PROJECT 2
Integer Linear Programming

Submit your project through *WISEflow*. The submission deadline is Monday October 11th, at 14:00 hrs. The project can be done individually or in a group of at most 2 students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) compressed in a single file (.zip). Include all files needed to run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submitted). In addition, provide a written report with your model formulations and the answers to the questions required in each part. The formulation of your models can be typed in a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Provide a short description (no more than two sentences, e.g. "#demand fulfillment") for every objective function and constraint in your formulations. All model formulations in this project, either involving continuous and/or integer variables, must be **linear**. Expected (not required) length of your report: Part A one page, Part B about four pages, Part C about four pages.

Part A (20%)

In the article "Optimizing the Norwegian Natural Gas Production and Transport", the authors describe the use of a decision model in a real-world application. The current debate in Norway has paid large attention to environmental concerns around the oil and gas industry. As an expert in decision modelling, you have been asked to modify the model formulated in the appendix to address some new situations, which consider environmental aspects. Your formulations must be linear and may involve new definitions (e.g. of variables), new expressions (e.g. in the objective function and/or constraints), the modification of some expressions in the original formulation, etc. These must be formulated in mathematical terms (not in AMPL code). The new situations are described below (each situation is independent from each other).

1. Suppose that there are two components called $c2$ and $c3$ that should not be transported through the same pipelines (because this could potentially produce undesirable consequences). Therefore, for every pair of nodes i and j in the network, you should assure that if there is flow of component $c2$ from node i to node j, there cannot be flow of component $c3$ from node i to node j (and likewise, if there is flow of component $c3$ from node i to node j, there cannot be flow of component $c2$ from node i to node j). Which modification(s) would you introduce in the model to address this new scenario? Which consequence(s) do you expect this will cause in the optimal objective value?

2. Suppose that the original model has an optimal objective value equal to F^*. Suppose that you have been asked to assess a new scenario, in which instead of maximizing flow to the markets, the objective function should minimize the total emissions. In this new scenario, any production field may be closed down. However, to secure a certain level of demand satisfaction with respect to the original scenario, the total flow to the markets in this new scenario should be at least a 90% of the optimal objective value of the original model. There is data available on the amount of emissions generated by each of the production fields. The data state that, if the production field g remains open, it generates a fixed amount of emissions equal to A_g, plus an amount B_g per each unit of flow departing from node g. In contrast, a production field that closes down will not produce any emission, but cannot supply any flow either. Which modification(s) would you introduce in the model to address this new scenario?

Part B (40%)

Bandy is a team winter sport played on ice, similar to ice hockey. In Norway, bandy is played by both male and female players in several age categories. In this problem, we will study the case of the Women's *Eliteserien* in the forthcoming 2021/22 season. The aim of the problem is to formulate a schedule of matches that could help league organizers to arrange this season tournament.[1]

There are seven teams participating in the league: Drammen Bandy, Høvik, Ready, Solberg, Stabæk, Ullevål IL, and NTNUI. For the ease of writing, we will refer to these teams by the following abbreviations: DRAM, HOVI, READ, SOLB, STAB, ULLE, and NTNU. Each team has its own home venue. Most of these are located in the Eastern area of the country, close to Olso, except for NTNU whose home venue is located in Trondheim. The tournament schedule should comply with a double round robin format, that is, every team must play against every other team one home match and one away match (i.e. every pair of teams meets twice during the season, once at the home venue of each of them).

The season spans from mid November until mid February, as it is shown in the calendar in Figure 1. The matches can be played on Tuesdays or during the weekends (see the yellow dates in the figure). On each Tuesday, every team can play at most one match. On each weekend, every team can play at most one match, except for NTNU, which can play at most two matches. However, if NTNU is scheduled two matches on the same weekend, both matches have to be either at home or away (i.e., it cannot happen that one of the matches is at home and the other match is away). There are good reason for this, as NTNU players are mostly students who prefer to play during weekends (during the week they are busy with lectures and course work), and traveling from Trondheim to Oslo is time-consuming and costly.

In some special dates marked in the calendar of Figure 1, there cannot be any match. These dates correspond to: week 51 in 2021 (Christmas), the weekend of week 52 in 2021 (close to New Year's Eve), and the weekend of week 1 in 2022 and the whole week 2 in 2022 (because the 2022 Women's Bandy World Championship will be played in Sweden during January 9th-16th, and the Norwegian national team wants to perform well there). Thus, there are in total 11 weeks available to schedule the *Eliteserien*, but in two of these 11 weeks the weekends are not available. In addition, the teams have specified some dates in which they cannot have a home match, because their home venues will be used for other purposes. These dates are specified for each team in Figure 1 (for example, NTNU cannot play a home match on Tuesday of week 4 in 2022, and neither SOLB nor STAB can play a home match on the weekend of that week).

To secure a balanced distribution of home matches during the season, the league organizers require that during each month, every team should play at least one home match (a *month* here refers to a calendar month, that is, November, December, January, and February).

In addition to all the considerations above, the league organizers have other wishes, which we study independently in tasks 1 and 2, and simultaneously in task 3 below.

1. To keep activity during midweek, it is good when a week has at least two matches on Tuesdays. For NTNU, on the other hand, it is good to play during the weekends. This motivates us to define the concept of "Super Week". A *Super Week* is a week in which there are at least two matches on Tuesday and NTNU plays two matches during the weekend. The wish of the organizers is to maximize the number of *Super Weeks* during the season. Formulate an integer linear model to schedule the league, using this wish as optimization criterion. Implement and solve the model in AMPL, using the solver ***cplex***. How many *Super Weeks* are obtained? Outline the optimal schedule that you found, either graphically, as a table, commented in words, or through some other type of output that can be readable for an external person. You may use AMPL or any other means to write this output (even doing it manually is fine). Imagine you are communicating the solution to the league organizers, who are not necessarily familiar with mathematical programming, so they want to observe the final schedule instead of a raw display with the optimal value of your variables.

2. For every team, except NTNU, we define now the concept of "Tough Week". A team has a *Tough Week* if it has two matches during the week (that is, it plays on Tuesday and on the weekend of the same week). The wish of the organizers is to minimize the sum of the number of *Tough Weeks* over all teams (except NTNU) during the season. Formulate an integer linear model to schedule the league, using this wish as optimization criterion[2]. Implement and solve the model in AMPL, using the solver ***cplex***. How many

[1] The problem described here is a modified and simplified version of the actual problem, adapted for pedagogical purposes.

[2] Note that on a same week there might be more than one team with a *Tough Week*, and we are interested in the sum

Tough Weeks are scheduled in total? How many *Tough Weeks* are scheduled for each team? Outline the optimal schedule that you found (using the same readable format that you used in task 1).

3. Can you find a schedule that fulfils the optimal number of *Super Weeks* that you obtained in task 1 and also the optimal number of *Tough Weeks* that you obtained in task 2? If yes, outline such a schedule. If not, what is the maximum number of *Tough Weeks* that you can schedule while fulfilling the optimal number of *Super Weeks* that you obtained in task 1? And what is the maximum number of *Super Weeks* that you can schedule while fulfilling the optimal number of *Tough Weeks* that you obtained in task 2?

Note: When solving integer linear models, it is interesting to observe how the algorithms of the solvers approach the optimal solution during the optimization process and how much time they take. For this purpose, when using *cplex*, you can add in your .run file the following lines anywhere before the statement *solve*:
option solver cplex;
option cplex_options 'mipdisplay=4';
option show_stats 1;

	Monday		Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Nov 2021	15	Week No 46	16 Venue unavailable this Tuesday: NTNU	17	18	19	20 Venue unavailable this Weekend: NTNU, DRAM	21
	22	Week No 47	23 Venue unavailable this Tuesday: NTNU	24	25	26	27 Venue unavailable this Weekend: HOVI	28
	29	Week No 48	30 Venue unavailable this Tuesday: NTNU	1	2	3	4 Venue unavailable this Weekend: NTNU, DRAM	5
Dec 2021	6	Week No 49	7 Venue unavailable this Tuesday: NTNU	8	9	10	11 Venue unavailable this Weekend: READ	12
	13	Week No 50	14 Venue unavailable this Tuesday: NTNU	15	16	17	18 Venue unavailable this Weekend: HOVI	19
	20	Week No 51	21 NO MATCHES THIS WEEK	22	23	24	25	26
	27	Week No 52	28 All venues available	29	30	31	1 NO MATCHES THIS WEEKEND	2
Jan 2022	3	Week No 1	4 All venues available	5	6	7	8 NO MATCHES THIS WEEKEND	9
	10	Week No 2	11 NO MATCHES THIS WEEK	12	13	14	15	16
	17	Week No 3	18 Venue unavailable this Tuesday: NTNU	19	20	21	22 Venue unavailable this Weekend: READ	23
	24	Week No 4	25 Venue unavailable this Tuesday: NTNU	26	27	28	29 Venue unavailable this Weekend: SOLB, STAB	30
	31	Week No 5	1 Venue unavailable this Tuesday: NTNU, STAB	2	3	4	5 Venue unavailable this Weekend: STAB, ULLE	6
Feb 2022	7	Week No 6	8 Venue unavailable this Tuesday: NTNU, ULLE	9	10	11	12 All venues available	13

Figure 1: Calendar features of the league season.

over all teams. Thus, for example, if DRAM plays on Tuesday and on the weekend of week 46 and HOVI also plays on Tuesday and on the weekend of week 46, this contributes with 2 to the objective function.

Part C (40%)

The company BANPetrol buys several types of crude oils daily at its two refineries (R1 and R2). The cost of purchasing one unit of crude oil j on day t is $C_{j,t}^{CRU}$. The crude oil is converted to components through a series of processes in the refineries. A crude oil which goes through these processes provides different components. The proportions in which these components are obtained vary from one to another refinery, because of the different technology and conditions under which the conversion occurs. We will refer by $Q_{r,j,b}$ to the amount of component b obtained from processing one unit of crude oil j at the refinery r. Under normal conditions, the maximum processing capacity of crude oil per day at the refineries is 650 units at R1 and 750 units at R2. It is possible to increase the daily capacity by 200 units at R1 and 100 units at R2, but this implies a fixed cost. If the extra capacity is utilized at R1 on a given day, the fixed cost to pay on that day is equal to \$12000. If the extra capacity is utilized at R2 on a given day, the fixed cost to pay on that day is equal to \$10000.

The cost of processing one unit of crude oil j at the refinery r is $C_{r,j}^{DIS}$. The crude oil is ready for being processed within the same day of purchase or, alternatively, it can be stored at the corresponding refinery. The cost of storing one unit of any type of crude oil is C^{INVI} per day. However, it is not possible to store components at the refineries. Therefore, all the components obtained from processing the crude oils are sent to another facility of the company that we will refer as the hub. Assume that the components sent from the refineries on day t are received at the hub on day $t+1$, due to some lead time of transportation. The cost of transporting one unit of any component from a refinery to the hub is C^{TRA1}. Once the components arrive at the hub, they are ready to be mixed for generating final products. The mix occurs according to predefined recipes. The recipe for producing one unit of product p needs $N_{b,p}$ units of component b. The cost of producing one unit of product p is C_p^{PRO}. It is possible to store components in the hub at a daily cost of C^{INVB} per unit. In contrast, the final products cannot be stored here and, instead, are sent to depots. The cost of transporting one unit of any product from the hub to depot d is C_d^{TRA2}. Assume that what is produced during one day arrives at the depots in the beginning of the following day.

Once the products arrive at the depots, they are ready to be shipped to the markets. The cost of shipping one unit of any product from depot d to market k is $C_{d,k}^{TRA3}$. Alternatively, the products may be stored at the depots. The cost of storing one unit of any type of product at depot d is C_d^{INVP} per day.

There is a maximum demand limit for product p in market k on day t, which we will refer by $\delta_{p,k,t}$. The price of one unit of product p in all markets is S_p. Assume that it is possible to fulfil demand of a same market partly from different depots. Also, assume that what is shipped from the depots one day arrives to the markets the following day, except for the *NorthTown* market which takes an extra day. A strategic requirement is that the shipments to *NorthTown* must be performed only from depot D1. On every day when a shipment of a positive amount of products (in total) departs from D1 to *NorthTown*, the company must pay a fixed cost equal to $CNorth$. Also, every shipment to *NorthTown* must contain (in total, adding up all products) a minimum amount referred as $qMinNorth$. An operational limitation also states that there cannot be two consecutive days with shipments to *NorthTown* (assume that this requirement is in terms of when a shipment to *NorthTown* starts and that there was no shipment on the day preceding the relevant planning horizon).

Figure 2 illustrates the different stages of the supply chain of BANPetrol. The file "Proj2PartC.dat" contains data for this problem. Note the set T contains 11 days, including from day 0 to day 10. The day 0 is only relevant for initial conditions (you may eventually delete it from the set T in the data file if your code does not need it). Note demand on day 10 is satisfied by shipping on day 9 (or day 8 in case of *NorthTown*). It is not possible to make a profit by shipping products that would arrive at the destination market after day 10.

1. Formulate a mixed integer linear programming model for the company, including decisions on procurement, production, transportation, storage and sales (use only continuous and binary variables). The objective is to maximize the total profit over the planning horizon.

   Implement the model in AMPL and solve it using the solver ***gurobi*** and the data instance contained in the file "Proj2PartC.dat" (you may modify the file according to your own definitions). Note the set of time periods (expressed in days) is defined as $T = \{0, 1, 2, \ldots, 10\}$, although the relevant decisions are from periods 1 to 10. The initial inventory of saleable products at the depots (that is, at the end of period $t = 0$), is given in the two columns under the header $Izero_{p,d}$ in Table 1. Assign value zero to any other initial inventory or initial flow variable that you may require in your implementation. However, the company would like to have at least some minimum quantity of components and products on inventory at the end of the planning horizon, in order to anticipate future demand. The final inventory of components *distilA* and *distilB* must be at least 100 units each, while for components *ISO* and *POL* it must be at least 400 units each. The minimum inventory of products for each depot at the end of the planning horizon is given in Table 1 under the header $Ifinal_{p,d}$. Note the inventory costs are incurred per any unit stored at the end of each day (including day 10).

	$Izero_{p,d}$		$Ifinal_{p,d}$	
	D1	D2	D1	D2
premium	210	210	125	100
regular	400	470	200	250
distilF	54	100	50	30
super	200	135	30	60

Table 1: Initial inventory and minimum final inventory quantities of each saleable product at each depot.

a) What is the optimal profit? When do the refineries operate at normal capacity and when do they operate with extra capacity? When do the shipments to *NorthTown* start?
b) Is there unsatisfied demand for some product(s)? If so, briefly explain why this occurs.
c) How much inventory of crude oils, components and final products is left at the end of the planning horizon? If there is any difference among them, briefly discuss why this occur.

2. The prices of the crude oils in the data file "Proj2PartC.dat" remain constant over the days ($117 y $105 for *CrA* and *CrB*, respectively). Suppose an increase of the prices is forecasted to occur on day 6. According to this new forecast, the price per unit of *CrA* on day 6 and later will be $120, while the price per unit of *CrB* in these days will be $107. Solve the model for this new scenario. What is the impact of this change in the optimal objective value? How is the inventory of crude oils stored at the end of each day affected? Briefly discuss how the optimal decisions have changed with respect to the original scenario.

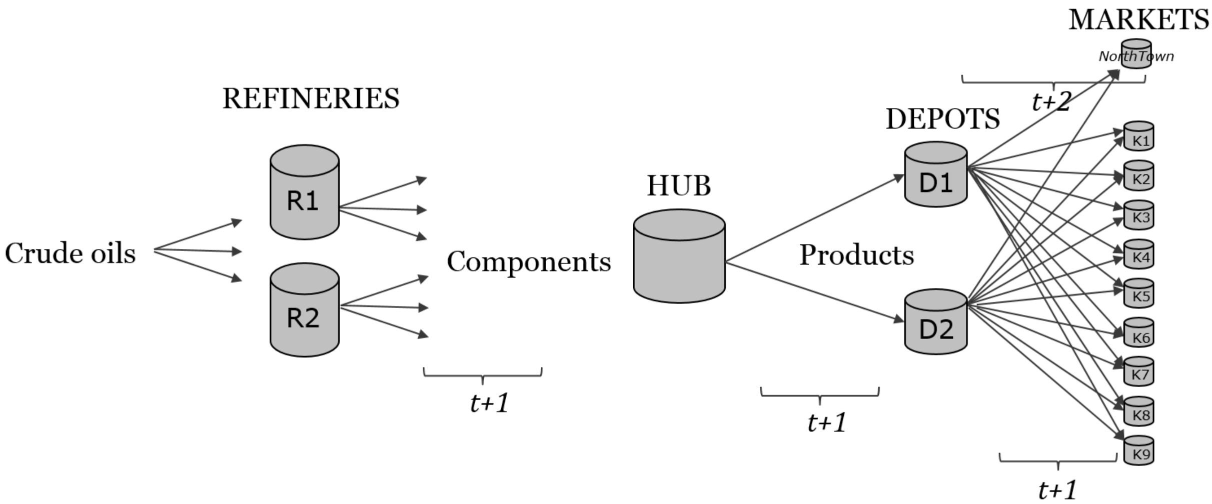

Figure 2: Illustration of the supply chain of the company BANPetrol.