PART A

1. Formulation of the linear model

Indexes and sets:

$p \in P$: set of pollutants
$i \in I$: set of regions

Parameters:

$R_{p,i}$: reduction (in tons) of pollutant p by using fertilizer in one km^2 in region i
C_i: costs per km^2 of using fertilizer in region i
G_p: required pollutant reduction of pollutant p

Decision variables:

x_i: area (in km^2)in which fertilizer is used in region i

Objective function:

The sum of fertilizer costs per km^2 for each region multiplied with the respective number of km^2, in which the fertilizer is used, shall be minimized as per the objective function:

$$\min Costs = \sum_{i \in I} C_i x_i$$

Constraints:

For each pollutant, the achieved reduction (i.e. the sum of reductions achieved in each region i) must be higher than or equal to the reduction required by the government:

$$\sum_{i \in I} x_i R_{p,i} \geq G_p \quad \forall p \in P \tag{1}$$

Non-negativity constraint for the area in km^2, in which the fertilizer is used:

$$x_i \geq 0 \quad \forall i \in I \tag{2}$$

Solution:

Optimal solution: area (in km^2), in which the fertilizer is used, for each region:

- Region L1: 161.54
- Region L2: 0
- Region L3: 30.77

Objective value: minimal costs (in $) of reducing the pollution by the required amounts:

- $\min Costs = 4{,}146.15$

2. Satisfaction of constraints and slack variables

(1) Emission reduction constraint:

constraint	body	lower bound	slack variable
$R_{Pollutant\ P1} \geq 35$	35	35	0
$R_{Pollutant\ P2} \geq 40$	40	40	0

For constraints of the type *f(x) ≥ lower bound*, the slack variable is the difference between the actual value in the solution ("body") and the lower bound as formulated in the constraint. It can be interpreted as how much the current solution exceeds the lower bound.

In the table above you can see the slack variables for the emission reduction constraint, which belongs to the type *f(x) ≥ lower bound*.

For *both pollutants,* the slack variable in the optimal solution is zero. Therefore, the optimal solution does not exceed the lower bound of the constraint for those two pollutants. This means that in the optimal solution the company does not reduce the emission of *the pollutant* more than required.

(2) Non-negativity constraint:

constraint	body	lower bound	slack variable
$x_{Region\ L1} \geq 0$	161.54	0	161.54
$x_{Region\ L2} \geq 0$	0	0	0
$x_{Region\ L3} \geq 0$	30.77	0	30.77

For constraints of the type *f(x) ≥ lower bound*, the slack variable is the difference between the actual value in the solution ("body") and the lower bound as formulated in the constraint. It can be interpreted as how much the current solution exceeds the lower bound.

In the table above you can see the slack variables for the non-negativity constraint, which belongs to the type *f(x) ≥ lower bound*.

For the regions L1 and L3, the slack variable is greater than zero, i.e. it exceeds the lower boundary (= 0) by the area, in which the fertilizer is used in the optimal solution.

For region L2, the slack variable is zero because the area, in which the fertilizer is used in the optimal solution is zero and therefore does not exceed the lower boundary of zero.

3. Sensitivity of the optimal solution to changed emission reduction requirements

From analyzing the slack variables of the emission reduction requirement constraint, we already know that the optimal solution does not exceed the lower boundary (i.e. minimum emission reduction) for both pollutants. The optimal solution results in the minimum emission reduction required by the regulation. Therefore, the optimal solution is sensitive towards a further restriction of the requirement for both pollutants.
To measure the sensitivity, we can extract the shadow price and the range, in which the shadow price is constant:

pollutant	shadow price	interval low	current	interval high
P1	69.23	30	35	56
P2	43.08	25	40	46.67

In the table above you can see the shadow prices for both pollutants in the current optimal solution as well as the interval for which the effect of changes in the emission reduction requirement can be determined using the shadow price. *interval low* is the lower limit of this interval and *interval high* is the upper limit of this interval.
It can be interpreted as follows: If the emission reduction requirement of pollutant P1 is between 30 and 56 tons, an increase of the requirement by one ton would lead to an increase of the optimal objective value by $69.23.

Therefore, if the requirement for the reduction of P1 would be 50 instead of 35, the costs would increase by

$$(50 - 35) * \$69.23 = 15 * \$69.23 = \$1{,}038.45$$

The optimal objective value (i.e. the costs) would now be $\$4{,}146.15 + \$1{,}038.45 = \$5{,}184.60$.

4. Effect of changes of the cost coefficients

The sensitivity of the optimal solution to the cost coefficients can be measured with the allowable range. The allowable range shows the smallest and the largest values of the coefficient for which the optimal value stays the same.

Region	Allowable range low	Current value	Allowable range high
L1	15	19	20.35
L2	20.69	26	∞
L3	31.55	35	44.33

The table above shown the allowable range for the cost coefficient for all three regions, rounded to two decimal places.
For each region (given that the cost coefficients for the other regions stay the same), cost values in the range between *Allowable range low* and *Allowable range high* will lead to the same optimal value.

For region L2, the allowable range is [20.69, ∞]. Therefore, if the cost of using the new fertilizer in one km^2 would be reduced to $23, the optimal decision will remain the same because $23 is within the allowable range.

PART B

1. Formulation of the linear model and conclusion regarding most profitable supplier

Indexes and sets:

$i \in I$: set of crude oils
$j \in J$: set of gasolines
$s \in S$: set of suppliers
$k \in K$: set of quality attributes

Parameters:

$A_{i,s}$: barrels of crude oil i available per week from supplier s
$C_{i,s}^{PUR}$: purchase price of one barrel of crude oil from supplier s in $
M_s: minimum value of purchases for supplier s per week
C_j^{PROD}: production costs of one barrel of gasoline j in $
P_j: sales price for one barrel of gasoline j in $
T_j: maximum production capacity of gasoline j per week
D_j: minimum demand for gas j per week
$R_{i,k}$: units of attribute k contributed per barrel of crude oil i
$L_{j,k}$: minimum allowed units of attribute k ber barrel of gas j
$U_{j,k}$: maximum allowed units of attribute k ber barrel of gas j

Decision variables:

$x_{i,s,j}$: barrels of crude oil i from supplier s used for production of gasoline j per week
y_j: barrels of gasoline j produced per week

Objective function:

The sum of revenues per barrel for each gasoline j minus the sum of purchasing costs for each crude oil minus the sum of production costs for each gasoline:

$$\max profit = \sum_{j \in J} P_j y_j - \sum_{i \in I} \sum_{s \in S} \sum_{j \in J} C_{i,s}^{PUR} x_{i,s,j} - \sum_{j \in J} C_j^{PROD}$$

Constraints:

Material availability (weekly purchase limit for each crude oil j):

$$\sum_{j \in J} x_{i,s,j} \leq A_{i,s} \quad \forall\, i \in I, s \in S \qquad (1)$$

Minimum purchase agreement (minimum total weekly purchase value for each supplier):

$$\sum_{i \in I} \sum_{j \in J} x_{i,s,j} C_{i,s}^{PUR} \geq M_s \quad \forall\, s \in S \qquad (2)$$

Production capacity (production limitation per week):

$$y_j \leq T_j \quad \forall j \in J \tag{3}$$

Material continuity (total barrels of crude oil used is equal to the total barrels of gasoline produced):

$$\sum_{i \in I} \sum_{s \in S} x_{i,s,j} = y_j \quad \forall j \in J \tag{4}$$

Octane quality (octane mix of crude oils must fit the requirement of the resulting gasoline):

$$\sum_{i \in I} \sum_{s \in S} x_{i,s,j} \geq L_{j,k} y_j \quad \forall j \in J, k \in K \tag{5}$$

Sulfur content (sulfur mix of crude oils must fit the requirement of the resulting gasoline; k should not be equal to 'Octane' → because there is no maximum octane content limit for all gasolines, but we must avoid multiplying with infinity in the constraint):

$$\sum_{i \in I} \sum_{s \in S} R_{i,k} x_{i,s,j} \leq U_{j,k} y_j \quad \forall j \in J, k \in K : k \neq 'Octane' \tag{6}$$

Market demand (minimum market demand must be satisfied):

$$y_j \geq D_j \quad \forall j \in J \tag{7}$$

Non-negativity constraint for the number of barrels of gasoline produced:

$$x_{i,s,j} \geq 0 \quad \forall i \in I, s \in S, j \in J \tag{7}$$

Solution:

Supplier S1:

	G1	G2	G3	G4	Sum crude oil
C1	9,000	0	5,000	6,000	**20,000**
C2	0	20,000	0	0	**20,000**
C3	0	13,776.5	0	0	**13,776.5**
C4	0	0	0	0	**0**
Sum gasoline	**9,000**	**33,776.5**	**5,000**	**6,000**	**53,776.5**

Supplier S2:

	G1	G2	G3	G4	Sum crude oil
C1	0	0	0	0	**0**
C2	0	4,903.69	0	0	**4,903.69**
C3	0	0	0	0	**0**
C4	0	11,319.9	5,000	6,000	**22,319.9**
Sum gasoline	**0**	**16223.59**	**5,000**	**6,000**	**27,223.59**

The tables above show the values for $x_{i,j,s}$ for each supplier (with crude oils C1, C2, C3 and C4 in the rows and gasolines G1, G2, G3, and G4 in the columns) in the optimal solution. The sum of columns (i.e. sum of inputs for each gasoline) is equal to the value y_j (i.e. number of produced barrels for each gasoline).

Objective value: maximum profit of the refinery:

- $\max profit = \$4{,}851{,}472.06$

Conclusion regarding most profitable product:

If we display the slack variable for the minimum purchase value constraint (constraint (2)), we see that the slack variable for supplier S1 is greater than zero and the slack variable for supplier S2 is zero. Therefore, the sourcing from supplier S2 in the optimal decision is at its minimum, whereas the oil refinery purchases more (i.e. higher value) from supplier S1 than it is obliged to based in the minimum purchase value. This indicates that supplier S1 is overall more profitable for the oil refinery.

The only other constraint, which influences the amounts purchased from the different suppliers, is the material availability constraint (constraint (1)). We see in the optimal solution that the oil refinery does not reach the maximum of the available crude oil amounts of supplier S2 for any of the crude oil types. For supplier S1 on the other hand, the refinery uses the maximum available amounts of crude oil C1 and C2. Therefore, the material availability is not the reason for the refinery purchasing only the minimum value from supplier S2. The reason must be that supplier S1 is overall more profitable than supplier S2.

2. Changes in minimum demand quantity for G1 and G2

To determine for which type of gasoline we would recommend increasing the capacity, we compute the shadow price for the production capacity constraint. The shadow price indicates the amount by which the optimal objective value changes if the constraint limitation is changed by one. Furthermore, we determine the lower bound and the upper bound of this shadow price in order to be able to say by how much the optimal objective value would change if we increased the constraint limitation by 5,000 units.

Gasoline	Shadow price	Lower bound	Current value	Upper bound
G1	0	9,000	50,000	∞
G2	57	36,223.5	50,000	66,223.5
G3	0	10,000	45,000	∞
G4	0	12,000	45,000	∞

The table shows that there is only one positive shadow price greater than 0. Therefore, the optimal objective value would change only if we increase the production capacity of gasoline G2 (or gasoline type A respectively). The upper bound of the shadow prices in the table shows, that the shadow price stays the same for all gasoline types, if the production capacity is increased by 5,000 since an increase of the current value by 5,000 is still within this bound for all gasolines. Therefore, it only makes sense to increase the production capacity of gasoline G2 (or gasoline type A) by 5,000 barrels per week.

The shadow price in this case is 57. Therefore, the optimal objective value (i.e. profit) increases by $57 if we increase the production capacity by one barrel per week. If we increase the production capacity of this gasoline by 5,000 barrels per week, the new capacity would be 55,000 barrels per week and would still be within the boundaries of the shadow price of $57. Therefore, the profit per week would increase by

$\$57 * 5{,}000 = \$285{,}000$ per week.

Therefore, after 52 weeks, the additional profit would be

$\$285{,}000 * 52 = \$14{,}820{,}000$.

Therefore, the additional profit exceeds the investment amount of $14,500,000 and the production capacity increase is recommended.

PART C

1. Formulation of the linear model

Indexes and sets:

$r \in R$: set of regions, in which the company can obtain salmon
$f \in F$: set of production facilities
$k \in K$: set of markets, in which the company can sell the salmon

Parameters:

$C^{TRA1}_{r,f}$: shipping costs per ton of salmon from region r to production facility f in $
$C^{TRA2}_{f,k}$: shipping costs per ton of salmon from production facility f to market k in $
A_r: available tons of salmon in region r per week
T: production capacity at each facility in tons per month
D_k: demand of salmon in market k in tons per week

Decision variables:

$x_{r,f,k}$: shipment of salmon from region r through production facility f to market k in tons per month

Objective function:

Transportation costs should be minimized:

$$\text{min costs} = \sum_{r \in R}\sum_{f \in F}\sum_{k \in K} C^{TRA1}_{r,f} x_{r,f,k} + \sum_{r \in R}\sum_{f \in F}\sum_{k \in K} C^{TRA2}_{f,k} x_{r,f,k}$$

Constraints:

Supply limit (monthly sourcing limit of salmon for each region r):

$$\sum_{f \in F}\sum_{k \in K} x_{r,f,k} \le A_r \quad \forall r \in R \qquad (1)$$

Production capacity constraint (maximum production capacity in production facilities shall not be exceeded):

$$\sum_{r \in R}\sum_{k \in K} x_{r,f,k} \le T \quad \forall f \in F \qquad (2)$$

Demand constraint (demand in markets must be satisfied):

$$\sum_{r \in R}\sum_{f \in F} x_{r,f,k} = D_k \quad \forall k \in K \qquad (3)$$

Non-negativity constraint:

$$x_{r,f,k} \geq 0 \quad \forall\, r \in R, f \in F, k \in K \tag{4}$$

Optimal solution:

$x_{r,f,k}$ for region R1:

	F1	F2
K1	0	0
K2	0	0
K3	0	17
K4	0	0
K5	0	22
K6	0	15
K7	0	20
K8	0	12
K9	0	0
K10	0	0
K11	0	19
K12	0	0
K13	0	25
K14	0	7
K15	0	0

$x_{r,f,k}$ for region R2:

	F1	F2
K1	15	0
K2	21	0
K3	0	0
K4	15	0
K5	0	0
K6	0	0
K7	0	0
K8	0	0
K9	15	0
K10	18	0
K11	0	0
K12	28	0
K13	0	0
K14	15	0
K15	23	0

min costs = \$10,002

2. Modifications: 20% demand increase & penalty costs for allowed unsatisfied demand

First, we implement the 20% demand increase (in all markets) only. Instead of manually changing all market demands in the data file in AMPL, the market demand increase is implemented by changing the demand constraint and keeping the data is in 1.:

New Demand constraint (demand in markets must be satisfied):

$$\sum_{r \in R} \sum_{f \in F} x_{r,f,k} = 1.2 * D_k \quad \forall\, k \in K$$

The result in AMPL for this new scenario is that the model is infeasible:

```
ampl: include 'C:\Users\mariu\OneDrive - Norges Handelshøyskole\M.Sc. Business Analytics\3. S
CPLEX 20.1.0.0: sensitivity
CPXobjsa failed; CPLEX Error  1260: Sensitivity analysis not available for current status.
.
CPXrhssa failed; CPLEX Error  1260: Sensitivity analysis not available for current status.
.
CPLEX 20.1.0.0: infeasible problem.
2 dual simplex iterations (0 in phase I)
constraint.dunbdd returned
2 extra dual simplex iterations for ray
```

This means that the constraints in the model are now conflicting. The demand, which must be satisfied in the markets, cannot be satisfied anymore. The reason could be that the available supply in the regions is not sufficient anymore or because the production capacities cannot handle the demand anymore.

Before the market demand increase, the overall sum of demand over all markets was 287 tons. The available supply of salmon was 200 tons for each region, 400 tons in total. The production capacity for each facility was 150 tons, 300 tons in total. Due to the market increase of 20%, the overall market demand increased from 287 tons to 344.4 tons. Therefore, it exceeded the total production capacity of the company, and the demand could not be satisfied.

Introduction of penalty costs for unsatisfied demand:

Allowing unsatisfied demand in the markets and introducing respective penalty costs leads to the following changes in the model:

1. **Introduction of a new parameter:**

   C_k^{PEN}: penalty costs per ton of unsatisfied demand in market k in $

2. **Objective function:**

   Sum of transportation costs and penalty costs should be minimized:

$$\text{min costs} = \sum_{r \in R} \sum_{f \in F} \sum_{k \in K} C_{r,f}^{TRA1} x_{r,f,k} + \sum_{r \in R} \sum_{f \in F} \sum_{k \in K} C_{f,k}^{TRA2} x_{r,f,k} + \sum_{k \in K} C_k^{PEN} * (1.2 * D_k - \sum_{r \in R} \sum_{f \in F} x_{r,f,k})$$

   (Remark: D_k still describes the initial market demands, not considering the 20% demand increase. Therefore, D_k is multiplied with 1.2 in the objective function)

3. **Change of demand constraint:**

   Since the defined demand in the market must not be satisfied anymore, the shipped quantities must not be equal to the demand anymore but can be less:

$$\sum_{r \in R} \sum_{f \in F} x_{r,f,k} \leq 1.2 * D_k \quad \forall\, k \in K$$

   (Remark: D_k still describes the initial market demands, not considering the 20% demand increase. Therefore, D_k is multiplied with 1.2 in the objective function)

Optimal solution

$x_{r,f,k}$ for region R1:

	F1	F2
K1	0	0
K2	0	0
K3	0	20.4
K4	0	0
K5	0	26.4
K6	0	18
K7	0	24
K8	0	14.4
K9	0	0
K10	0	0
K11	0	0
K12	0	0
K13	0	30
K14	0	0
K15	0	0

$x_{r,f,k}$ for region R2:

	F1	F2
K1	18	0
K2	25.2	0
K3	0	0
K4	18	0
K5	0	0
K6	0	0
K7	0	0
K8	0	0
K9	18	0
K10	21.6	0
K11	0	0
K12	0	0
K13	0	0
K14	0	0
K15	0	0

min costs = $9,578.4

Unsatisfied demand:

In order to determine, which markets have an unsatisfied demand, the slack variables for the demand constraint can be computed.

For constraints of the type *f(x) ≤ upper bound*, the slack variable is the difference between the actual value in the solution ("body") and the upper bound as formulated in the constraint. It can be interpreted as how much the current solution does not "utilize" the upper limit of the constraint.

	Demand.body	Demand.ub	Demand.slack
K1	18	18	0
K2	25.2	25.2	0
K3	20.4	20.4	0
K4	18	18	0
K5	26.4	26.4	0
K6	18	18	0
K7	24	24	0
K8	14.4	14.4	0
K9	18	18	0
K10	21.6	21.6	0
K11	0	22.8	22.8
K12	0	33.6	33.6
K13	30	30	0
K14	0	26.4	26.4
K15	0	27.6	27.6

For markets K11, K12, K14 and K15, the slack variable in the demand constraint is greater than zero. Therefore, there is unsatisfied demand in those markets. The unsatisfied demand is equal to the slack variable ("Demand.slack") in the table: 22.8 tons in K11, 33.6 tons in K12, 26.4 tons in K14 and 27.6 tons in K15.

Facility capacity utilization

In order to determine, if the production capacity in the facilities is fully utilized, we compute the slack variable of the production capacity constraint (analog to the demand constraint above).

	ProdCapacity.body	ProdCapacity.ub	ProdCapacity.slack
F1	100.8	150	49.2
F2	133.2	150	16.8

For both facilities, the slack variable is greater than zero. Therefore, both facilities do not fully utilize their production capacities. The slack variable indicates the difference between their maximum production capacity and their actual production volume in the optimal solution.

3. Modification: New production facility instead of allowing unsatisfied demand

This problem will be solved by creating two new models. In the first model, we assume that the new production facility will be built in region R1. Therefore, we add the transport costs from the regions to the facility and the transport costs from the facility to the markets to the data accordingly. Furthermore, we add the facility as a subscript to the parameter of the production capacity per facility f and add the capacity of the new facility to the data.

The optimal objective value of this model with the new facility in region R1 is $12,356.

Now, the same procedure can be done, and a second model can be created, this time assuming that the new facility will be built in region R2. The cost data must be added to the model accordingly.

The optimal objective value of this model with the new facility in region R2 is $11,738.

Therefore, the model with the facility in region R2 results in an optimal solution with lower total costs. It is recommended to open the new facility in region R2. The optimal shipping plan would be as follows:

$x_{r,f,k}$ for region R1:

	F1	F2	F3
K1	0	0	0
K2	0	0	0
K3	0	20.4	0
K4	0	0	0
K5	0	26.4	0
K6	0	18	0
K7	0	24	0
K8	0	14.4	0
K9	0	0	0
K10	0	0	0
K11	0	16.8	0
K12	0	0	0
K13	0	30	0
K14	0	0	0
K15	0	0	0

$x_{r,f,k}$ for region R2:

	F1	F2	F3
K1	18	0	0
K2	25.2	0	0
K3	0	0	0
K4	18	0	0
K5	0	0	0
K6	0	0	0
K7	0	0	0
K8	0	0	0
K9	18	0	0
K10	0	0	21.6
K11	6	0	0
K12	33.6	0	0
K13	0	0	0
K14	26.4	0	0
K15	4.8	0	22.8

The shipping plan shows that only markets K10 and K15 will be served by the new facility.

PART D

1. Infeasibility and unboundedness

a) Infeasibility

A linear programming model is infeasible if at least two restricting constraints are incompatible. Therefore, the combination of two constraints can be checked if they could be conflicting

1. Constraint (2) (resource availability constraint) and constraint (3) (max. demand constraint): The constraints cannot be conflicting. Both constraints induce an upper limit to the production quantity of the products, but no lower limit. k and $d_{i,t}$ are defined as positive parameters. Therefore, there will always be solutions which satisfy both constraints.
2. Constraint (2) (resource availability constraint) and constraint (4) (minimum increase of 10% of production of product p from one period to another): Constraint (4) makes sure that the decision variable (for each product i) in a time period p is 10% higher than the decision variable (for the same product i) in the previous time period. Therefore, the constraint ensures that the production quantities increase with time. Constraint (2) on the other hand induces an upper limit to the production as the resources are limited. However, even if the time horizon of the problem is very high, the initial production quantity of each product p can be selected as such a small number so that the resource availability constraint can still be met after the time horizon. Therefore, the constraints can never lead to an infeasible result.
3. Constraint (2) (resource availability constraint) and constraint (5) (non-negativity of decision variable): Those two constraints can never be incompatible. Constraint (2) induces an upper limit to the production quantity of the products due to the resource limit k. As k is defined as a positive parameter, there will always be solutions of the decision variable which satisfy the resource availability constraint and the non-negativity constraint.
4. Constraint (3) (max. demand constraint) and constraint (4) (minimum increase of 10% of production of product p from one period to another): Constraint (4) makes sure that the decision variable (for each product i) in a time period p is 10% higher than the decision variable (for the same product i) in the previous time period. Therefore, the constraint ensures that the production quantities increase with time. Constraint (3) on the other hand induces an upper limit to the production as demand for the products cannot be exceeded. However, even if the time horizon of the problem is very high, the initial production quantity of each product p can be selected as such a small number so that the demand limit constraint can still be met after the time horizon. Therefore, the constraints can never lead to an infeasible result.
5. Constraint (3) (max. demand constraint) and constraint (5) (non-negativity of decision variable): Those two constraints can never be incompatible. Constraint (3) induces an upper limit to the production quantity of the products due to the demand limit $d_{i,t}$. As $d_{i,t}$ is defined as a positive parameter, there will always be solutions of the decision variable which satisfy the demand limit constraint and the non-negativity constraint.
6. Constraint (4) (minimum increase of 10% of production of product p from one period to another) and constraint (5) (non-negativity of decision variable): There will always be solutions which satisfy both constraints. Constraint (4) makes sure that the production quantity for each product increases by at least 10% in each time period. Constraint (5) defines non-negativity for the production quantity of each product in each time period. Therefore, it cannot be incompatible with constraint (4).

Therefore, with the given constraints and definitions in the problem, the problem cannot be infeasible.

b) Unboundedness

A linear programming model is unbounded, if it is feasible but the objective function is infinite (i.e. it can be made arbitrarily favorable). The objective function in this model consists of the coefficient $p_{i,t}$ and the decision variable $x_{i,t}$. The parameter $p_{i,t}$ describes the revenue of one unit of product i in time period t. It is set as a positive parameter and can therefore not be unbounded (i.e. infinity). Therefore, the model could only be unbounded, if $x_{i,t}$ is unbounded.

A lower bound of $x_{i,t}$ is included in the model by the non-negativity constraint. Therefore, $x_{i,t}$ cannot be unbounded (direction to negative infinity).
Constraint (2) and (3) set an upper bound for the production quantity $x_{i,t}$. The respective upper limits are defined as positive parameters, i.e. not infinity. Therefore, $x_{i,t}$ cannot be unbounded (direction to positive infinity).

Therefore, the model cannot be unbounded.

2. Modification of parameter $d_{i,t}$

a) The feasibility of the problem is independent from parameter $d_{i,t}$ as long as it is defined as a positive parameter. The only constraint, which is affected by the modification of parameter $d_{i,t}$, is constraint (3). There is no other constraint which restricts the decision variable in a way that it is incompatible with constraint (3). Therefore, the model could not become unfeasible.

b) Reducing the maximum demand of the first product in period 1 from $d_{1,1}$ to $0.9d_{1,1}$ has a restricting effect on the decision variable $x_{i,t}$. The allowed range of the decision variable in period 1 for product 1 becomes less because the maximum demand for this combination becomes less. Therefore, the number possible values for $x_{i,t}$ becomes less. Not every feasible solution in the original problem will also be feasible in the modified problem.

   Therefore, z_2^* will be less than or equal to (≤) 650. It could be less than 650, if the initial optimal solution included a value for $x_{1,1}$ of more than $0.9d_{1,1}$. It could be equal to 650 if the initial optimal solution included a value for $x_{1,1}$ of less than or equal to $0.9d_{1,1}$

3. Conclusion about demand satisfaction

Since the slack variables associated to constraints (2) are positive for all t ∈ T (i.e. resource available per period was never used at the fullest), another constraint must have always limited the production quantity of the products. The only constraint, which qualifies for being responsible for limiting the production quantity are constraint (3) (max. demand constraint) and constraint (4) (minimum increase of 10% of production of product p from one period to another).