Derek Haynes
Math 303
Discussion Posts Module 2

Markus Pomper
Sep 7, 2019

Suppose that A is a 3×3matrix and $\mathbf{b}$ and $\mathbf{c}$are vectors in $\mathbb{R}^3$.

If the equation $A\vec{x} = \vec{b}$has a unique solution, what can you tell about the solutions to $A\mathbf{x} = \mathbf{c}$?

Reply

Derek Haynes
Sep 7, 2019

I'm not sure I'm understanding this. If the system of equations from Ax has a unique solution, that means there are no free variables for X and there for, C could not be a a solution to Ax ?

I'm referencing Ch 1.2 pg 21 Theorem 2.

Reply

Stephanie Chichester
Sep 9, 2019

I have in my notes that the geometric interpretation of one free variable is a line and the geometric interpretation of two free variables is a plane. If the 3×3 matrix is an augmented matrix, and it has a unique non-trivial solution, then $x_3$would be free - but do we have enough information to know if $x_2$is free? This answer would allow us to determine if vectors b ⇀ and c⇀ lie in a line or a plane.

Reply

Stephanie Chichester
Sep 9, 2019

Also, another question in the same vein: The lecture asks us to try section 1.5 #1-4 on our own. For #4, I got the following matrix: $\begin{bmatrix} 1 & 0 & -20 & 0 \\ 0 & 1 & -13 & 0 \end{bmatrix}$

My first thought was that there are no free variables here. If true, this would mean there is not a non-trivial solution to the system. However, I have in my notes that if a system has just one equation then x_2 and x_3 are free so it follows that x_3 must be free in the above matrix, signaling the existence of a unique non-trivial solution. Agree?

Reply

Markus Pomper
Sep 10, 2019

It is not stated in the problem that A is an augmented matrix of any sort. A is a 3×3 matrix. We are considering the equations $A\vec{x} = \vec{b}$ and $A\mathbf{x} = \mathbf{c}$. If we are told that $A\vec{x} = \vec{b}$ has a unique solution, what can we say about the pivot points of A?

Reply

Derek Haynes
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Derek Haynes
Sunday

1.5.4 Given:

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix}, b = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$$

find a vector x whose image under T is b. and determine whether x is unique.

Solution:

step1. Place in Augmented Matrix

$$\begin{bmatrix} 1 & -3 & 2 & 6 \\ 0 & 1 & -4 & -7 \\ 3 & -5 & -9 & -9 \end{bmatrix}$$

step 2. Row Reduce

$$\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Answer

$$x = \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$$

'x' is unique since there are no free variables in vector x. So there is exactly 1 'x' whose image is b.
Edited by Derek Haynes on Sep 15 at 6:04pm

View in discussion

Derek Haynes
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Markus Pomper
Sep 7, 2019

Let $\vec{q}_1, \vec{q}_2, \vec{q}_3$ and $\vec{v}$ represent vectors in $\mathbb{R}^5$ and let x_1, x_2 andx_3 be scalars.

Write the vector equation $x_1\vec{q}_1, +x_2\vec{q}_2 + x_3\vec{q}_3 = \vec{v}$ as a matrix equation.

Reply

Derek Haynes
Sep 7, 2019

I believe this is the solution: Ax = b form

$$\begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = v$$

Reply

Derek Haynes
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Discussion Posts Module 2

Derek Haynes
Sep 6, 2019

Question 1.5.5

Write the solution set of the given homogenous system in parametric vector form:

given:

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 0 \\ -4x_1 - 9x_2 + 2x_3 &= 0 \\ -3x_2 - 6x_3 &= 0 \end{aligned}$$

Solution:

$$\begin{bmatrix} 1 & 4 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$x_1 = 5x_3$

$x_2 = -2x_3$

$x_3 is free$

Therefor

$$x = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

Edited by Derek Haynes on Sep 21 at 12:48pm

Reply

Derek Haynes
12:25pm

Posting a useful link to LaTeX package symbols. I've been using this when typing markdown and in the discussion editor.

[Link]

Derek Haynes
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Derek Haynes
Sep 6, 2019

Question 1.5.15

Follow the method in example 3 to describe the solutions of the following system in parametric vector form. Also, give a geometric description of the solution set

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3 \end{aligned}$$

Solution:

$$\begin{bmatrix} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 & -5 & -2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow \begin{aligned} x_1 - 5x_3 &= -2 \\ x_2 + 2x_3 &= 1 \\ 0 &= 0 \end{aligned}$$

Thus

$$\begin{aligned} x_1 &= -2 + 5x_3 \\ x_2 &= 1 - x_3 \\ 0 &= 0 \end{aligned}$$

X = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 + 5x_3 \\ 1 - 2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

$X = \rho + x_3 v$

$X = \rho + tv$

Derek Haynes
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Derek Haynes
2:41pm

Questions 1.8.13 from Lecture recommended practice.

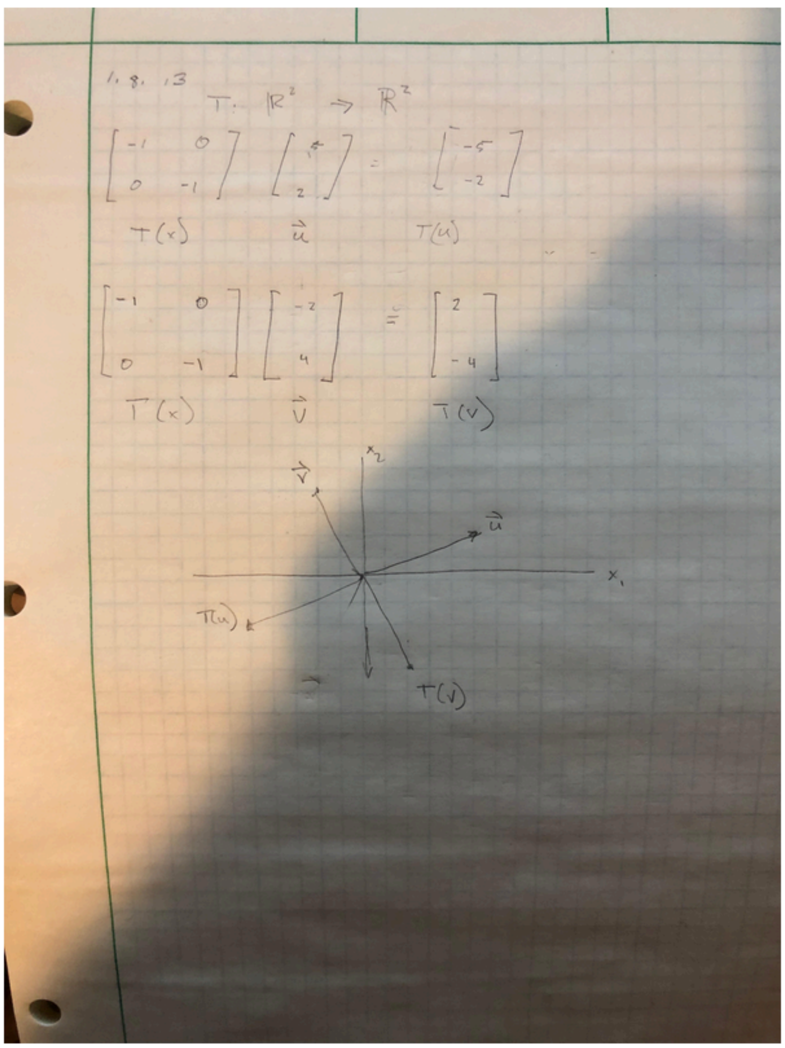
Use a rectangular coordinate system to plot $u = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and their images under the given transformation T. Describe geometrically what T does to each vector **x** in $\mathbb{R}^2$.

Given:

$$T(x) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Solution:

From the drawing, the image of the Transformation is a reflection through the origin.



Edited by Derek Haynes on Sep 22 at 2:50pm