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DS LAB1: Problem Statement

During our classroom discussions we have explored the idea of basis vectors, change of basis and its utility.
Write a sample script (preferably user interactive!) for a unique 2D data associated with a binary set and change its basis to complex exponential to study the signature of the data in the changed basis form. Explore the perfect and partial reconstruction samples of the original data from the signatures. Also investigate the rotation, translate and scale variance properties of the basis. Does the order and point of beginning affect the signature? Hence, comment on isotropic/non-isotropic properties of the basis. Theoretically verify the properties under investigation.
Compile the script, output figures and theoretical verification in one (pdf) file for the submission. Kindly make sure that submissions are not through drive links or colabs!

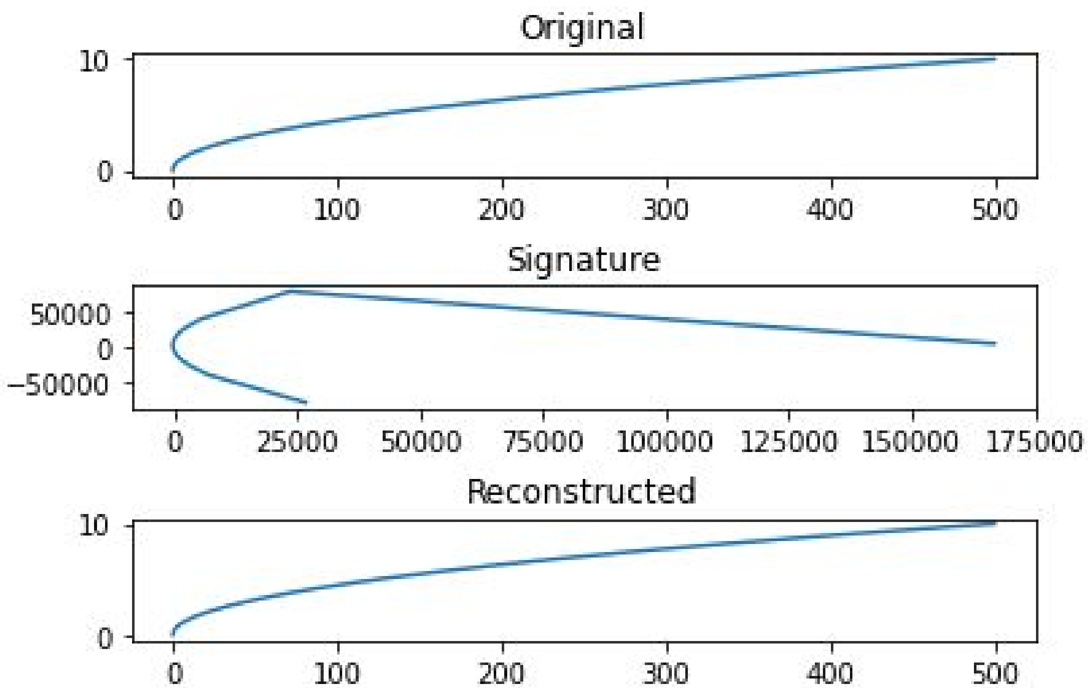
Sample Script

```
import numpy as np
import matplotlib.pyplot as plt

def func():
    x1 = np.linspace(0,10,1000)
    y = np.dot(x1*x1,5)
    f = y + x1*1j
    return f

total = 1000
percent = int(input("The percentage of points to be considered: "))
N = int((percent/100)*total)


def signature(f,N):
    complex_expo = [[0 for a in range(1000)] for b in range(1000)]
    np.array(complex_expo)
    for k in range(0,1000):
        for n in range(0,1000):
            complex_expo[k][n] =np.exp(-2j*np.pi*n*k/len(f))
    sign = np.dot(f,complex_expo)
    return f, sign,N
```

```
def reconstruction(sign,N):
    sign_partial = sign[0:N]
    inv_complex_expo = [[0 for a in range(N)] for b in range(N)]
    np.array(inv_complex_expo)
    for k in range(0,N):
        for n in range(0,N):
            inv_complex_expo[k][n] =np.exp(2j*np.pi*n*k/N)
    recon = (np.dot(inv_complex_expo,sign_partial))/N
    return recon


def plot(f,sign,*args):
    if args:
        recon = args[0]
        fig,axs = plt.subplots(3)
        axs[0].plot(f.real,f.imag)
        axs[0].set_title("Original")
        axs[1].plot(sign.real,sign.imag)
        plt.subplots_adjust(hspace=0.9)
        axs[1].set_title("Signature")
        axs[2].plot(recon.real,recon.imag)
        plt.subplots_adjust(hspace=0.9)
        axs[2].set_title("Reconstructed")
    else:
        fig,axs = plt.subplots(2)
        axs[0].plot(f.real,f.imag)
        axs[0].set_title("Operated")
        axs[1].plot(sign.real,sign.imag)
        plt.subplots_adjust(hspace=0.9)
        axs[1].set_title("Signature")

f = func()
f, s, N= signature(f,N)
r = reconstruction(s, N)
plot(f, s,r)
```

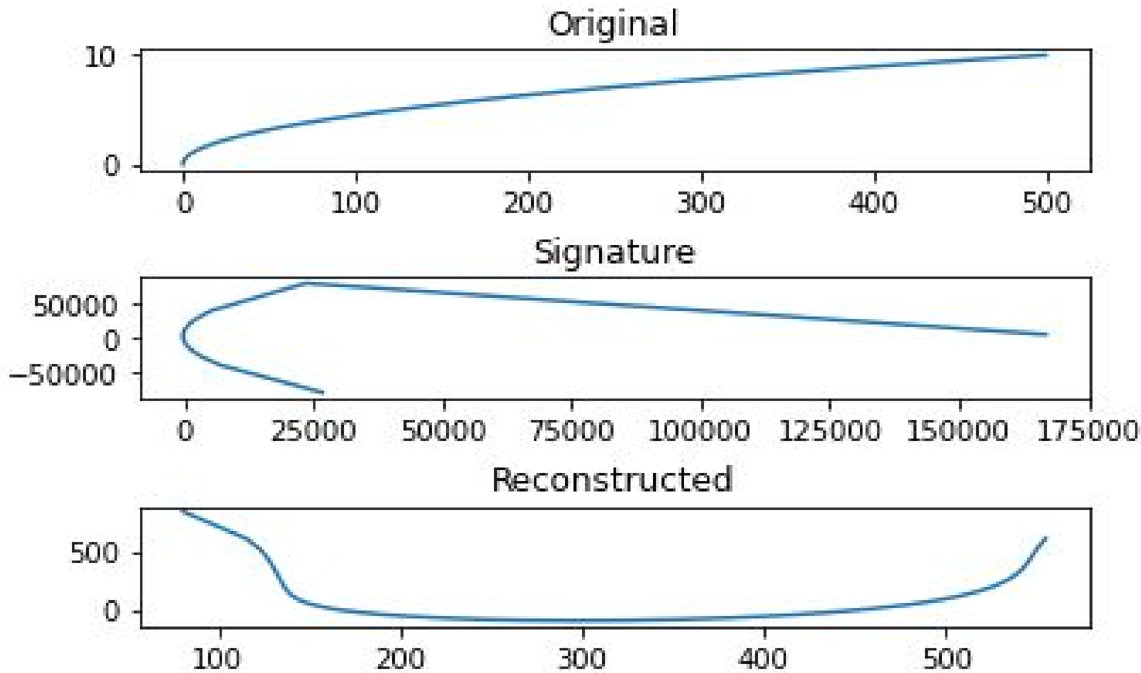
```
ch = int(input("Operation to be performed : 1. Rotation   2. Translation   3. Scaling  :-  "))
if ch==1:
    def rotate_p(point, origin, angle):
        radians = np.deg2rad(angle)
        x,y = point
        x1, y1 = origin
        x2 = (x - x1)
        y2 = (y - y1)
        cos_rad = np.cos(radians)
        sin_rad = np.sin(radians)
        r1 = x1 + cos_rad * x2 + sin_rad * y2
        r2 = y1 + -sin_rad * x2 + cos_rad * y2
        return r1, r2

    def rotate(f):
        angle = input ("Enter the angle in degrees: ")
        for i in range(1000):
            pt=(f[i].real, f[i].imag)
            orig=(0,0)
            pt_r=rotate_p(pt,orig,angle)
            np.array(pt_r)
            fx = np.asarray(pt_r)
            f[i] = fx[0]+fx[1]*1j
        return f

    f_ro=rotate(f)
    f, s, N= signature(f_ro,N)
    plot(f,s)

elif (ch==3):
    scale = int(input("Enter the scaling factor: "))
    f_scale= np.dot(f,scale)
    f_scaled, s, N= signature(f_scale,N)
    plot(f_scaled,s)

elif (ch==2):
    translate = int(input("Enter the translation degree: "))
    f_tran= f + translate
    f_trans, s, N= signature(f_tran,N)
    plot(f_trans,s)

else:
    print("Invalid choice!")
```

Complete Reconstruction (100%)



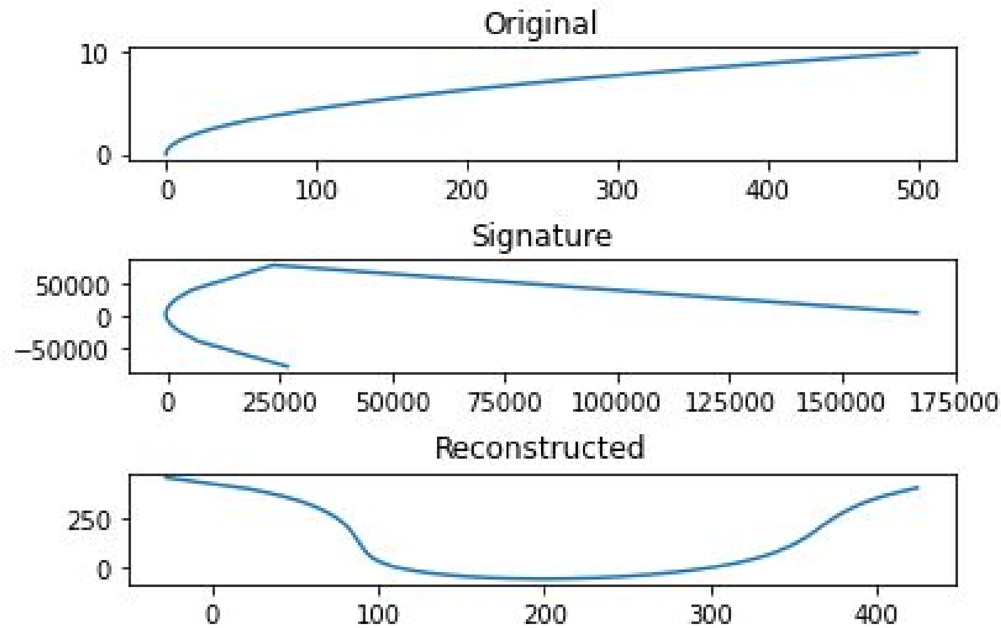
On considering all the points from the signature and reconstruction it provides the above output. Also, we can observe that the reconstructed plot is similar to the original input plot. Thus in case of complete reconstruction we obtain the original input from the signature.

Partial Reconstruction (60%)



In case of partial reconstruction we are able to observe that the reconstructed plot is different from the original. This is due to the fact that the reduced number of points considered in the complex exponential domain are not sufficient to derive the original plot.

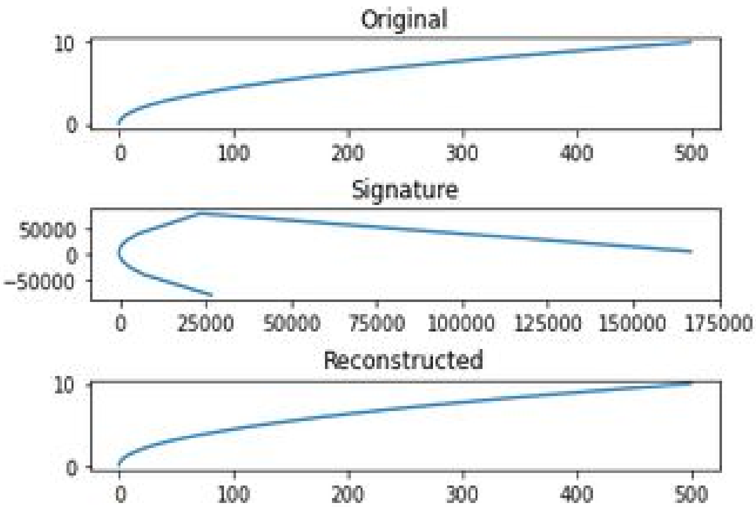
Partial Reconstruction (90%)

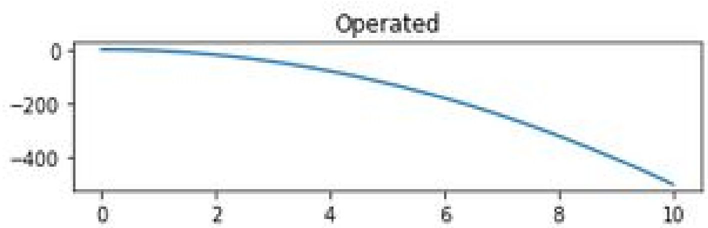


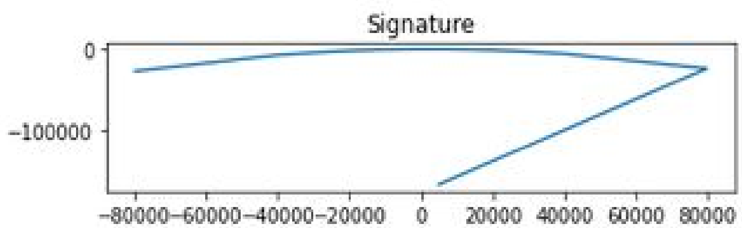
As compared to the 60% partial reconstruction example here we observe that the reconstructed plot tends to shift a bit upwards, that is, a bit closer to the original plot. Yet, in this case as well the reconstructed plot is not similar to the original exactly due to the same reason for the lesser number of points used in the changed basis we get a different plot from the original input.

Rotation Property

```
The percentage of points to be considered: 100
Operation to be performed : 1. Rotation   2. Translation   3. Scaling  :-  1
Enter the angle in degrees: 90
```



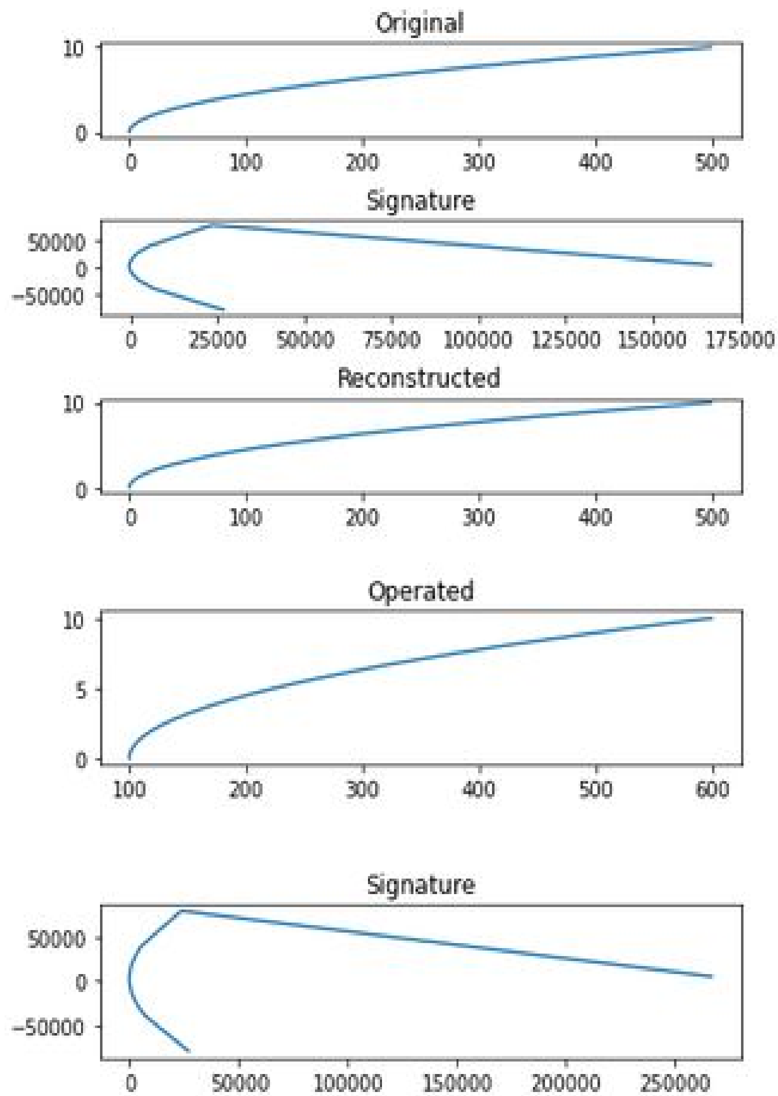




We observe that on rotating the original basis, the signature also gets changed depending on the degree of rotation. Hence, the changed basis is dependent on the degree of rotation of the original basis.

Translation Property

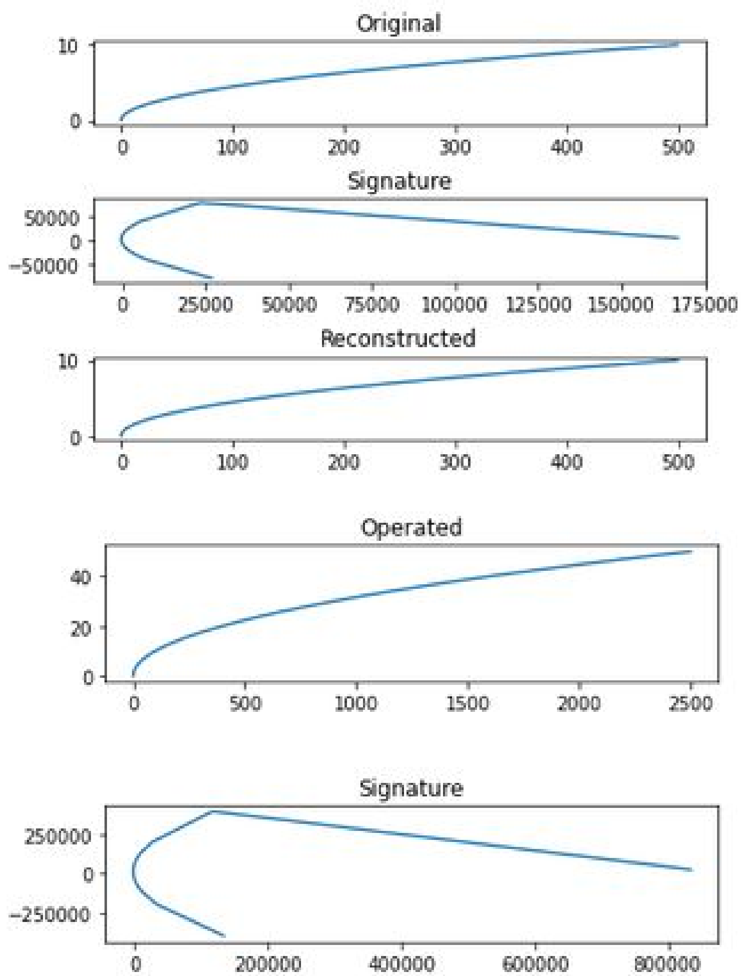
```
The percentage of points to be considered: 100
Operation to be performed : 1. Rotation   2. Translation   3. Scaling  :-  2
Enter the translation degree: 100
```



We see that the signature also gets translated due to translation in the original basis. That is the complex exponential basis is dependent on the translation of the original basis.

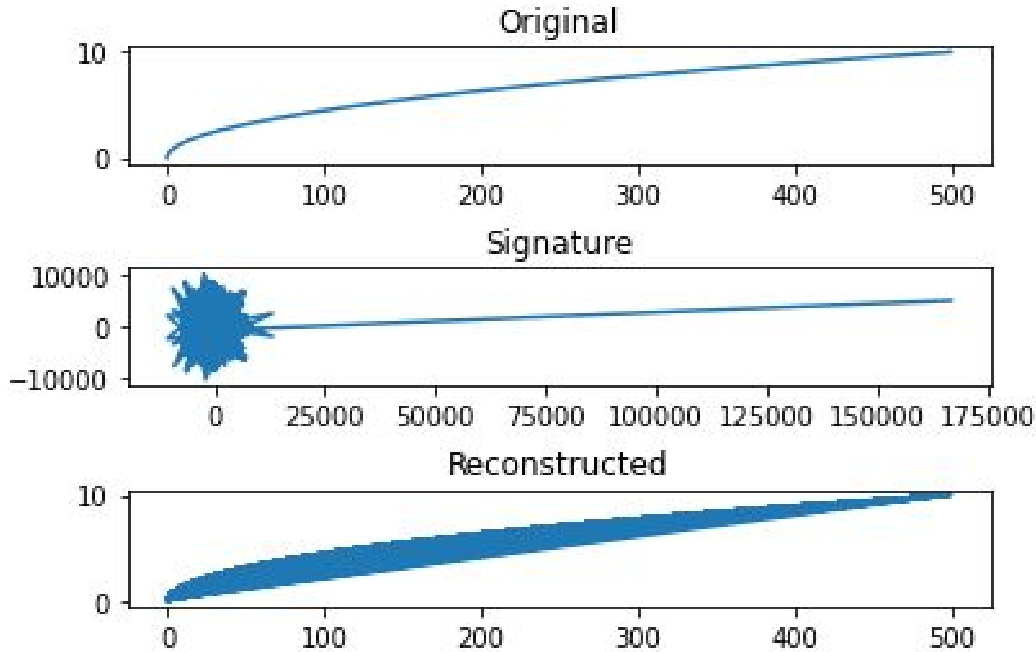
Scaling Property

```
The percentage of points to be considered: 100
Operation to be performed : 1. Rotation   2. Translation   3. Scaling  :-  3
Enter the scaling factor: 5
```



Here the signature also gets scaled, by similar proportionality as that of the scaling factor, in the complex exponential basis as the input in the original basis. Thus, scaling also affects the signature in the changed basis.

Order of Points



From the above example where the order of points is changed, we can clearly observe that the signature gets affected by the order of the points.

Theoretical Verification

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Theoretical Verification.

→ In the given problem statement we are changing basis to complex exponential form

Consider a point from our original basis,

$$a(k) = x(k) + i\,y(k)$$

In order to change from original (x, y) to complex exponential basis.

$$A(u) = \sum_{k=0}^{N-1} a(k)\, e^{-j 2\pi k u/N}$$

where N is the total number of points / elements. Also, in order to get the original data, considering all points from the complex domain (signature) we term it as complete reconstruction,

$$a(k) = \frac{1}{N} \sum_{u=0}^{N-1} A(u)\, e^{j 2\pi k u/N}$$

In case of partial reconstruction, i.e, taking only some points from the signature, we get,

$$\hat{a}(k) = \sum_{u=0}^{m-1} A(u)\, e^{j 2\pi k u/m} \quad ; m < N$$

→ We can evidently observe that the order of points is significant while deriving the signature by change of basis as the points and the weights are taken into dot product sequentially. Hence the properties are non-isotropic.

→ The properties that we need to investigate are, rotation, scaling & translation.

→ In case of rotation
- In original basis (x, y).
We use,

$$r_1 = x + x(k)\cos\theta + i\, y(k)\sin\theta$$

$$r_2 = y + x(k)(-\sin\theta) + i\, y(k)\cos\theta$$

i.e., x & y are changed to r_1 & r_2.

Now, $a(k') = r_1(k) + i\, r_2(k)$.

When we try to change the basis to complex exponential we get,

$$A(u) = \sum_{k=0}^{N-1} a(k')\, e^{-2\pi k u / N}$$

It is clearly evident that the signature in the changed basis also gets rotated as observed from the above equation.

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→ In case of translation
- We add a constant to the original $a(k) = x(k) + i\,y(k)$ to obtain $a(k') = x(k) + iy(k) + c$
When we change the basis, the signature also gets translated as,

$$A(u) = \sum_{k=0}^{N-1} a(k')\, e^{-j2\pi k u/N}$$

So, the signature also gets translated.

→ In case of scaling
- We multiply the original $a(k)$ by a constant or scaling factor (s) to get $a(k') = s(x(k) + iy(k))$
When we change the basis,

$$A(u) = \sum_{k=0}^{N-1} a(k')\, e^{-2\pi k u/N}$$

$$= s \sum_{k=0}^{N-1} a(k)\, e^{-2\pi k u/N}$$

i.e., the signature also gets ~~scale~~ scaled by the same factor.

Calculations

★ Calculations

→ consider the following points.

$$f = [1+2j,\ 2,\ 3+7j,\ 4+9j]$$

Now, weights for change of basis are,

$$W = \sum_{k=0}^{N-1} f\, e^{-j2\pi ku/N}$$

we get,

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

To change the basis to,

$$F = f \cdot W$$

So,

$$F = [10+18j,\ -11-3j,\ -20,\ 7-7j] \rightarrow 1$$

→ F is the data that we get in the complex exponential basis.

In case of complete reconstruction to get back f.

So,

$$W\_exp = \sum_{u=0}^{N-1} f\, e^{+j2\pi ku/N}$$

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solving this we get,

$$W\_exp = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

Now,

$$f = W\text{-}exp \cdot F$$

So,

$$f = [1+2j,\ 2,\ 3+7j,\ 4+9j]$$

∴ We get the original data after complete reconstruction.

→ In case of horizontal ~~scaling~~ translation
Let, $t = 5$ be the ~~scaling~~ value of translation along x-axis.

So,

$$f_i' = [6+2j,\ 7,\ 8+7j,\ 9+9j].$$

After applying change of basis we get.

$$F_i' = [30+18j,\ -11-3j,\ -2,\ 7-7j] \quad \to (2)$$

From (1) & (2), we observe that translation leads to change in the signature.

→ In case of scaling
Let the scaling factor be $s = 4$.
So,
$f_2 = [4+8j, 8, 12+28j, 16+36j]$

We observe that the data in the changed basis,

$F_2 = [40+7j, -44-12j, -8, 28-28j]$ → ③

From ① & ③, we observe that the signature in the changed basis also gets scaled by the same factor.

Conclusion

By performing this experiment the conceptualization about the change of basis was implemented by obtaining its signature in the complex exponential basis. Also, various properties such as rotation, translation and scaling were observed on the input dataset. And, the concept of reconstruction from the changed basis was done considering partial as well as complete reconstruction. All the experimental results obtained have been attached along with their interpretation and theoretical verification.