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Bridge to Abstract Math
Midterm Part 2

Problem 1

Use a truth table to determine whether $(\sim P) \wedge Q$ and $Q \Rightarrow \sim P$ are equivalent or not

Solution:

Solution:

P	Q	$\sim P$	$\sim P \wedge Q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

P	Q	$\sim P$	$Q \Rightarrow \sim P$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

The 4th column in both tables are not equal, therefore the statements are `not` equivalent

Problem 2

Let $a, b,$ and c be integers, Prove that if a divides $b - 1$ and $c + 1$, then a divides $bc + 1$.

Solution

suppose that a divides $b - 1$ and $c + 1$

then,

$b - 1 = ak$ and $c + 1 = al$ (Where $k4andl$$ are integers)

then.

$b = ak + 1$ and $c = al - 1$

and $bc + 1 = (ak + 1)(al - 1) + 1$

After some algebraic simplification...

$(ak + 1)(al - 1) + 1 = a(akl - k + l)$. (Where $akl - k + l$ is an integer)

furthermore $a(akl - k + l)$ is divisible by a

and since $bc + 1 = a(akl - k + l)$

proves $bc + 1$ is divisible by a

Problem 3

For all sets X and Y in a universe U, prove that $(X \cap Y)^c = X^c \cup Y^c$

Solution:

i) Suppose $x \in (X \cap Y)^c$ then $x \notin X \cap Y$

furthermore $x \notin X$ or $x \notin Y$

then $x \in X^c$ or $x \in Y^c$

then $x \in X^c \cup Y^c$

therefore if $x \in (X \cap Y)^c$ then $x \in X^c \cup B^c$

ii) Suppose $x \in X^c \cup Y^c$ then $x \in X^c$ or $x \in Y^c$

then $x \notin X$ or $x \notin Y$

furthermore $x \notin X \cap Y$

then $x \in (X \cap Y)^c$

therefore if $x \in X^c \cup Y^c$ then $x \in (X \cap Y)^c$

proving

$(X \cap Y)^c = X^c \cup Y^c$

Problem 4

Let $P,$ $Q,$ R and S be sets. Prove that $(P \times Q) \cap (R \times S) = (P \cap R) \times (Q \cap S)$.

Solution:

Suppose $(P \times Q) \cap (R \times S) = (P \cap R) \times (Q \cap S)$

then $(x, y) \in (P \times Q) \cap (R \times S)$ iff $(x, y) \in (P \times Q)$ and $(x, y) \in (R \times S)$

iff $(x \in P$ and $y \in Q)$ and $(x \in R$ and $y \in S)$

iff $x \in P \cap R$ and $y \in Q \cap S$

iff $(x, y) \in (P \cap R) \times (Q \cap S)$

Proving $(P \times Q) \cap (R \times S) = (P \cap R) \times (Q \cap S)$

Problem 5

Let $\mathscr{A} = \{A_i : i \in \mathbb{N}\}$ be an indexed family of sets with the property that $A_1 \supseteq A_2 \supseteq A_3, \ldots . \supseteq A_i \supseteq . \ldots$

Find $\cap_{i=1}^{20} A_i$. Justify your answer giving a rigorous proof.

Suppose $\mathscr{A} = \{A_i : i \in \mathbb{N}\}$

Given the property that

$A_i \supseteq A_{i+1}$ meanst that Ai is a super set of A_{i+1}

That is A_i contains $Ai + 1$ and some more values ...

Furthermore, lets suppose that

$A_1 = \{A_2, B_1\}$ (where B_i is a a random set}

$A_2 = \{A_3, B_2\}$

$A_3 = \{A_4, B_3\}$

$A_4 = \{A_5, B_4\}$

$\vdots$

$A_{20} = \{A_{21}, B_{20}\}$

Then

$A_1 \cap A_2 = A_2$ as $(A_1 \supseteq A_2)$

$A_2 \cap A_3 = A_2$ as $(A_2 \supseteq A_3)$

$A_1 \cap A_3 = A_3$ as $(A_1 \supseteq A_2 \supseteq A_3)$

furthermore

$A_1 \cap A_2 \cap A_3 \cap A_4 \cap . \ldots A_{20} = A_{20}$

Thus

$\cap_{i=1}^{20} A_i = A_{20}$

Problem 6

Use the Principle of Mathematical Induction (PMI) to prove that $2^{2n} - 1$ is divisible by 3 for all $n \in \mathbb{N}$.

Solution:

i) Base case for $n = 1$

$2^{2(1)} - 1 = 3$

3 is divisible by 3

Thus, the result is true for n = 1.

ii) Suppose that the result is true for some variable k (where n = k) and is divisible by 3. Then we want to prove that it is also true when n = k+1.

Assume $2^{2k} - 1 = 3n, n \in \mathbb{N}$

For the Left Hand Side

$2^{2(k+1)} - 1 = 2^{2k+2} - 1$

$= 2^2 k \cdot 2^2 - 1$

$= 2^2 k \cdot 4 - 1$

$= 3 \cdot 2^{2k} + (2^{2k} - 1)$

$= 3 \cdot 2^{2k} + 3n$

$= 3(2^{2k} + n)$

$= 3p$ where $p = (2^{2k} + n)$ and $p \in \mathbb{N}$

therfore, for n = k+1 the reulst is true

and furthermore, by the PMI, for all n in $\mathbb{N}$, $2^{2n} - 1$ is divisble by 3.

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