Generating Discrete Random Numbers Using Poisson Distribution

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1 Problem Description

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event. The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume. In this assignment, we have to generate N number of random variables using the Poisson Distribution. We also have to show the curve plotting the probability distribution and observed frequencies in fraction (frequency/N) i.e. observed probability. For this problem, the parameter $\lambda = 1$.

2 Definitions

A discrete random variable X is said to have a Poisson distribution with parameter $\lambda > 0$, if, for $k = 0, 1, 2, \cdots$ the probability mass function of X is given by,

$$Mass, p(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & \text{if } x \epsilon \{0, 1, 2, \cdots\} \\ \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$Distribution, F(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & \text{if } x < 0 \\ \\ e^{-\lambda} \sum_{i=0}^{\lfloor x \rfloor} 2, & \text{otherwise} \end{cases} \tag{2}$$

3 Code

Below is the Python code to simulate this problem.

```python
#!/home/heisenberg/anaconda3/bin/python

import matplotlib.pyplot as plt
import math
from random import seed
from random import random
from random import uniform

# This function is to plot a single curve

def plotSingleGraph(x, y):
    plt.plot(x, y)
    plt.xlabel('X')
    plt.ylabel('CDF')
    plt.title('Poisson Distribution')
    plt.show()

# This function calculates the CDF or F(x) of the Poisson Distribution

def getCDF(x, Lambda):
    if x < 0:
        return 0
    else:
        return math.exp(-1.0 * Lambda) * sum([(pow(Lambda, i) / math.factorial(i)) for i in range(math.floor(x+1))])

# This function calculates the PDF or p(x) of the Poisson Distribution

def getPx(x, Lambda):
    if x < 0:
        return 0
    else:
        return math.exp(-1.0 * Lambda) * math.pow(Lambda, x) / math.factorial(x)


# This function calculates the Frequencies and Cummulative Frequencies of random numbers between 0 and 1.
def getExperimentalValues(cdfs):
    discreteValues = []

    seed(1)

    for _ in range(1000):
        # generate random numbers between 0-1
        value = random()

        discreteVal = 0

```

```python
        # Finding the smallest number which is larger than 'value'. It is actually upper bound.
        # It can be solved using both Bruteforce and Binary Search. I used the latter.

        lo = 0
        hi = len(cdfs)-1
        while lo <= hi:
            mid = int(lo + (hi-lo)/2)

            if value > cdfs[mid]:
                lo = mid+1
            else:  # value <= cdfs[mid]
                hi = mid-1
                discreteVal = mid

        discreteValues.append(discreteVal)

    freqencies = [0 for i in range(21)]

    for val in discreteValues:
        freqencies[val] = freqencies[val] + 1

    for i in range(len(freqencies)):
        freqencies[i] = freqencies[i] / 1000

    noncumulative = [i for i in freqencies]

    for i in range(1, len(freqencies)):
        freqencies[i] = freqencies[i] + freqencies[i-1]

    return discreteValues, freqencies, noncumulative


# In this function I calculated p(x), F(x), frequencies/N, cumulative frequencies/N and plot them.

def main():
    ####### Theoratical values #######
    x = [i for i in range(0, 21)]
    Lambda = 1

    ######### p(x) ###########

    Px = [getPx(i, Lambda) for i in x]
    plt.plot(x, Px, label="x vs P(x)")

    ########## F(x) ###########
    cdfs = [getCDF(i, Lambda) for i in x]
    plt.plot(x, cdfs, label="x vs F(x)")

    ######## Experimental values #########
    # print(cdfs)

```

```
    discreteValues, cummulative_freq, noncumulative_freq =
   getExperimentalValues(
        cdfs)
    plt.plot(x, cummulative_freq, label="x vs cummulative frequencies")
    plt.plot(x, noncumulative_freq, label="x vs non-cummulative
   frequencies")

    ############# Plot them ###############
    plt.xlabel('x - axis')
    plt.ylabel('y - axis')
    plt.title('Poisson')
    plt.legend()
    plt.show()


if __name__ == '__main__':
    main()
```

Listing 1: Python Code

4 Results

Give a schematic of the experimental setup(s) used in the experiment (see figure **??**). Give the description of abbreviations either in the figure caption or in the text. Write a description of what is going on.

and eventually arrived to the balanced photodiode as seen in the figure **??**.

5 Results

In this section you will need to show your experimental results. Use tables and graphs when it is possible. Table 1 is an example.

Table 1: Every table needs a caption.

x (m)	V (V)
0.0044151	0.0030871
0.0021633	0.0021343
0.0003600	0.0018642
0.0023831	0.0013287

Analysis of equation **??** shows ...

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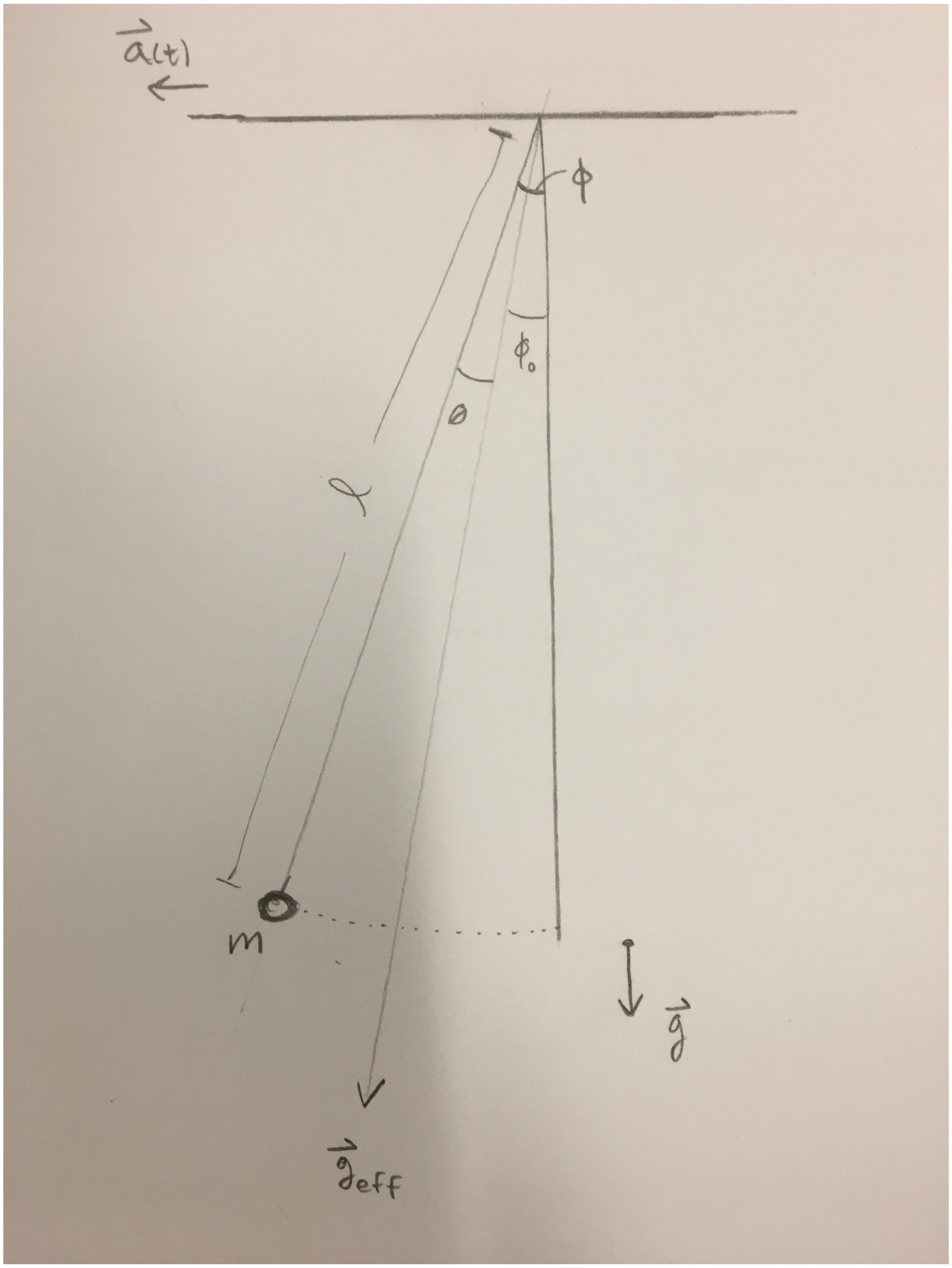

Figure 1: Pendulum that starts at rest in an accelerating frame. If the acceleration is not constant then the apparent vertical, and thus ϕ_0 will change with time

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For example, it is easy to conclude that the experiment and theory match each other rather well if you look at Fig. **??** and Fig. **??**.

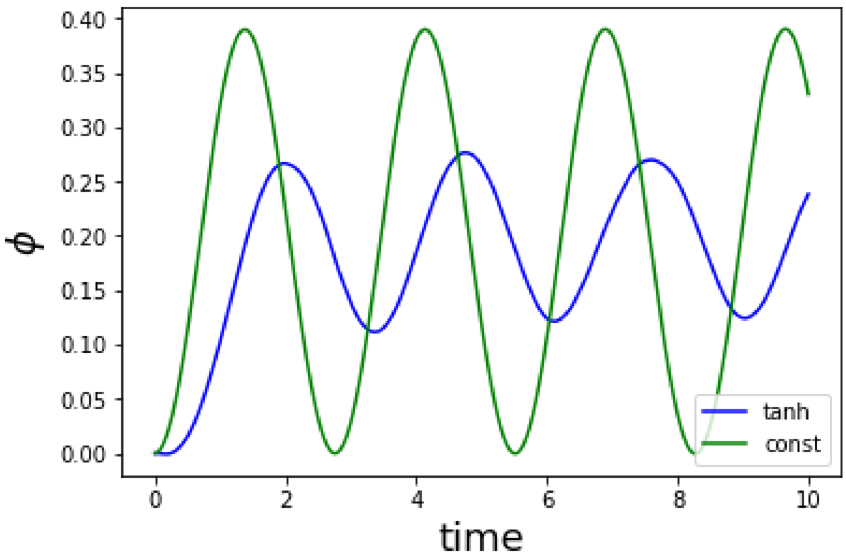

Figure 2: Hyperbolic tangent acceleration vs immediate constant acceleration. The slow approach to the same asymptotic value of 2 meters per second per second induces a lag in the oscillation and also diminishes the amplitude of oscillation.

6 Conclusions

Here you briefly summarize your findings. Did you learn any new physics? Was everything as expected?

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7 Future Work

Since you had limited time to work on this project, what questions are left outstanding? What would be your next steps?

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References

[1] A. C. Melissinos and J. Napolitano, *Experiments in Modern Physics*, (Academic Press, New York, 2003).

[2] N. Cyr, M. Têtu, and M. Breton, IEEE Trans. Instrum. Meas. **42**, 640 (1993).

[3] *Expected value*, available at `http://en.wikipedia.org/wiki/Expected_value`.