$/person/mile

$/lb

$ FB → A

$ FB → DP

Not % having car

Fraction 0-1

with car

No car

$$\text{Min } Z = \sum_{i\in I} c_i y_i + \sum_{j\in J} c_j \tilde{x}_j + \sum_{j\in J} c_{ij}(1-\alpha_j) pop_j x_{ij} + \sum_{j\in J} P^C s_j^C + \sum_{j\in J} P^{NC} s_j^{NC} \quad (1)$$

lb lb

poor people with car

A Shortage Penalty

B

subject to

$$e^{FB} + \sum_{i\in I} e_i^A \le B \quad (2)$$

Param. Budget (lbs)

$$y_i \le q_i^A + e_i^A \quad \forall i \in I \quad (3)$$

Capacity of i

$$\sum_{j\in J} \tilde{x}_j + \sum_{i\in I} y_i \le q^{FB} + e^{FB} \quad (4)$$

100k

$P^C < P^{NC}$

$$\sum_{j\in J} \tilde{x}_j + \sum_{i\in I} y_i \le S \quad (5)$$

Donation Received by FB

100k

lb

$$y_i \ge \sum_{j\in J} (1-\alpha_j) pop_j x_{ij} \quad \forall i \in I \quad (6)$$

people lbs

$$\sum_{i\in I} pop_j (1-\alpha_j) x_{ij} + s_j^C = d_j^C \quad \forall j \in J \quad (7)$$

~~30~~ demand of poor w/ car

lbs

$$70 \quad \tilde{x}_j + s_j^{NC} = d_j^{NC} \quad \forall j \in J \quad (8)$$

100 ~~30~~ 30

$$\left| \frac{s_j^C + s_j^{NC}}{d_j^C + d_j^{NC}} - \frac{s_k^C + s_k^{NC}}{d_k^C + d_k^{NC}} \right| \le \theta_1 \quad \forall k \in J, j \in J \setminus \{k\}, \; j \ne k \quad (9)$$

(lbs)

$$0 \le x_{ij} \le 1 \quad \forall i \in I, j \in J \quad (10)$$

→ what fraction of poor people from $DP_j$ go to Agency $i$

$\Phi_{DP_j}$

$$e^{FB}, e_i^A, y_i, \tilde{x}_j, s_j^C, s_j^{NC} \ge 0 \quad \forall i \in I, j \in J. \quad (11)$$

$\Phi_{DP_k}$

$$\left| \frac{s_j^C}{d_j^C} - \frac{s_j^{NC}}{d_j^{NC}} \right| \le \theta_2 \quad \forall j \in J \quad (12)$$

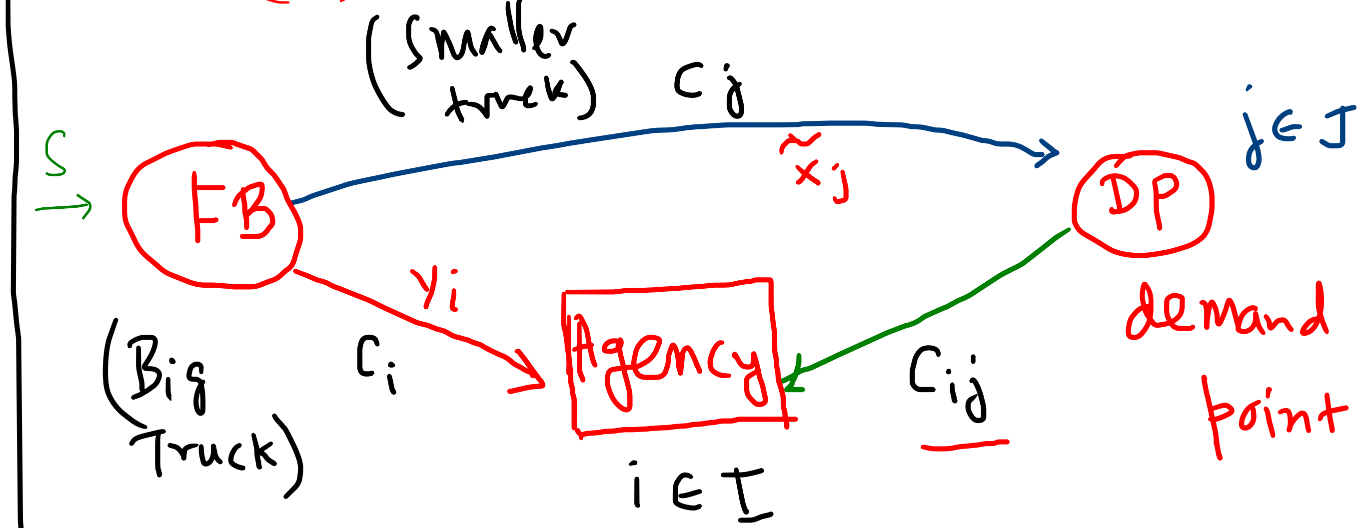
$\theta_1 = 0$

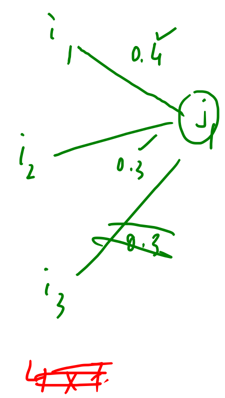
$\theta_2 = 0$

1 person need ~4 lb/day

We are doing it weekly.

(4 × 7) → 30




$/lb/mile

Transp Costs

($/lb) $c_i = 0.1 \times d_i$ (mile)

$c_j = 0.3 \times d_j$

$c_{ij} = 0.5 \times d_{ij}$

$i$ → Agency $\quad i = 1, \ldots, 9$

$j$ → DP $\quad j = 1, \ldots, 85$

$(1-\alpha_j) * pop_j$ = # of people who have car in DP $j \in J$

→ Excel "DemandPoints" column 'N'

$pop_j$ → population needing food assistance in DP $j \in J$

→ Excel Col "L"

* $P^C$ → unmet demand penalty for "with-car" people (Assume 100.)

* $P^{NC}$ → Unmet demand penalty for Carless (Assume 500)

Set $q^{FB} = 100{,}000$ lbs

$B = 10{,}000$ lbs (This is a guess; we will change it later)

$q_i^A :=$ For now, assume these:

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $q_i^A$ | 48,000 | 40,000 | 50,000 | 42,000 | 49,000 | 51,000 | 40,000 | 43,000 | 45,000 |

$S = 100{,}000$ lbs

$\left.\begin{array}{l}\theta_1\\ \theta_2\end{array}\right\}$ We will vary them systematically between 0–1. (Do a sensitivity analysis)

$d_j^{NC}$ → Col. "M" (# ppl)

$d_j^C$ → Col. "N" (# ppl.)

(9) and (12) need linearization as follows:

$$\left|\frac{s_j^C + s_j^{NC}}{d_j^C + d_j^{NC}} - \frac{s_k^C + s_k^{NC}}{d_k^C + d_k^{NC}}\right| \le \theta_1 \qquad \forall k \in J, j \in J \setminus \{k\} \qquad (9)$$

constant

$$(-\theta_1) \le \left(\frac{s_j^C + s_j^{NC}}{d_j^C + d_j^{NC}} - \frac{s_k^C + s_k^{NC}}{d_k^C + d_k^{NC}}\right) \le (\theta_1) \qquad \forall\, j,k$$

Part (a)

$$\Rightarrow \frac{s_j^C + s_j^{NC}}{d_j^C + d_j^{NC}} - \frac{s_k^C + s_k^{NC}}{d_k^C + d_k^{NC}} \le \theta_1$$

(a)

$$\Rightarrow \frac{1}{(d_j^C + d_j^{NC})}\left(s_j^C + s_j^{NC}\right) - \frac{1}{(d_k^C + d_k^{NC})}\left(s_k^C + s_k^{NC}\right) \le \theta_1 \qquad \forall\, j,k \ (j \ne k)$$

Constants

Part (b)

$$\frac{s_j^C + s_j^{NC}}{d_j^C + d_j^{NC}} - \frac{s_k^C + s_k^{NC}}{d_k^C + d_k^{NC}} \ge -\theta_1$$

(b)

$$\Rightarrow \frac{1}{(d_k^C + d_k^{NC})}\left(s_k^C + s_k^{NC}\right) - \frac{1}{(d_j^C + d_j^{NC})}\left(s_j^C + s_j^{NC}\right) \le \theta_1 \qquad \forall\, j,k \ (j \ne k)$$

Thus, you won't code (9), but above pair of constraints: (a), (b).

$\theta_1 = 0.2$ $\theta_2 = 0.2$

XLS → Py → Report (.txt / .xls)

$0 \rightarrow 1$ — 9 — 9 — 9 —

0, 0.2, 0.4, 0.6, 0.8, 1.0

| $\theta_1$ | $\theta_2$ |
|---|---|
| 0 | 0 |
| | 0.2 |
| | 0.4 |
| | 0.6 |
| | 0.8 |
| | 1.0 |
| 0.2 | 0 |
| | 0.2 |
| | ⋮ |
| | 1.0 |

⋮

for

for

⋮

for

→ Σ

(1)

| $\theta_1$ | $\theta_2$ | OFV | Cost1 | . . . | Costs | . . . |
|---|---|---|---|---|---|---|

(2) $B \rightarrow 100{,}000$ ⋮ Budget = [ ]

(3) S

(4) PC

(5) PNC

$$\min \underline{C}^T \underline{x}$$
$$\underline{A}\,\underline{x} \geq \underline{b}$$

$$\left|\frac{s_j^C}{d_j^C} - \frac{s_j^{NC}}{d_j^{NC}}\right| \le \theta_2 \quad \forall j \in J$$

Similarly, $-\theta_2 \le \left(\frac{s_j^C}{d_j^C} - \frac{s_j^{NC}}{d_j^{NC}}\right) \le \theta_2 \quad \forall j \in J$

$$\Rightarrow \boxed{\left(\frac{1}{d_j^C}\right) * s_j^C - \left(\frac{1}{d_j^{NC}}\right) * s_j^{NC} \le \theta_2 \quad \forall j} \text{ --- (i)}$$

$$\Rightarrow \boxed{\left(\frac{1}{d_j^{NC}}\right) s_j^{NC} - \left(\frac{1}{d_j^C}\right) s_j^C \le \theta_2 \quad \forall j} \text{ --- (ii)}$$

$\therefore$ Instead of (12), code this pair.