Derek Haynes

Probability Theory

Homework 6

Exercise 3.5

Given:

Suppose that the discrete random variable X has cumulative distribution function given by:

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{3}, & 1 \leq x < \frac{4}{3} \\ \frac{1}{2}, & \frac{4}{3} \leq x < \frac{3}{2} \\ \frac{3}{4}, & \frac{3}{2} \leq x < \frac{9}{5} \\ 1, & x \geq \frac{9}{5} \end{cases}$$

Find the possible values and the probability mass function of X

Solution:

$P(X = 1) = \frac{1}{3}$

$P(X = \frac{4}{3}) = \frac{1}{6}$

$P(X = \frac{3}{2}) = \frac{1}{4}$

$P(X = \frac{9}{5}) = \frac{1}{4}$

Exercise 3.7

Given:

Suppose that the continuous random variable X has a cumulative distribution function given by

$$F(x) = \begin{cases} 0, & x < \sqrt{2} \\ x^2 - 2, & \sqrt{2} \leq x < \sqrt{3} \\ 1, & \sqrt{3} \leq x. \end{cases}$$

a) Find the smallest interval $[a, b]$, such that of $P(a \leq X \leq b) = 1$

Solution:

The smallest intervale is $[\sqrt{2}, \sqrt{3}]$ since

$F(\sqrt{2}) = 0$ and $F(\sqrt{3}) = 1$.

b) Find $P(X = 1.6)$.

Solution: 0 since X is a continuous random variable.

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 By fact 3.7 ... a random variable X has a density fucntion f, then point values
have a probability of zero
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c) Find $P(1 \leq X \leq \frac{3}{2})$

Solution:

$P(1 \leq X \leq \frac{3}{2}) = F(\frac{3}{2}) - F(1)$

$= (\frac{3}{2})^2 - 0 = \frac{1}{4}$

d) Find the probability density function of X

$$f(x) = \begin{cases} 2x, & \sqrt{2} \leq x \leq \sqrt{3} \\ 0, & else \end{cases}$$

Exercise 3.19

Let $Z \sim Bin(10, \frac{1}{3})$.

Find the value of its cumulative distribution function at 2 and at 8.

$P(z = 0) = \binom{10}{0}(1 - \frac{1}{3})^1 0 = 0.01734$

$P(z = 1) = \binom{10}{1}(\frac{1}{3})^1(\frac{2}{3})^9 = 0.086$

$P(z = 2) = \binom{10}{2}(\frac{1}{3})^2(\frac{2}{3})^8 = 0.195$

$P(z \leq 2) = P(0) + P(1) + P(2) = 0.29834$

$P(z \leq 8) = 1 - [P(x = 9) + P(x = 10)]$

$P(1 - [\binom{10}{9}(\frac{1}{3})^9(\frac{2}{3}) + \binom{10}{10}(\frac{1}{3})^{10}]$

$P(z \leq 8) = 0.99964$

Exercise 3.32

Given:

Let X be a continuous random variable with density function

$$f_X(x) = \begin{cases} \frac{1}{2}x^{\frac{-3}{2}}, & 1 < x < \infty \\ 0, & else \end{cases}$$

a) Find $P(X > 10)$.

Solution:

$\int_{10}^{\infty} \frac{1}{2}x^{\frac{-3}{2}}\, dx$

= $\frac{1}{2}\int_{10}^{\infty} x^{\frac{-3}{2}}\, dx$

= $-X^{\frac{-1}{2}}|_{10}^{\infty}$

= $-\frac{1}{\sqrt{\infty}} + \frac{1}{\sqrt{10}}$

= 0.3162276

b) Find the cumulative distribution function Fx of X

$$F_X(x) = \begin{cases} -x^{\frac{-1}{2}}, & 1 < x < \infty \\ 0, & else \end{cases}$$

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