Derek Haynes
Module 4
Bridge to Abstract Math

2.1

5.b.

True or False

$\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$

Solution: `True`

List all possible subsets of ... $\{\emptyset, \{\emptyset\}\}$

$\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

it is obvious that $\{\emptyset\}$ belongs to the above set

5.l

$\{\{4\}\} \subseteq \{1, 2, 3, \{4\}\}$

Solution: `True`

Listing the first few elements in the powerset for $\{1, 2, 3, \{4\}\}$

$\{\emptyset, 1, 2, 3, \{\{4\}\}, \dots\}$

it is obvious that $\{\{4\}\}$ is a subset of $\{1, 2, 3, \{4\}\}$

8.

Prove if $A \subseteq B \; and \; B \subseteq C, \; then \; A \subseteq C.$

Solution:

Suppose $A \subseteq B$

then $\forall x \in A \Rightarrow x \in B$

furthermore suppose $B \subseteq C$

then $\forall x \in B \Rightarrow x \in C$

Since every element of A is contained by B and every element of B is contained by C

it goes that A is contained by C

hence,

$A \subseteq C$

16.

List all the proper subsets for the following set $\{\emptyset, \{\emptyset\}\}$

Solution:

$\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

2.2

7.e

Prove $A \cap A = A$

Solution:

By part b of Theorem 2.2 ($A \cap B \subseteq A$)

We assume $A \cap A \subseteq A$

then $x \in A \cap A$ and $x \in A$ (definition of subset)

which shows $A \subseteq A \cap A$

and when combined with $A \cap A \subseteq A$ from the beginning

gives $A \cap A = A$

7.o

Prove $A \subseteq B$ if and only if $A \cup B = B$

Solution:

i)Suppose that $A \subseteq B$

then $\forall x \in A \Rightarrow x \in B$

which means all elements of A are in B by the definition of a union operator

Therefore $A \cup B = B$

ii) Suppose $A \cup B = B$

then $x \in A$

furthermore $x \in A \cup B$

and given that $A \cup B = B$ then $x \in B$

that is

$x \in A \Rightarrow x \in B$

Therefore $A \subseteq B$

Then $A \subseteq B$ if and only if $A \cup B = B$

7.r

Prove $A \subseteq B \; then \; A \cap C \subseteq B \cap C$

Solution:

Suppose $A \subseteq B$

Then $\forall x \in A \Rightarrow x \in B$

And furthermore, Suppose $x \in A \cap C$

Then $x \in A$ and $x \in C$

Then $x \in B$ and $x \in C$

Then $x \in B \cap C$

Therefore $x \in A \cap C \Rightarrow x \in B \cap C$

Therefore

$A \cap C \subseteq B \cap C$

8.f

Prove the remaining parts of Theorem 2.2.2:

Let U be the universe, and let A and B be subsets of U.

Prove $A \cap B = \emptyset$ iff $A \subseteq B^c$

Solution:

i) Suppose $A \cap B = \emptyset$

then $x \in A$

and $x \notin B$

$x \in B^c$

that is $x \in A \Rightarrow x \in B^c$

Therefore $A \subset B^c$

ii) Suppose $A \subseteq B^c$

Therefore $x \in A \Rightarrow x \in B^c$

$x \notin B$

that is

$A \cap B = \emptyset$

Therefore

$A \cap B = \emptyset$ iff $A \subseteq B^c$

8.h

Prove $(A \cap B)^c = A^c \cup B^c$

Solution:

i) Suppose $x \in A^c \cup B^c$

then $x \in A^c$ or $x \in B^c$

then $x \notin A$ or $x \notin B$

then $x \notin A \cup B$

then $x \in (A \cap B)^c$

ii) Suppose $x \in (A \cap B)^C$

then $x \notin A \cap B$

then $x \notin A \; or \; x \notin B$

then $x \in A^c \; or \; x \in B^c$

then $x \in A^c \cup B^c$

Therefore,

$(A \cap B)^c = A^c \cup B^c$

9a.

Let A, B and C bet sets.

Prove that $A \subseteq B$ if and only if $A - B = \emptyset$

i) Suppose $A \subseteq B$

and further that $A - B \neq \emptyset$

then $x \in A - B$

then $x \in A$ and $x \notin B$

This contradicts $A \subseteq B$ (because $\forall x \in A \Rightarrow x \in B$)

Therefore $A - B = \emptyset$

ii) Suppose that $A - B = \emptyset$

then suppose that B does not contain A

Then $\exists x \; P(x \in A \; and \; x \notin B)$

Then $x \in A - B$ is a contradition to $A - B = \emptyset$

Therefore $A \subseteq B$

10b.

Let A, B, C and D be sets.

Prove that $C \subseteq A \; and \; D \subseteq B \, , then \; C \cup D \subseteq A \cup B$

Solution:

Suppose that $C \subseteq A$

then $\forall x \in C \Rightarrow x \in A$

and suppose $D \subseteq B$

then $\forall y \in D \Rightarrow y \in B$

So

$x, y \in C \cup D$ and $x, y \in A \cup B$

Therefore

$C \subseteq A \; and \; D \subseteq B \, , then \; C \cup D \subseteq A \cup B$

15c.

$(AxB) \cap (CxD) = (A \cap C)x(B \cap D)$

Solution:

$(a, b) \in (AXB) \cap (CxD) \Leftrightarrow (a, b) \in AxB \; and \; (a, b) \in CxD$

iff $(a \in Aandb \in B)$ and $(a \in Candb \in D)$

iff $a \in A \cap C$ and $b \in B \cap D$

iff $(a, b) \in (A \cap C)x(B \cap D)$

Therefore...

$(AxB) \cap (CxD) = (A \cap C)x(B \cap D)$

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