Q5.1 $L(x) = a\underbrace{(x_1+x_2)^2}_{f_1} + b\underbrace{(x_1-x_2)^2}_{f_2}$

for f_1,

$$\nabla f_1 = \left[\frac{\partial f_1}{\partial x_1} \quad \frac{\partial f_1}{\partial x_2}\right] = \left[2(x_1+x_2) \quad 2(x_1+x_2)\right]$$

and Hessian is given by,

$H_1 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, which is positive semidefinite

$\lambda = 0$ or 4

similarly, for f_2

$$\nabla f_2 = \left[\frac{\partial f_2}{\partial x_1} \quad \frac{\partial f_2}{\partial x_2}\right] = \left[2(x_1-x_2) \quad -2(x_1-x_2)\right]$$

$H_2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$, which is positive semi definite (p.s.d)

$\lambda = 0$ or 4

Since Hessians are p.s.d. f_1 and f_2 are convex.

and using the property of linear transforms,

$af_1(x) + bf_2(x)$ is also convex.