

# 因果推断文献综述

罗珊珊

北京工商大学 数学与统计学院

January, 2025

# 目录

- ① 随机化试验 (Randomized Controlled Trial)
- ② 观察性研究 (Observational Studies)
- ③ 工具变量 (Instrumental Variable)
- ④ 阴性对照变量 (Proximal Casual Inference)
- ⑤ 主分层分析 (Principal Stratification Analysis)
- ⑥ 中介分析 (Mediation Analysis)
- ⑦ 非随机缺失 (Nonignorable Missing Data)

# 随机化试验

## Randomized Controlled Trial

## 随机化试验下相关问题 I

### ► Interference, SUTVA 假设不成立时



Hudgens, M. G., Halloran, M. E. (2008). Toward causal inference with interference. *Journal of the American Statistical Association*, **103**(482), 832–842.



Tchetgen, E. J. T., Van der Weele, T. J. (2012). On causal inference in the presence of interference. *Statistical Methods in Medical Research*, **21**, 55–75.



Liu, L., Hudgens, M. G. (2014). Large sample randomization inference of causal effects in the presence of interference. *Journal of the American Statistical Association*, **109**(505), 288–301.



Liu, L., Hudgens, M. G., Becker-Dreps, S. (2016). On inverse probability-weighted estimators in the presence of interference. *Biometrika*, **103**(4), 829–842.

## 随机化试验下相关问题 II



Athey, S., Eckles, D., Imbens, G. W. (2018). Exact p-values for network interference. *Journal of the American Statistical Association*, **113**(521), 230–240.



Park, C., Kang, H. (2021). Assumption-Lean Analysis of Cluster Randomized Trials in Infectious Diseases for Intent-to-Treat Effects and Network Effects. *Journal of the American Statistical Association*, **118**, 1195–1206.

### ► Noncompliance, 非依从



Imbens, G. W., Angrist, J. D. (1994). Identification and estimation of local average treatment effects. *Econometrica*, **62**, 467–475.



Angrist, J. D., Imbens, G. W., Rubin, D. B. (1996). Identification of causal effects using instrumental variables. *Journal of the American Statistical Association*, **91**, 444–455.

## 随机化试验下相关问题 III



Imai, K., Jiang, Z., Malani, A. (2020). Causal inference with interference and noncompliance in two-stage randomized experiments. *Journal of the American Statistical Association*, **116**(534), 632–644.

### ► Covariate adjustment in RCT



Zhao, A., Ding, P. (2022). To Adjust or not to Adjust? Estimating the Average Treatment Effect in Randomized Experiments with Missing Covariates. *Journal of the American Statistical Association*, **119**(545), 450–460.



Ding, P. (2024). *A first course in causal inference*. CRC Press. (Chapters 3-9)

### ► Rerandomization



Ding, P. (2024). *A first course in causal inference*. CRC Press. (Chapters 3-9)

# 观察性研究


## Observational Studies


# 观察性研究中的相关问题 I


## ► 基础知识

 Ding, P. (2024). *A first course in causal inference*. CRC Press. (Chapters 10-15)

## ► 稳健估计

 Robins, J. M., Rotnitzky, A., Zhao, L. P. (1994). Estimation of regression coefficients when some regressors are not always observed. *Journal of the American Statistical Association*, **89**, 846–866.

 Lunceford, J. K., Davidian, M. (2004). Stratification and weighting via the propensity score in estimation of causal treatment effects: A comparative study. *Statistics in Medicine*, **23**, 2937–2960.

 Bang, H., Robins, J. M. (2005). Doubly robust estimation in missing data and causal inference models. *Biometrics*, **61**, 962–973



## 观察性研究中的相关问题 II



Kang, J. D. Y., Schafer, J. L. (2007). Comment: Demystifying double robustness: A comparison of alternative strategies for estimating a population mean from incomplete data. *Statistical Science*, **22**, 523–539.



Cao, W., Tsiatis, A. A., Davidian, M. (2009). Improving efficiency and robustness of the doubly robust estimator for a population mean with incomplete data. *Biometrika*, **96**(3), 723–734.



Tan, Z. (2010). Bounded, efficient and doubly robust estimation with inverse weighting. *Biometrika*, **97**, 661–682.



Chernozhukov, V., Chetverikov, D., Demirer, M., Duflo, E., Hansen, C., Newey, W., Robins, J. (2018). Double/debiased machine learning for treatment and structural parameters. *The Econometrics Journal*, **21**(1), C1–C68.

## 观察性研究中的相关问题 III



Pan, Y., Zhao, Y. Q. (2020). Improved doubly robust estimation in learning optimal individualized treatment rules. *Journal of the American Statistical Association*, **116**(533), 283–294.

### ► 迁移学习，数据融合



Yang, S., Kim, J. K., Song, R. (2020). Doubly robust inference when combining probability and non-probability samples with high dimensional data. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, **82**(2), 445–465.



Yang, S., Ding, P. (2020). Combining multiple observational data sources to estimate causal effects. *Journal of the American Statistical Association*, **115**(531), 1540–1554.



Li, X., Miao, W., Lu, F., Zhou, X.-H. (2023). Improving efficiency of inference in clinical trials with external control data. *Biometrics*, **79**(1), 394–403.

## 观察性研究中的相关问题 IV



Wu, L., Yang, S. (2023). Transfer learning of individualized treatment rules from experimental to real-world data. *Journal of Computational and Graphical Statistics*, **32**(3), 1036–1045.



Chu, J., Lu, W., Yang, S. (2023). Targeted optimal treatment regime learning using summary statistics. *Biometrika*, **110**(4), 913–931.








Kallus, N., Mao, X. (2024). On the role of surrogates in the efficient estimation of treatment effects with limited outcome data. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, to appear.







Wu, P., Luo, S., Geng, Z. (2024). On the comparative analysis of average treatment effects estimation via data combination. *Journal of the American Statistical Association*, to appear.

### ► Optimal treatment regime (OTR); Policy learning

## 观察性研究中的相关问题 V

-  Murphy, S. A. (2003). Optimal dynamic treatment regimes. *Journal of the Royal Statistical Society, Series B*, **65**, 331–355. (A-learning)
-  Qian, M., Murphy, S. A. (2011). Performance guarantees for individualized treatment rules. *The Annals of Statistics*, **39**, 1180. (Q-learning)
-  Zhang, Y., Laber, E. B., Tsiatis, A., Davidian, M. (2015). Using decision lists to construct interpretable and parsimonious treatment regimes. *Biometrics*, **71**, 895–904.
-  Zhao, Y.-Q., Zeng, D., Rush, A. J., Kosorok, M. R. (2012). Estimating individualized treatment rules using outcome weighted learning. *Journal of the American Statistical Association*, **107**, 1106–1118.
-  Zhao, Y.-Q., Zeng, D., Laber, E. B., Kosorok, M. R. (2015). New statistical learning methods for estimating optimal dynamic treatment regimes. *Journal of the American Statistical Association*, **110**, 583–598.

## 观察性研究中的相关问题 VI

-  Chu, J., Lu, W., Yang, S. (2023). Targeted optimal treatment regime learning using summary statistics. *Biometrika*, **110**(4), 913–931.
-  Li, C., Zeng, D., Zhu, W. (2024). A robust covariate-balancing method for learning optimal individualized treatment regimes. *Biometrika*, to appear.
-  Pan, Y., Zhao, Y. Q. (2020). Improved doubly robust estimation in learning optimal individualized treatment rules. *Journal of the American Statistical Association*, **116**(533), 283–294.
-  Guo, W., Zhou, X. H., Ma, S. (2021). Estimation of optimal individualized treatment rules using a covariate-specific treatment effect curve with high-dimensional covariates. *Journal of the American Statistical Association*, **116**(533), 309–321.

## 观察性研究中的相关问题 VII

### ► 混杂调整



Craycroft, J. A., Huang, J., Kong, M. (2020). Propensity score specification for optimal estimation of average treatment effect with binary response. *Statistical Methods in Medical Research*, **29**(12), 3623–3640.



Luo, S., Min, J., Li, W., Wang, X., Geng, Z. (2025). A comparative analysis of different adjustment sets using propensity score based estimators. *Computational Statistics & Data Analysis*, **203**, 108079.

工具变量


Instrumental Variable


# 工具变量的相关问题 I

## ► 基础知识

 Ding, P. (2024). *A first course in causal inference*. CRC Press. (Chapters 21-25)

## ► 非参平均因果作用识别

 Wang, L., Tchetgen Tchetgen, E. (2018). Bounded, efficient and multiply robust estimation of average treatment effects using instrumental variables. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **80**(3), 531–550.

 Cui, Y., Tchetgen Tchetgen, E. (2020). A semiparametric instrumental variable approach to optimal treatment regimes under endogeneity. *Journal of the American Statistical Association*, **116**(533), 162–173.



## 工具变量的相关问题 II



Sun, B., Cui, Y., Tchetgen Tchetgen, E. (2022). Selective machine learning of the average treatment effect with an invalid instrumental variable. *Journal of Machine Learning Research*, **23**(204), 1–40.

### ► Optimal treatment regime (OTR); Policy learning



Cui, Y., Tchetgen Tchetgen, E. (2020). A semiparametric instrumental variable approach to optimal treatment regimes under endogeneity. *Journal of the American Statistical Association*, **116**(533), 162–173.



Qiu, H., Carone, M., Sadikova, E., Petukhova, M., Kessler, R. C., Luedtke, A. (2021). Optimal individualized decision rules using instrumental variable methods. *Journal of the American Statistical Association*, **116**(533), 174–191.

## 工具变量的相关问题 III

### ► 高维（无效）工具变量



Lin, W., Feng, R., Li, H. (2015). Regularization methods for high-dimensional instrumental variables regression with an application to genetical genomics. *Journal of the American Statistical Association*, **110**(509), 270–288.



Kang, H., Zhang, A., Cai, T. T., Small, D. S. (2016). Instrumental variables estimation with some invalid instruments and its application to Mendelian randomization. *Journal of the American Statistical Association*, **111**(513), 132–144.



Guo, Z., Kang, H., Cai, T. T., Small, D. S. (2018). Confidence intervals for causal effects with invalid instruments by using two-stage hard thresholding with voting. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, **80**(4), 793–815.

## 工具变量的相关问题 IV



Windmeijer, F., Farbmacher, H., Davies, N., Davey Smith, G. (2019). On the use of the lasso for instrumental variables estimation with some invalid instruments. *Journal of the American Statistical Association*, **114**(527), 1339–1350.



Windmeijer, F., Liang, X., Hartwig, F. P., Bowden, J. (2021). The confidence interval method for selecting valid instrumental variables. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, **83**(4), 752–776.

### ► 无效工具变量



Sun, B., Cui, Y., Tchetgen Tchetgen, E. (2022). Selective machine learning of the average treatment effect with an invalid instrumental variable. *Journal of Machine Learning Research*, **23**(204), 1–40.

## 工具变量的相关问题 V



Liu, Z., Ye, T., Sun, B., Schooling, M., Tchetgen, E. T. (2023). Mendelian randomization mixed-scale treatment effect robust identification and estimation for causal inference. *Biometrics*, **79**, 2208–2219.



Dukes, O., Richardson, D. B., Shahn, Z., Robins, J. M., Tchetgen Tchetgen, E. J. (2024). Using negative controls to identify causal effects with invalid instrumental variables. *Biometrika*, to appear.

### ► 个体因果作用



Vuong, Q., Xu, H. (2017). Counterfactual mapping and individual treatment effects in nonseparable models with binary endogeneity. *Quantitative Economics*, **8**(2), 589–610.

## 工具变量的相关问题 VI



Feng, Q., Vuong, Q., Xu, H. (2020). Estimation of heterogeneous individual treatment effects with endogenous treatments. *Journal of the American Statistical Association*, **115**(529), 231–240.

### ► 迁移学习；两样本孟德尔随机化



Sun, B., Miao, W. (2022). On semiparametric instrumental variable estimation of average treatment effects through data fusion. *Statistica Sinica*, **32**, 569–590.



Zhao, Q., Wang, J., Spiller, W., Bowden, J., Small, D. S. (2019). Two-sample instrumental variable analyses using heterogeneous samples. *Statistical Science*, **34**(2), 317–333.



Zhao, Q., Wang, J., Hemani, G., Bowden, J., Small, D. S. (2020). Statistical inference in two-sample summary-data Mendelian randomization using robust adjusted profile score. *Annals of Statistics*, **48**, 1742–1769.

## 工具变量的相关问题 VII



Shuai, K., Luo, S., Li, W., He, Y. (2024). Identifying causal effects using instrumental variables from the auxiliary population. *Statistica Sinica*, To appear.

### ► 缺失数据



Chen, H., Geng, Z., Zhou, X. H. (2009). Identifiability and estimation of causal effects in randomized trials with noncompliance and completely nonignorable missing data. *Biometrics*, **65**, 675–682.

### ► 孟德尔随机化



VanderWeele, T. J., Tchetgen Tchetgen, E. J., Cornelis, M., Kraft, P. (2014). Methodological challenges in Mendelian randomization. *Epidemiology*, **25**, 427.



Bowden, J., Davey Smith, G., Burgess, S. (2015). Mendelian randomization with invalid instruments: effect estimation and bias detection through Egger regression. *International Journal of Epidemiology*, **44**(2), 512.

## 工具变量的相关问题 VIII



Bowden, J., Spiller, W., Del Greco M, F., Sheehan, N., Thompson, J., Minelli, C., Davey Smith, G. (2018). Improving the visualization, interpretation and analysis of two-sample summary data Mendelian randomization via the radial plot and radial regression. *International Journal of Epidemiology*, **47**, 1264–1278.

### ► 双向因果作用



Li, S., Ye, T. (2024). A focusing framework for testing bi-directional causal effects with GWAS summary data. *Journal of the Royal Statistical Society: Series B*.



Xie, F., Yao, Z., Xie, L., Zeng, Y., Geng, Z. (2024). Identification and estimation of the bi-directional MR with some invalid instruments. In *Proceedings of the 38th International Conference on Neural Information Processing Systems (NeurIPS)*, Vancouver, Canada.





## 阴性对照变量

# Negative Controls, Proximal Casual Inference



# 阴性对照下相关问题 I

## ► 阴性对照识别，基础知识

-  Miao, W., Geng, Z., Tchetgen Tchetgen, E. (2018). Identifying causal effects with proxy variables of an unmeasured confounder. *Biometrika*, **105**, 987–993.
-  Shi, X., Miao, W., Nelson, J. C., Tchetgen Tchetgen, E. (2020). Multiply robust causal inference with double negative control adjustment for categorical unmeasured confounding. *Journal of the Royal Statistical Society: Series B*, **82**, 521–540.
-  Miao, W., Shi, X., Li, Y., Tchetgen Tchetgen, E. J. (2024). A confounding bridge approach for double negative control inference on causal effects. *Statistical Theory and Related Fields*, **8**(4), 262–273.
-  Cui, Y., Pu, H., Shi, X., Miao, W., Tchetgen Tchetgen, E. (2023). Semiparametric proximal causal inference. *Journal of the American Statistical Association*, **119**(546), 1348–1359.

## 阴性对照下相关问题 II

### ► 阴性对照稳健估计



Shi, X., Miao, W., Nelson, J. C., Tchetgen Tchetgen, E. (2020). Multiply robust causal inference with double negative control adjustment for categorical unmeasured confounding. *Journal of the Royal Statistical Society: Series B*, **82**, 521–540.



Cui, Y., Pu, H., Shi, X., Miao, W., Tchetgen Tchetgen, E. (2023). Semiparametric proximal causal inference. *Journal of the American Statistical Association*, **119**(546), 1348–1359.

### ► 中介分析存在未知混杂的识别性



Dukes, O., Shpitser, I., Tchetgen Tchetgen, E. J. (2023). Proximal mediation analysis. *Biometrika*, **110**(4), 973–987.

## 阴性对照下相关问题 III

### ▶ 溢出效应存在未知混杂的识别性



Egami, N., Tchetgen Tchetgen, E. J. (2024). Identification and estimation of causal peer effects using double negative controls for unmeasured network confounding. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, **86**(2), 487–511.

### ▶ 死亡删失存在未知混杂的识别性



Luo, S., Li, W., Miao, W., He, Y. (2024). Identification and estimation of causal effects in the presence of confounded principal strata. *Statistics in Medicine*, 2024; 43(22): 4372-4387.

### ▶ 数据融合/无效工具变量 (NCE)

## 阴性对照下相关问题 IV



Dukes, O., Richardson, D. B., Shahn, Z., Robins, J. M., Tchetgen Tchetgen, E. J. (2024). Using negative controls to identify causal effects with invalid instrumental variables. *Biometrika*, to appear.

### ► binary outcome (regression view) Proximal Causal Inference






Liu, J., Park, C., Li, K., Tchetgen Tchetgen, E. J. (2024). Regression-based proximal causal inference. *American Journal of Epidemiology*, to appear.

### ► 混杂桥函数的非参估计




Li, W., Miao, W., Tchetgen Tchetgen, E. (2023). Nonparametric inference about mean functionals of nonignorable nonresponse data without identifying the joint distribution. *Journal of the Royal Statistical Society: Series B*, **85**(3), 913–935.

## 阴性对照下相关问题 V


-  Zhang, J., Li, W., Miao, W., Tchetgen Tchetgen, E. (2023). Proximal causal inference without uniqueness assumptions. *Statistics & Probability Letters*, **198**, 109836.
-  Kallus, N., Mao, X., Uehara, M. (2021). Causal inference under unmeasured confounding with negative controls: A minimax learning approach. *arXiv preprint arXiv:2103.14029*.
-  Ghassami, A., Ying, A., Shpitser, I., Tchetgen Tchetgen, E. (2022). Minimax kernel machine learning for a class of doubly robust functionals with application to proximal causal inference. In *International Conference on Artificial Intelligence and Statistics* (pp. 7210–7239). PMLR.


## 阴性对照下相关问题 VI


### ► 阴性对照 OTR

 Qi, Z., Miao, R., Zhang, X. (2023). Proximal learning for individualized treatment regimes under unmeasured confounding. *Journal of the American Statistical Association*, **119**(546), 915–928.

### ► 找阴性对照变量，因果发现

 Shi, X., Miao, W., Tchetgen, E. T. (2020b). A selective review of negative control methods in epidemiology. *Current Epidemiology Reports*, **7**, 190–202.

 Xie, F., Chen, Z., Luo, S., Miao, W., Cai, R., Geng, Z. (2024). Automating the selection of proxy variables of unmeasured confounders. In *Proceedings of the 41st International Conference on Machine Learning (ICML)*, Vienna, Austria.

 Kummerfeld, E., Lim, J., Shi, X. (2024). Data-driven Automated Negative Control Estimation (DANCE): search for, validation of, and causal inference with negative controls. *Journal of Machine Learning Research*, **25**(229), 1–35.

## 阴性对照下相关问题 VII

### ► 选择偏差, Test-Negative Design



Schnitzer, M. E. (2022). Estimands and estimation of COVID-19 vaccine effectiveness under the test-negative design: Connections to causal inference. *Epidemiology (Cambridge, Mass.)*, **33**(3), 325–333.



Li, K. Q., Shi, X., Miao, W., Tchetgen Tchetgen, E. (2024). Double negative control inference in test-negative design studies of vaccine effectiveness. *Journal of the American Statistical Association*, **119**(547), 1859–1870.

# 主分层分析

## Principal Stratification Analysis





# 主分层分析下相关问题 I


## ► 基础知识


 Ding, P. (2024). *A first course in causal inference*. CRC Press. (Chapter 26)

## ► 主分层因果作用的界

 Zhang, J. L., Rubin, D. B. (2003). Estimation of causal effects via principle stratification when some outcomes are truncated by ‘death’. *Journal of Educational and Behavioral Statistics*, **28**, 353–368.

 Imai, K. (2008). Sharp bounds on the causal effects in randomized experiments with “truncation-by-death”. *Statistics & Probability Letters*, **78**, 144–149.

 Lee, D. S. (2009). Training, wages, and sample selection: Estimating sharp bounds on treatment effects. *The Review of Economic Studies*, **76**, 1071–1102.






 Long, D. M., Hudgens, M. G. (2013). Sharpening bounds on principal effects with covariates. *Biometrics*, **69**, 812–819.

## 主分层分析下相关问题 II

### ► 主分层因果作用的识别性

-  Ding, P., Geng, Z., Yan, W., Zhou, X.-H. (2011). Identifiability and estimation of causal effects by principal stratification with outcomes truncated by death. *Journal of the American Statistical Association*, **106**, 1578–1591.
-  Zhang, J. L., Rubin, D. B., Mealli, F. (2009). Likelihood-based analysis of causal effects of job-training programs using principal stratification. *Journal of the American Statistical Association*, **104**, 166–176.
-  Jiang, Z., Ding, P., Geng, Z. (2016). Principal causal effect identification and surrogate end point evaluation by multiple trials. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **78**, 829–848.
-  Ding, P., Lu, J. (2017). Principal stratification analysis using principal scores. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **79**, 757–777.

## 主分层分析下相关问题 III

-  Wang, L., Richardson, T. S., Zhou, X.-H. (2017). Causal analysis of ordinal treatments and binary outcomes under truncation by death. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **79**, 719–735.
-  Wang, L., Zhou, X.-H., Richardson, T. S. (2017). Identification and estimation of causal effects with outcomes truncated by death. *Biometrika*, **104**, 597–612.
-  Jiang, Z., Ding, P. (2021). Identification of causal effects within principal strata using auxiliary variables. *Statistical Science*, **36**, 493–508.
-  Jiang, Z., Yang, S., Ding, P. (2022). Multiply robust estimation of causal effects under principal ignorability. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **84**(4), 1423–1445.
-  Luo, S., Li, W., He, Y. (2023). Causal inference with outcomes truncated by death in multiarm studies. *Biometrics*, **79**(1), 502–513.

## 主分层分析下相关问题 IV



Luo, S., Li, W., Miao, W., He, Y. (2024). Identification and estimation of causal effects in the presence of confounded principal strata. *Statistics in Medicine*, 2024; 43(22): 4372-4387.

### ► 归因分析



Shingaki, R., Kuroki, M. (2021). Identification and estimation of joint probabilities of potential outcomes in observational studies with covariate information. In *Proceedings of the 35th International Conference on Neural Information Processing Systems*, 26475–26486.



Lu, Z., Geng, Z., Li, W., Zhu, S., Jia, J. (2023). Evaluating causes of effects by posterior effects of causes. *Biometrika*, **110**(2), 449–465.

## 主分层分析下相关问题 V



Kawakami, Y., Shingaki, R., Kuroki, M. (2023). Identification and estimation of the probabilities of potential outcome types using covariate information in studies with non-compliance. In *Proceedings of the AAAI Conference on Artificial Intelligence*, **37**(10), 12234–12242.



Li, W., Lu, Z., Jia, J., Xie, M., Geng, Z. (2024). Retrospective causal inference with multiple effect variables. *Biometrika*, **111**(2), 573–589.

### ► 公平性评价



Imai, K., Jiang, Z., Greiner, J., Halen, R., Shin, S. (2023). Experimental evaluation of algorithm-assisted human decision-making: Application to pretrial public safety assessment (with discussion). *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, **186**, 167–189.

## 主分层分析下相关问题 VI



Imai, K., Jiang, Z. (2023). Principal fairness for human and algorithmic decision-making. *Statistical Science*, **38**, 317–328.



Ben-Michael, E., Greiner, D. J., Huang, M., Imai, K., Jiang, Z., Shin, S. (2024). Does AI help humans make better decisions? A methodological framework for experimental evaluation. *arXiv preprint*, arXiv:2403.12108.

### ► 敏感性分析



Schwartz, S., Li, F., Reiter, J. P. (2012). Sensitivity analysis for unmeasured confounding in principal stratification settings with binary variables. *Statistics in Medicine*, **31**(10), 949–962.



Ding, P., Lu, J. (2017). Principal stratification analysis using principal scores. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **79**, 757–777.

## 主分层分析下相关问题 VII

### ► Optimal treatment regime (OTR); Policy learning



Ben-Michael, E., Imai, K., Jiang, Z. (2024). Policy learning with asymmetric counterfactual utilities. *Journal of the American Statistical Association*, **2024**, 1–14.

# 中介分析

## Mediation Analysis




## 中介分析下相关问题 I


### ► 基础知识

 Ding, P. (2024). *A first course in causal inference*. CRC Press. (Chapters 27-28)

### ► 稳健估计

 Tchetgen Tchetgen, E. J., Shpitser, I. (2012). Semiparametric theory for causal mediation analysis: efficiency bounds, multiple robustness, and sensitivity analysis. *Annals of Statistics*, **40**(3), 1816.

### ► 多（高维）中介变量

 VanderWeele, T., Vansteelandt, S. (2014). Mediation analysis with multiple mediators. *Epidemiologic Methods*, **2**, 95–115.

 Daniel, R. M., De Stavola, B. L., Cousens, S., Vansteelandt, S. (2015). Causal mediation analysis with multiple mediators. *Biometrics*, **71**, 1–14.

## 中介分析下相关问题 II



Xia, F., Chan, K. C. G. (2022). Decomposition, identification and multiply robust estimation of natural mediation effects with multiple mediators. *Biometrika*, **109**(4), 1085–1100.

### ► 存在未知混杂的识别问题





Guo, Z., Small, D. S., Gansky, S. A., Cheng, J. (2018). Mediation analysis for count and zero-inflated count data without sequential ignorability and its application in dental studies. *Journal of the Royal Statistical Society Series C: Applied Statistics*, **67**, 371–394.





Yuan, Y., Qu, A. (2024). De-confounding causal inference using latent multiple-mediator pathways. *Journal of the American Statistical Association*, **119**(547), 2051–2065.

## 中介分析下相关问题 III

-  Shuai, K., Liu, L., He, Y., Li, W. (2023). Mediation pathway selection with unmeasured mediator-outcome confounding. *arXiv preprint*, arXiv:2311.16793.
-  Dukes, O., Shpitser, I., Tchetgen Tchetgen, E. J. (2023). Proximal mediation analysis. *Biometrika*, **110**(4), 973–987.

### ► 缺失数据

-  Li, W., Zhou, X.-H. (2017). Identifiability and estimation of causal mediation effects with missing data. *Statistics in Medicine*, **36**, 3948–3965.
-  Zuo, S., Ghosh, D., Ding, P., Yang, F. (2024). Mediation analysis with the mediator and outcome missing not at random. *Journal of the American Statistical Association*, to appear.

非随机缺失

Nonignorable Missing Data

## 缺失数据下相关问题 I

- 非随机缺失的识别, shadow variable

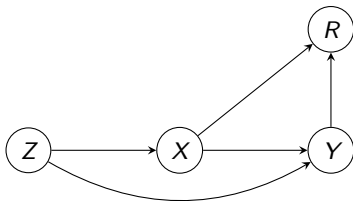







图: (a)  $Z \perp\!\!\!\perp R \mid (Y, X)$ ; (b)  $Z \not\perp\!\!\!\perp Y \mid (R = 1, X)$



Ma, W. Q., Geng, Z., Hu, Y. H. (2003). Identification of graphical models for nonignorable nonresponse of binary outcomes in longitudinal studies. *Journal of Multivariate Analysis*, **87**(1), 24–45.

## 缺失数据下相关问题 II

-  D'Haultfœuille, X. (2010). A new instrumental method for dealing with endogenous selection. *Journal of Econometrics*, **154**, 1–15.
-  Wang, S., Shao, J., Kim, J. K. (2014). An instrumental variable approach for identification and estimation with nonignorable nonresponse. *Statistica Sinica*, **24**, 1097–1116.
-  Miao, W., Tchetgen Tchetgen, E. J. (2016). On varieties of doubly robust estimators under missingness not at random with a shadow variable. *Biometrika*, **103**, 475–482.
-  Li, W., Zhou, X.-H. (2017). Identifiability and estimation of causal mediation effects with missing data. *Statistics in Medicine*, **36**, 3948–3965.
-  Yang, S., Wang, L., Ding, P. (2019). Causal inference with confounders missing not at random. *Biometrika*, **106**(4), 875–888.

## 缺失数据下相关问题 III



Zhao, J., Ma, Y. (2022). A versatile estimation procedure without estimating the nonignorable missingness mechanism. *Journal of the American Statistical Association*, **117**(540), 1916–1930.



Li, W., Miao, W., Tchetgen Tchetgen, E. (2023). Nonparametric inference about mean functionals of nonignorable nonresponse data without identifying the joint distribution. *Journal of the Royal Statistical Society: Series B*, **85**(3), 913–935.



Zuo, S., Ghosh, D., Ding, P., Yang, F. (2024). Mediation analysis with the mediator and outcome missing not at random. *Journal of the American Statistical Association*, to appear.

## 缺失数据下相关问题 IV

### ► 非随机缺失的识别, instrumental variable

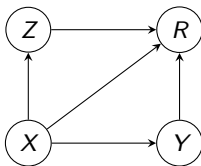


图: (a)  $Z \not\perp R \mid X$ ; (b)  $Z \perp Y \mid X$



Tchetgen Tchetgen, E. J., Wirth, K. E. (2017). A general instrumental variable framework for regression analysis with outcome missing not at random. *Biometrics*, **73**, 1123–1131.



Sun, B., Liu, L., Miao, W., et al. (2018). Semiparametric estimation with data missing not at random using an instrumental variable. *Statistica Sinica*, **28**, 1965–1983.



## 缺失数据下相关问题 V



Liu, L., Miao, W., Sun, B., Robins, J., Tchetgen Tchetgen, E. (2020).

Identification and inference for marginal average treatment effect on the treated with an instrumental variable. *Statistica Sinica*, **30**(3), 1517.

### ► 非随机缺失的识别，参数模型



Heckman, J. J. (1979). Sample selection bias as a specification error.

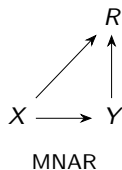
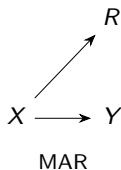
*Econometrica*, **47**, 153–161.



Miao, W., Ding, P., Geng, Z. (2016). Identifiability of normal and normal mixture models with nonignorable missing data. *Journal of the American Statistical Association*, **111**, 1673–1683.

# 随机缺失相关知识

## 缺失机制



- 随机缺失 (Missing at random, MAR):  $R \perp\!\!\!\perp Y \mid X$ ;  
(缺失只依赖于完全观测到的协变量, 不依赖于缺失值本身)
- 非随机缺失 (Missing not at random, MNAR):  $R \not\perp\!\!\!\perp Y \mid X$ ;  
(缺失是依赖于缺失值的)

## 随机缺失 (MAR)

- 基于模型  $E(Y | X, R = 1) = m(X; \beta)$  的回归估计 (REG)

$$\hat{\mu}_{\text{reg}} = \hat{E} \left\{ m(X_i; \hat{\beta}) \right\}$$

- 基于倾向评分模型  $\text{pr}(R = 1 | X) = \pi(X; \alpha)$  的逆概率加权估计 (IPW)

$$\hat{\mu}_{\text{ipw}} = \hat{E} \left\{ \frac{R_i}{\pi(X_i; \hat{\alpha})} Y_i \right\}$$

- 双稳健估计 (AIPW):

$$\begin{aligned} \hat{\mu}_{\text{aipw}} &= \hat{E} \left\{ \frac{R_i}{\pi(X_i; \hat{\alpha})} Y_i + \left\{ 1 - \frac{R_i}{\pi(X_i; \hat{\alpha})} \right\} m(X_i; \hat{\beta}) \right\} \\ &= \hat{E} \left\{ m(X_i; \hat{\beta}) + \frac{R_i}{\pi(X_i; \hat{\alpha})} \left\{ Y_i - m(X_i; \hat{\beta}) \right\} \right\} \end{aligned}$$

未完待续...