Compressible Navier Stokes Equations With Constant Viscosity And Thermal Conductivity.

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The compressible Navier-Stokes equations solved by `CNS` are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0, \tag{1}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) + \nabla p = \nabla \cdot \boldsymbol{\tau}, \tag{2}$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + p)\boldsymbol{u}] = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\boldsymbol{\tau} \cdot \boldsymbol{u}), \tag{3}$$

where ρ is the density, $\boldsymbol{u}$ is the velocity, p is the pressure, E is the specific energy density (kinetic energy plus internal energy), $\boldsymbol{\tau}$ is the viscous stress tensor, λ is the thermal conductivity, and T is the temperature. The viscous stress tensor is given by

$$\tau_{ij} = \eta \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \nabla \cdot \boldsymbol{u} \right), \tag{4}$$

where η is the shear viscosity. In `CNS`, we assume that λ and η are constants.

The `CNS` algorithm is based on finite-difference methods. For first derivatives with respect to spatial coordinates, the following standard 8th-order stencil is employed,

$$\left.\frac{du}{dx}\right|_i \approx \frac{\alpha(u_{i+1} - u_{i-1}) + \beta(u_{i+2} - u_{i-2}) + \gamma(u_{i+3} - u_{i-3}) + \delta(u_{i+4} - u_{i-4})}{\Delta x}, \tag{5}$$

where α, β, γ, δ are constants denoted in the code by `ALP`, `BET`, `GAM`, and `DEL`, respectively. For double derivatives, the following 8th-order stencil is employed,

$$\left.\frac{d^2 u}{dx^2}\right|_i \approx \frac{c_0 u_i + c_1(u_{i+1} + u_{i-1}) + c_2(u_{i+2} + u_{i-2}) + c_3(u_{i+3} + u_{i-3}) + c_4(u_{i+4} + u_{i-4})}{(\Delta x)^2}, \tag{6}$$

where c_0, c_1, c_2, c_3, and c_4 are constans denoted in the code by `CENTER`, `OFF1`, `OFF2`, `OFF3`, and `OFF4`, respectively.

In `CNS`, the `U` variable is a vector with five components: ρ, ρu, ρv, ρw, and ρE. Here, u, v, and w are the velocity in x, y, and z-direction. The `Q` variable is a vector with 6 components: ρ, u, v, w, p, and T. Given `U`, the `ctoprim` subroutine computes `Q`. The `hypterm` subroutine updates `U` according to the left-hand side of Euqations (1)–(3), whereas the `diffterm` subroutine treats the right-hand side. A 3rd-order Runge-Kutta scheme is used for advancing in time.