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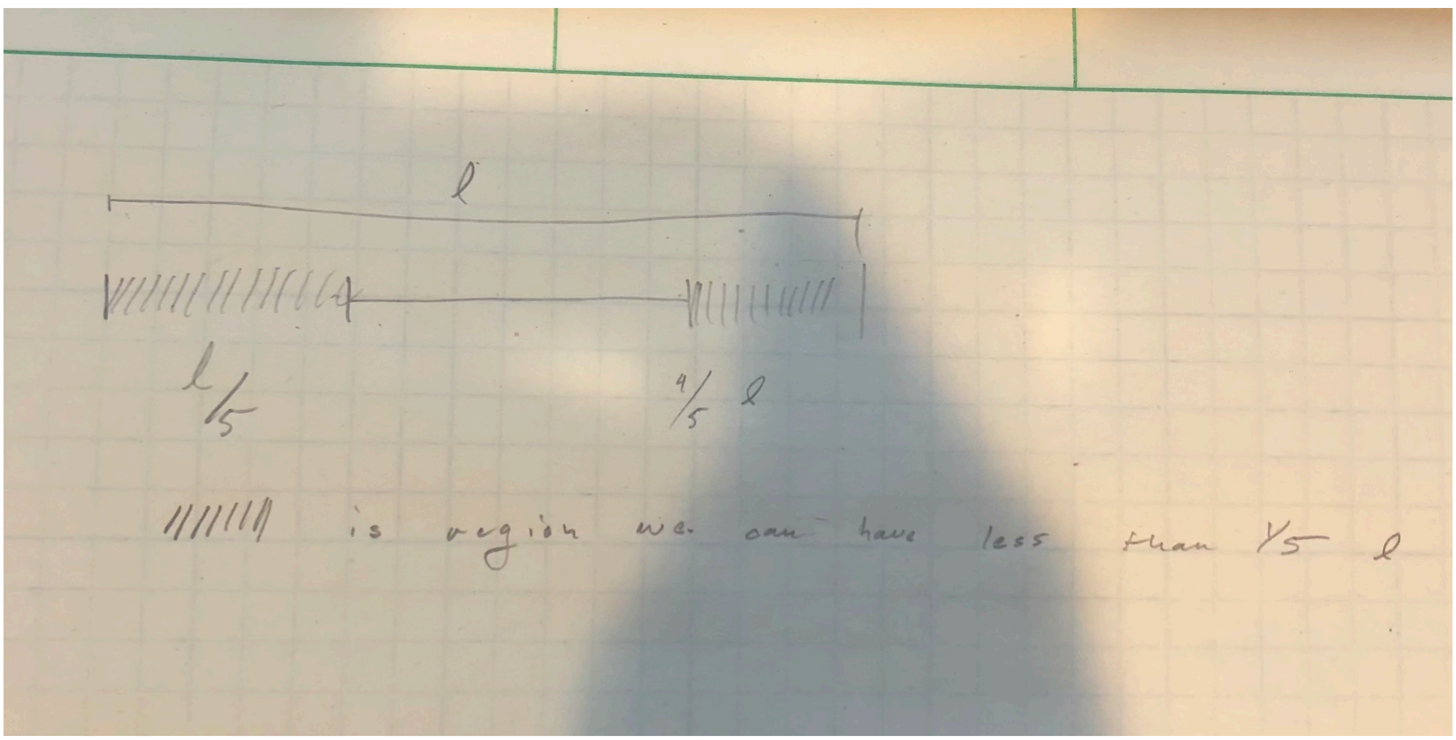
Math 463

HW #2

1.9

Given: We break a stick at a uniformly chose random location. Find the probability that the shorter piece is less than $\frac{1}{5}th$ of the original.

Solution:

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Answer
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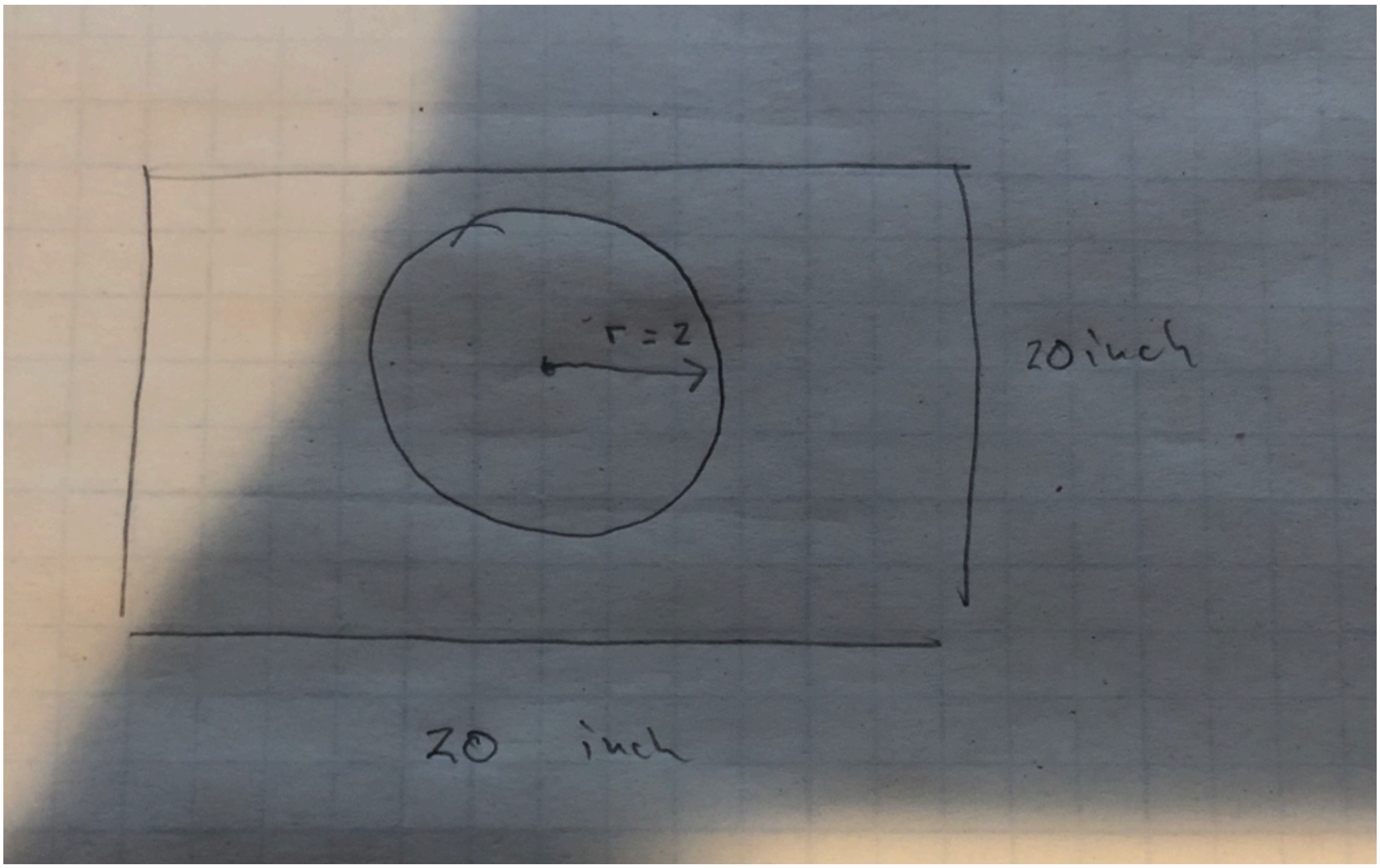
$$P(A) = \frac{\frac{l}{5}+\frac{l}{5}}{l} = \frac{\frac{2l}{5}}{l} = \frac{2}{5}$$

1.11

Given: We throw a dart at a square shaped board of side length 20 inches. Assume that the dart hits the board at a uniformly chosen random point. Find the probability that the dart is within 2 inches of the center of the board

Solution:



Area of a Circle $= \pi r^2$

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Answer
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P(within 2 inches of center of board) = $\frac{\pi(2)^2}{20\cdot20} = \frac{4\pi}{400} = \frac{\pi}{100}$

1.13 Given: At a certain school, 25% of the students wear a watch and 30% wear a bracelet. 60% of the student wear neither a watch nor a bracelet.

A) One of the students is chosen at random. What is the probability that this student is wearing a watch or bracelet?

B) What is the probability that this student is wearing both a watch and a bracelet?

$P(A) = 0.25 \ P(B) = 0.30$

Solution A:

$P(A \cup B)$ = probability of watch or bracelet

$P(A \cup B) = 1.0 - 0.60 = 0.40$

Solution B:

$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.25 + 0.30 - 0.40 = 0.15$

1.14

Given Assume that $P(A) = 0.4$ and $P(B) = 0.7$ Making no further assumptions on A and B, show that:

$P(A \cap B)$ satisfies $0.1 < P(A \cap B) \leq 0.4$

Knowing $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

also...

$0 \leq P(A \cup B) \leq 1$

$0 \leq P(A) + P(B) - P(A \cap B) \leq 1$

$0 \leq 0.4 + 0.7 - P(A \cap B) \leq 1$

$0 \leq 1.1 - P(A \cap B) \leq 1$

$\rightarrow P(A \cap B) \geq 1.1 - 1$

$P(A \cap B) \geq 0.1$

also...

$P(A \cup B) \geq P(B)$

$P(A) + P(B) - P(A \cap B) \geq P(B)$

$P(A \cap B) \leq P(A)$

$P(A \cap B) \leq 0.4$

Therefore ...

$0.1 \leq P(A \cap B) \leq 0.4$

1.15

Given: An urn contains 4 balls: 1 white, 1 green and 2 red. We draw 3 balls with replacement. Find the probability that we did not see all three colors.

A) Define the event W = $\{$ white ball did not appear $\}$ and similiary for G and R. Use inclusion-exclusion

B) Compute the probability by considering the complement of the event that we did not see all three colors.

Solution:

A)

Event White ball did not appear $P(W) = \frac{3}{4}\frac{3}{4}\frac{3}{4} = \frac{27}{64}$

Event Green ball did not appear $P(G) = \frac{3}{4}\frac{3}{4}\frac{3}{4} = \frac{27}{64}$

Event Red ball did not appear $P(R) = \frac{2}{4}\frac{2}{4}\frac{2}{4} = \frac{8}{64}$

$P(W \cup G \cup R) = P(W) + P(G) + P(R) - P(W \cap G) - P(W \cap R) - P(G \cap R) + P(W \cap G \cap R)$

$P(W \cap G) = \frac{2}{4}\frac{2}{4}\frac{2}{4} = \frac{8}{64}$

$P(W \cap R) = \frac{1}{4}\frac{1}{4}\frac{1}{4} = \frac{1}{64}$

$P(G \cap R) = \frac{1}{4}\frac{1}{4}\frac{1}{4} = \frac{1}{64}$

$P(W \cap G \cap R) = 0$

$P(W \cup G \cup R) = \frac{27}{64} + \frac{27}{64} + \frac{8}{64} - \frac{8}{64} - \frac{1}{64} - \frac{1}{64} + 0$

= $\frac{13}{16}$

B)

We can chose 1 ball of each color $\binom{1}{1}\binom{1}{1}\binom{2}{1} = 2$ and the three can be arranged 3! ways so the total number of all color outcomes is $3! * 2 = 12$

Thus

P(all colors) = $\frac{12}{64}$

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by complement law
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P(not all three colors) = 1 - P(all colors) = $1 - \frac{12}{64} = \frac{13}{16}$

In [ ]: