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Probability Theory

HW 9

Exercise 4.2

Given: The probability of getting a single pair in a poker hand of 5 cards is approximately 0.42. Find the approximate probability that out of 1000 poker hands, there will be at leas 450 with a single pair.

Solution:

$$P(X_{1000} \geq 450) = 1 - P(X_{1000} < 450)$$

Therefore ...

$$\mu = n \cdot p = 420$$

$$\sigma^2 = np(1 - p) = 243.6$$

$$1 - P(Z < \frac{450-420}{\sqrt{243.6}})$$

$$1 - P(z < 1.922) = 1 - 0.9726 = 0.0274$$

Exercise 4.3

Given: Approximate the probability that out of 300 die rolls we get exactly 100 numbers that are multiple of 3

Solution:

$S_n \sim Bin(300, 1/3)$

$np = 100$

$\sigma^2 = np(1-p) = \frac{200}{3}$

$P(99.5 \leq S_n \leq 100.5)$

`using CLT`

$= P(\frac{99.5-100}{\sqrt{66.6667}} \leq Z \frac{\leq 100.5-100}{\sqrt{66.6667}})$

$= P(-0.06 \leq Z \leq 0.06)$

$= P(Z \leq 0.06) - P(Z \leq -0.06)$

$= \Phi(0.06) - \Phi(-.06)$

$= \Phi(0.06) - (1 - \Phi(0.06))$

$= \Phi(0.06) + \Phi(0.06) - 1$

$= 0.5239 + 0.5239 - 1 = 0.0478$

Exercise 4.4

Given: Liz is standing on the real number line at position 0. She rolls a die repeatedly. If the roll is 1 or 2 she takes one step to the right (in the positive direction). If the roll is 3,4,5,06 she takes two steps to the right. Let X_n be Liz's position after n rolls of the die. Estimate the probability that X_{90} is at least 160 steps.

Also, by using the Continuity Correction, estimate the probability that X_{90} is at least 160.

Xn = number of steps taken up to and including the nth die roll

Sn = number of $1, 2$s in n rolls.

$Sn \sim Bin(160, \frac{1}{3})$

$Xn = 1Sn + 2(n - Sn) = 2n - Sn$

$\mu = np = \frac{90}{3} = 30$

$\sigma^2 = np(1 - p) = 20$

$P(X_{90} \geq 160) = P(180 - S_{90} \geq 160) = P(-S_{90} \geq -20)$

$P(S_{90} < 20)$

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using CLT
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$P(Z < \frac{20-30}{\sqrt{2}0})$

$P(Z < -2.23)$

$\Phi(-2.23) = 1 - \Phi(2.23) = 1 - .9871 = 0.0129$

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by continuity correction
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$P(159.5 \leq X_{90} \leq 160.5)$

$P(159.5 \leq 180 - S_{90} \leq 160.5)$

$P(-20.5 \leq -S_{90} \leq -19.5)$

$P(20.5 \geq S_{90}) + P(S_{90} \geq 19.5)$

$= P(19.5 \leq S_{90} \leq 20.5)$

$= P(\frac{19.5-30}{\sqrt{2}0} \leq Z \leq \frac{20.5-30}{\sqrt{2}0})$

$= P(-2.34 \leq Z \leq -2.12)$

$= \Phi(-2.12) - \Phi(-2.34)$

$= (1 - \Phi(2.12)) - (1 - \Phi(2.34))$

$= 0.0074$

Exercise 4.21

Given: In a game, you win 10 dollars with probability $\frac{1}{20}$ and lose 1 dollar with probability $\frac{19}{20}$ Approximate the probability that you lost less than 100 dollars after the first 200 games.

Solution:

$S_n =$ number of wins after n games

$X_n =$ amount of winnings after the first n games

Therefore $X_n = 10Sn - (n - Sn) = 11Sn - n$

and $X_{200} = 11S_{200} - 200$

$\mu = np = 10$

$\sigma^2 = np(1-p) = 9.5$

$P(X_{200} > -100) = P(11S_{200} - 200 > -100)$

$= P(S_{200} > \frac{100}{11})$

$= P(S_{200} > 9.091)$

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continuity correction applied
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$= P(S_{200} > 9.5)$

$= P(Z > \frac{9.091-10}{\sqrt{9}.5})$

$= P(Z > -0.162)$

$= 1 - P(Z < -0.162)$

$= 1 - \Phi(-0.162)$

$= 1 - (1 - \Phi(0.162))$

$= 0.5636$

Exercise 4.5

Consider the setup of Exercise 4.4. Find the limits below and explain your answer.

a) Find $lim_{n\to\infty}P(X_n > 1.6n)$

solution:

S_n ~ $Bin(n, \frac{2}{3})$

$= lim_{n\to\infty}P(X_n > 1.6n)$

$= lim_{n\to\infty}P(S_n + n > 1.6n)$

$= lim_{n\to\infty}P(S_n > 0.6n)$

$= lim_{n\to\infty}P(\frac{S_n}{n} - \frac{2}{3} > 0.6 - \frac{2}{3})$

$= lim_{n\to\infty}P(\frac{S_n}{n} - \frac{2}{3} > -\frac{1}{15})$

$\geq lim_{n\to\infty}P(\frac{S_n}{n} - \frac{2}{3} < \frac{1}{15}) = 1$

b) Find $lim_{n\to\infty}P(X_n > 1.7n)$

S_n ~ $Bin(n, \frac{2}{3})$

$X_n = S_n + n$

$= lim_{n\to\infty}P(S_n + n > 1.7n)$

$= lim_{n\to\infty}P(S_n > 0.7n)$

$= lim_{n\to\infty}P(\frac{S_n}{n} > 0.7)$

$= lim_{n\to\infty}P(\frac{S_n}{n} - \frac{2}{3} > 0.7 - \frac{2}{3})$

$\leq lim_{n\to\infty}P(|\frac{S_n}{n} - \frac{2}{3}| > 0.7 - \frac{2}{3}) = 0$

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