PROJECT 1
Linear Programming

Submit your project through *WISEflow*. The submission deadline is Monday October 11th, at 14:00 hrs. The project can be done individually or in a group of at most 2 students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) compressed in a single file (.zip). Include all files needed to run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submitted). In addition, provide a written report as a pdf file with your model formulations and the answers to the questions required in each part. The formulation of your models can be typed in a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Recall using the solver *cplex* to solve the models. Provide a short description (no more than two sentences, e.g. "demand fulfillment") for every objective function and constraint in your formulations. Expected (not required) length of your report: Parts A, D one page each; Parts B, C between 2 and 3 pages each. All model formulations in this project must be linear.

Part A

A farming company grows vegetables in three locations within a same region (L1, L2, and L3). Due to the use of chemical fertilizers, the company emits two types of pollutants (P1 and P2) into the soil of this region. The regional government is promoting the use of a new type of fertilizers, in order to reduce the soil pollution. It costs \$19 to use the new fertilizer in one km^2 at L1, and per each km^2 in which it is used the amount of pollutant P1 is reduced by 0.15 ton and the amount of pollutant P2 by 0.20 ton. It costs \$26 to use the new fertilizer in one km^2 at L2, and per each km^2 in which it is used the amount of pollutant P1 is reduced by 0.05 ton and the amount of pollutant P2 by 0.40 ton. It costs \$35 to use the new fertilizer in one km^2 at L3, and per each km^2 in which it is used the amount of pollutant P1 is reduced by 0.35 ton and the amount of pollutant P2 by 0.25 ton. The regional government wants to reduce the total amount of pollutant P1 in the region by at least 35 tons and the amount of pollutant P2 by at least 40 tons. The company needs to fulfil this condition by incorporating the new fertilizers in part of its locations. Assume the locations owned by the company are large enough to satisfy the requirement of the government (that is, there is no upper limit on the amount of km^2 where the new type of fertilizer can be used).

1. Formulate a linear programming model to decide the amount of km^2 per location where the company should incorporate the new fertilizers, such that the optimal solution minimizes the total cost. Implement the model in AMPL and solve it. What is the optimal solution? What is the optimal objective value?

2. Briefly discuss how the constraints are satisfied in the optimal solution, based on the slack variables.

3. How sensitive is the optimal cost to the targets of the government? In particular, if the target of P1 increases to 50 tons can you conclude what is the effect in the cost without running the model again?

4. Investigate the effect of changes of the cost coefficients on the optimal solution. How sensitive are the decisions to the accuracy in these coefficients? In particular, if the cost of using the new fertilizer in one km^2 at L2 would be reduced to \$23, would your optimal decisions remain the same?

Part B

Consider an oil refinery company producer of 4 types of gasoline (G1, G2, G3 and G4). Each gasoline is produced by blending 4 types of crude oil (C1, C2, C3 and C4). The crude oils are obtained from two different suppliers $S1$ and $S2$.

The weekly purchases from $S1$ of crude oil C1 and C2 is limited to 20,000 barrels each, while the purchases of crude C3 is limited by 30,000 barrels per week. Supplier $S1$ has no offer of crude C4.

The weekly purchases from $S2$ of crude oil C2 and C3 is limited to 25,000 barrels each, while the purchases of crude C4 is limited by 40,000 barrels per week. Supplier $S2$ has no offer of crude C1.

The purchase prices per barrel obtained from supplier $S1$ are \$115 for C1, \$100 for C2 and \$105 for C3. The purchase prices per barrel obtained from supplier $S2$ are \$102 for C2, \$107 for C3 and \$112 for C4. The refinery has an agreement of minimum purchases with both suppliers. The agreement states that the total weekly purchases from each of these two suppliers must have a minimum value of \$3,000,000.

The gasolines have been classified in two types: G1 and G2 are of type A, and G3 and G4 are of type B. The production costs per barrel of gasolines are \$8 for gasolines of type A and \$10 for gasolines of type B. The sales prices per barrel are \$160 for G1, \$170 for G2, \$195 for G3 and \$200 for G4.

The production capacity for gasolines of type A in total is limited to 50,000 barrels per week, while the capacity for gasolines of type B in total is limited to 45,000 barrels per week.

There are minimum demand quantities that must be satisfied for each product. These quantities are: 9000 barrels of G1; 7000 barrels of G2; 10,000 barrels of G3; and 12,000 barrels of G4 per week. Any production above these minimum quantities is also sold in the market of gasolines.

The crude oils differ in quality, so the gasoline quality is dependent on the proportion of each crude oil in blending (assume linearity of yield). The quality of crude oils and gasolines is measured by two characteristics: average octane rating, and sulfur content (in percentage). Table 1 shows the quality of the crude oils and Table 2 shows the minimum average octane rating and maximum sulfur content required for each type of gasoline.

Crude	Octane rating	Sulfur content
C1	98	0.2%
C2	90	2.0%
C3	92	1.5%
C4	96	0.4%

Table 1: Quality of crude oils in octane rating and sulfur content.

Gasoline	Octane rating	Sulfur content
G1	88	2.0%
G2	92	1.5%
G3	96	0.3%
G4	97	0.3%

Table 2: Minimum average octane rating and maximum sulfur content of the different types of gasoline.

1. Formulate a linear programming model to determine a weekly blending plan such that the refinery maximizes profit. Implement the model in AMPL and solve it. What is the optimal blending plan and how much profit it provides? Without explicit use of data on costs and prices, but only the rest of the data and the optimal solution, could you conclude which supplier is overall the most profitable for the refinery?

2. The refinery is evaluating to increase the production capacity by 5,000 barrels per week either for gasolines of type A or type B. Without running a new model, for which type of gasoline would you recommend to increase the capacity? In particular, if the increase of capacity requires an investment of \$14,500,000 and the refinery would like to obtain the investment return in at most one year (52 weeks), would you recommend to increase capacity? (Assume the parameter values remain stable during this year).

Part C

A seafood company obtains salmon from two regions. In Region R1 the company can obtain as many as 200 ton per month, and in Region R2 it can also obtain as many as 200 ton per month. From these regions, the salmon is sent to any of the two production facilities of the company, denoted by F1 and

F2. The cost of transporting 1 ton from R1 to F1 and F2 is \$15 and \$12, respectively. The cost of transporting 1 ton from R2 to F1 and F2 is \$10 and \$13, respectively.

The production capacity at each facility is 150 ton per month. After the salmon is processed at the facilities, the final product is ready to be shipped to the markets (assume the total production in ton equals the number of ton processed). The company perceives demand from 15 markets, denoted as K1,K2,...,K15. The monthly demands of the markets are shown in Table 3 and the costs of shipping 1 ton between facilities and markets are shown in Table 4.

K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15
15	21	17	15	22	15	20	12	15	18	19	28	25	22	23

Table 3: Demand from each market expressed in ton.

$	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15
F1	9	9	21	21	42	42	90	87	18	24	48	30	75	24	33
F2	84	51	15	60	21	24	27	21	96	84	51	84	12	60	114

Table 4: Costs of shipping 1 ton between facilities and markets.

1. Formulate a linear programming model to minimize the monthly transport costs in meeting demands of all markets. Implement the model in AMPL and solve it. What is the optimal shipping plan and how much does it cost?

2. Suppose for the incoming month the company anticipates an increase of 20% in the demand from each market. Run your model for this new scenario. What do you obtain? Assume the marketing and operation managers have agreed on allowing for unsatisfied demand, but at a certain penalty cost (due to the negative impact in reputation for the company). The penalization cost has been defined according to priorities the company has on the markets. Per each ton of unsatisfied demand in markets K1,K2,...,K5, the penalty cost is \$100; for markets K6,K7,...,K10, it is \$50 per ton; and for markets K11,K12,...,K15 it is \$25 per ton. Formulate a linear programming model to minimize the monthly costs (including transportation and penalty costs) in this new situation. Implement the model in AMPL and solve it. What is the optimal shipping plan and how much does it cost? If there are customers with unsatisfied demand, identify them and report the unsatisfied demand for each of them. Is the total capacity of the facilities used at its maximum?

3. Instead of allowing unsatisfied demand, the company is evaluating to build a new production facility in one of the two regions. The capacity of this new facility will be of only 50 ton per month. The cost of transporting 1 ton from the potential new facility and the markets is shown in Table 5. If the new

$	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15
R1	72	64	72	120	72	24	96	100	108	48	88	128	48	68	76
R2	52	72	144	96	140	80	168	168	84	20	160	108	152	64	60

Table 5: Costs of shipping 1 ton between the potential new facility and markets.

facility is built in region R1, the salmon obtained from region R1 will be transported at no cost to the facility, while the salmon obtained from region R2 will be transported to the new facility at a cost of \$20 per ton. Likewise, if the new facility is built in region R2, the salmon obtained from region R2 will be transported at no cost to the facility, while the salmon obtained from region R1 will be transported to the new facility at a cost of \$20 per ton. Evaluate these two alternatives. Where would you recommend to open the new facility? What would be the optimal shipping plan and how much would it cost? Which customers would be served from the new facility?

Part D

Consider the following linear programming model:

$$
\begin{aligned}
\max z &= \sum_{i\in I}\sum_{t\in T} p_{i,t}x_{i,t} && (1)\\
\text{s.t.} \quad \sum_{i\in I} r_i x_{i,t} &\leq k \quad \text{for all } t\in T && (2)\\
x_{i,t} &\leq d_{i,t} \quad \text{for all } i\in I, t\in T && (3)\\
x_{i,t} - 1.1x_{i,t-1} &\geq 0 \quad \text{for all } i\in I, t\in T : t\geq 2 && (4)\\
x_{i,t} &\geq 0 \quad \text{for all } i\in I, t\in T, && (5)
\end{aligned}
$$

where I and T are sets; $p_{i,t}$, r_i, k, $d_{i,t}$ are positive parameters (for all $i\in I$, $t\in T$); and $x_{i,t}$ are decision variables. You may interpret this model as a representation of a problem where: I is a set of products; T is a set of time periods in a planning horizon; the decision variables are the production quantities of each product $i\in I$ in each time period $t\in T$; the revenue associated to one unit of product i in period t is $p_{i,t}$; the objective is to maximize the total revenue; the production of one unit of product i requires r_i units of resource; the total resource available in each period is k; it is possible to satisfy demand of up to $d_{i,t}$ for product i in period t; the production of product i must increase by at least 10% from one period to the next period.

1. a) Could this model be infeasible?
   b) Could this model be unbounded?
2. Suppose you have solved the model for a particular instance of data which is feasible and results in an optimal objective value $z^* = 650$. In another instance of data that you would like to evaluate, the values of all parameters remain the same except for the maximum demand of the first product in period 1, which is reduced by 10% (that is, to $0.9d_{1,1}$).
   a) Could the new model be infeasible?
   b) Assume that you run the new model and obtain an optimal objective value z_2^*. Could z_2^* be equal to 650? Could z_2^* be greater than 650? Could z_2^* be less than 650?
3. Consider again the original model with the original data. Suppose the production plan has been executed according to the optimal solution to the model. At the end of the planning horizon, an operator tells you that the resource available per period was never used at the fullest. As you are an expert in decision modelling, you conclude that the slack variables associated to constraints (2) are positive for all $t\in T$. The day after you have a meeting with the marketing department, where the results about the satisfaction of the demand quantities $d_{i,t}$ will be discussed. What could you say about the results on demand satisfaction over the time periods of the planning horizon? Elaborate your answer as explicitly as possible in terms of the parameters of the model.