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Math 303

Mod 2 HW 1.8

1.8.6

Given: Find a vector X whose image under T is b, and determine x in unique.

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -4 & 5 \\ 0 & 1 & 1 \\ -3 & 5 & -4 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 9 \\ 3 \\ -6 \end{bmatrix}$$

Solution:

Step 1. Create the Augmented Matrix and Row Reduce

$$\begin{bmatrix} 1 & -2 & 1 & 1 \\ 3 & -4 & 5 & 9 \\ 0 & 1 & 1 & 3 \\ -3 & 5 & -4 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

`Answer` x is not unique since there is a free variable with x_3. There are infinitely many solutions

1.8.10

Given: Find all x in $\mathbb{R}^4$ that are mapped into the zero vector by the transformation of $x \rightarrow Ax$

$$A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}$$

Solution:

Step 1 :

$$[A0] = \begin{bmatrix} 1 & 3 & 9 & 2 & 0 \\ 1 & 0 & 3 & -4 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ -2 & 3 & 0 & 5 & 0 \end{bmatrix}$$

Step 2. Row Reduce the Augmented Matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step3.

From this we have

$x_1 + 3x_3 = 0$

$x_2 + 2x_3 = 0$

x_3 is free

$x_4 = 0$

Thus we have 3 equations and 4 variables with x_3 as a free variable

$x_1 = -3x_3$

$x_2 = -2x_3$

$x_3 = x_3$

$x_4 = 0$

$$\texttt{Answer } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3x_3 \\ -2x_3 \\ x_3 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

1.8.16

Given: Use a rectangular coordinate system to plot $u = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and their images under the given transformation T. Describe geometrically what T does to each vector x in $\mathbb{R}^2$

$$T(x) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Solution:

For Vector u

$$T(x) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

For Vector v

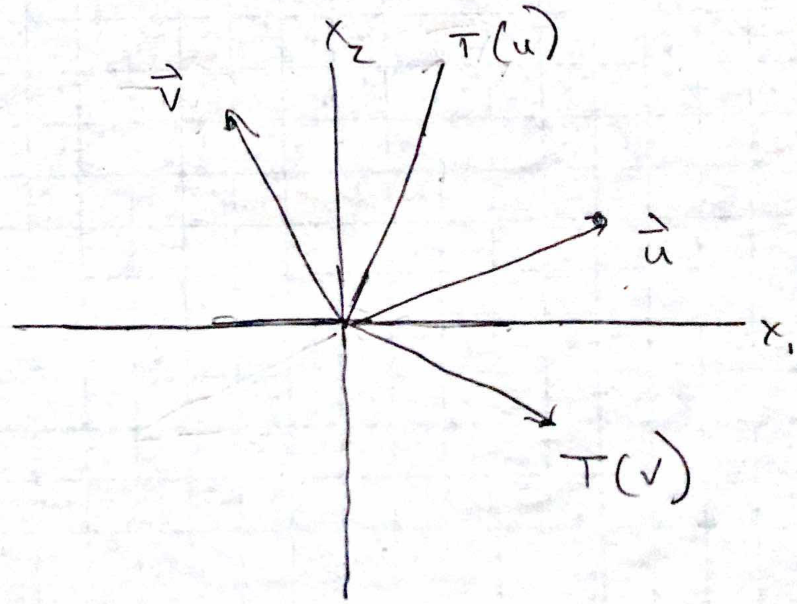
$$T(x) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

1.8.16

$T: \mathbb{R}^2 \to \mathbb{R}^2$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 5 \\ 2 \end{bmatrix}}_{u} = \underbrace{\begin{bmatrix} 2 \\ 5 \end{bmatrix}}_{T(u)}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} -2 \\ 4 \end{bmatrix}}_{v} = \underbrace{\begin{bmatrix} 4 \\ -2 \end{bmatrix}}_{T(v)}$$



1.8.17

Given: Let T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation that maps $u = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ into $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and maps $v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

Use the fact that T is linear to find the images under T of $3u$, $2v$, and $3u + 2v$

Solution:

$$T(3u) = 3(Tu) = 3 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$T(2v) = 2(Tv) = 2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

$$T(3u + 2v) = 3(Tu) + 2T(v) = \begin{bmatrix} 6 \\ 3 \end{bmatrix} + \begin{bmatrix} -2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 9 \end{bmatrix}$$

1.8.33

Given: show that the transformation T defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear

Solution:

If T is a linear transformation, then $T(0) = 0$ for all vectors u, v in the domain of T.

If we set $x_1 = x_2 = 0$

Then $T(0, 0) = (2(0) - 3(0), (0) + 4, 5(0))$

$= (0, 4, 0)$

And Therefore, $T(0, 0) \neq 0$ and is not a linear Transformation

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