EN3563 ROBOTICS
Mini Project – 4-DoF Robot Arm

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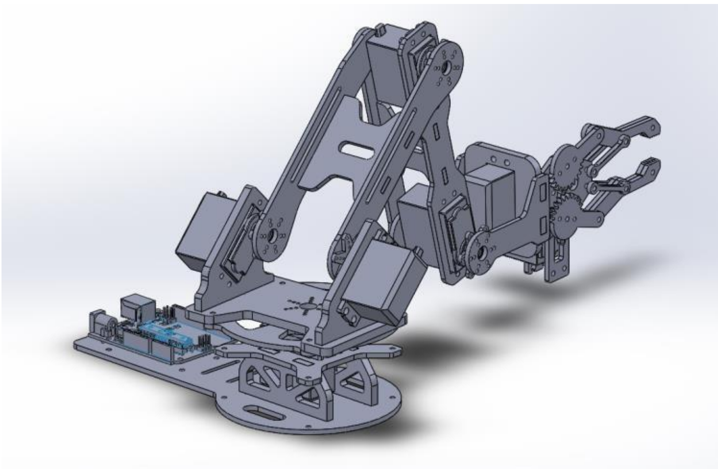
DH Table



Denavit- Hartenberg (DH) Table for the above arm is as follows.

Joint	a_i	α_i	d_i	θ_i
1	0	90	10	θ1
2	12.5	0	0	θ2
3	12.5	0	0	θ3
4	15	0	0	θ4

Forward kinematics

Using the above DH parameters, we calculated the homogeneous transformation matrices for each joint. These matrices describe the position and orientation of each link relative to its preceding link, ultimately leading to the end-effector transformation matrix.

$$^{n-1}T_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \cos\alpha_n & \sin\theta_n \sin\alpha_n & r_n \cos\theta_n \\ \sin\theta_n & \cos\theta_n \cos\alpha_n & -\cos\theta_n \sin\alpha_n & r_n \sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} R & T \\ 0\,0\,0 & 1 \end{pmatrix}$$

Transformation matrices

The transformation matrices for individual joints are:

$$T_1^0 = \begin{bmatrix} cos\,\theta_1 & 0 & sin\,\theta_1 & 0 \\ sin\,\theta_1 & 0 & -cos\,\theta_1 & 0 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} cos\,\theta_2 & -sin\,\theta_2 & 0 & 12.5\,cos\,\theta_2 \\ sin\,\theta_2 & cos\,\theta_2 & 0 & 12.5\,sin\,\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} cos\,\theta_3 & -sin\,\theta_3 & 0 & 12.5\,cos\,\theta_3 \\ sin\,\theta_3 & cos\,\theta_3 & 0 & 12.5\,sin\,\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} cos\,\theta_4 & -sin\,\theta_4 & 0 & 15\,cos\,\theta_4 \\ sin\,\theta_4 & cos\,\theta_4 & 0 & 15\,sin\,\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combined Transformation Matrices

By multiplying the above matrices sequentially, we derived the overall transformations:

$$T_2^0 = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & 12.5c_1c_2 \\ s_1c_2 & -s_1s_2 & -c_1 & 12.5s_1c_2 \\ s_2 & c_2 & 0 & 12.5s_2 + 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & 12.5c_1[c_{23} + c_2] \\ s_1c_{23} & -s_1s_{23} & -c_1 & 12.5s_1[c_{23} + c_2] \\ s_{23} & c_{23} & 0 & 12.5[s_2 + s_{23}] + 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} c_1c_{234} & -c_1s_{234} & s_1 & c_1[12.5c_{23} + 12.5c_2 + 15c_{234}] \\ s_1c_{234} & -s_1s_{234} & -c_1 & s_1[12.5c_{23} + 12.5c_2 + 15c_{234}] \\ s_{234} & c_{234} & 0 & 12.5[s_2 + s_{23}] + 15s_{234} + 10] \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End-Effector Coordinates

The position of the end effector is given as:

X = $c_1[12.5c_{23} + 12.5c_2 + 15c_{234}]$

Y = $s_1[12.5c_{23} + 12.5c_2 + 15c_{234}$

Z = $12.5[s_2 + s_{23}] + 15s_{234} + 10]$

Jacobian Matrix

The Jacobian matrix was derived to determine the relationship between the joint velocities and the end-effector velocities. The calculated Jacobian matrix is:

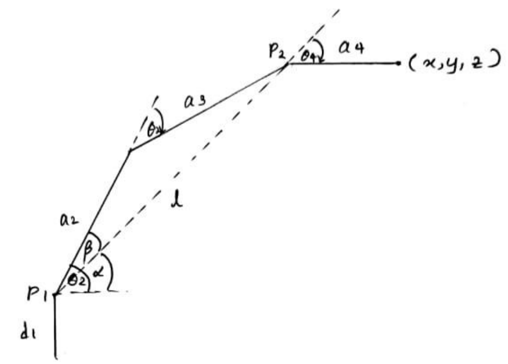
Jacobian Matrix Calculated Jacobian matrix is as follows.

$$\begin{bmatrix} -s_1(12.5c_2 + 12.5c_{23} + 15c_{234}) & -c_1(12.5s_2 + 12.5s_{23} + 15s_{234}) & -c_1(12.5s_{23} + 15s_{234}) & -15c_1s_{234} \\ c_1(12.5c_2 + 12.5c_{23} + 15c_{234}) & -s_1(12.5s_2 + 12.5s_{23} + 15s_{234}) & -s_1(12.5s_{23} + 15s_{234}) & -15s_1s_{234} \\ 0 & 12.5c_2 + 12.5c_{23} + 15c_{234} & 12.5c_{23} + 15c_{233} & 15c_{234} \\ 0 & s_1 & s_1 & s_1 \\ 0 & -c_1 & -c_1 & -c_1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Inverse Kinematics

Inverse kinematics was calculated to determine the joint angles (θ1, θ2, θ3, θ4) for a given end-effector position and orientation. To simplify the process, the end effector was constrained to remain horizontal. This involved solving a set of nonlinear equations to match the target position () with the end-effector coordinates derived from forward kinematics.

The results were validated to ensure that the joint configurations fell within their allowable ranges. The solution ensures precise positioning for desired tasks.



$$\tan\theta_1 = \frac{y}{x} \Rightarrow \theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$$

$$P_1 = (x - a_4\cos\theta_1,\ y - a_4\sin\theta_1,\ z)$$

$$P_2 = (0, 0, d_1)$$

$$l = |P_1 - P_2|$$

$$\cos\theta_3 = \frac{l^2 - a_2^2 - a_3^2}{2a_2a_3} \Longrightarrow \theta_3 = \pm\cos^{-1}\left(\frac{l^2 - a_2^2 - a_3^2}{2a_2a_3}\right)$$

$$\beta = \cos^{-1}\left(\frac{a_2^2 + l^2 - a_3^2}{2a_2 l}\right)$$

$$\gamma = \tan^{-1}\left(\frac{z - d_1}{\sqrt{(x - a_4\cos\theta_1)^2 + (y - a_4\sin\theta_1)^2}}\right)$$

$$\boxed{\theta_2 = \gamma \overset{(-)}{+} \beta} \qquad -\theta_4 = \theta_2 + \theta_3 \Rightarrow \boxed{\theta_4 = -(\theta_2 + \theta_3)}$$