

. A continued fraction representation for the Golden Ratio ϕ is given by (as a sequence)

$$x_1 = 1 + \frac{1}{1}, \quad x_2 = 1 + \frac{1}{1 + \frac{1}{1}}, \quad x_3 = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}, \quad \dots \quad \phi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}$$

The sequence $\{x_n\}$ converges to $\phi = 1.6180339887498948482045868343656\dots$ as the number of iterations goes to infinity.

- (a) Write a C or C++ function `double mygold(int n)`, that evaluates the n -th order iterated value of ϕ i.e. x_n , using the above algorithm. [4]
- (b) The Golden Ratio may be used to compute the Fibonacci numbers using the relation below:

$$f_n = \frac{1}{2\phi - 1} (\phi^{n+1} - (1 - \phi)^{n+1}), \quad \text{for } n = 2, 3, \dots$$

This is an amazing equation. The right-hand side involves powers and quotients of irrational numbers, but the result is a sequence of integers.

Write a C or C++ function `double myfibo(n)` that evaluates the n -th order Fibonacci number using the above algorithm and approximating to the nearest integer value. [4]

Code Solution:

```
#include <iostream>

#include <cstdlib>

#include <cmath>

using namespace std;

double mygold(int n)
{
    double prod = 1.0;

    for(int i=0; i<n; i++) {
        prod = 1 + 1 / prod;
    }
```

```
    return prod;
}

double myfibo(int n)
{
    return (pow(mygold(50), n+1) - pow(1 - mygold(50), n+1)) / (2 * mygold(50) - 1);
}

int main()
{
    cout << "Phi = " << mygold(50) << endl;

    for(int n=1; n<=11; n++) {
        cout << n << ": " << myfibo(n) << endl;
    }

    return 0;
}
```