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# The complete census of orientable cusped hyperbolic 3-manifolds, up to 10 tetrahedra

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University of Illinois, Urbana-Champaign

October 2025

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| Crossings | Knots |
|:---------:|:------|
| 3 | $3_1$ |
| 4 | $4_1$ |
| 5 | $5_1$ $5_2$ |
| 6 | $6_1$ $6_2$ $6_3$ |

Table: The census of knots up to 6 crossings

| Crossings | Knots |      |      |
|-----------|-------|------|------|
| 3         | $3_1$ |      |      |
| 4         | $4_1$ |      |      |
| 5         | $5_1$ | $5_2$ |     |
| 6         | $6_1$ | $6_2$ | $6_3$ |

Table: The census of knots up to 6 crossings

For orientable cusped hyperbolic manifolds: replace crossings with tetrahedra.

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| Tetrahedra | Orientable cusped hyperbolic manifolds | | | | |
|---|---|---|---|---|---|
| 2 | m003 | m004 | | | |
| 3 | m006 | m007 | m009 | m010 | m011 |
| | m015 | m016 | m017 | m019 | |

Table: The census of orientable cusped hyperbolic manifolds up to 3 tetrahedra

For orientable cusped hyperbolic manifolds: replace crossings with tetrahedra.

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Table: Timeline of census of orientable cusped hyperbolic manifolds

| Tetrahedra | Name convention | Year | Contributer(s) |
|:----------:|:---------------:|:----:|:--------------:|
| 2 - 5 | m003 $\sim$ m412 | 1989 | Hildebrand & Weeks |

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| 6 | s000 $\sim$ s961 | 1999 | Callahan, |
| 7 | v0000 $\sim$ v3551 | | Hildebrand & Weeks |

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| 7 | v0000 $\sim$ v3551 | | Hildebrand & Weeks |
| 8 | t00000 $\sim$ t12845 | 2010 | Thistlethwaite |

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| 7 | v0000 $\sim$ v3551 | | Hildebrand & Weeks |
| 8 | t00000 $\sim$ t12845 | 2010 | Thistlethwaite |
| 9 | o9_00000 $\sim$ o9_44249 | 2014 | Burton |

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| 8 | t00000 $\sim$ t12845 | 2010 | Thistlethwaite |
| 9 | o9_00000 $\sim$ o9_44249 | 2014 | Burton |
| 10 | o10_000000 $\sim$ o10_150729 | 2025 | L. |

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The completeness of the census was not confirmed until Burton's work in 2014.

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Table: Timeline of census of orientable cusped hyperbolic manifolds

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The completeness of the census was not confirmed until Burton's work in 2014.

## Burton, 2014

There are precisely 75,956 cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

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3-step outline of creating a complete census:

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3-step outline of creating a complete census:

1. Generate all candidates

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3-step outline of creating a complete census:

1 Generate all candidates

- for hyperbolic 3-manifolds: triangulations of 3-manifolds

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3-step outline of creating a complete census:

1. Generate all candidates
   - for hyperbolic 3-manifolds: triangulations of 3-manifolds
2. Decide the eligibility of each candidate

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3-step outline of creating a complete census:

1. Generate all candidates
   - for hyperbolic 3-manifolds: triangulations of 3-manifolds
2. Decide the eligibility of each candidate
   - for hyperbolic 3-manifolds: minimality and hyperbolicity of the triangulations

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3-step outline of creating a complete census:

1. Generate all candidates
   - for hyperbolic 3-manifolds: triangulations of 3-manifolds
2. Decide the eligibility of each candidate
   - for hyperbolic 3-manifolds: minimality and hyperbolicity of the triangulations
3. Deduplicate the eligible candidates

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| Step | Candidates | Discarded | Eligible |
|---|---|---|---|
| Candidate generation | 8,373,308 | 0 | 0 |

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| Step | Candidates | Discarded | Eligible |
|------|-----------|-----------|----------|
| Candidate generation | 8,373,308 | 0 | 0 |
| Greedy nonminimality | 1,946,782 | 6,426,526 | 0 |

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| Step | Candidates | Discarded | Eligible |
|------|------------|-----------|----------|
| Candidate generation | 8,373,308 | 0 | 0 |
| Greedy nonminimality | 1,946,782 | 6,426,526 | 0 |
| Certify hyperbolicity $(r = 1)$ | 1,072,874 | 0 | 873,908 |

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| Step | Candidates | Discarded | Eligible |
|---|---|---|---|
| Candidate generation | 8,373,308 | 0 | 0 |
| Greedy nonminimality | 1,946,782 | 6,426,526 | 0 |
| Certify hyperbolicity $(r = 1)$ | 1,072,874 | 0 | 873,908 |
| Exhaustive nonminimality $(h = 2)$ | 698,650 | 374,224 | 0 |

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| Step | Candidates | Discarded | Eligible |
|:---:|:---:|:---:|:---:|
| Candidate generation | 8,373,308 | 0 | 0 |
| Greedy nonminimality | 1,946,782 | 6,426,526 | 0 |
| Certify hyperbolicity $(r = 1)$ | 1,072,874 | 0 | 873,908 |
| Exhaustive nonminimality $(h = 2)$ | 698,650 | 374,224 | 0 |
| Special planes (non-hyperbolicity) | 33,807 | 664,843 | 0 |

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| Step | Candidates | Discarded | Eligible |
|------|-----------|-----------|----------|
| Candidate generation | 8,373,308 | 0 | 0 |
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| Certify hyperbolicity ($r = 1$) | 1,072,874 | 0 | 873,908 |
| Exhaustive nonminimality ($h = 2$) | 698,650 | 374,224 | 0 |
| Special planes (non-hyperbolicity) | 33,807 | 664,843 | 0 |
| Certify hyperbolicity ($r = 60$) | 3269 | 0 | 30,538 |

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| Step | Candidates | Discarded | Eligible |
|------|-----------|-----------|----------|
| Candidate generation | 8,373,308 | 0 | 0 |
| Greedy nonminimality | 1,946,782 | 6,426,526 | 0 |
| Certify hyperbolicity $(r = 1)$ | 1,072,874 | 0 | 873,908 |
| Exhaustive nonminimality $(h = 2)$ | 698,650 | 374,224 | 0 |
| Special planes (non-hyperbolicity) | 33,807 | 664,843 | 0 |
| Certify hyperbolicity $(r = 60)$ | 3269 | 0 | 30,538 |
| Exhaustive nonminimality $(h = 5$ and $6)$ | 6 | 3263 | 0 |

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| Step | Candidates | Discarded | Eligible |
|------|-----------|-----------|----------|
| Candidate generation | 8,373,308 | 0 | 0 |
| Greedy nonminimality | 1,946,782 | 6,426,526 | 0 |
| Certify hyperbolicity ($r = 1$) | 1,072,874 | 0 | 873,908 |
| Exhaustive nonminimality ($h = 2$) | 698,650 | 374,224 | 0 |
| Special planes (non-hyperbolicity) | 33,807 | 664,843 | 0 |
| Certify hyperbolicity ($r = 60$) | 3269 | 0 | 30,538 |
| Exhaustive nonminimality ($h = 5$ and 6) | 6 | 3263 | 0 |
| Certify hyperbolicity ($r = 1000$) | 0 | 0 | 6 |

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| Step | Candidates | Discarded | Eligible |
|------|------------|-----------|----------|
| Candidate generation | 8,373,308 | 0 | 0 |
| Greedy nonminimality | 1,946,782 | 6,426,526 | 0 |
| Certify hyperbolicity ($r = 1$) | 1,072,874 | 0 | 873,908 |
| Exhaustive nonminimality ($h = 2$) | 698,650 | 374,224 | 0 |
| Special planes (non-hyperbolicity) | 33,807 | 664,843 | 0 |
| Certify hyperbolicity ($r = 60$) | 3269 | 0 | 30,538 |
| Exhaustive nonminimality ($h = 5$ and 6) | 6 | 3263 | 0 |
| Certify hyperbolicity ($r = 1000$) | 0 | 0 | 6 |
| Total | 8,373,308 | 7,468,856 | 904,452 |

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Deduplication: Group by canonical triangulations computed with
SnapPea/SnapPy

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Deduplication: Group by canonical triangulations computed with
SnapPea/SnapPy (float accuracy issue might occur)

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Deduplication: Group by canonical triangulations computed with
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## Burton's method of seperation

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Deduplication: Group by canonical triangulations computed with
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## Burton's method of seperation

- By algebraic invariants

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Deduplication: Group by canonical triangulations computed with
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### Burton's method of seperation

- By algebraic invariants
  - $\{\text{orientability}, H_1 = \pi_1^{\text{ab}}, S_2^{\text{ab}}(\pi_1), \ldots, S_{11}^{\text{ab}}(\pi_1)\}$

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Deduplication: Group by canonical triangulations computed with SnapPea/SnapPy (float accuracy issue might occur)

## Burton's method of seperation

- By algebraic invariants
  - $\{\text{orientability}, H_1 = \pi_1^{ab}, S_2^{ab}(\pi_1), \ldots, S_{11}^{ab}(\pi_1)\}$

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### Burton's method of seperation

- By algebraic invariants
  - $\{\text{orientability}, H_1 = \pi_1^{ab}, S_2^{ab}(\pi_1), \ldots, S_{11}^{ab}(\pi_1)\}$

$S_i^{ab}(G)$: the multi-set of abelianisations of index $i$ subgroups of $G$,
one subgroup for each conjugacy class.

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Deduplication: Group by canonical triangulations computed with SnapPea/SnapPy (float accuracy issue might occur)

## Burton's method of seperation

- By algebraic invariants
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## Our method of seperation

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Deduplication: Group by canonical triangulations computed with
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## Burton's method of seperation

- By algebraic invariants
  - $\{\text{orientability}, H_1 = \pi_1^{ab}, S_2^{ab}(\pi_1), \ldots, S_{11}^{ab}(\pi_1)\}$

$S_i^{ab}(G)$: the multi-set of abelianisations of index $i$ subgroups of $G$,
one subgroup for each conjugacy class.

## Our method of seperation

- By verified computations (implemented in SnapPy by M. Goerner)

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## Burton's method of seperation

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$S_i^{\text{ab}}(G)$: the multi-set of abelianisations of index $i$ subgroups of $G$, one subgroup for each conjugacy class.

## Our method of seperation

- By verified computations (implemented in SnapPy by M. Goerner)

With verified computation, the canonical triangulation produced is a complete invariant of cusped hyperbolic 3-manifolds.

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The price of verified computation:

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The price of verified computation:

- Need to do algebra on the number field of shapes.

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The price of verified computation:

- Need to do algebra on the number field of shapes.

### BoGwang Jeon

$$\deg\{\text{shape field}\} \propto \frac{1}{\min\{\text{length of geodesics}\}}$$

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The price of verified computation:

- Need to do algebra on the number field of shapes.

### BoGwang Jeon

$$\deg\{\text{shape field}\} \propto \frac{1}{\min\{\text{length of geodesics}\}}$$

The verified computation of 16/608,918 of orientable eligible triangulations ended up OOM (with 60GB RAM).

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- Using (volume[:60], $H_1$), the 16 triangulations were separated from other triangulations ($\leq 10$ tet.) and into 6 groups;

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The price of verified computation:

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- Within each group: all reprsent the same manifold via unverified canonical triangulations.

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The price of verified computation:

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The verified computation of $16/608{,}918$ of orientable eligible triangulations ended up OOM (with 60GB RAM).

- Using (volume[:60], $H_1$), the 16 triangulations were separated from other triangulations ($\leq 10$ tet.) and into 6 groups;
- Within each group: all reprsent the same manifold via unverified canonical triangulations.

### L. 2025

There are precisely 150,730 orientable cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

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Census of exceptional Dehn fillings

### Exceptional Dehn filling

A Dehn filling on a cusped hyperbolic 3-manifold is *exceptional* if the resulting manifold is not hyperbolic.

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Census of exceptional Dehn fillings

### Exceptional Dehn filling

A Dehn filling on a cusped hyperbolic 3-manifold is *exceptional* if the resulting manifold is not hyperbolic.

### 6-Theorem (Agol, 2000)

Let $M$ be a 1-cusped hyperbolic 3-manifold. If a slope $(p, q)$ has length larger than 6, then $M(p, q)$ is hyperbolic.

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Census of exceptional Dehn fillings

### Exceptional Dehn filling

A Dehn filling on a cusped hyperbolic 3-manifold is *exceptional* if the resulting manifold is not hyperbolic.

### 6-Theorem (Agol, 2000)

Let $M$ be a 1-cusped hyperbolic 3-manifold. If a slope $(p, q)$ has length larger than 6, then $M(p, q)$ is hyperbolic.

### Corollary

There are only finitely many exceptional Dehn fillings on a 1-cusped hyperbolic 3-manifold.

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### Dunfield, 2019

There are precisely 205,822 exceptional Dehn fillings on 1-cusped
hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

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### Dunfield, 2019

There are precisely 205,822 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

### L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

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### L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

| Step | Candidates | Discarded | Confirmed |
|------|------------|-----------|-----------|
| Candidate generation | 800,447 | 0 | 0 |

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### L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

| Step | Candidates | Discarded | Confirmed |
|---|---|---|---|
| Candidate generation | 800,447 | 0 | 0 |
| Certify hyperbolicity | 444,549 | 355,898 | 0 |

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### L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

| Step | Candidates | Discarded | Confirmed |
|---|---|---|---|
| Candidate generation | 800,447 | 0 | 0 |
| Certify hyperbolicity | 444,549 | 355,898 | 0 |
| Combinatorial recognition | 12,280 | 4647 | 427,622 |

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### L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

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|------|-----------|-----------|-----------|
| Candidate generation | 800,447 | 0 | 0 |
| Certify hyperbolicity | 444,549 | 355,898 | 0 |
| Combinatorial recognition | 12,280 | 4647 | 427,622 |
| Essential $S^2$ or torus | 4 | 0 | 12,276 |

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### L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

| Step | Candidates | Discarded | Confirmed |
|---|---|---|---|
| Candidate generation | 800,447 | 0 | 0 |
| Certify hyperbolicity | 444,549 | 355,898 | 0 |
| Combinatorial recognition | 12,280 | 4647 | 427,622 |
| Essential $S^2$ or torus | 4 | 0 | 12,276 |
| Identified to be m135(1,3) | 0 | 4 | 0 |

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### L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

| Step | Candidates | Discarded | Confirmed |
|---|---|---|---|
| Candidate generation | 800,447 | 0 | 0 |
| Certify hyperbolicity | 444,549 | 355,898 | 0 |
| Combinatorial recognition | 12,280 | 4647 | 427,622 |
| Essential $S^2$ or torus | 4 | 0 | 12,276 |
| Identified to be m135(1,3) | 0 | 4 | 0 |
| Total | 800,447 | 360,549 | 439,898 |

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### L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

| Step | Candidates | Discarded | Confirmed |
|---|---|---|---|
| Candidate generation | 800,447 | 0 | 0 |
| Certify hyperbolicity | 444,549 | 355,898 | 0 |
| Combinatorial recognition | 12,280 | 4647 | 427,622 |
| Essential $S^2$ or torus | 4 | 0 | 12,276 |
| Identified to be m135(1,3) | 0 | 4 | 0 |
| Total | 800,447 | 360,549 | 439,898 |

(In progress) Classification of each filling & Verify conjectures

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A conjecture on Thurston norm:

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A conjecture on Thurston norm:

### Conjecture (Dunfield-Friedl-Jackson, 2024)

For any cusped hyperbolic 3-manifold $M$ with $b_1(M) = 1$,

$$x(M) = \frac{1}{2} \deg \tau_{2,\rho}(M)$$

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- 143,919 manifolds with $b_1(M) = 1$ in the 10-tet census
- Successfully computed $x(M)$ for all but 21 of them
- The above equality holds for all the computed ones

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## Homological spheres

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### Homological spheres

By Dehn fillings on 1-cusped manifolds in the 10-tetrahedra census

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## Homological spheres

By Dehn fillings on 1-cusped manifolds in the 10-tetrahedra census

- Integral: 10,729

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## Homological spheres

By Dehn fillings on 1-cusped manifolds in the 10-tetrahedra census

- Integral: 10,729
- Rational: 1,899,926

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## Homological spheres

By Dehn fillings on 1-cusped manifolds in the 10-tetrahedra census

- Integral: 10,729
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- The above include *almost all* homological spheres with systole $\geq 0.163$ obtainable in this way (did not deduplicate)

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They can be used to test the *L*-space conjecture

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## The one cusped manifold with totally geodesic surfaces

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o10_14425: kLwLLAQkccgghihijjjdegwtplldjo_aBBaaBBa

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The only one in the orientable cusped census up to 10-tet.

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## The one cusped manifold with totally geodesic surfaces

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The only one in the orientable cusped census up to 10-tet.
In particular, it is the smallest among all.

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The 10-tetrahedra census is available for installation on GitHub.



Figure: QR code to the GitHub repository

Alternatively:

shana-y-li.github.io $\rightarrow$ Research $\rightarrow$ Projects $\rightarrow$ snappy_10_tets