$Derek\ Haynes$
$Bridge\ to\ Abstract\ Math$
$Module\ 9$

Section 3.3 Partitions

2.

For the given set A, determine whether $\mathscr{P}$ is a partion of A.

(c) $A = \{1, 2, 3, 4, 5, 6, 7\}$, $\mathscr{P} = \{\{1, 3\}, \{5, 6\}, \{2, \}, \{7\}\}$

Solution:

$\mathscr{P}$ is a partition of A because it satisfies all three conditions of the defintion of a partition

7.

(c) Describe the equivalence relation on $\mathbb{R}$ with the partition $\{(-\infty, 0), \{0\}, (0, \infty)\}$

Solution:

The relation is

reflexive because xRx

symmetric because xRy and yRx

and it is transitive $x, y, z \in \mathbb{R}$ where xRy and yRz therefore xRz.

9.

List the order pairs in the equivalence relation on $A = \{1, 2, 3, 4, 5\}$ associated with the following partitions:

(b) $\{\{1\}, \{2\}, \{3, 4\}, \{5\}\}$

Solution:

$\{(1, 1), (2, 2), (3, 4), (4, 3), (4, 4), (5, 5)\}$

10.

(b) (Completing a part of the proof of Theorem 3.3.2): Suppose $\mathscr{P}$ is a partition of A and suppose that xQy if there exists $C \in \mathscr{P}$ such that $x \in$ and $y \in C$. Prove that Q is reflexive on A

Solution:

Let $x \in A$.

As $\mathscr{P}$ is a partition of A

then $\cup_{C \in \mathscr{P}} C = A$

therefore $x \in \cup_{C \in \mathscr{P}} C$ because $x \in A$.

So there exists a $C \in P$ such that $x \in C$

therefore $x \in C$ and $x \in C$,

showing that xQx

and Q is reflexive

4.1 Functions as Relations

6(d)

Let R = $\{(x, y) = \mathbb{N} \times \mathbb{N} : 2x^2 - y = 1\}$ Prove that R is a function with domain $\mathbb{N}$

Solution:

Suppose that x is a natural number.

Then $2x^2 - 1$ is an integer

and $(x, 2x^2 - 1) \in R$

so R is a relation with domain $\mathbb{N}$

ii)

Suppose that (x, y) and $(x, z) \in R$

Then $2x^2 - 1 = y$ and $2x^2 - 1 = z$

Hence, $y = z$

From i) and ii) we conclude that x is in the domain set and y in the codomain such that $(x, y) \in R$.

Therefore R defines a function with domain $\mathbb{N}$

7.

Complete the proof of Theorem 4.1.1. That is, prove that if (i) $Dom(f) = Dom(g)$ and (ii) for all $x \in Dom(f), f(x) = g(x), then f = g$.

Solution:

Suppose that $x \in Dom(f)$.

Then $(x, y) \in f$ for some y, and because f = g.

we have $(x, y) \in g$.

Therefore $x \in Dom(g)$.

Hence

$Dom(f) \subseteq Dom(g)$

Similarly, $x \in Dom(g)$

Then $(x, y) \in g$ for some y, because g = f.

then we have $(x, y) \in f$

Therefore $x \in Dom(f)$

This shows that $Dom(g) \subseteq Dom(f)$

Proving condition (i) of 4.1.1

Addtionally, since $f(x) = g(x), forall$x \in Dom(f)$

therefore $g(x) = f(x) = y$

hence $(x, y) \in g$

and for all x, $g(x) = f(x)$ so, $f = g$

Hence (ii) is proved.

9(d).

Let the Universe be $\mathbb{R}$ and let A = $[1, 3)$

Find $X_A(2) - X_A(0.2)$

Solution:

$A = [1, 3)$ is closed on 1 and open at 3$

The characteristic funtion,

$$X_A(x) = \begin{cases} 1 & \text{if x} \in A \\ 0 & \text{if x} \notin A \end{cases}$$

Then

$X_A(1) = 1$ and $X_A(0.2) = 0$

then $X_A(1) - X_A(0) = 1 - 0 = 1$

10(d)

Let U be the universe. Suppose that $A \subseteq U$ with $A \neq \emptyset$, and $A \neq U$. Let X_A be the characteristic function of A.

Find:

$\{x \in U : X_A(x) \leq 1\}$

Solution:

Suppose $\{x \in U : X_A(x) \leq 1\}$

Then $x \in U$

and as $X_A(x) \leq 1$

Therefore,

$x \in A, X_A(x) = 1$

$x \in A, X_A(x) = 0$

Furthermore

$\{x \in U : X_A(x) \leq 1\}$ = $\{x \in U : x \in A\} \cup \{x \in U : x \notin A\}$

= $A \cup (U - A)$ `Theorem 2.2.2`

= U

Hence,

$\{x \in U : X_A(x) \leq 1\} = U$

13 (d)

For the canonical map $f : \mathbb{Z} \to \mathbb{Z}_6$ find all pre-images of $\bar{1}$

f is the canonical map for the relation on congruence modulo 6 on $\mathbb{Z}, the preimages of$\bar 1$

are any x such that $f(x) = \bar{1} = \{6k + 1\}$ where k is an integer

Therefore, the pre-mianges of $\bar{1}$ are ..., -17,-11,-5, 1, 7, 13,19, ...

In [ ]: