Q2. Yes.

It is a direct result of Extreme value theorem.

Theorem

for $f: A \to \mathbb{R}$, if f is continuous and A is compact thus f takes on a global minimum in A. where $A \subset \mathbb{R}^N$

Reference: week 3 slides: slide 26