Experiment with Exponential Distribution and Central Limit Theorem

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Overview

In this report we will simulate a exponential distribution with lambda equals to 0.2, 1000 times to compare with central limit theorem. Followings are the 3 main aspects we're going to discuss about.

- 1. Compare sample mean to the theoretical mean of the distribution.
- 2. Compare variance to the theoretical variance of the distribution.
- 3. Is the distribution is approximately normal?

Simulation

create a list named "m" and a list named "v" to record the mean and variance of every run, there are totally 1000 run.

```
m = c()
v = c()
for(i in 1:1000){
    myRun <- rexp(40, 0.2)
    m[i] = mean(myRun)
    v[i] = var(myRun)
}</pre>
```

Sample Mean versus Theoretical Mean

Find Sample mean and compare to the theoretical mean

give the sample mean named "sm" and theoretical mean named "tm" as following:

```
sm = mean(m)
tm = 1/0.2
sm

## [1] 4.971347
tm
```

[1] 5

Hypothesis Testing about the mean

```
Ho: sm = tm Ha: sm != tm

t.test(x=m, mu=tm, alternative="two.sided")

##

## One Sample t-test

##

## data: m

## t = -1.1516, df = 999, p-value = 0.2498

## alternative hypothesis: true mean is not equal to 5

## 95 percent confidence interval:

## 4.922521 5.020172

## sample estimates:

## mean of x

## 4.971347
```

According to the result, we failed to reject the null hypothesis. That's say, we have 95% condidentce to say the sample maen equals to the theoretical mean.

Sample Variance versus Theoretical Variance

Find sample variance and compare to theoretical variance

give the sample variance mean named "sv" and theoretical variance as "tv" as following:

```
sv = mean(v)
tv = (1/0.2)^2
sv

## [1] 25.10752
tv
## [1] 25
```

Hypothesis Testing about the variance

```
Ho: sv = tv Ha: sv != tv

t.test(x=v,mu=tv,alternative="two.sided")
```

```
##
## One Sample t-test
##
## data: v
## t = 0.29805, df = 999, p-value = 0.7657
## alternative hypothesis: true mean is not equal to 25
```

```
## 95 percent confidence interval:
## 24.39960 25.81544
## sample estimates:
## mean of x
## 25.10752
```

According to the result, we failed to reject the null hypothesis. That's say, we have 95% condidentce to say the sample variance equals to the theoretical variance.

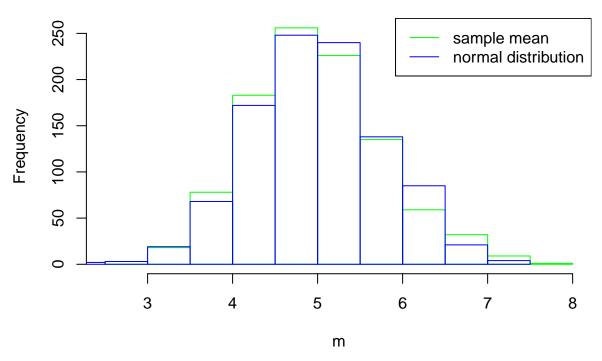
Distribution

Graph comparison

standard deviation of sample mean named "s". We show the histgram of sample mean and compare to the normal distribution mean as 5, variance as 0.6

```
s = sd(m)
p1 <- hist(m,border="green",main="Compare Sample Mean to Normal")
myNorm <- rnorm(1000,mean=5,sd=s)
p2 <- hist(myNorm,border="blue",add=T)
legend('topright',legend=c("sample mean","normal distribution"),lty=1,col=c("green","blue"))</pre>
```

Compare Sample Mean to Normal



According to the histgram, we can approxiately say the sample mean of the 1000 try of 40 exponential distribution is normal distributed.