

# The complete census of orientable cusped hyperbolic 3-manifolds, up to 11 tetrahedra

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**2** Extending the census to 11 tetrahedra

**3** Applications

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## 1 Introduction

## 2 Extending the census to 11 tetrahedra

## 3 Applications

Crossings	Knots		
3	$3_1$		
4	$4_1$		
5	$5_1$	$5_2$	
6	$6_1$	$6_2$	$6_3$

Table: The census of knots up to 6 crossings

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minimal number of crossings in planar diagrams of knots

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minimal number of crossings in planar diagrams of knots



minimal number of tetrahedra in triangulations of manifolds

Tetrahedra	Orientable cusped hyperbolic 3-manifolds					
2	m003	m004				
3	m006	m007	m009	m010	m011	
	m015	m016	m017	m019		

**Table:** The census of orientable cusped hyperbolic manifolds up to 3 tetrahedra

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### Burton, 2014

There are precisely 75,956 cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

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## L. 2025

There are precisely 150,730 and 505,352 orientable cusped hyperbolic 3-manifolds whose minimal ideal triangulations consist of 10 and 11 tetrahedra respectively.

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3-step outline of creating a complete census:

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  - for hyperbolic 3-manifolds: triangulations of 3-manifolds
- 2 Decide the eligibility of each candidate
  - for hyperbolic 3-manifolds: minimality and hyperbolicity of the triangulations
- 3 Group eligible candidates into isomorphism classes and certify the distinctness of each class

For 10 tetrahedra census:

Step	Candidates	Discarded	Kept
Candidate generation	8,373,308	0	0

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Exhaustive nonminimality ( $h = 5$ and 6)	6	3263	0

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Exhaustive nonminimality ( $h = 5$ and 6)	6	3263	0
Certify hyperbolicity ( $r = 1000$ )	0	0	6
Total	8,373,308	7,468,856	904,452

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Certify hyperbolicity ( $r = 10000$ )	0	0	196
Total	27,794,289	25,171,728	2,622,561



## Epstein & Penner, 1988

There is a canonical way to give a cusped hyperbolic 3-manifold a polygonal cellular structure, using its geometric structure.

## Epstein & Penner, 1988

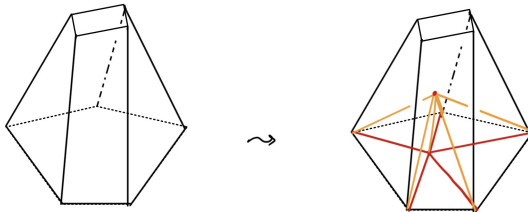
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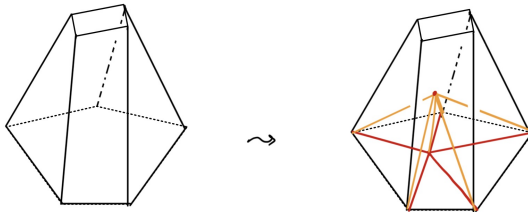
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The triangulation obtained is called the *canonical triangulation*.

The canonical triangulation is a complete invariant of cusped hyperbolic 3-manifolds.

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## Fact

If two cusped hyperbolic 3-manifolds have the same unverified canonical triangulations, they are isometric to each other.

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This was used to group the candidates into isomorphism classes.

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$S_i^{\text{ab}}(G)$ : the multi-set of abelianisations of index  $i$  subgroups of  $G$ , one subgroup for each conjugacy class.



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Drawback: Expensive to compute & Need to go back and forth

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Verified computation always gives correct canonical triangulations, serving as a complete invariant.

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For 11-tetrahedra triangulations: 203/2,622,561 uncomputed.  
Handled using  $(\text{volume}[:80], H_1, S_2^{ab})$ .

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## Census of exceptional Dehn fillings

### Exceptional Dehn filling

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### 6-Theorem (Agol, 2000)

Let  $M$  be a 1-cusped hyperbolic 3-manifold. If a slope  $(p, q)$  has length larger than 6, then  $M(p, q)$  is hyperbolic.

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A Dehn filling on a cusped hyperbolic 3-manifold is *exceptional* if the resulting manifold is not hyperbolic.

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Let  $M$  be a 1-cusped hyperbolic 3-manifold. If a slope  $(p, q)$  has length larger than 6, then  $M(p, q)$  is hyperbolic.

### Corollary

There are only finitely many exceptional Dehn fillings on a 1-cusped hyperbolic 3-manifold.

## Dunfield, 2019

There are precisely 205,822 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

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Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0

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Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	439,933	360,514	0

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Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	439,933	360,514	0
Regina census lookup	11,540	24	428,369

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Regina census lookup	11,540	24	428,369
Essential $S^2$ or torus	11	0	11,529



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Total	800,447	360,549	439,898

These reveal exactly 1849 knot exteriors in 10-tetrahedra census

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Step	Candidates	Discarded	Confirmed
Candidate generation	2,548,823	0	0

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Step	Candidates	Discarded	Confirmed
Candidate generation	2,548,823	0	0
Regina census lookup	1,249,311	10,799	1,288,713

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Step	Candidates	Discarded	Confirmed
Candidate generation	2,548,823	0	0
Regina census lookup	1,249,311	10,799	1,288,713
Certify hyperbolicity	52,242	1,197,069	0

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These reveal exactly 4673 knot exteriors in 11-tetrahedra census

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In particular, they are the smallest among all.

The 10-tetrahedra census comes automatically with SnapPy 3.3.  
The 11-tetrahedra census is available for installation on GitHub.



Figure: QR code to the GitHub repository

Alternatively:

shana-y-li.github.io → Code → snappy\_11\_tets