

## Number Theory.

# NUMBER THEORY

**1.Regular number:** A regular number is an integer of the form  $2^i 3^j 5^k$  where  $i, j, k$  is non negative integer. The regular number are also called 5-smooth indicating that there greatest prime factor is at most 5.

**Example:** 1,2,3,4,5,6,8,9,10,12,15,16,18..

**2.Complex number:** A complex number is a number of form  $a+ib$  where  $a$  &  $b$  are real number and  $i$  is the imaginary unit satisfying  $i^2=-1$ . Here  $a$  is the real part and  $b$  is the imaginary part.

Example:  $3+4i, -5+3i$

**3.Real Number:** A real number include all rational numbers such as the integers 5 and the fraction  $4/3$  and all the irrational numbers such as  $\sqrt{2}$ .

Transcendental numbers such as  $\pi$  ( $\pi=3.141592..$ )

Type equation here.

**4.Prime Number:** Prime number is a natural number that have no positive divisor other than 1 and itself. That means it's can have exactly two positive divisor 1 and the number itself.

Example: 2,3,5,7,11,13,17,19,23,29,31,

**5.Composite Number:** Composite number is positive integer that can be formed by multiplying together two smaller positive integer.

Example: 4,6,8,9,10,12,14,15,16,18,20

**6.Powerful Number:** A powerful number is a product of a square and root that is number  $m=a^2 b^3$  here  $a$  &  $b$  is a positive integer.

Example: 1,4,8,9,16,25,27,32,36

**7.Friendly Number:** Friendly Number are two or more natural numbers with a common abundancy the ratio between sum of divisors of the number and the number itself. A number n is friendly number if they exist  $m \neq n$  such that

$\frac{\partial(m)}{m} = \frac{\partial(n)}{n}$  the smallest friendly number is 6 and friendly pair(6,28) with abundancy.

$$\frac{\partial(6)}{6} = \frac{(1 + 2 + 3 + 6)}{6} = 2 \text{ the same as}$$

$$\frac{\partial(28)}{28} = \frac{(1 + 2 + 4 + 7 + 14 + 28)}{28} = 2$$

Example:( 6,120),(12,28),(24,60),(56,120),(102,216)

**8.Solitary Number:** A number that is not a part of any friendly pair is called solitary number.

**Example:**(18,39),(45,78),(49,26),(130,370)

**9.Amicable Number:** Amicable number are two different number as related that the sum of the proper divisor of each is equal the other number.

The Smallest pair of (220,284)

$220 = 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110$  of which the sum is 284.

And  $284 = 1 + 4 + 71 + 141$  of which the sum is 220.

**Other example** (6232,6268)

**10.Perfect Number:** A perfect number is a positive integer that is equal to the sum of that's proper positive divisor .the formula of even perfect number is  $2^{p-1}(2^p - 1)$  Where p is the prime number

**Example:**

$$p=2: 2^{2-1}(2^2 - 1) = 6$$

$$P=3: 2^{3-1}(2^3 - 1) = 28$$

$$p = 5: 2^{5-1}(2^5-1) = 496$$

**Fibonacci Number:** The Fibonacci number are the number in the following integer sequence called the Fibonacci number and the characteristic by the fact that every number after the first two number is the sum of the two preceding ones.

In mathematics term the sequence  $F_n$  of fibonacci number is defined by the recurrence relation.

$$F_n = F_{n-1} + F_{n-2} \text{ where } F_1=1, F_2=1$$

**Example:** 1,1,2,3,5,8,13,21,34,55,89,144

**12.Lucas number:** Like the Fibonacci number lucas number is defined to be sum of it's two intermediate previous term thereby forming a Fibonacci sequence . First two lucas numbers are  $L_0 = 2$  and  $L_1 = 1$ .

The lucas number may be defined as follows

$$L_n = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ L_{n-1} + L_{n-2} & \text{if } n > 1 \end{cases}$$

*The sequences of Lucas number is 2,1,3,4,7,11,18,29,47,76,123*

**13.Octagonal Number:** A Octagonal number is figurate number that represent an octagon . The Octagon number for n is given by the formula  $3n^2 - 2n$

where  $n > 0$

**Example:** 1,8,21,40,65,96,133,176,225

**14.Central Octagonal Number:** A Octagonal number is centred figurate number that represent an octagon with a dot in the centre and all others dots surrounding the centre dots in successive octagonal layers as the figure



Thus the  $n^{th}$  centred original number is given by formula

$$(2n - 1)^2 = 4n^2 - 4n + 1$$

**Example:** 1,9,25,49,81,121..

**15.Smooth Number:** A smooth number is a integer which factor completely into small prime number . A positive integer called B-smooth .if none of its prime factorization  $2^2 3^4 5$  therefore 1620 is 5 smooth because none of it prime factor are greater than 5.

**16.rough Number:** A K-rough number is a positive integer whose prime factors are all greater than or equal to k . K roughness has alternately been defined as requiring all prime factor to strictly exceed.

Example:

- Every Odd positive integer(1,3,5) is 3 rough
- Every positive integer that is congruent to 1 or 5 mod 6 is 5 rough.
- Every positive integer is 2-rough since all it's positive factors being prime number exceed 1.

**17.Sphenic number:** A sphenic number is a positive integer that has the product of three distinct prime numbers. Sphenic number is the product of pqr where p,q,&r three distinct prime number.

The smallest sphenic prime number is  $30=2.3.5$

The product of three smallest prime.

**Others Example:** 30,42,66,70,78,102,105,110

**18.Pronic Number:** A pronic number is a number which is the product of two consietive integers that is a number of the form  $n(n+1)$  they are also called along numbers.

Heteromcie Numbers or rectangular number

**Example:** 0,2,6,12,20,30,42,56,72..

**19. Automorphic Number:** An automorphic number is a number whose square "ends" in the same digit as the number itself

**Example:**

$$5^2 = 25$$

$$6^2 = 36$$

$$25^2 = 625$$

$$76^2 = 5776$$

$$890625^2 = 793212890625$$

**20. Kaprekar Number:** A Kaprekar number for a given base is a non-negative integer the representation of whose square in that base can be split into two parts that add up to the original number again.

297 is a Kaprekar number for base 10.

Because  $297^2 = 88209$  which can be split into 88 and 209 and  $88 + 209 = 297$

**Others Example:** 1, 9, 45, 55, 99, 297, 703, 999

**21. Cabtaxi Number:** In mathematics the  $n^{th}$  cabtaxi number typically denoted  $\text{cabtaxi}(n)$  is defined as the smallest positive integer that can be written as the sum of two positive or negative or 0 in  $n$  ways. Such kind of number exists for all  $n$ .

**Example:**

$$\text{cabtaxi}(1) = 1 = 1^3 \mp 0^3$$

$$\text{Cabtaxi}(2) = 91 = 3^3 + 4^3 = 6^3 - 5^3$$

$$\text{Cabtaxi}(3) = 728 = 6^3 + 8^3 = 9^3 - 1^3$$

**22. Square Number:** In mathematics a square number is a integer that is the square of an integer. The number m is square number if and only if one can compose a square of m equal square.

**Example:**  $0^2 = 0, 1^2 = 1, 5^2 = 25, 6^2 = 36, 10^2 = 100, 19^2 = 361, 31^2 = 961$

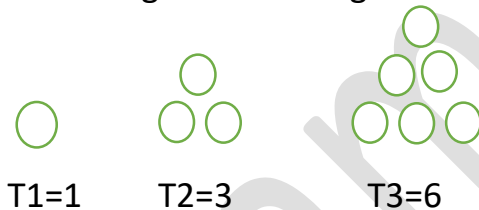
**23. Polite number:** Polite number is a positive integer that can be written as the sum of two or more consecutive positive integers. Polite number have also been called staircase numbers.

**Example:** 3,5,7,9,10,11,12,13,14,15,17

**24. perfect cube number:** a cube number or a perfect cube number which is the cube of an integer.

**Example:**  $1^3 = 1, 2^3 = 8, 6^3 = 125, 7^3 = 343, 15^3 = 3375, 24^3 = 13824, 30^3 = 27000$

**25. Triangular Number:** A triangular number counts the object that can form an equilateral triangle .as the diagram of the figure



The nth triangular number is the number is the number os dots composing a triangle with the n dots m a side and is equal to the sum of the n natural number from 1 to n.

The triangular number are given lay following explicit formular :

$$T_n = \sum_{k=1}^n K = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} = \frac{n+1}{2}$$

Here  $\binom{n+1}{2}$  is a bionomial coeficient

Example: 0,1,3,6,10,15,21,28,36

**26. tetrahedral Number:** A Tetrahedral number is a figarate number that represents a pyramid with the triangular base and three sides called a tetrahedral

The  $n^{th}$  tetrahedral number is the sum of 1<sup>st</sup>  $n^{th}$  triangular number.

The formular for the nth tetrahedral number is

$$T_n = \frac{n(n+1)(n+2)}{6} = \frac{n^3}{3!}$$

Example:1,4,10,20,35,56,84,120,165 ..

(Armstrong ,Factorial ,palindrome ,ugly)Number