$$\text{Min } \lambda_0\Big(\sum_{i\in I}\sum_{l\in L} F_{il}Y_{il} + \sum_{i\in I} F_i^C Y_i^C\Big) + \lambda_1 \sum_{i\in I}\sum_{j\in J} C_{ij}X_{ij} + \lambda_2 \sum_{i\in I}\sum_{j\in J} \Pi\delta_{ij}^- + \lambda_3\rho$$

subject to

MP

$$\sum_{i\in L} Y_{il} + Y_i^C + Y_i^{NC} = 1 \qquad \forall i \in I \qquad (6)$$

$$\sum_{i\in I} X_{ij} + V_j = \overline{POP_j} \qquad \forall j \in J \qquad (7)$$

$$\sum_{j\in J} X_{ij} + U_i = (Q_i Y_i^{NC} + \sum_{l\in L} Q_{il} Y_{il}) \qquad \forall i \in I \qquad (8)$$

MP

$$X_{ij} \le a_{ij}(\overline{POP_j}) \qquad \forall i \in I, \forall j \in J \qquad (9)$$

$$Y_i^C = 0 \qquad \forall i \in I_e \qquad (10)$$

$$X_{ij} - \widehat{X_{ij}} = \delta_{ij}^+ - \delta_{ij}^- \qquad \forall i \in I, \forall j \in J \qquad (11)$$

$$U_i \le \alpha_i(Q_i Y_i^{NC} + \sum_{l\in L} Q_{il} Y_{il}) \qquad \forall i \in I \qquad (12)$$

$$V_j \le \beta(\overline{POP_j}) \qquad \forall j \in J \qquad (13)$$

$$\frac{\sum_{i\in I} C_{ij} X_{ij}}{\overline{POP_j}} \le \rho \qquad \forall j \in J \qquad (14)$$

$$\rho \ge 0 \qquad (15)$$

$$X_{ij}, U_i, V_j, \delta_j^+, \delta_j^- \ge 0, \text{INTEGER} \qquad \forall i \in I, \forall j \in J \qquad (16)$$

$$Y_{il}, Y_i^C, Y_i^{NC} \in \{0,1\} \qquad \forall i \in I. \qquad (17)$$

MP

$$\text{Min } \lambda_0\Big(\sum_i\sum_l F_{il}y_{il} + \sum_i F_i^C y_i^C\Big) + \phi$$

St

$$\sum_l y_{il} + y_i^C + y_i^{NC} = 1 \quad \forall i$$

$$y_i^C = 0 \quad \forall i \in I_e$$

$$\phi \ge 0 \text{ (AUX VAR.)}$$

$\phi \ge$ fn. of y_i^C, y_i^{NC}, y_{il}

Construct BD cut:

$\phi \ge$ ...

y_i^C, y_i^{NC}, y_{il}

PSP

$$Z_{PSP} = \text{Min } \lambda_1 \sum_{i,j} C_{ij}x_{ij} + \lambda_2 \sum_{i,j} \Pi\,\delta_{ij}^- + \lambda_3\rho$$

St (7), (8)*, (9), (11), (12)*, (13), (14)

$\{ x_{ij} \ge 0, \ \rho \ge 0, \ \ldots$

DSP:

$Z_{DSP} = \ldots$

A

B

C

D

$Z_{PSP}^* = Z_{DSP}^*$

Comes from

$Z_{LB} = Z_{MP}^*$

$Z_{UB} =$

$$\%\,gap = \left(\frac{Z_{UB} - Z_{LB}}{Z_{LB}}\right) \times 100\%$$

if gap $\le$ 2% Stop.

Loop

Iter 1:

Construct & Solve MP → ($\bar{y}$ vector) → $LB = Z_{MP}^*$

Constr. + Solve PSP → ... DSP → Create BD cut → to get UB

Plug-in $\bar{y}$ vector in the Whole Model.

calcUB($\bar{y}$)

$y_i^C \leftarrow \hat{y}_i^C$

Fixing variables

Z_{UB} = Z value from whole model, after fixing y's

"Minimization"