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HW 1.3

Math 303

1.3.12

Given:

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, a_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

determine if $x_1a_1 + x_2a_2 + x_3a_3 = b$

rewriten as vector equations:

$$\begin{aligned} x_1 + 2x_3 &= -5 \\ -2x_1 + 5x_2 &= 11 \\ 5x_2 + 8x_3 &= -7 \end{aligned}$$

rewriten as an augmented matrix

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 0 & 5 & 8 & -7 \end{bmatrix}$$

Row reduce by

1. Add $2 * R_1 + R_2$
2. Add $-1 * R_1 + R_3$
3. $\frac{1}{4} * R_3$
4. $-4R_3 + R_2$
5. $-2R_3 + R_1$
6. $\frac{1}{5} R_2$

to get reduced aug. matrix of

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 9/5 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Solution:

$x_1 = -1, x_2 = 9/5, x_3 = -2$

$$-1 * \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + 9/5 * \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} + -2 * \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

In [ ]:

1.3.14

Given:

$$A = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}, b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

Determine if b is a linear combination of the matrix A

First create the augmented matrix

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix}$$

Then reduce

1. $R_1 + R_2$
2. $\frac{1}{11}R_3$
3. $-7R_3 + R_2$
4. $6R_3 + R_1$
5. $\frac{1}{3}R_3$
6. $2R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{245}{33} \\ 0 & 1 & 0 & \frac{-41}{33} \\ 0 & 0 & 1 & \frac{-2}{11} \end{bmatrix}$$

Solution

$$x_1 = \frac{245}{33}, x_2 = \frac{-41}{33}, x_3 = \frac{-2}{11}$$

$$\frac{245}{33}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \frac{-41}{33}\begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix} + \frac{-2}{11}\begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

In [ ]:

1.3.16

Given :

$$v_1 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

Solution:

The $span\{v_1, v_2\}$ plane of $x_1v_1 + x_2v_2 = b$

1. when x_1 = 1 and x_2 = 0

b = $\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$

1. x_1 = 0 and x_2 = 1

b = $\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$

1. x_1 = -1 and x_2 = 0

b = $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$

1. when x_1 = 0 and x_2 = -1

b = $\begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$

1. when x_1 = 1 and x_2 = 1

b = $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

In [ ]: