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Math 303

Module 2 Sec 1.4
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1.4.7

Given:

$$x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ 7 \end{bmatrix}$$

Use the definition of Ax to write the matrix equation as a vector equation

Solution:

$$\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

1.4.10

given:

$$\begin{aligned} 8x_1 - x_2 &= 4 \\ 5x_1 + 4x_2 &= 1 \\ x_1 - 3x_2 &= 2 \end{aligned}$$

Write as a Vector equation then as a Matrix equation.

Solution:

Vector Equation

$$x_1 \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Matrix Equation

$$\begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

1.4.12

Given :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

Write the Augmented matrix for the linear system that corresponds to the matrix equation Ax = b. Then Solve the system and write the solution as a vector

Solution:

The augmented matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix}$

Step 1. $R_2 \longrightarrow R_2 + 3R_1$

Step 2. $R_2 \longrightarrow \frac{1}{5}R_2$

Step 3. $R_1 \longrightarrow -2R_2 + R_1$

Step 4. $R_3 \longrightarrow -5R_2 + R_3$

Step 5. $R_3 \longrightarrow \frac{-1}{2}R_3$

Step 6. $R_1 \longrightarrow R_1 + R_3$

Step 7. $R_1 \longrightarrow -R_3 + R_2$

Augmented matrix

$$\begin{bmatrix} 1 & 0 & 0 & \frac{3}{5} \\ 0 & 1 & 0 & -\frac{4}{5} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \\ 1 \end{bmatrix}$$

1.4.14

Given:

$$u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix} and A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$$

Determine is in the subset of $\mathbb{R}^3$ spanned by the columns of A?

Solution:

The Augmented matrix is

[AU] = $\begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix}$

in Reduced Echolon form it becomes

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The linear system is not consisten since the last row has the form $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$ therefore vector $\boldsymbol{U}$ is not in the span of A

1.4.18

Given:

B = $\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$

Solution

Row Reduce B to form

B = $\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

According to Theorem 4 The columns of B do not span $\mathbb{R}^4$ because there isn't a pivot position in each row.

Additionally Since only 3 of the rows contain pivot positions and one of the rows has the form $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$ the system is in consistent and the matrix equation Bx = y doesn't have a solution for each y in R4

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In [ ]:
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