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Math 303

Sec 2 HW 1.5
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1.5.4

given:

$$\begin{aligned} -5x_1 + 7x_2 + 9x_3 &= 0 \\ x_1 - 2x_2 + 6x_3 &= 0 \end{aligned}$$

Determine if the system has a nontrivial solution:

Solution:

[A 0] = $\begin{bmatrix} -5 & 7 & 9 & 0 \\ 1 & -2 & 6 & 0 \end{bmatrix}$

Step 1. $R_2 \rightarrow R_1$

Step 2. $R_2 \rightarrow 5R_1 + R_2$

Step 3. $R_2 \rightarrow \frac{-1}{3} R_2$

Step 4. $R_1 \rightarrow 2R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & -20 & 0 \\ 0 & 1 & -13 & 0 \end{bmatrix}$$

X_3 is a free variable, therefore, $Ax = 0$ has a nontrivial solution.

1.5.6

Given:

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 0 \\ x_1 + 4x_2 - 8x_3 &= 0 \\ -3x_1 - 7x_2 + 9x_3 &= 0 \end{aligned}$$

Write the solution set of the given homogeneous system in parametric vector form

Solution:

Step 1. Let A be the matrix of coefficients of the system and row reduce the augmented matrix [A 0] into echelon form

$$= \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 2. Solve for the basic variables x_1, x_2, x_3, with x_3 being a free

$$\begin{aligned} x_1 + 4x_3 &= 0 \\ x_2 - 3x_3 &= 0 \\ 0 &= 0 \end{aligned}$$

Step 3. Solve for the basic variables x_1, x_2

$$x_1 = -4x_3$$

$$x_2 = 3x_3$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4x_3 \\ 3x_3 \\ x_3 \end{bmatrix}$$

so $x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$

1.5.12

given:

$$\begin{bmatrix} 1 & 5 & 2 & -6 & 9 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Describe all solutions of $Ax = 0$ in parametric vector form, where A is row equivalent to the given matrix.

Solution:

$$[A0] = \begin{bmatrix} 1 & 5 & 2 & -6 & 9 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 1. $R_3 \rightarrow 8R_3 + R_2$

Step 2. $R_1 \rightarrow -2R_2 + R_1$

$$\begin{bmatrix} 1 & 5 & 0 & 8 & 1 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

X_2, X_4, X_5 **are free variables**

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -5x_2 - 8x_4 - x_5 \\ x_2 \\ 7x_4 - 4x_5 \\ x_4 \\ x_5 \\ 0 \end{bmatrix} = \begin{bmatrix} -5x_2 \\ x_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -8x_4 \\ 0 \\ 7x_4 \\ x_4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -x_5 \\ 0 \\ -4x_5 \\ 0 \\ x_5 \\ 0 \end{bmatrix} =$$

$$x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ 0 \\ 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ -4 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$x = x_2 u + x_4 v + x_5 w$

In [ ]:

1.5.16

given:

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ x_1 + 4x_2 - 8x_3 &= 7 \\ -3x_1 - 7x_2 + 9x_3 &= -6 \end{aligned}$$

Solution:

$$\begin{bmatrix} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{bmatrix}$$

Step 1. Apply row operations to get in reduced echelon form:

$$\begin{bmatrix} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 2.System of Equations

$x_1 = -4x_3 - 5$

$x_2 = 3 + 3x_3$

x_3 is free variable

Step 3. Translate into parametric form

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4x_3 - 5 \\ 3x_3 + 3 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$$

$$= u + x_3 v$$

From the solution in Exersize 1.5.6: $x = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} = x_3 w$

We can see that the line $x = u + x_3 v$ is parallel to $x = x_3 w$

In [ ]:

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