Q5.2 Since $L(x)$ is convex, its local minimum is global minimum. Also since it's convex the local minimum resides at the stationary pt.

$$L(x) = a(x_1+x_2)^2 + b(x_1-x_2)^2$$

$$\frac{\partial L}{\partial x_1} = 2a(x_1+x_2) + 2b(x_1-x_2) = 0 \quad (1)$$

$$\frac{\partial L}{\partial x_2} = 2a(x_1+x_2) - 2b(x_1-x_2) = 0 \quad (2)$$

$$(1)+(2) = 4a(x_1+x_2) = 0$$

$$\Rightarrow x_1 + x_2 = 0 \quad (3)$$

$$(1)-(2) = 4b(x_1-x_2) = 0$$

$$x_1 = x_2 \quad (4)$$

Using (3) and (4), global minimum at $(0,0)$