DATE

1) Find $L(k)$ or find ~~Laplace~~ Laplace transformer of k, k is Constant

$$L(f(t)) = \int_0^\infty e^{-st} f(t)\, dt$$

$$L(k) = \int_0^\infty e^{-st} . k\, dt$$

$$= k\left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= k\left[0 - \left(\frac{1}{-s}\right)\right]$$

$$L(k) = k\left[\frac{1}{s}\right]$$

$L(1) = 1/s$
$L(2) = 2/s$
$L(5) = 5/s$

2) Find $L(e^{at})$, if $(s-a)>0$

$$L(e^{at}) = \int_0^\infty e^{-st} . e^{at}\, dt$$

$$= \int_0^\infty e^{-st+at}\, dt$$

$$= \int_0^\infty e^{t(-s+a)}\, dt$$

$$= \left[\cancel{(-s+a)}\ \frac{e^{t(-s+a)}}{-s+a}\right]_0^\infty$$

$$= \left[0 - \left(\frac{1}{-s+a}\right)\right]$$

$$= \frac{-1}{-(s-a)} \Rightarrow \frac{1}{s-a} = L(e^{at})$$

DATE

2) Find $L(e^{-at})$ if $S+a>0$

$$L(e^{-at}) = \int_0^\infty e^{-St} \cdot e^{-at} \, dt$$

$$= \int_0^\infty e^{-St+(-at)} \, dt$$

$$= \int_0^\infty e^{-t(S+a)} \, dt$$

$$= \left[\frac{e^{-(S+a)t}}{-(S+a)} \right]_0^\infty$$

$$= \left[0 - \left(\frac{1}{-(S+a)} \right) \right]$$

$$= \left[\frac{1}{S+a} \right]$$

3. Find $L(\text{Sin } at)$

$$L(f(t)) = \int_0^\infty e^{-St} f(t) \, dt$$

$$L(\text{Sin } at) = \int_0^\infty e^{-St} \cdot \text{Sin } at \, dt$$

$\Rightarrow \int e^{ax} \text{ Sin } bx \, dx = \dfrac{e^{ax}}{a^2+b^2} (a \text{ Sin } bx - b \text{ Cos } bx)$

$\int e^{ax} \text{ Cos } bx \, dx = \dfrac{e^{ax}}{a^2+b^2} (a \text{ Cos } bx + b \text{ Sin } bx)$

$$L(\text{Sin } at) = \left[\frac{e^{-St}}{S^2+a^2} (-S \text{ Sin } at - a \text{ Cos } at) \right]_0^\infty$$

$$L(\text{Sin } at) = \left[0 - \frac{1}{S^2+a^2} (-a) \right]$$

$$L(\text{Sin } at) = \left[\frac{a}{S^2+a^2} \right]$$

Note $L(\sin at) = \frac{a}{a^2+s^2}$

$L(\cos at) = \frac{s}{s^2+a^2}$

$L(\cosh at) = \frac{s}{s^2-a^2}$

$L(\sinh at) = \frac{a}{s^2-a^2}$

Formula:

1) $L(k) = k/s \; ; k>0$

2) $L(e^{at}) = 1/s-a$

3) $L(e^{-at}) = \frac{1}{s+a}$

~~4. L(Sin~~

4) $L(t^n) = \frac{n!}{s^{n+1}}$

i) $L(t^2) = \frac{2!}{s^3}$ $\frac{6!}{}$

Linear Property

Laplace operator L is linear

(i.e)

i) $L(a\,f(t) + b\,g(t)) = a\,L(f(t)) + b\,L(g(t))$

ii) $L(a\,f(t) - b\,g(t)) = a\,L(f(t)) - b\,L(g(t))$

DATE

Find

$$L(e^{-5t}) = \frac{1}{s+5}$$

$$L(e^{-5t}) = \int_0^\infty e^{-st} \cdot e^{-5t}\, dt$$

$$= \int_0^\infty e^{(-s-5)t}\, dt$$

$$= \left[\frac{e^{-(s+5)t}}{-(s+5)}\right]_0^\infty$$

$$= \left[0 - \left(\frac{1}{-(s+5)}\right)\right]$$

$$= \left[\frac{1}{s+5}\right]$$

$$L(\cos bt) = \frac{s}{s^2+b^2}$$

$$L(e^{10t}) = \frac{1}{s-10}$$

$$L(\sin 2t) = \frac{2}{4+s^2}$$

$$L(\cosh 4t) = \frac{s}{s^2-16}$$

$$L(\sinh 3t) = \frac{3}{s^2-9}$$

$\cancel{L(\cosh at)} =$ $L(t^6) = \frac{6!}{s^7}$

Find

i) $L(1-4\sin 3t) = L(1) - 4L(\sin 3t)$

$$= \frac{1}{s} - 4 \times \frac{3}{9+s^2}$$

$$= \frac{1}{s} - \frac{12}{s^2+9}$$

ii) $L(\sin^2 5t) = L\left(\frac{1-\cos 2(5t)}{2}\right)$

$$= L\left(\frac{1-\cos 10t}{2}\right)$$

$$= \frac{1}{2}L(1) - \frac{1}{2}L(\cos 10t)$$

$$= \frac{1}{2}\frac{1}{s} - \frac{1}{2} \times \frac{s}{s^2+100}$$

$$= \frac{1}{2}\frac{1}{s} - \frac{s}{2(s^2+100)}$$

$$= \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+100}\right)$$

iii) $L(\sin 5t \cos 2t)$

$\sin(A+B) = \sin A \cos B + \cancel{\cos A \sin B}$

$\sin(A-B) = \sin A \cos B - \cancel{\cos A} \sin B$

$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

$$\sin 5t \cos 2t = \frac{1}{2}\left[\sin(5t+2t) + \sin(5t-2t)\right]$$

$$= \frac{1}{2}\left[\sin(7t) + \sin(3t)\right]$$

$$L\left[\frac{1}{2}\sin 7t + \frac{1}{2}\sin 3t\right] =$$

DATE

$= \frac{1}{2} L(\sin 7t) + \frac{1}{2} L(\sin 3t)$

$= \frac{1}{2}\left[\frac{7}{s^2+49}\right] + \frac{1}{2}\left[\frac{3}{s^2+9}\right]$

$= \frac{1}{2}\left[\frac{7}{s^2+49} + \frac{3}{s^2+9}\right]$

4 $L(\cos(at+b))$

$L[\cos at \cos b - \sin at \sin b]$

$L(\cos at \cos b) - L(\sin at \sin b)$

$\cos b \; L(\cos at) - \sin b \; L(\sin at)$

$\cos b\left[\frac{s}{s^2+a^2}\right] - \sin b\left[\frac{a}{s^2+a^2}\right]$

$4 \times 3 \times 2$

$L[(t+2t^2)^2] =$

$2(t)(2t^2)^2$

$\frac{24 \times 4}{96}$

$L\,4(t^2+4t^4+4t^3)$

$L(t^2) + 4L(t^4) + 4L(t^3)$

$\frac{2!}{s^3} + 4 \times \frac{4!}{s^5} + 4\frac{3!}{s^4}$

$\frac{2}{s^3} + \frac{96}{s^5} + \frac{24}{s^4}$

$L(\cos^3 5t)$ $\quad \theta = 5t$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$4\cos^3\theta = \cos 3\theta + 3\cos\theta$

$\cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}$

$L\left(\frac{\cos 15t + 3\cos 5t}{4}\right)$

$\frac{1}{4} L(\cos 15t) + \frac{3}{4} L(\cos 5t)$

$\frac{1}{4}\left[\frac{s}{s^2+15^2}\right] + \frac{3}{4}\left[\frac{s}{s^2+5^2}\right]$

First Shifting theorem :-

If $L(f(t)) = F(s)$, then $L(e^{-at} f(t)) = F(s)_{s \to s+a}$ (or) $(L f(t))_{s \to s+a}$ (or) $F(s+a)$

1) Find the $L(e^{-at} \cos 3t)$

By 1st Shifting Theorem

$$L(e^{-at} f(t)) = L(f(t))_{s \to s+a}$$

$$= L(\cos 3t)_{s \to s+a}$$

$$= \left(\frac{s}{s^2+9}\right)_{s \to s+a}$$

$$= \frac{s+a}{(s+a)^2+9}$$

2) $L(e^{-t} t^3)$

$$L(e^{-t} t^3) = L(t^3)_{s \to s+1}$$

$$= \cancel{3}\left(\frac{3!}{s^4}\right)_{s \to s+1}$$

$$= \cancel{3}\left(\frac{6}{(s+1)^4}\right)$$

DATE

3. $$L(e^{-2t} \cos^2 t) = L(\cos^2 t)_{s \to s+2}$$

$$= L\left[\frac{1+\cos 2t}{2}\right]_{s+2}$$

$$= L\left(\frac{1}{2}\right)_{s \to s+2} + L\left(\frac{\cos 2t}{2}\right)_{s \to s+2}$$

$$= \left(\frac{1}{2s}\right)_{s \to s+2} + \frac{1}{2}\left(\frac{s}{s^2+4}\right)_{s \to s+2}$$

$$= \frac{1}{2(s+2)} + \frac{1}{2}\left(\frac{s+2}{(s+2)^2+4}\right)$$

$$= \frac{1}{2s+4} + \frac{1}{2}\left(\frac{s+2}{s^2+4+4s+4}\right)$$

$$= \frac{1}{2s+4} + \frac{1}{2}\left(\frac{s+2}{s^2+4s+8}\right)$$

$$= \frac{1}{2}\left[\frac{1}{s+2} + \frac{s+2}{s^2+4s+8}\right]$$

4. $$L(e^{-3t} \cosh t) = L(\cosh t)_{s \to s+3}$$

$$= \left(\frac{s}{s^2-1}\right)_{s \to s+3}$$

$$= \frac{s+3}{(s+3)^2-1}$$

5) $L(e^{-t}\sin t\cos t) = L(\sin t\cos t)_{s\to s+1}$

$\sin(A+B) = \sin A\cos B + \cos A\cancel{\sin B}$
$\sin(A-B) = \sin A\cos B - \cos\cancel{A}\sin B$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$A = t \quad B = t$

$\frac{1}{2}[\sin 2t]$

$\Rightarrow L\left(\frac{1}{2}\sin 2t\right)_{s\to s+1}$

$\Rightarrow \frac{1}{2}\left[\frac{2}{s^2+4}\right]_{s\to s+1}$

$\Rightarrow \frac{1}{2}\left(\frac{2}{(s+1)^2+4}\right)$

$= \frac{1}{(s+1)^2+4}$

6) $L(e^{-6t}\sin 7t) = L(\sin 7t)_{s\to s+6}$

$= \frac{7}{s^2+49}$

$= \frac{7}{(s+6)^2+49}$

$= \frac{7}{s^2+36+12s+49}$

$= \frac{7}{s^2+12s+85}$

DATE

① $L(f(t)) = F(s)$

$$L(f(at)) = \frac{1}{a} F\left(\frac{s}{a}\right)$$

② Transform of $t \cdot f(t)$:

If $L(f(t)) = F(s)$, then

$$L(t f(t)) = \frac{-d}{ds} F(s) \text{ or } \frac{-d}{ds} L(f(t))$$

Note

$$L(t^2 f(t)) = (-1)^2 \frac{d^2}{ds^2} F(s) \text{ or}$$

$$(-1)^2 \frac{d^2}{ds^2} L(f(t))$$

$$d\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

1 Find $L(t \cos at)$:

$$L(t f(t)) = \frac{-d}{ds} L(f(t))$$

$$L(t \cos at) = \frac{-d}{ds} L(\cos at)$$

$$= \frac{-d}{ds}\left[\frac{s}{s^2+a^2}\right]$$

$$= -\left[\frac{s^2+a^2-2s^2}{(s^2+a^2)^2}\right]$$

$u = s$ $\quad v = s^2+a^2$

$u' = 1$ $\quad v' = 2s$

$$= -\left[\frac{-s^2+a^2}{(s^2+a^2)^2}\right]$$

$$= -\left[\frac{a^2-s^2}{(s^2+a^2)^2}\right]$$

DATE

$D[x^n] = nx^{n-1}$

2. $L(t \sin 3t) = -\frac{d}{ds} L(\sin 3t)$

$= -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$= -3 \frac{d}{ds}\left[\frac{1}{s^2+9}\right]$

$= -3 \frac{d}{ds} (s^2+9)^{-1-1}$

$= -3\left[-1(s^2+9)^{-2}\right]$

$= \cancel{-3[-1(1)]}$

$= \cancel{-3(-3)}$

$= \cancel{3}$

$= -3\left[-1 \frac{1}{(s^2+9)^2}\right]$

$= \frac{3}{(s^2+9)^2}$

3. $\cancel{L(t^2 e^{3t})} = \cancel{L(t^2)_{s \to s+3}}$

$L(t^2 e^{3t}) =$

WKT

$L(t^2 f(t)) = (-1)^2 \frac{d^2}{ds^2} L(f(t))$

$= (-1)^2 \frac{d^2}{ds^2} L(e^{3t})$

$= \frac{d^2}{ds^2}\left[\frac{1}{s-3}\right]$

$= \cancel{d^2}$

$$\left[\begin{array}{ll} u = 1 & v = s-3 \\ u' = 0 & v' = 1 \end{array}\right]$$

DATE

$$\frac{d}{ds}\left[\frac{-1}{(S-3)^2}\right]$$

$$\left[\begin{array}{ll} u = +1 & V = (S-3)^2 \\ u = 0 & V' = 2(S-3)' \\ & \quad = 2(S-3) \end{array}\right]$$

$$= -\frac{d}{ds}\left[\frac{-2(S-3)}{(S-3)^4}\right]$$

$$= \frac{2(S-3)}{(S-3)^4}$$

$$= \frac{2}{(S-3)^3}$$

4. $L(t e^{-t} \sin t) =$

$$L(t \cdot f(t)) = -\frac{d}{ds} L(f(t))$$

$$= -\frac{d}{ds} L(e^{-t} \sin t)$$

$$= -\frac{d}{ds} L(\sin t)_{s \to s+1}$$

$$= -\frac{d}{ds}\left[\frac{1}{1+s^2}\right]_{s \to s+1}$$

$$= -\frac{d}{ds}\left[\frac{1}{1+(s+1)^2}\right]$$

$=$

$$\left[\begin{array}{ll} u = 1 & V = 1+(S+1)^2 \\ u' = 0 & V' = 2(S+1) \end{array}\right]$$

$$= -\left[\frac{-2(S+1)}{[1+(S+1)^2]^2}\right]$$

$$L(t e^{-t} \sin t) = \frac{2(s+1)}{[1+(s+1)^2]^2}$$

PAGE

Transform of $\frac{f(t)}{t}$

if $\lim_{t \to 0} \frac{f(t)}{t}$ exist, and if $L(f(t)) = F(s)$,

then $L\left(\frac{f(t)}{t}\right) = \int_s^\infty L(f(t))\, ds$.

1) Find $L\left(\frac{1-e^t}{t}\right)$

$$L\left(\frac{f(t)}{t}\right) = \int_s^\infty L(f(t))\, ds$$

$$L\left(\frac{1-e^t}{t}\right) = \int_s^\infty L(1-e^t)\, ds$$

$$= \int_s^\infty L(1) - L(e^t)\, ds$$

$$= \int_s^\infty \frac{1}{s} - \frac{1}{s-1}\, ds$$

$$= \Big[\log s - \log(s-1)\Big]_s^\infty$$

$$= \left[\log \frac{s}{s-1}\right]_s^\infty$$

$$= \log \frac{\cancel{s}}{\cancel{s}(1-1/s)}\Bigg]_s^\infty$$

$$= \log\left[\frac{1}{1-\frac{1}{s}}\right]_s^\infty$$

$$= \log\left[\frac{1}{1-0}\right] - \log \frac{1}{1-1/s}\Big]$$

$$= \log 1 - \log \frac{s}{s-1}$$

$$= -\log\left(\frac{s}{s-1}\right)$$

DATE

2) $L\left(\frac{e^{-at}-e^{-bt}}{t}\right)$

$$L\left(\frac{e^{-at}-e^{-bt}}{t}\right) = \int_s^\infty L(e^{-at}-e^{-bt})\,ds$$

$$= \int_s^\infty L(e^{-at}) - L(e^{-bt})\,ds$$

$$= \int_s^\infty \frac{1}{s+a} - \frac{1}{s+b}\,ds$$

$$= \left[\log s+a - \log s+b\right]_s^\infty$$

$s+a+\cancel{b}-\cancel{b}$

$$= \left[\log\left(\frac{s+a}{s+b}\right)\right]_s^\infty$$

$$= \left[\log\left(\frac{s+a+b-b}{s+b}\right)\right]_s^\infty$$

$$= \left[\log \frac{\cancel{(s+b)} + (a-b)}{\cancel{(s+b)}\ s+b}\right]_s^\infty$$

$$= 0 - \log\frac{(a-b}{s+b}$$

DATE

$$f(t) = \begin{cases} k & 0<t<a \\ -k & a<t<2a \end{cases}$$

1) $P = 2a$

$$L(f(t)) = \frac{1}{1-e^{-Ps}} \int_0^P e^{-st} f(t)\,dt$$

$$= \frac{1}{1-e^{-2as}} \int_0^{2a} e^{-st} f(t)\,dt$$

$$= \frac{1}{1-e^{-2as}} \left[\int_0^a e^{-st} k\,dt + \int_a^{2a} e^{-st} (-k)\,dt \right]$$

$$= \frac{1}{1-e^{-2as}} \left[k \left[\frac{e^{-st}}{-s} \right]_0^a - k \left[\frac{e^{-st}}{-s} \right]_a^{2a} \right]$$

$$= \frac{1}{1-e^{-2as}} \left[\left(\frac{-ke^{-st}}{s} \right)_0^a + k \left[\frac{e^{-st}}{s} \right]_a^{2a} \right]$$

$$= \frac{1}{1-e^{-2as}} \left[\frac{-ke^{-sa}}{s} + \frac{k}{s} + \frac{ke^{-s2a}}{s} - \frac{ke^{-sa}}{s} \right]$$

$$= \frac{1}{1-e^{-2as}} \left(\frac{-2ke^{-as}}{s} + \frac{k}{s} + \frac{ke^{-2as}}{s} \right)$$

$$= \frac{1}{1-e^{-2as}} \left(\frac{-2ke^{-as} + k + ke^{-2as}}{s} \right)$$

$$= \frac{k}{1-e^{-2as}} \left(\frac{-2e^{-as} + 1 + (e^{-as})^2}{s} \right)$$

$$= \frac{k}{1-e^{-2as}} \left(\frac{(1-e^{-as})^2}{s} \right)$$

$$= \frac{k}{1-(e^{-as})^2} \left(\frac{(1-e^{-as})^2}{s} \right)$$

$$= \frac{k}{(1-e^{-as})(1+e^{-as})} \left[\frac{(1-e^{-as})^2}{s} \right]$$

$$= \frac{k(1-e^{-as})}{(1+e^{-as})s}$$

DATE

2) L of $f(t) = \begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$ with $f(t+2a) = f(t)$

$P = 2a$

$$L(f(t)) = \cancel{\frac{1}{1-e^{-2as}}\int} \frac{1}{1-e^{-Ps}} \int_0^P e^{-st} f(t)\, dt$$

$$= \frac{1}{1-e^{-2as}} \int_0^{2a} e^{-st} f(t)\, dt$$

$$= \frac{1}{1-e^{-2as}} \left[\int_0^a e^{-st} \underbrace{t}_{I_1}\, dt + \int_a^{2a} e^{-st} \underbrace{(2a-t)}_{I_2}\, dt \right]$$

$$= \frac{1}{1-e^{-2as}} \left[\cancel{e^{-st}} \left[t\left[\frac{e^{-st}}{-s}\right] + \frac{1}{s^2}\left[e^{-st}\right] \right]_0^a \right.$$

$u = t \quad V = e^{-st}$

$$u\left[\frac{e^{-st}}{-s}\right] + \int t \cdot \left[\frac{e^{-st}}{-s}\right]$$

$$u\left[\frac{e^{-st}}{-s}\right] - \frac{1}{s}\left[\cancel{t \cdot e^{-st}} \; \frac{e^{-st}}{-s} \right]$$

$u = 2a - t \quad V = e^{-st}$

$$(2a-t)\frac{e^{-st}}{-s} + \int 1 \cdot \frac{e^{-st}}{-s}$$

$$(2a-t)\frac{e^{-st}}{-s} + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]$$

$$(2a-t)\frac{e^{-st}}{-s} - \frac{1}{s^2} e^{-st}$$

$$+ \left[-(2a-t)\frac{e^{-st}}{s} - \frac{1}{s^2} e^{-st} \right]_a^{2a}$$

$$= \frac{1}{1-e^{-2as}} \left(\left[\frac{-ae^{-sa}}{s} + \frac{1}{s^2}(e^{-sa}) - \left(-\cancel{\frac{0}{s}} + \frac{1}{s^2} \right) \right] + \left[-\frac{e^{-s2a}}{s^2} - \left(-\frac{ae^{-sa}}{s} - \frac{e^{-sa}}{s^2} \right) \right] \right)$$

$$= \frac{1}{1-e^{-2as}} \left[\frac{-ae^{-sa}}{\cancel{s}} + \frac{e^{-sa}}{s^2} + \cancel{\frac{0}{s}} - \frac{1}{s^2} - \frac{e^{-2sa}}{s^2} + \frac{ae^{-sa}}{\cancel{s}} + \frac{e^{-sa}}{s^2} \right]$$

$$= \frac{1}{1-e^{-2a}} \left[\frac{2e^{-sa}}{s^2} + \cancel{\frac{0}{s}} - \frac{1}{s^2} - \frac{e^{-2sa}}{s^2} \right]$$

Find the L(half wave rectifier given by $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases}$ where $f(t+2\pi) = f(t)$

Given $P = 2\pi$

$$L(f(t)) = \frac{1}{1-e^{-PS}} \int_0^P e^{-St} f(t)\, dt$$

$$= \frac{1}{1-e^{-2\pi S}} \int_0^{2\pi} e^{-St} f(t)\, dt$$

$$= \frac{1}{1-e^{-2\pi S}} \left[\int_0^{\pi} e^{-St} \sin t\, dt + \int_{\pi}^{2\pi} e^{-St}\, 0\, dt \right]$$

X

$\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$

$\cancel{= \frac{1}{1-e^{-2\pi S}} \left[-\cos t \right]}$

$u = e^{-St}$ $\quad V = \sin t$

$u' = -e^{-St}$ $\quad V =$

$e^{-St}\frac{[-\cos t]}{S} + \int \frac{-e^{-St}}{S}$

$a = -S \quad b = 1$

$$= \frac{1}{1-e^{-2\pi S}} \left[\frac{e^{-St}}{(-S)^2+1^2} \left(-S \sin t - 1(\cos t) \right) \right]_0^{\pi}$$

$$= \frac{1}{1-e^{-2\pi S}} \left[\frac{e^{-St}}{S^2+1^2} \left(-S \sin t - \cos t \right) \right]_0^{\pi}$$

$$= \frac{1}{1-e^{-2\pi S}} \left[\frac{e^{-S\pi}}{S^2+1^2} (1) - \frac{1}{S^2+1^2} (-1) \right]$$

DATE

$$= \frac{1}{1-e^{-2\pi s}}\left[\frac{e^{-s\pi}}{s^2+1} + \frac{1}{s^2+1^2}\right]$$

$$= \frac{1}{1-(e^{-s\pi})^2}\left[\frac{e^{-s\pi}+1}{s^2+1}\right]$$

$$= \frac{1}{(1+e^{-s\pi})(1-e^{-s\pi})}\left[\frac{e^{-s\pi}+1}{s^2+1}\right]$$

$$= \frac{1}{(1-e^{-s\pi})(s^2+1)}$$

Inverse Laplace Transform

If $L\{f(t)\} = F(s)$, then $f(t)$ is Called inverse Laplace Transform of $F(s)$, and is denoted by

$$f(t) = L^{-1}(F(s))$$

Standard result

$L(1) = \frac{1}{s}$ $\qquad 1 = L^{-1}\left(\frac{1}{s}\right)$

$L(e^{at}) = \frac{1}{s-a}$ $\qquad e^{at} = L^{-1}\left(\frac{1}{s-a}\right)$

$L(e^{-at}) = \frac{1}{s+a}$ $\qquad e^{-at} = L^{-1}\left(\frac{1}{s+a}\right)$

$L(\cos at) = \frac{s}{s^2+a^2}$ $\qquad \cos at = L^{-1}\left(\frac{s}{s^2+a^2}\right)$

$L(\sin at) = \frac{a}{s^2+a^2}$ $\qquad \sin at = L^{-1}\left(\frac{a}{s^2+a^2}\right)$

$L(\sinh at) = \frac{a}{s^2-a^2}$ $\qquad \sinh at = L^{-1}\left(\frac{a}{s^2-a^2}\right)$

$4 \; L(\cosh at) = \frac{s}{s^2-a^2}$ $\qquad \cosh at = L^{-1}\left(\frac{s}{s^2-a^2}\right)$

$L(t^n) = \frac{n!}{s^{n+1}}$ $\qquad t^n = L^{-1}\left(\frac{n!}{s^{n+1}}\right)$

$\frac{t^n}{n!} = L^{-1}\left(\frac{1}{s^{n+1}}\right)$

i) $L^{-1}\left(\frac{1}{s-4}\right)$

e^{4t}

ii) $L^{-1}\left(\frac{s}{s^2-9}\right)$

$\cosh 3t$

iii) $L^{-1}\left(\frac{1}{s^4}\right) = \frac{t^3}{3!}$

iv) $L^{-1}\left(\frac{1}{s-3} + \frac{1}{s} + \frac{s}{s^2-4}\right)$

$L^{-1}\left(\frac{1}{s-3}\right) + L^{-1}\left(\frac{1}{s}\right) + L^{-1}\left(\frac{s}{s^2-4}\right)$

$e^{3t} + 1 + \cosh 2t$

v) $L^{-1}\left(\frac{1}{s^2} + \frac{1}{s+4} + \frac{1}{s^2+4} + \frac{s}{s^2-9}\right)$

$t + e^{-4t} + \frac{\sin 2t}{2} + \cosh 3t$

DATE

Note:

$$L^{-1}(F(s+a)) = e^{-at} L^{-1}(F(s))$$
$$L^{-1}(F(s-a)) = e^{at} L^{-1}(F(s))$$

Find

i) $L^{-1}\left(\frac{3}{s+2}\right)$

$3 L^{-1}\left(\frac{1}{s+2}\right)$

$3e^{-2t}$

ii) $L^{-1}\left(\frac{2s}{s^2-16}\right)$

$2L^{-1}\left(\frac{s}{s^2-16}\right)$

$2 \cosh 4t$

1) Find $L^{-1}\left(\frac{1}{(s+1)^2}\right)$

$$L^{-1}(F(s+a)) = e^{-at} L^{-1}(f(s))$$

$$L^{-1}\left(\frac{1}{(s+1)^2}\right) = e^{-t} L^{-1}\left(\frac{1}{s^2}\right)$$

$$= e^{-t} \cdot t$$

2) $L^{-1}\left(\frac{s-3}{(s-3)^2+4}\right)$

$$= L^{-1}\left(\frac{s-3}{(s-3)^2+2^2}\right)$$

$$= e^{3t} L^{-1}\left(\frac{s}{s^2+2^2}\right)$$

$$= e^{3t} \cos 2t$$

$s = s-3$

$s = s-a$

$s = s+a$

e^{3t} $L^{-1}\left(\frac{s}{s^2+4}\right)$

$16s^2$

4 4

$s^2+4s+4s+16$

$s(s+4)\ 4(s+4)$

$L^{-1}\left(\frac{1}{s^2+8s+16}\right)$

$= L^{-1}\left(\frac{1}{(s+4)^2}\right) \Rightarrow e^{-4t} L^{-1}\left(\frac{1}{s^2}\right)$

$$= e^{-4t} \cdot t$$

4 $L^{-1}\left(\frac{s}{(s+2)^2}\right)$

$s = s+a$

$s = s+2$

$s-2$

$= e^{-2t} L^{-1}\left(\frac{s-2}{s^2}\right)$

$= e^{-2t}\left[L^{-1}\left(\frac{\cancel{s}}{\cancel{s^2}_s} - \frac{2}{s^2}\right)\right]$

$= e^{-2t}\left[L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{2}{s^2}\right)\right]$

$= e^{-2t}\left[(1) - 2t\right]$

$= e^{-2t} - 2e^{-2t} \cdot t$

~~(or)~~

5) $L^{-1}\left(\frac{s}{(s+1)^2+4}\right)$

$s = s+1$

$s-1 = s$

$s =$

$s+2 = s$

~~L^{-1}~~

$\cancel{e^{-t} L^{-1}\left(\frac{s-1}{s^2+2^2}\right)}$

$\cancel{e^{-t}\left[L^{-1}\left(\frac{s}{s^2+2^2}\right) - L^{-1}\left(\frac{1}{s^2+2^2}\right)\right]}$

$\cancel{e^{-t}\left[\cos 2t - \right.}$

$s = s+1$

$L^{-1}\left(\frac{s+1-1}{(s+1)^2+4}\right)$

$L^{-1}\left(\frac{(s+1)}{(s+1)^2+4}\right) - L^{-1}\left(\frac{1}{(s+1)^2+4}\right)$

$L^{-1}\left(\frac{(s+1)}{(s+1)^2+4}\right) - L^{-1}\left(\frac{1}{(s+1)^2+4}\right)$

$\cancel{L^{-1}\left(\frac{1}{s+5}\right)}$

$e^{-t} L^{-1}\left(\frac{s}{s^2+2^2}\right) - e^{-t} L^{-1}\left(\frac{1}{s^2+4}\right)$

$\left[e^{-t}\cos 2t - e^{-t}\left(\frac{\sin 2t}{2}\right)\right]$

Inverse laplase transformer of trignometric & logrithmic

Note

$$L^{-1}(F'(s)) = -t\, L^{-1}(F(s))$$
$$L^{-1}(F(s)) = -\frac{1}{t}\, L^{-1}(F'(s)) \checkmark$$

Find $L^{-1}(\cot^{-1}(s))$

$$F'(s) = \frac{-1}{1+s^2}$$

$$\Rightarrow -\frac{1}{t}\, L^{-1}\left(\frac{-1}{1+s^2}\right)$$

$$\Rightarrow \frac{1}{t} \cdot \sin t$$

$L^{-1}(\cot^{-1}(s+1))$

$$F'(s) = \frac{-1}{1+(s+1)^2}$$

$s = s+1$

$$\Rightarrow \frac{-1}{t}\, L^{-1}\left(\frac{-1}{1+(s+1)^2}\right)$$

$$\Rightarrow \frac{+1}{t}\, L^{-1}\left(\frac{1}{1+(s+1)^2}\right)$$

$$\Rightarrow \frac{1}{t}\, e^{-t}\, L^{-1}\left(\frac{1}{1+s^2}\right)$$

$$\Rightarrow \frac{1}{t}\, e^{-t} \cdot \sin t$$

$\frac{-a}{s^2}$

DATE

$u = a \quad v = s$

$u' = 0 \quad v' = 1$

3) $L^{-1}\left(\tan^{-1}\left(\frac{a}{s}\right)\right)$

$F'(s) = \frac{1}{1+\left(\frac{a}{s}\right)^2}\left[a \cdot \frac{-1}{s^2}\right] \Rightarrow \frac{-a}{s^2+a^2}$

$\frac{1}{1+\frac{a^2}{s^2}}\left(\frac{-a}{s^2}\right)$

$= \cancel{\frac{1}{1+\frac{a^2}{s^2}}} \Rightarrow \cancel{\frac{s^2}{s^2+a^2}}$

$s = s+a$

$\frac{-a}{s^2+a^2}$

$\Rightarrow \cancel{\frac{-1}{t} L^{-1}\left(\frac{s^2+a^2-a^2}{s^2+a^2}\right)}$

$\Rightarrow \cancel{\frac{-1}{t} L^{-1}\left(\frac{s^2+a^2}{s^2+a^2} - \frac{a^2}{s^2+a^2}\right)}$

$\Rightarrow \cancel{\frac{-1}{t}\left[L^{-1}(1) - L^{-1}\left(\frac{a^2}{s^2+a^2}\right)\right]}$

$\Rightarrow \cancel{\frac{-1}{t}\left[L^{-1}(1) - \right.}$

$\Rightarrow \frac{-1}{t} L^{-1}\left(\frac{-a}{s^2+a^2}\right)$

$\Rightarrow \frac{+1}{t} \sin at$ //

4) $L^{-1}\left(\log\left(\frac{s+1}{s-1}\right)\right)$

$F'(s) = \log\left(\frac{s+1}{s-1}\right)$

$= \log(s+1) - \log(s-1)$

$= \frac{1}{s+1} - \frac{1}{s-1}$

$\Rightarrow \frac{(s-1) - (s+1)}{(s+1)(s-1)}$.

DATE

$$\Rightarrow \frac{-1}{t} L^{-1}\left(\frac{(s-1)-(s+1)}{(s+1)(s-1)}\right)$$

$$\Rightarrow \frac{-1}{t}\left[L^{-1}\left(\frac{1}{s+1}\right) - L^{-1}\left(\frac{1}{s-1}\right)\right]$$

$$\Rightarrow \frac{-1}{t}\left[e^{-t} - e^{t}\right]$$

Inverse Laplose transform using partial fraction method.

1) Find $L^{-1}\left(\frac{1}{(s+1)(s+3)}\right) \rightarrow$ Using partial fraction method

$$L^{-1}\left(\frac{A}{s+1} + \frac{B}{s+3}\right)$$

$$\frac{A}{s+1} + \frac{B}{s+3} = \frac{(s+3)A + B(s+1)}{(s+1)(s+3)}$$

$$= \frac{sA+3A+Bs+B}{\cancel{(s+1)(s+3)}} = \frac{1}{\cancel{(s+1)(s+3)}}$$

$$1 = As+3A+Bs+B$$

$\cancel{0 = As+Bs}$ $\quad$ $\cancel{1 = 3A}$

$\cancel{0 = (A+B)s}$

$\cancel{s = 0}$

$s = -1$

$1 = -A+3A-\cancel{B}+\cancel{B}$

$1 = +2A$

$\boxed{A = 1/2}$

$s = -3$

$1 = -3A+3A-3B+B$

$1 = 0-2B$

$\frac{1}{-2} = B \Rightarrow \boxed{B = -1/2}$

Sub A & B in ①

$$\cancel{\frac{1}{(s+1)(s+3)}} = \cancel{\frac{1}{2}(s+3)} - \cancel{\frac{1}{2}(s+1)}$$

DATE

$$\frac{1}{(s+1)(s+3)} = \frac{1}{2(s+1)} + \frac{-1}{2(s+3)}$$

$$L^{-1}\left(\frac{1}{(s+1)(s+3)}\right) = L^{-1}\left(\frac{1}{2(s+1)}\right) - L^{-1}\left(\frac{1}{2(s+3)}\right)$$

$$= \frac{1}{2}L^{-1}\left(\frac{1}{s+1}\right) - \frac{1}{2}L^{-1}\left(\frac{1}{s+3}\right)$$

$$= \frac{1}{2}e^{-t} + \frac{-1}{2}e^{-3t}$$

$13s^2$

2. $L^{-1}\left(\frac{1-s}{(s+1)(s^2+4s+13)}\right)$

(15)

$$L^{-1}\left(\frac{A}{s+1} + \frac{Bx+c}{s^2+4s+13}\right)$$

$$\frac{1-s}{(s+1)(s^2+4s+13)} = \frac{(s^2+4s+13)A + Bs + c(s+1)}{(s+1)(s^2+4s+13)}$$

$$1-s = As^2 + 4As + 13A + Bs^2 + Bs + Cs + C$$

$0 = A+B$

$A = -B$

$-1 = 4A+B+C$

$-1 = 4(-B)+B+C$

$-1 = -4B+B+C$

$-1 = -3B+C$

$-1+3B = C$

$-1+3\left(\frac{-1}{5}\right) = C$

$-1-\frac{3}{5} = C$

$\frac{-5-3}{5} = C$

$C = -8/5$

$1 = 13A + C$

$1 = 13(-B) + (-1+3B)$

$1 = -13B - 1 + 3B$

$1 = -10B - 1$

$2 = -10B$

$\boxed{B = \frac{-1}{5}}$

$\boxed{A = \frac{1}{5}}$

$C = -8/5$

classmate

DATE

$$\frac{A}{s+1} + \frac{Bs+c}{s^2+4s+13} = \frac{1}{5(s+1)} + \frac{-\frac{1}{5}s - \frac{8}{5}}{s^2+4s+13}$$

$$= \frac{1}{5(s+1)} - \frac{1}{5}\frac{s}{(s^2+4s+13)} - \frac{8}{5(s^2+4s+13)}$$

$$L^{-1}\left(\frac{1-s}{(s+1)(s^2+4s+13)}\right) = L^{-1}\left(\frac{1}{5(s+1)}\right) - \frac{1}{5}L^{-1}\left(\frac{s}{s^2+4s+13}\right) - \frac{8}{5}L^{-1}\left(\frac{1}{s^2+4s+13}\right)$$

$$= \frac{1}{5}e^{-t} - \frac{1}{5}L^{-1}\left[\frac{s+8}{s^2+4s+4+9}\right]$$

$a^2 = s^2 \quad a = s$

$\cancel{2ab = 2(s)(\sqrt{13})\,b}$

$\cancel{b^2 = 13}$ $\quad \cancel{2ab = 2sb}$

$\cancel{b = \sqrt{13}}$ $\quad 2ab = 4s \quad (s+\frac{1}{4})^2 \quad \frac{13\times4}{s^2}\ 1$

$2sb = 4s \quad s^2+4s+13+\frac{1}{4}-\frac{1}{4}$

$b = 2$

$b^2 = 4 \quad s^2+4s+\frac{s}{4}+\frac{1}{4}$

$(s+\frac{s}{4})^2 \quad s^2+4+4s+\frac{1}{4}$

$s = s+2$

$$= \frac{1}{5}e^{-t} - \frac{1}{5}L^{-1}\left[\frac{(s+2)+6}{(s+2)^2+9}\right]$$

$$= \frac{1}{5}e^{-t} - \frac{1}{5}L^{-1}\left(\frac{s+2}{(s+2)^2+9} + \frac{6}{(s+2)^2+9}\right)$$

$$= \frac{1}{5}e^{-t} - \frac{1}{5}L^{-1}\left(\frac{s}{s^2+3^2}\right) - \frac{6}{5}L^{-1}\left[\frac{1}{(s+2)^2+3^2}\right]$$

$$= \frac{1}{5}e^{-t} - \frac{1}{5}e^{-2t}\cos 3t - \frac{1}{5}e^{-2t}L^{-1}\left(\frac{6}{s^2+3^2}\right)$$

$$= \frac{1}{5}e^{-t} - \frac{e^{-2t}}{5}\cos 3t - \frac{1}{5}e^{-2t}\,6\left(\frac{\sin 3t}{3}\right)$$

DATE

$$L^{-1}\left(\frac{5s^2-15s-11}{(s+1)(s-2)^3}\right)$$

$$\frac{A}{s+1}+\frac{B}{(s-2)}+\frac{C}{(s-2)^2}+\frac{D}{(s-2)^3}=\frac{5s^2-15s-11}{(s+1)(s-2)^3}$$

$$5s^2-15s-11=\frac{A(s-2)^3+B(s+1)(s-2)^2+C(s+1)(s-2)+D(s+1)}{(s+1)(s-2)^3}$$

A($s=-1$

$A(-3)^3+B(0)+C(0)+D(0)=5+15-11$

$-27A=9$

$\boxed{A=-1/3}$

B $s=2$

$A(0)^3+B(0)+(0)+D(3)=20-30-11$

$3D=-21$

$D=\frac{-21}{3}$

$D=-7$

$A(s^3-8+3(s)(2)^2-3(s^2)(2))+B(s+1)(s^2+4-4s)$
$+C(s^2-2s+s-2)+Ds+D$

$A(s^3-8+12s-6s^2)+B(s^3+4s-4s^2+s^2+4-4s)$
$+C(s^2-s-2)+Ds+D$

$A(s^3-8+12s-6s^2)+B(s^3-3s^2+4)+C(s^2-s-2)$
$+D(s+1)$

$0=A+B$

$0=\frac{-1}{3}+B$

$B=\frac{1}{3}$

DATE

$5 = -6A - 3B + C$

$5 = -6\left(\frac{-1}{3}\right) - 3\left(\frac{1}{3}\right) + C$

$5 = \frac{6}{3} - \frac{3}{3} + C$

$S = S-2 \qquad S = S-3$

$5 = 1 + C$

$\boxed{C = 4}$

$L^{-1}\left(\frac{5s^2 - 15s - 11}{(s+1)(s-2)^3}\right) = L^{-1}\left(\frac{-1}{3(s+1)} + \frac{1}{3(s-2)} + \frac{4}{(s-2)^2} - \frac{7}{(s-3)^3}\right)$

$= \frac{-1}{3} L^{-1}\left(\frac{1}{s+1}\right) + \frac{1}{3} L^{-1}\left(\frac{1}{s-2}\right) + 4L^{-1}\left(\frac{1}{(s-2)^2}\right) - 7L^{-1}\left(\frac{1}{(s-3)^3}\right)$

$= \frac{-1}{3} e^{-t} + \frac{1}{3} e^{2t} + 4e^{2t} L^{-1}\left(\frac{1}{s^2}\right) - 7e^{3t} L^{-1}\left(\frac{1}{s^3}\right)$

$= \frac{-1}{3} e^{-t} + \frac{1}{3} e^{2t} + 4e^{2t} t - 7e^{3t}\left(\frac{t^2}{2}\right)$

DATE

Convolution

If $f(t)$ and $g(t)$ are 2 fun defined for $t \geq 0$, then Convolution of $f(t)$ & $g(t)$ is defined as

$$f(t) * g(t) = \int_0^t f(u)\, g(t-u)\, du$$

Convolution Theorem:

If $f(t)$ & $g(t)$ are fun defined for $t \geq 0$, then

$$L(f(t) * g(t)) = L(f(t)) \cdot L(g(t))$$

$$L(f(t) * g(t)) = F(s) \cdot G(s)$$

Note

$$L^{-1}(F(s) \cdot G(s)) = f(t) * g(t)$$
$$L^{-1}(F(s) \cdot G(s)) = L^{-1}(F(s)) * L^{-1}(G(s))$$

1) Using Convolution theorem, find $L^{-1}\left(\frac{1}{s^2(s+3)}\right)$

$$L^{-1}(F(s) \cdot G(s)) = L^{-1}(F(s)) * L^{-1}(G(s))$$

$$L^{-1}\left(\frac{1}{s^2} \cdot \frac{1}{s+3}\right) = L^{-1}\left(\frac{1}{s^2}\right) * L^{-1}\left(\frac{1}{s+3}\right)$$

$$= t * e^{-3t}$$

$$= \int_0^t u \cdot e^{-3(t-u)}\, du$$

$$= \int_0^t u \cdot e^{-3t} \cdot e^{3u}\, du$$

$\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{6}$ $\frac{2}{6} \cdot \frac{3}{6} \times \frac{1}{6}$

DATE

$$= e^{-3t} \int_0^t u \cdot e^{3u} \, du$$

~~u=v~~

$$= e^{-3t} \left[u \left[\frac{e^{3u}}{3} \right] - \int \frac{e^{3u}}{3} du \right]_0^t$$

$$= e^{-3t} \left[\frac{e^{3u}(u)}{3} - \frac{1}{3} \frac{e^{3u}}{3} \right]_0^t$$

$$= e^{-3t} \left[\frac{ue^{3u}}{3} - \frac{e^{3u}}{9} \right]_0^t$$

$$= e^{-3t} \left[\frac{te^{3t}}{3} - \frac{e^{3t}}{9} - \left(-\frac{e^0}{9} \right) \right]$$

$$= e^{-3t} \left[\frac{te^{3t}}{3} - \frac{e^{3t}}{9} + \frac{1}{9} \right]$$

$$= \frac{t}{3} - \frac{1}{9} + \frac{1}{9} e^{-3t}$$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos A-B = \cos A \cos B + \sin A \sin B$

$2 \cos A \cos B$

$\frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

2) $L^{-1}\left(\frac{s^2}{(s^2+a^2)^2} \right)$

$$\frac{s^2}{(s^2+a^2)^2} = \cancel{\frac{Ax+B}{s^2+a^2}}$$

$$L^{-1}\left(\frac{s}{s^2+a^2} \cdot \frac{s}{s^2+a^2} \right) = L^{-1}\left(\frac{s}{s^2+a^2} \right) * L^{-1}\left(\frac{s}{s^2+a^2} \right)$$

$$= \cos at * \cos at$$

$$= \int_0^t \cos au \cdot \cos a(t-u) \, du$$

$$= \int_0^t \frac{1}{2}\left[\cos(au+at-au) + \cos(au-at+au) \right] du$$

$$= \int_0^t \frac{1}{2}\left[\cos at + \cos(2au-at) \right] du$$

DATE

$$= \frac{1}{2}\cos at\,[u]_0^t + \frac{1}{2}\int_0^t \cos(2au - at)\,du$$

$$= \frac{1}{2}\cos at\,(t) + \frac{1}{2}\left[\frac{\sin(2au-at)}{2a}\right]_0^t$$

$$= \frac{t}{2}\cos at + \frac{1}{2}\left[\frac{\sin(2at-at)}{2a} - \frac{\sin(-at)}{2a}\right]$$

$$= \frac{t}{2}\cos at + \frac{1}{2}\left[\frac{\sin at}{2a} + \frac{\sin at}{2a}\right]$$

$$= \frac{t}{2}\cos at + \frac{1}{2}\left[\frac{2\sin at}{2a}\right]$$

$$= \frac{t}{2}\cos at + \frac{1}{2}\frac{\sin at}{a}$$

$$= \frac{1}{2}\left[t\cos at + \frac{\sin at}{a}\right]$$

3) Using Convolution theorem find $L^{-1}\left(\frac{1}{(s+a)(s+b)}\right)$

By Convolution theorem.

$$L^{-1}(F(s).\,G(s)) = L^{-1}(F(s)) * L^{-1}(G(s))$$

$$L^{-1}\left(\frac{1}{(s+a)}\cdot\frac{1}{(s+b)}\right) = L^{-1}\left(\frac{1}{(s+a)}\right) * L^{-1}\left(\frac{1}{(s+b)}\right)$$

$$= e^{-at} * e^{-bt}$$

$$= \int_0^t e^{-au}\cdot e^{-b(t-u)}\,du$$

$$= \int_0^t e^{-au-b(t-u)}\,du$$

DATE

$$= \int_0^t e^{-au-bt+bu}\, du$$

$$= \int_0^t e^{-au+bu} \cdot e^{-bt}\, du$$

$$= e^{-bt} \int_0^t e^{-au+bu}\, du$$

$$e^{-bt} \int_0^t e^{u(b-a)}\, du$$

$$= e^{-bt} \left[\frac{e^{u(b-a)}}{b-a} \right]_0^t$$

$$= e^{-bt} \left[\frac{e^{t(b-a)}}{b-a} - \frac{1}{b-a} \right]$$

4) Using Convolution theorem find $L^{-1}\left(\frac{1}{(s^2+1)^2}\right)$

$$L^{-1}\left(\frac{1}{s^2+1} \cdot \frac{1}{s^2+1}\right) = L^{-1}\left(\frac{1}{s^2+1}\right) * L^{-1}\left(\frac{1}{s^2+1}\right)$$

$$= \sin t * \sin t$$

$$= \int_0^t \sin u \cdot \sin(t-u)\, du$$

$u = \sin(t-u)$ $\quad A = u \quad B = t-u$

$= \cos$

$\cancel{\sin(A+B)} = \cancel{\sin A \sin}$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$- \qquad - \qquad -$$

$$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$$

$$\frac{-1}{2}\left[\cos(A+B) - \cos(A-B)\right] = \sin A \sin B$$

classmate PAGE

DATE

$$\int_0^t \frac{-1}{2}\left[\cos(u+t-u) - \cos(u-t+u)\right] du$$

$$\int_0^t \frac{-1}{2}\left[\cos t - \cos(2u-t)\right] du$$

$$= \cancel{\frac{-1}{2}\left[\sin t - \frac{\sin(2u-t)}{2}\right]}$$

$$\frac{-1}{2}\int_0^t \cos t - \cos(2u-t)\, du$$

$$= \frac{-1}{2}\int_0^t \cos t + \frac{1}{2}\int_0^t \cos(2u-t)\, du$$

$$= \frac{-1}{2}\cos t\,[u]_0^t + \frac{1}{2}\left[\frac{\sin(2u-t)}{2}\right]_0^t$$

$$= \frac{-1}{2}\cos t\,(t) + \frac{1}{2}\left[\frac{\sin t}{2} - \frac{\sin(-t)}{2}\right]$$

$$= \frac{-1}{2}\cos t\,(t) + \frac{1}{2}\left[\frac{\sin t}{2} + \frac{\sin t}{2}\right]$$

$$= \frac{-t}{2}\cos t + \frac{1}{2}\left[\frac{\cancel{2}\sin t}{\cancel{2}}\right]$$

$$= \frac{1}{2}\left[-t\cos t + \sin t\right]$$

DATE

Application of Laplose transform for Solving differential equation

Solution of linear ODE of 2nd order with Constant Coefficient using Laplose tranform.

i) $L(y'(t)) = s\,L(y(t)) - y(0)$

ii) $L(y''(t)) = s^2\,L(y(t)) - s(y(0)) - y'(0)$

i) $\frac{d^2y}{dt^2} + y = 2$ given $y(0) = 0$ $y'(0) = 1$

Given

$$y'' + y = 2$$

$$y''(t) + y(t) = 2$$

Laplose on b/s

$$L(y''(t)) + L(y(t)) = L(2)$$

$$L(y''(t)) = s^2\,L(y(t)) - s\,y(0) - y'(0)$$

$$s^2 L(y(t)) - s\,y(0) - y'(0) + L(y(t)) = L(2)$$

$$s^2 L(y(t)) - s(0) - 1 + L(y(t)) = \frac{2}{s}$$

$$\cancel{s}\quad L(y(t))[s^2+1] = \frac{2}{s} + 1$$

$$L(y(t))(s^2+1) = \frac{2+s}{s}$$

$$L(y(t)) = \frac{s+2}{s(s^2+1)}$$

$$y(t) = L^{-1}\left(\frac{s+2}{s(s^2+1)}\right)$$

DATE

$$\frac{s+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$$\frac{s+2}{s(s^2+1)} = \frac{A(s^2+1) + Bs+C\,(s)}{s(s^2+1)}$$

$$s+2 = As^2+A+Bs^2+Cs$$

$0 = A+B$ $\quad 1 = C \quad 2 = A$

$A = -B \quad B = -2$

$$L^{-1}\left(\frac{s+2}{s(s^2+1)}\right) = L^{-1}\left(\frac{2}{s} - \frac{2s+1}{s^2+1}\right)$$

$$= L^{-1}\left(\frac{2}{s}\right) - L^{-1}\left(\frac{2s+1}{s^2+1}\right)$$

$$= 2L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{2s}{s^2+1}\right) - L^{-1}\left(\frac{1}{s^2+1}\right)$$

$$= 2 - 2\cos t - \sin t$$

$$y(t) = 2 - 2\cos t - \sin t \;//$$

2. Solve

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 2y = 0 \quad \text{given } y = \frac{dy}{dt} = 1 \text{ at } t=0$$

$$y'' - 2y' + 2y = 0$$

$$y''(t) - 2y'(t) + 2y(t) = 0$$

Laplace on b/s

$$L(y''(t)) - L(2y'(t)) + L(2y(t)) = L(0)$$

$$s^2 L(y(t)) - s(y(0)) - y'(0) - 2[s\,L(y(t)) - y(0)]$$
$$+ 2L(y(t)) = L(0)$$

$$\underline{s^2 L(y(t))} - s\underset{\downarrow\atop 1}{\underline{y(0)}} - y'(0) - \underline{2s\,L(y(t))} + 2y(0) + 2L(y(t))$$
$$= 0$$

$\frac{1}{s+1}$ $\frac{s^2+s}{s+1}$ $\frac{s^2+s-3s}{s+1}$ s^2-2s-3

DATE

$s^2 L(y(t)) - 2s\,L(y(t)) - s\,y(0) + 2y(0)$

$L(y(t))[s^2-2s+2] = s+1-2$

$L(y(t))\,[s^2-2s+2] = s-1$

$L(y(t)) = \frac{s-1}{s^2-2s+2}$

$y(t) = L^{-1}\left(\frac{s-1}{s^2-2s+2}\right)$

$y(t) = L^{-1}\left(\frac{s-1}{(s-1)^2+1^2}\right)$

$y(t) = e^t L^{-1}\left(\frac{s}{s^2+1^2}\right)$

$y(t) = e^t \cos t$

$2s^2$

1 2

$a = s$

$2ab = -2s$

$2sb = 2s$

$b = -1$

$b^2 = 1$

~~$s^2+2s+2+1-1$~~

$s^2+2s+1-1+2$

$(s-1)^2+1^2$

$s+1 = a$

3. Using laplace Transform $y''-3y'+2y = e^{-t}$ $\quad y(0)=1$, $y'(0)=0$

$y''-3y'+2y = e^{-t}$

$y''(t)-3y'(t)+2y(t) = e^{-t}$

Laplace on b/s

$L(y''(t)) - 3L(y'(t)) + 2L(y(t)) = L(e^{-t})$

$s^2L(y(t)) - s(y(0)) - y'(0) - 3[s\,L(y(t)) - y(0)] + 2L(y(t)) = \frac{1}{s+1}$

$s^2L(y(t)) - s(y(0)) - y'(0) - 3s\,L(y(t)) + 3y(0) + 2L(y(t)) = \frac{1}{s+1}$

$(s^2-3s+2)\,L(y(t)) = \frac{s-3}{\cancel{s+1}} + \frac{1}{s+1}$

$L(y(t)) = \frac{s^2-2s-3}{(s+1)(s^2-3s+2)}$

DATE

$$y(t) = L^{-1}\left(\frac{s^2-2s-3}{(s+1)(s^2-3s+2)}\right)$$

$2s^2$

$-1 \; -2$

$s^2-s-2s+2$

$s(s-1)$

$-2(s-1)$

$$\frac{s^2-2s-3}{(s+1)(s^2-3s+2)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-3s+2}$$

$$s^2-2s-3 = A(s^2-3s+2) + B(s+1)$$

$1 = A$ $\quad$ $-2 = -3$ $\quad$ $-2 = -3A+B$

$-2 = -3(1) + B$

$B = -2+3$ $\quad$ $B = -2+3$

$B = 1$

$-3 = 2A + B$

$-3 = 2(1) + 1$

$-3 = 2+1$

-3

$$s^2-2s-3 = A(s^2-3s+2) + Bs+C(s+1)$$

$$s^2-2s-3 = \underline{As^2} - \underline{3As} + \underline{2A} + \underline{Bs^2} + \underline{Bs} + \underline{Cs} + C$$

$1 = A+B$ $\quad$ $-2 = -3A+B+C$ $\quad$ $-3 = 2A+C$

$A = -B$ $\quad$ $-2 = -3(-B)+B-3+2B$ $\quad$ $-3 = -2B+C$

$A = \frac{-1}{6}$ $\quad$ $-2 = 3B+B-3+2B$ $\quad$ $C = -3+2$

$-2 = 6B-3$ $\quad$ $C = -3+$

$-2+3 = 6B$

$B = 1/6$ $\quad$ $C = -3+$

$C = -9$

$$\frac{-1}{6}(s^2-3s+2) + \frac{1}{6}s + \frac{-8}{3}(s+1)$$

$C =$

DATE

$\cancel{s = 1}$

$\cancel{1 - 2 - 3 =}$

$$\frac{s^2 - 2s - 3}{(s+1)(s-1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s-2}$$

$$\frac{s^2 - 2s - 3}{(s+1)(s-1)(s-3)} = \frac{A(s-1)(s-2) + B(s+1)(s-2) + C(s+1)(s-1)}{(s+1)(s-1)(s-2)}$$

$$s^2 - 2s - 3 = A(s^2 - 2s - s + 2) + B(s^2 - 2s + s - 2) + C(s^2 - s + s - 1)$$

$1 = A + B + C$ $\qquad -2 = -3A - B$ $\qquad -3 = 2A - 2B - C$

$\cancel{-2 + B = -3A}$

$-2 + 3A = -B$

$2 - 3A = B$