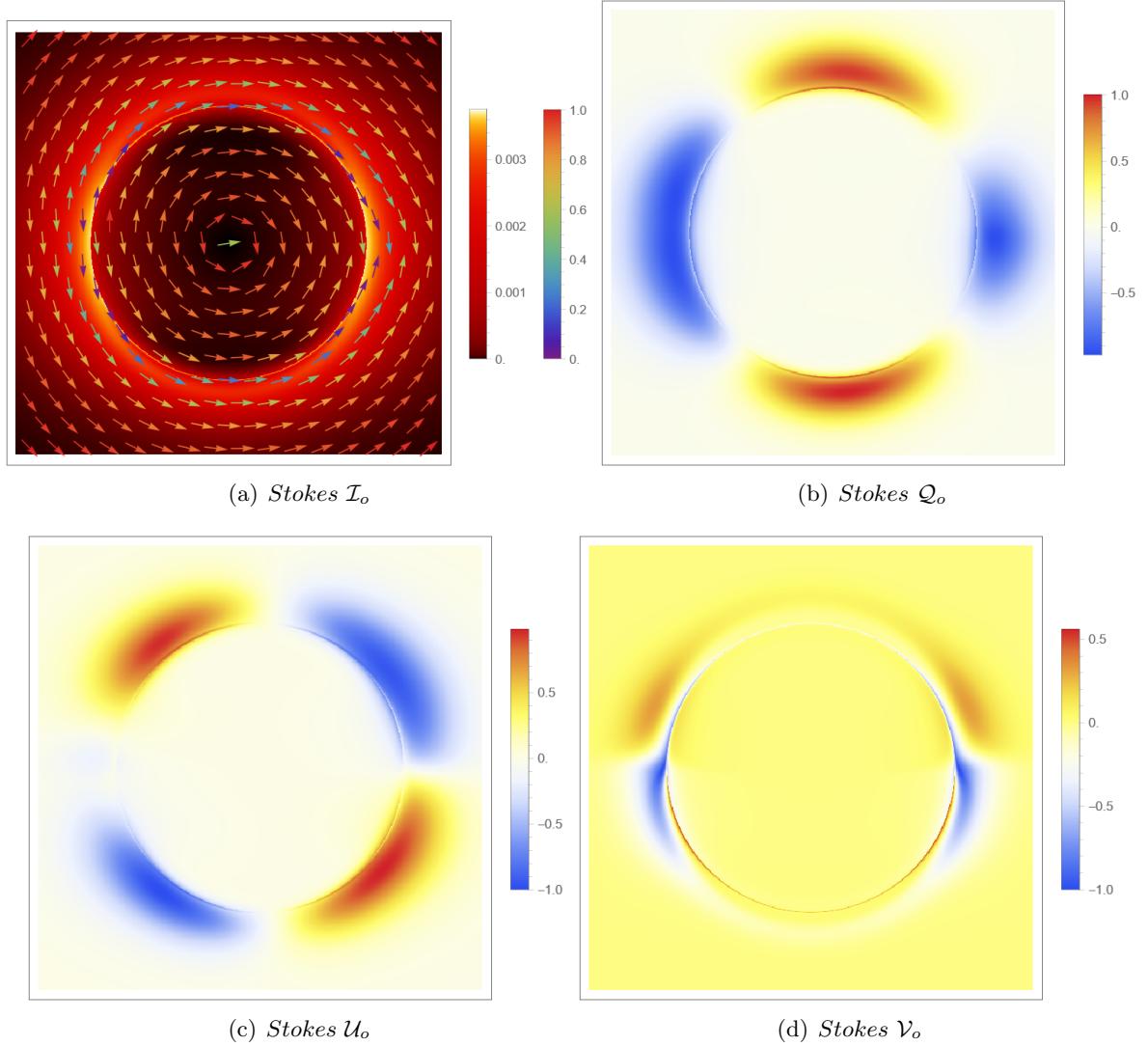


Figure 13: The resulting Stokes parameters \mathcal{I}_o , \mathcal{Q}_o , \mathcal{U}_o , \mathcal{V}_o under the BAAF disk model. The dynamics of the accretion flow is infalling motion, with fixed parameters $\eta = 2$, $\theta_o = 85^\circ$, 230 GHz.

dark region expand, accompanied by a corresponding change in the EVPA distribution. Across each column, increasing the inclination angle causes the EVPA to deviate slightly from the purely azimuthal orientation, while the polarized intensity within the dark region becomes stronger. It is noteworthy that, in thin-disk models, the dark region corresponds to the black hole horizon, from which no light reaches the observer, resulting in an absence of polarization within this region. In contrast, for the present geometrically thick disk, gravitational lensing enables emission from regions above and below the equatorial plane to obscure the horizon contour, producing polarization vectors distributed over the entire image plane.

6 Summary

In this work, we investigated the imaging properties of spherically symmetric black holes in the Konoplya–Zhidenko (KZ) spacetime under the presence of geometrically thick accretion flows. We first reviewed the basic properties of the KZ black hole, including the numerical determination of the event horizon and the dependence of the photon sphere on the deformation parameter η . In the small-deformation regime, we derived an approximate analytic expression for the photon ring.

We then considered two representative models of thick accretion flows: a phenomenological RIAF model and the analytical BAAF model. The synchrotron radiation from thermal electrons in the magnetofluid was computed by numerically integrating the null geodesics and radiative transfer equations to obtain the corresponding black hole images.

For the RIAF model, we first analyzed the isotropic emission case with different fluid motions—namely, orbiting, infalling, and combined motions—and several observing frequencies. The resulting images show that, as the deformation parameter η increases, both the photon ring and the inner dark region expand in size. For polar observers, the brightness distribution is nearly axisymmetric, whereas for larger inclination angles, the brightness becomes asymmetric and the central dark region splits into two parts due to off-equatorial emission. The morphology and contrast of the image are also sensitive to the flow dynamics and observing frequency: orbiting flows tend to blur the photon ring, making higher-order and primary images less distinguishable, while changes in observing frequency can either enhance or suppress the image contrast depending on the model parameters.

Next, we examined the case of anisotropic synchrotron emission in the RIAF model with a toroidal magnetic field and infalling flow. Compared to the isotropic case, the brightness distribution becomes noticeably asymmetric for high-inclination observers. The photon ring appears vertically elongated and slightly elliptical, reflecting the geometry of the underlying magnetic field configuration.

Finally, we analyzed the imaging and polarization properties of the BAAF disk, which assumes

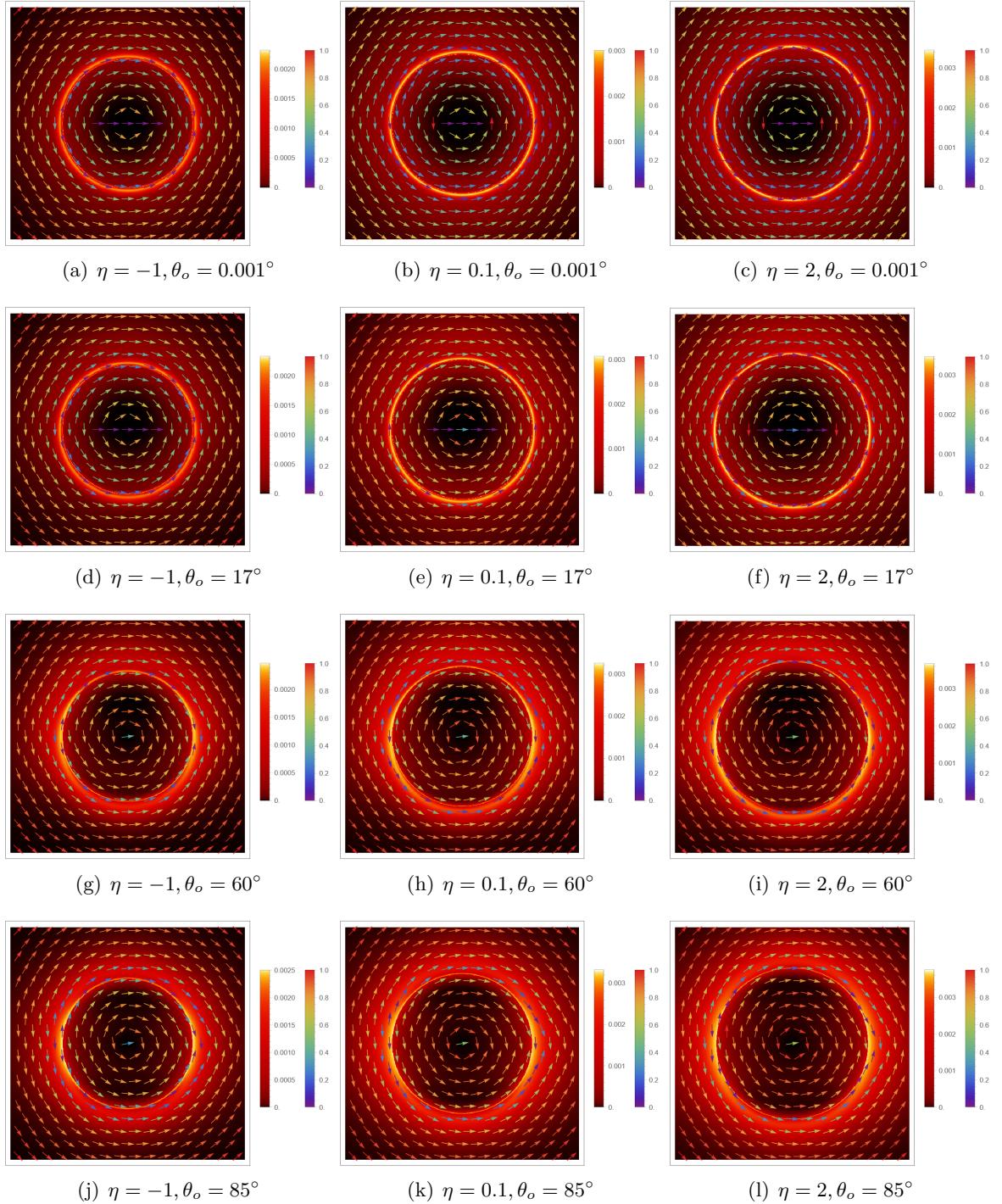


Figure 14: Polarized images of the KZ black hole in the BAAF model with anisotropic emission. The accretion flow follows the infalling motion, the observing frequency is 230 GHz.

an infalling steady-state flow with $u_\theta \equiv 0$. The resulting images show a narrower photon ring and a darker central region compared to the phenomenological model. This difference arises because, under certain parameter choices, the BAAF disk in the conical approximation is geometrically thinner than the RIAF disk in some regions. For the polarized images, the polarization intensity closely follows the total brightness distribution, with stronger polarization in brighter regions. Both the polarization degree and orientation exhibit clear dependence on the deformation parameter and viewing angle, suggesting that the polarization signatures of KZ black holes can effectively probe the intrinsic spacetime structure. It is also worth noting that, unlike thin-disk models where the inner shadow remains unpolarized, in thick disks the gravitational lensing of off-equatorial emission allows polarization vectors to appear across the entire image plane.

Acknowledgments

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