INTRODUCTION TO AC CIRCUITS

AC CIRCUITS $\longrightarrow$ deals with AC Current & AC Voltage.

(i) What is AC current? (மாறுதிசை மின்னோட்டம் / ஆ. மின்னோட்டம்

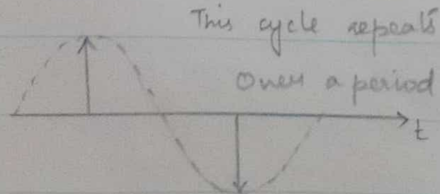
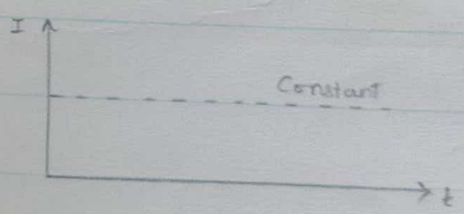
(ii) What is AC voltage? (மாறுதிசை மின்னழுத்தம்)

(iii) What are the advantages of A.C.

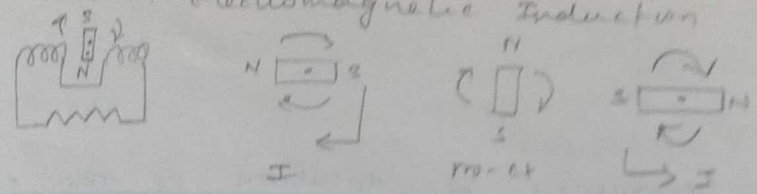
(i) ALTERNATING CURRENT (AC)

When the current flowing in the circuit varies in magnitude (அளவும்) as well as direction (திசையும்) (angle) periodically (with respect to time), it is called alternating current.

Difference between A.C. & D.C.

A.C.	D.C.
1. Magnitude & direction of the current varies with respect to time. → Positive (நேர்திசை) direction → Negative (எதிர்) (reverse) direction	The magnitude & direction of the current is constant irrespective of the time. → Uni direction
	

Law : Electromagnetic Induction



(ii) ALTERNATING VOLTAGE : (

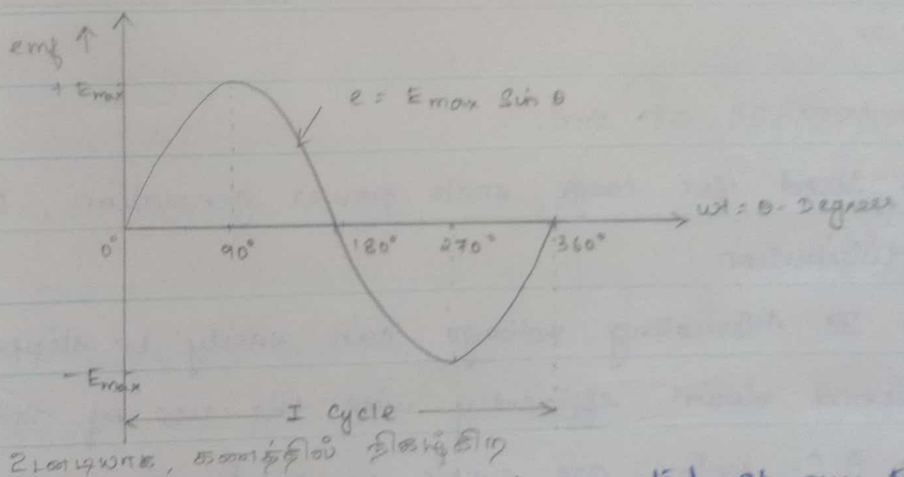
→ When a conductor is rotated in a magnetic field, an alternating emf is generated in the conductor.

→ By changing the magnetic field within the stationary coil - emf can be generated.

→ The emf depends on (i) Strength of the magnetic field

(ii) Number of turns in the coil

(iii) The speed at which the coil or the magnetic field rotates.



* The instantaneous value of emf generated at any time 't' is

$$\boxed{e = E_{max} \sin \omega t} \rightarrow (1)$$

* The instantaneous value of the induced alternating current is given by,

$$\boxed{i = I_{max} \sin \omega t} \rightarrow (2)$$

Note : Instantaneous Value : Value of the sine wave at any instant of the cycle.

(சுற்று
அதிர்வெண்)

If 'f' is the frequency of rotation of the coil, (ie).
the number of cycles passed through per second, then
(நொடி)
(வினாடி)

$$\boxed{\omega = 2\pi f} \rightarrow (3)$$

Applying (3) in (1) & (2), we get

Instantaneous emf generated, $e = E_{max} \sin(2\pi f)t$

Sinusoidal Voltage $\rightarrow$ $\boxed{e = E_{max} \sin\left[2\pi \cdot \frac{1}{T}\right] \cdot t}$

Instantaneous value of Induced Current, i $\}$ $i = I_{max} \sin(2\pi f)t$

Sinusoidal Current $\rightarrow$ $\boxed{i = I_{max} \sin\left[\frac{2\pi}{T}\right] \cdot t}$

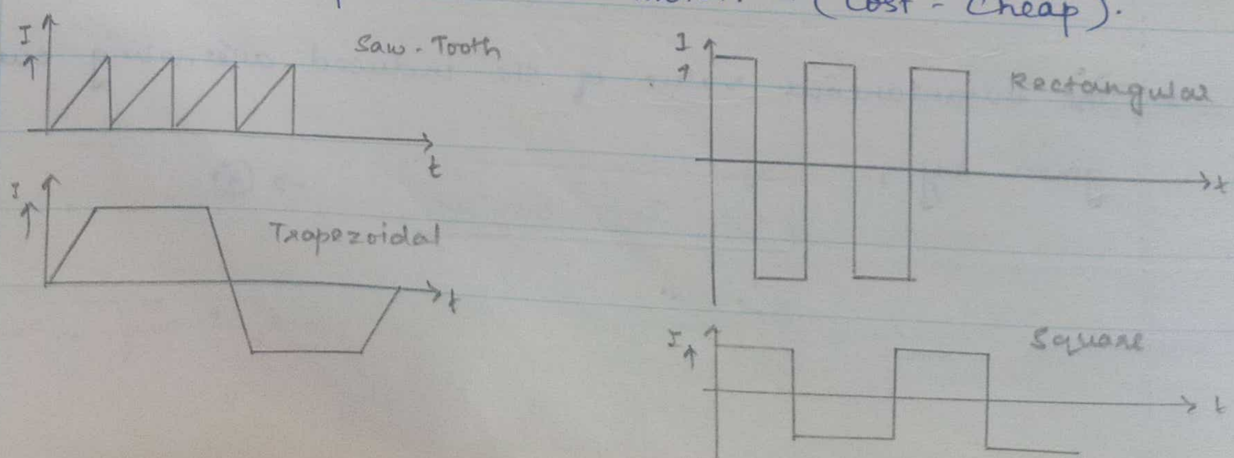
ADVANTAGES OF A.C. :

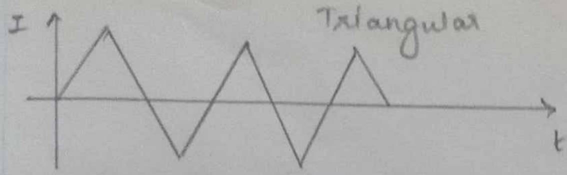
→ Used for large scale power generation, transmission & distribution.

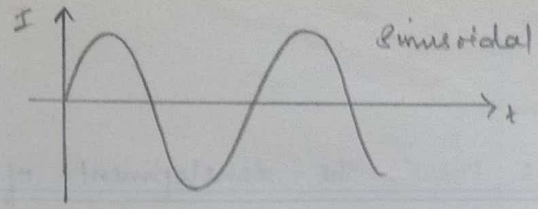
→ ~~It~~ Alternating voltage can easily be stepped up and stepped down efficiently with the use of transformers.

→ A.C. motors are simple in construction, efficient & more robust as compared to d.c. motor. & (Cost - Cheap).

Types of Waveforms







TERMINOLOGY :

1. WAVEFORM :

A waveform is a graph in which the instantaneous value of any quantity is plotted against time.

Alternating Waveform : This is a wave which reverses its direction at regularly recurring intervals.

Sinusoidal & Non-sinusoidal Waveform :

sinusoidal : It is an alternating waveform in which sine law is followed.

Non-sinusoidal : " " " is not followed

→ Types of waveforms

2. CYCLE :

One complete set of positive and negative halves constitute a cycle.

3. TIME PERIOD : (T) காலக் கட்டம்

* Time taken to complete one cycle.
* Time period = Reciprocal of frequency. $T = 1/f$
* Expressed in secs.

4. FREQUENCY (f) :

The number of cycles per second of an alternating quantity is known as frequency.

Unit : Cycles/second or Hertz

வீச்சம் / வீச்சு

5. AMPLITUDE OR PEAK VALUE :

The maximum positive or negative value of an alternating quantity is called the amplitude.

கட்டம்.

6. **PHASE:** The development of an ac quantity through different stages

7. **IN-PHASE:**

When two alternating quantities reach their maximum and minimum values simultaneously (at same time), then these quantities are said to be in phase with each other.

கோணம்

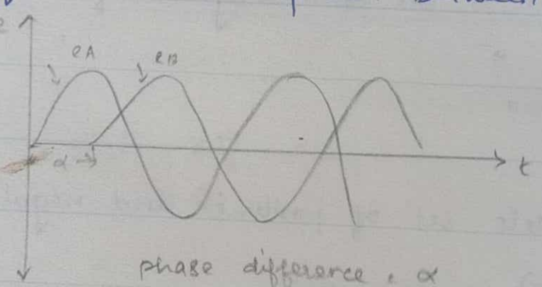
8. **PHASE ANGLE:**

It is an angular displacement between alternating quantities.

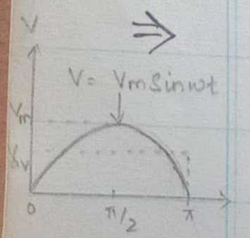
கட்ட வேறுபாடு

9. **PHASE DIFFERENCE:**

The difference in time period between two waveforms.



phase difference, α



Average value

⇒ AVERAGE VALUE: சராசரி மதிப்பு (Avg. ct)

The arithmetical average of all the values of an alternating quantity over one cycle is called as an average value.

$$\boxed{\text{Average value} = \frac{\text{Area under the curve}}{\text{Base period}}}$$

$$\left.\begin{array}{l}\text{Area of} \\ \text{Half cycle} \\ \text{Vtg wave}\end{array}\right\} = \int_0^{\pi} V \, d\theta$$

$$= \int_0^{\pi} V_m \sin\theta \, d\theta$$

$$= V_m \left[-\cos\theta\right]_0^{\pi}$$

$$= V_m \left[1 - (-1)\right]$$

$$\Rightarrow \boxed{2V_m = V_{avg}}$$

$$V_{av} = \frac{\text{AREA}}{\text{BASE}} = \frac{2V_m}{\pi} = 0.637\, V_m$$

DEFINITION:

The steady or direct current which transfer in a circuit the same charge as is transferred by the a.c. ~~during~~ one alternation in the same circuit in the same time.

$$\therefore \boxed{\begin{array}{l} V_{av} = \dfrac{2V_m}{\pi} = 0.637\, V_m \\ I_{av} = \dfrac{2}{\pi} I_m = 0.637\, I_m \end{array}}$$

சம அமைப்பொடு

For symmetrical waves:

These are one which has positive half cycle exactly equal to the negative half cycle.

$$\boxed{\left.\begin{array}{l}\text{Average value for} \\ \text{symmetrical waves}\end{array}\right\} = \frac{\text{Area under half the cycle}}{\text{Half the period.}}}$$

For unsymmetrical waves: செய்வொழுங்கு இல்லாத

Waves which are not symmetrical → unsymmetrical waves

$$\boxed{\left.\begin{array}{l}\text{Average value for} \\ \text{unsymmetrical waves}\end{array}\right\} = \frac{\text{Area over one Cycle}}{\text{Base Time for one cycle}}}$$

சராசரி வர்க்கமூலம்

⇒ ROOT MEAN SQUARE VALUE (R.M.S) / EFFECTIVE VALUE:

The R.M.S value of alternating current is defined as that நிலையான steady current (d.c) which when flowing through a given தடுப்பாற்றல்/எதிர்ப்பு (தடையம்) resistance for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistance for the same time.

$$\boxed{I_{rms} = \sqrt{\frac{\text{Area Under the Squared wave}}{\text{Period}}}}$$

பரப்பளவு

Sinusoidal current & voltage

$$I_{RMS} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$V_{RMS} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$\boxed{I_{rms} = \frac{I^2}{\pi}}$$

$$I^2 = \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta$$

For sinusoidal current,

$$I^2 = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left[\pi - 0 - \sin\frac{2\pi}{2} + 0\right]$$

$$I_{rms}^2 = \frac{I_m^2}{2}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

R.M.S. VALUE OF SYMMETRICAL WAVE:

$$\text{R.M.S Value} = \sqrt{\frac{\text{Area Under Half Cycle of Squared wave}}{\text{Half cycle period.}}}$$

Internal Resistance அகமின்தடுப்பு

$$\therefore \boxed{\begin{array}{l} I_{rms} = I_m/\sqrt{2} = 0.707\, I_m \\ V_{rms} = V_m/\sqrt{2} = 0.707\, V_m \end{array}}$$

Relation b/w Avg, RMS & max values can be expressed by two factors → i) Form factor (K_f); 2) Peak factor (K_p)

⇒ **FORM FACTOR: (K_f):** Indicates the shape of the form of the ac wave

→ Ratio of RMS value to the average value.

$$\boxed{\text{Form Factor } (K_f) = \frac{\text{R.M.S. Value}}{\text{Average Value}}} = \frac{0.707\, I_m}{0.637\, I_m} = 1.11 \quad \text{(sine wave)}$$

→ Greater the form factor, sharper is the wave. Δ^{le} wave, $K_f > 1.11$

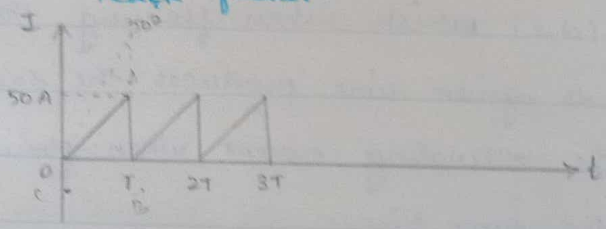
⇒ **PEAK FACTOR: (K_p): / CREST FACTOR**

Ratio of Peak value to the R.M.S. Value

$$\boxed{\text{Peak Factor } (K_p) \text{ (or) Crest factor} = \frac{\text{Peak Value}}{\text{RMS value}}}$$

② **PROBLEMS:**

1. Find out the average value of the sawtooth wave & R.M.S. value. Form factor + Peak factor.



$m = \frac{AB}{BC} = \frac{I_0}{T}$

$y = mx$

$m = \frac{50}{T}\, t$

Solution:

Given: Maximum value = 50A

Time period = T

(i) Average Value:

$$\text{Avg Value} = \frac{\text{Area under the Curve}}{\text{Base period}}$$

$\frac{1}{T}\int_0^T \frac{50}{T}\, t$

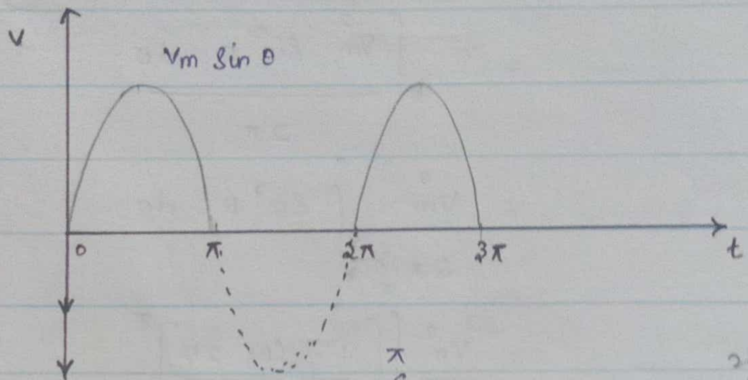
Area of Δ^{le} = $\frac{1}{2} \times b \times h = \frac{1}{2} \times T \times 50 = 25T$

$\therefore$ Avg Value $= \frac{25T}{T} = 25A$

$$\boxed{\text{Avg value} = 25A.}$$

I

=> RMS & AVERAGE VALUES OF HALF-WAVE RECTIFIED QUANTITY:



$$\therefore \text{Average value} = \frac{\int_0^{\pi} V_m \sin\theta \cdot d\theta + \int_{\pi}^{2\pi} V_m \sin\theta \cdot d\theta}{2\pi}$$

$\int_{\pi}^{2\pi} \to 0$

$\cos 0 = 1$
$\cos\pi = -1$
$\cos 2\pi = 1$

$\sin 0 = 0$
$\sin\pi = 0$
$\sin 2\pi = 0$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta \cdot d\theta + 0$$

$$= \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{V_m}{2\pi}\left[\underset{+(-1)}{-\cos\pi} + \underset{(+1)}{\cos 0^\circ}\right]$$

$$= \frac{V_m}{2\pi}\left[1+1\right] = \frac{V_m(\cancel{2})}{\cancel{2}\pi}$$

$$\boxed{V_{avg} = \frac{V_m}{\pi}} \rightarrow \text{AVERAGE VALUE}$$

$|||^{rly}$

$$\boxed{I_{avg} = \frac{I_m}{\pi}}$$

$\therefore$ RMS Value $= \sqrt{\dfrac{\text{Area of the squared curve}}{\text{Base period}}}$

$$= \frac{\int_0^{\pi} V_m^2 \sin^2\theta \cdot d\theta}{2\pi}$$

$$= \frac{V_m^2}{2\pi} \int_0^{\pi} \sin^2\theta \cdot d\theta$$

$$= \frac{V_m^2}{2\pi} \int_0^{\pi} \left[\frac{1 - \cos 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m^2}{4\pi} \left[\theta - \left(\frac{\sin 2\theta}{\cancel{2}}\right)\right]_0^{\pi}$$

$$= \frac{V_m^2}{4\pi} \left[[\theta]_0^{\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{\pi} \right]$$

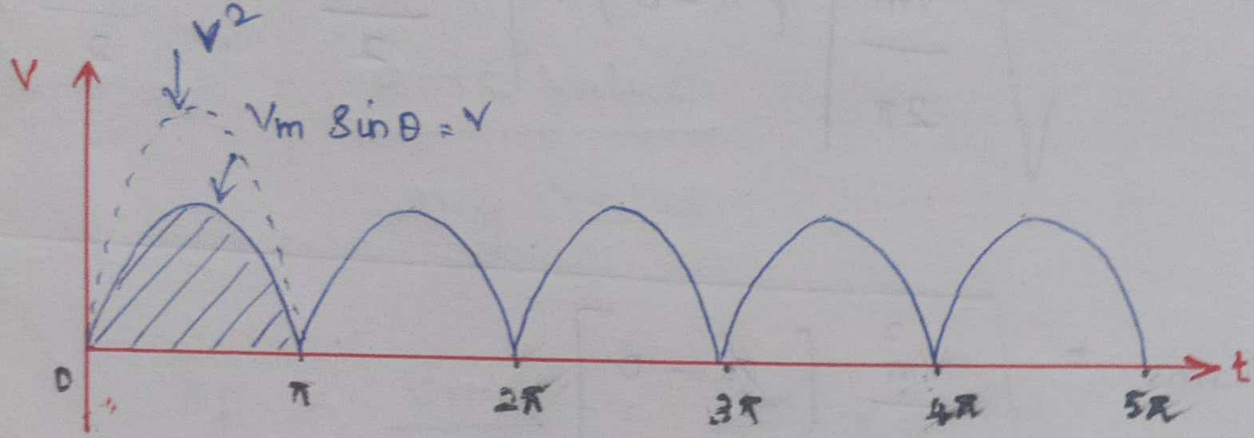
$$= \frac{V_m^2}{4\pi} \left[\pi - \frac{\sin 2\pi}{2} - \frac{\sin 0}{2}\right]$$

$\therefore$ RMS Value, $V_{rms} = \sqrt{\dfrac{V_m^2 \cancel{\pi}}{4\cancel{\pi}}} = \dfrac{V_m}{2}$ //

$$\boxed{\begin{array}{l} V_{rms} = \dfrac{V_m}{2} \\ \text{III}^{rly} \quad I_{rms} = \dfrac{I_m}{2} \end{array}}$$

$\rightarrow$ RMS VALUE OF HALF-WAVE RECTIFIED QUANTITY

II. RMS & AVG VALUE OF FULL WAVE RECTIFIER



Full-Wave-Rectified wave : symmetrical

$\therefore$ Base period $= \pi$

Vtg Eqn, $V = V_m \sin\theta$.

(i) RMS Value :

$$V_{rms} : \sqrt{\frac{\text{Area Under the Squared Curve (Half)}}{\text{Half Base Period (Time)}}}$$

$$= \sqrt{\frac{\int_0^{\pi} V_m^2 \sin^2\theta \, . \, d\theta}{\pi}}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\left[\frac{1 - \cos 2\theta}{2}\right] d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\left[\theta\right]_0^{\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{\pi}\right]}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi - 0) - \left[\frac{\sin 2\pi}{2} - \frac{\sin 2(0)}{2}\right]\right]}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\cancel{\pi}}\left[\cancel{\pi} - 0\right]}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

|||ly $\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$

(ii) Avg Value :

$$V_{avg} = \frac{\int_0^{\pi} V_m \sin \theta \cdot d\theta}{\pi}$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin \theta \cdot d\theta$$

$$= \frac{V_m}{\pi}\left[-\cos \theta\right]_0^{\pi} = \frac{V_m}{\pi}\left[-(\cos \pi - \cos 0\right.$$

$$= \frac{V_m}{\pi}(-(-2))$$

$$\boxed{V_{avg} = \frac{2V_m}{\pi}}$$, |||ly $\boxed{I_{avg} = \frac{2I_m}{\pi}}$

(iii) Form Factor (k_f)

$$k_f = \frac{\text{RMS Value}}{\text{Avg Value}}$$

$$k_f = \frac{V_{rms}}{V_{avg}} = \frac{\cancel{V_m}}{\sqrt{2}} \times \frac{\pi}{2\cancel{V_m}} = \frac{\pi}{2\sqrt{2}}$$

$$\boxed{k_f = 1.11}$$

(iv) Peak factor (k_p)

$$k_p = \frac{\text{Peak Value}}{\text{RMS Value}}$$

$$k_p = \frac{\cancel{V_m}}{\cancel{V_m}/\sqrt{2}} = 1.414$$

$$\boxed{k_p = 1.414}$$

IMPORTANT FORMULAS TO REMEMBER

(i) AVERAGE VALUE:

(i) Symmetrical Waves:

Average Value = $\frac{\text{Area Under the Curve (Half) (cycle)}}{\text{Base period (Half)}}$

Sinusoidal (Full wave)

$$\boxed{I_{avg} = \frac{2 I_m}{\pi}}$$

$$\boxed{V_{avg} = \frac{2 V_m}{\pi}}$$

(Half Wave)

$$\boxed{I_{avg} = \frac{I_m}{\pi}}$$

$$\boxed{V_{avg} = \frac{V_m}{\pi}}$$

(ii) Unsymmetrical Waves:

Average value = $\frac{\text{Area under one Complete Cycle}}{\text{Base Period}}$

(ii) RMS VALUE: / EFFECTIVE VALUE:

(i) Unsymmetrical Waves:

RMS Value = $\frac{\text{Area under the squared curve}}{\text{Base Period}}$

(ii) Symmetrical Waves:

RMS Value = $\frac{\text{Area Under Squared Half Curve (cycle)}}{\text{Half Base Period}}$

Sinusoidal: (Full wave)

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}} \; ; \; V_{rms} = \frac{V_m}{\sqrt{2}}}$$

(Half wave) (Rectified)

$$\boxed{I_{rms} = \frac{I_m}{2} \; , \; V_{rms} = \frac{V_m}{2}}$$

(iii) Form factor (K_f) = $\frac{\text{RMS Value}}{\text{Average Value}}$

(iv) Peak factor (K_p) (or) Creast factor = $\frac{\text{Peak Value}}{\text{RMS Value}}$

(v) If Delay angle is given, apply the limit from delay angle to the time period (one cycle) / half cycle
(unsymm) (symm)

$I = I_m \sin \omega t$

$V = V_m \sin \omega t$

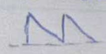
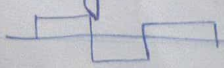
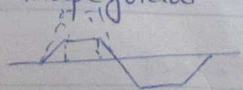
$\boxed{\omega = 2\pi f}$

eg: delay angle $\theta = \frac{\pi}{4}$

Time period $= \pi$

$\therefore \int_{\pi/4}^{\pi} I_m \sin \omega t \cdot dt$

$\boxed{f = \frac{1}{T} \text{ (Hz)}} \Rightarrow \boxed{T = \frac{1}{f} \text{ sec}}$

	(Area) AVG VALUE	(Squared Area) RMS VALUE
(i) Saw Tooth 	$\frac{1}{2} bh$	$\frac{1}{3} h^2 \cdot b$
(ii) Rectangle 	$l \times b$	$l^2 \times b$
(iii) Trapezoidal 	2 Δles + 1 Rec $2\left[\frac{1}{2} bh\right] + (l \times b)$	$\left(2\left[\frac{1}{3} h^2 \cdot b\right] + \left[l^2 \times b\right]\right)$