Derek Haynes

Math 303

Mod 2 HW 1.7

1.7.2

Given: Determine if the vectors are linearly independent. Justify each answer.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$ $$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

Solution: The vectors are `linearly independent` because there are 3 pivot points in the augmented matrix and no free - variables. Therefore Ax = 0 only has the trivial solution.

$$\begin{bmatrix} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -8 & 1 & 0 \\ 0 & 5 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{bmatrix}$$ $$\begin{bmatrix} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -8 & 1 & 0 \\ 0 & 5 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{bmatrix}$$

1.7.4

Given: Determine if the vectors are linearly independent. Justify each answer.

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -8 \end{bmatrix}$$ $$\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -8 \end{bmatrix}$$

Solution: The vectors are `linearly independent` because neither vector is a scalar multiple of each other

1.7.6

Given: Determine if the columns of the matrix form a linearly independent set. Justify each answer.

$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$
$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

Solution:

$$[A0] = \begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix}$$
$$[A0] = \begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix}$$

Step 1. Interchange R_1 R_1 with R_3 R_3

Step 2. $R_3 \rightarrow 4R_1 + R_3$ $R_3 \rightarrow 4R_1 + R_3$

Step 3. $R_4 \rightarrow -5R_1 + R_4$ $R_4 \rightarrow -5R_1 + R_4$

Step 4. $R_2 \rightarrow -1R_2$ $R_2 \rightarrow -1R_2$

Step 5. $R_3 \rightarrow 3R_2 + R_3$ $R_3 \rightarrow 3R_2 + R_3$

Step 6. Interchange R_3 R_3 and R_4 R_4

Step 7. Divide R_3 R_3 by 7

Step 8. $R_2 \rightarrow 4R_3 + R_2$ $R_2 \rightarrow 4R_3 + R_2$

Step 9 $R_1 \rightarrow -3R_3 + R_1$ $R_1 \rightarrow -3R_3 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

From the matrix you can see that the columns in the matrix are `linearly independent` because there are no free variables. It only has the trivial solution to make it homogenous

1.7.15

Given: Determine by inspection whether the vectors are linearly dependent

$$\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1, \\ -7 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1, \\ -7 \end{bmatrix}$$

Solution: The set contains 4 vectors, each of which only has 2 entries. So the set is `linearly dependent` by Theorem 8.

1.7.16

Given: Determine by inspection whether the vectors are linearly dependent

$$\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$$

Solution: The vectors are `linearly dependent` since one vector is a mutlipe of the other

1.7.17

Given: Determine by inspection whether the vectors are linearly dependent

$$\begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 5 \\ 4 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 5 \\ 4 \end{bmatrix}$$

Solution: Since the Zero vector is in the set, the set is `linearly dependent` according to Theorem 9.

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1.7.2

Given: Determine if the vectors are linearly independent. Justify each answer.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

Solution: The vectors are `linearly independent` because there are 3 pivot points in the augmented matrix and no free - variables. Therefore Ax = 0 only has the trivial solution.

$$\begin{bmatrix} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -8 & 1 & 0 \\ 0 & 5 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{bmatrix}$$

1.7.4

Given: Determine if the vectors are linearly independent. Justify each answer.

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -8 \end{bmatrix}$$

Solution: The vectors are `linearly independent` because neither vector is a scalar multiple of each other

1.7.6

Given: Determine if the columns of the matrix form a linearly independent set. Justify each answer.

$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

Solution:

$$[A0] = \begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix}$$

Step 1. Interchange R_1 with R_3

Step 2. $R_3 \to 4R_1 + R_3$

Step 3. $R_4 \to -5R_1 + R_4$

Step 4. $R_2 \to -1R_2$

Step 5. $R_3 \to 3R_2 + R_3$

Step 6. Interchange R_3 and R_4

Step 7. Divide R_3 by 7

Step 8. $R_2 \to 4R_3 + R_2$

Step 9 $R_1 \to -3R_3 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

From the matrix you can see that the columns in the matrix are `linearly independent` because there are no free variables. It only has the trivial solution to make it homogenous

1.7.15

Given: Determine by inspection whether the vectors are linearly dependent

$$\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1, \\ -7 \end{bmatrix}$$

Solution: The set contains 4 vectors, each of which only has 2 entries. So the set is `linearly dependent` by Theorem 8.

1.7.16

Given: Determine by inspection whether the vectors are linearly dependent

$$\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$$

Solution: The vectors are `linearly dependent` since one vector is a mutlipe of the other

1.7.17

Given: Determine by inspection whether the vectors are linearly dependent

$$\begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 5 \\ 4 \end{bmatrix}$$

Solution: Since the Zero vector is in the set, the set is `linearly dependent` according to Theorem 9.

In [ ]: