

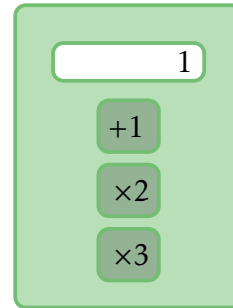
### 5.2.2 Primitive Calculator

**Primitive Calculator Problem**

Find the minimum number of operations needed to get a positive integer  $n$  from 1 by using only three operations: add 1, multiply by 2, and multiply by 3.

**Input:** An integer  $n$ .

**Output:** The minimum number of operations “+1”, “ $\times 2$ ”, and “ $\times 3$ ” needed to get  $n$  from 1.



You are given a calculator that only performs the following three operations with an integer  $x$ : add 1 to  $x$ , multiply  $x$  by 2, or multiply  $x$  by 3. Given a positive integer  $n$ , your goal is to find the minimum number of operations needed to obtain  $n$  starting from the number 1. Before solving the programming challenge below, test your intuition with our [Primitive Calculator](#) puzzle.

Let's try a greedy strategy for solving this problem: if the current number is at most  $n/3$ , multiply it by 3; if it is larger than  $n/3$ , but at most  $n/2$ , multiply it by 2; otherwise add 1 to it. This results in the following pseudocode.

```
GREEDYCALCULATOR( $n$ ):  
  numOperations  $\leftarrow$  0  
  currentNumber  $\leftarrow$  1  
  while currentNumber <  $n$ :  
    if currentNumber  $\leq n/3$ :  
      currentNumber  $\leftarrow$   $3 \times$  currentNumber  
    else if currentNumber  $\leq n/2$ :  
      currentNumber  $\leftarrow$   $2 \times$  currentNumber  
    else:  
      currentNumber  $\leftarrow$   $1 +$  currentNumber  
    numOperations  $\leftarrow$  numOperations + 1  
  return numOperations
```

**Stop and Think.** Can you find a number  $n$  such that

`GREEDYCALCULATOR( $n$ )`

produces an incorrect result?

**Input format.** An integer  $n$ .

**Output format.** In the first line, output the minimum number  $k$  of operations needed to get  $n$  from 1. In the second line, output a sequence of intermediate numbers. That is, the second line should contain positive integers  $a_0, a_1, \dots, a_k$  such that  $a_0 = 1$ ,  $a_k = n$  and for all  $1 \leq i \leq k$ ,  $a_i$  is equal to either  $a_{i-1} + 1$ ,  $2a_{i-1}$ , or  $3a_{i-1}$ . If there are many such sequences, output any one of them.

**Constraints.**  $1 \leq n \leq 10^6$ .

**Sample 1.**

Input:

1

Output:

0

1

**Sample 2.**

Input:

96234

Output:

14

1 3 9 10 11 22 66 198 594 1782 5346 16038 16039 32078 96234

Another valid output in this case is “1 3 9 10 11 33 99 297 891 2673 8019 16038 16039 48117 96234”.