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HW module 1

Math 303 Linear Algebra

1.1.12 Solve System of equations

Step 1: Convert system of equations into Augment Matrix

$$\begin{matrix} x_1 - 3x_2 + 4x_3 = -4 \\ 3x_1 - 7x_2 + 7x_3 = -8 \\ -4x_1 + 6x_2 - x_3 = 7 \end{matrix} = \begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{bmatrix}$$

Step 2: Replace row 2 by sum of itself and adding it to the multiple of row 1 (multiply by -2).

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & -2 & -5 & -4 \\ -4 & 6 & -1 & -8 \end{bmatrix}$$

Step 3: Replace row 3 by sum of itself and adding it to the multiple of row 1 (multiply by 4).

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & -2 & -5 & -4 \\ 0 & -6 & 15 & -9 \end{bmatrix}$$

Step 4: Swap row 2 with row 3

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & -6 & 15 & -9 \\ 0 & -2 & -5 & -4 \end{bmatrix}$$

Step 5: Add 1/3 x row 2 to row 3

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & -6 & 15 & -9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 6: Add 9x(row 3) to row 2

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & -6 & 15 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 7: Add 4x(row 3) to row 1

$$\begin{bmatrix} 1 & -3 & 4 & 0 \\ 0 & -6 & 15 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 8: Divide row 2 by - 6

$$\begin{bmatrix} 1 & -3 & 4 & 0 \\ 0 & 1 & -5/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 9 : Add 3 x (row 2) to row 1

$$\begin{bmatrix} 1 & 0 & -7/2 & 0 \\ 0 & 1 & -5/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution: Augmented Matrix is in reduced echelon form

$$\begin{bmatrix} 1 & 0 & -7/2 & 0 \\ 0 & 1 & -5/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.1.14 Solve the system of equations:

Step 1: Convert system of equations into Coefficient Matrix

$$\begin{matrix} x_1 - 3x_2 = 5 \\ -x_1 + x_2 + 5x_3 = 2 \\ x_2 + x_3 = 5 \end{matrix} = \begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Step 2: Add row 1 to row 2 and replace

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Step 3: Swap row 3 with row 2

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 5 & 7 \end{bmatrix}$$

Step 4: Add 2x(row 2) to row 3

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{bmatrix}$$

Step 5: Divide row 3 by 7

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Step 6: Subtract row 3 from row 2

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Step 7: Add 3x(row 2) to row 1

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Solution is (2, -1, 1)

1.1.30

Find the elementary row operation that transforms the first matrix into the second, and then find the reverse row operation that transforms the second matrix into the first.

$$\begin{bmatrix} 1 & 3 & -4 \\ 0 & -2 & 6 \\ 0 & -5 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -4 \\ 0 & 1 & -3 \\ 0 & -5 & 9 \end{bmatrix}$$

Solution

To transform matrix 1 to matrix 2 `divide row 2 by -2 for matrix 1`

To reverse the row operation from matrix 2 to matrix 1 `multiply row 2 in matrix 2, by -2`

In [ ]: