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Module 3
Bridge to Abstract Math

Section 1.5

3.

Let $x, \ y \ and \ z$ be integers. Write a proof by contraposition to show that

(d) if xy is even, then either $x \ or \ y$ is even.

Solution: `Proof by contraposition`

Assume neither $x \ or \ y$ are even

then x is an odd integer $x = 2k + 1$

and y is an odd integer $y = 2l + 1$

furthermore $xy = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1$ is an odd integer

Therefore xy is even if x or y is even.

5.

A circle has center $(2, 4)$.

(a) Prove that $(-1, 5) \ and \ (5, 1)$ are not both on the same circle.

Solution:

Assume that $(-1, 5) \ and \ (5, 1)$ are on the same circle:

Then the radius for point $(-1, 5)$ on a circle centered at $(2, 4)$ would be:

$r_1 = \sqrt{(-1-2)^2 + (5-4)^2} = \sqrt{10}$

and the radius for a point $(5, 1)$ on a circle centered at $(2, 4)$ would be:

$r_2 = \sqrt{(5-2)^2 + (1-4)^2} = \sqrt{18}$

$r_1 \neq r_2$ and therefore $(-1, 5) \ and \ (5, 1)$ are not on the same circle.

6.

Suppose $a \ and \ b$ are positive integers. Write a proof by contradiction to show that

(c) if a is odd, then $a + 1$ is even.

Solution: `proof by contradiction`

a is odd

and suppose that a+1 is odd as well,

then there exists and integer k such that

$a + 1 = 2k + 1$ (definition of an odd integer)

then $a = 2k$ is even

this contradicts that a is odd; therefore a+1 is is even.

7.

Suppose $a, \ b, \ c \ and \ d$ are positive integers. Prove the biconditional statement:

(a) ac divides bc if and only if a divides b.

Solution:

Suppose ac divides bc

then $bc = ac \cdot k$ so that ac is a k multiple of bc

and $b = a \cdot k$

also suppose that a divides b

therefore $b = a \cdot m$ where m is positive integer

then $bc = acm$

thus ac divides bc and proves the biconditional statement.

Section 1.6

1.

Prove that

(b) there exist integers m and n such that $15m + 12n = 3$

Solution: `proof by construction`

If $m = 1$ and $n = -1$

then $15m + 12n = 15 - 12 = 3$

Therefore 15m+12n = 3 is true when $m \ and \ n$ are the integers $1 \ and \ -1$

2.

Prove for all integers $a \ , \ b \ and \ c$

(a) if a divides $b - 1$ and a divides $c - 1$, then a divides $bc - 1$

Solution:

a divides $b - 1$ and a divides $c - 1$ therefore

$ak = b - 1$ and $al = c - 1$ (where k and l are arbitrary integers)

then ...

$bc = (ak - 1)(al - 1)$

furthermore

$bc - 1 = a^2kl - ak - al + 1 - 1 = a^2kl - ak - al = a(akl - k - l)$

therefore

a divides $bc - 1$

4.

Provide either a proof or a counter example for each of these statements:

(b) $(\forall \ x) \ (\exists \ y) \ (x + y = 0)$ (Universe of all reals)

Solution:

Assume $x + y = 0$

Then $x = -y$

so for all x there is a y that makes $x + y = 0$

In [ ]: