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The complete census of orientable cusped hyperbolic 3-manifolds, up to 10 tetrahedra

Shana Y. Li

University of Illinois, Urbana-Champaign

October 2025

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Crossings	Knots		
3	3_1		
4	4_1		
5	5_1	5_2	
6	6_1	6_2	6_3

Table: The census of knots up to 6 crossings

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Crossings	Knots
3	3_1
4	4_1
5	5_1 5_2
6	6_1 6_2 6_3

Table: The census of knots up to 6 crossings

The left column:

minimal number of crossings in planar diagrams of knots

Crossings	Knots		
3	3_1		
4	4_1		
5	5_1	5_2	
6	6_1	6_2	6_3

Table: The census of knots up to 6 crossings

The left column:

minimal number of crossings in planar diagrams of knots

↓

minimal number of tetrahedra in triangulations of manifolds

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Tetrahedra	Orientable cusped hyperbolic 3-manifolds				
2	m003	m004			
3	m006	m007	m009	m010	m011
	m015	m016	m017	m019	

Table: The census of orientable cusped hyperbolic manifolds up to 3 tetrahedra

The left column:

minimal number of crossings in planar diagrams of knots

↓

minimal number of tetrahedra in triangulations of manifolds

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Table: Timeline of census of orientable cusped hyperbolic 3-manifolds

Tetrahedra	Name convention	Year	Contributer(s)
2 - 5	m003 $\sim$ m412	1989	Hildebrand & Weeks

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Table: Timeline of census of orientable cusped hyperbolic 3-manifolds

Tetrahedra	Name convention	Year	Contributer(s)
2 - 5	m003 $\sim$ m412	1989	Hildebrand & Weeks
6	s000 $\sim$ s961	1999	Callahan,
7	v0000 $\sim$ v3551		Hildebrand & Weeks

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6	s000 $\sim$ s961	1999	Callahan,
7	v0000 $\sim$ v3551		Hildebrand & Weeks
8	t00000 $\sim$ t12845	2010	Thistlethwaite

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Table: Timeline of census of orientable cusped hyperbolic 3-manifolds

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7	v0000 $\sim$ v3551		Hildebrand & Weeks
8	t00000 $\sim$ t12845	2010	Thistlethwaite
9	o9_00000 $\sim$ o9_44249	2014	Burton

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9	o9_00000 $\sim$ o9_44249	2014	Burton
10	o10_000000 $\sim$ o10_150729	2025	L.

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Table: Timeline of census of orientable cusped hyperbolic 3-manifolds

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9	o9_00000 $\sim$ o9_44249	2014	Burton
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The completeness of the census was not confirmed until Burton's work in 2014.

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Table: Timeline of census of orientable cusped hyperbolic 3-manifolds

Tetrahedra	Name convention	Year	Contributer(s)
2 - 5	m003 $\sim$ m412	1989	Hildebrand & Weeks
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The completeness of the census was not confirmed until Burton's work in 2014.

Burton, 2014

There are precisely 75,956 cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

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3-step outline of creating a complete census:

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3-step outline of creating a complete census:

1. Generate all candidates

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3-step outline of creating a complete census:

1. Generate all candidates
   - for hyperbolic 3-manifolds: triangulations of 3-manifolds

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3-step outline of creating a complete census:

1 Generate all candidates
   - for hyperbolic 3-manifolds: triangulations of 3-manifolds
2 Decide the eligibility of each candidate

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3-step outline of creating a complete census:

1. Generate all candidates
   - for hyperbolic 3-manifolds: triangulations of 3-manifolds
2. Decide the eligibility of each candidate
   - for hyperbolic 3-manifolds: minimality and hyperbolicity of the triangulations

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3-step outline of creating a complete census:

1. Generate all candidates
   - for hyperbolic 3-manifolds: triangulations of 3-manifolds
2. Decide the eligibility of each candidate
   - for hyperbolic 3-manifolds: minimality and hyperbolicity of the triangulations
3. Deduplicate the eligible candidates

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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0

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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0

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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0
Certify hyperbolicity $(r = 1)$	1,072,874	0	873,908

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Extending the census to 10 tetrahedra
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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0
Certify hyperbolicity $(r = 1)$	1,072,874	0	873,908
Exhaustive nonminimality $(h = 2)$	698,650	374,224	0

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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0
Certify hyperbolicity $(r = 1)$	1,072,874	0	873,908
Exhaustive nonminimality $(h = 2)$	698,650	374,224	0
Special planes (non-hyperbolicity)	33,807	664,843	0

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Extending the census to 10 tetrahedra
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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0
Certify hyperbolicity ($r = 1$)	1,072,874	0	873,908
Exhaustive nonminimality ($h = 2$)	698,650	374,224	0
Special planes (non-hyperbolicity)	33,807	664,843	0
Certify hyperbolicity ($r = 60$)	3269	0	30,538

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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0
Certify hyperbolicity ($r = 1$)	1,072,874	0	873,908
Exhaustive nonminimality ($h = 2$)	698,650	374,224	0
Special planes (non-hyperbolicity)	33,807	664,843	0
Certify hyperbolicity ($r = 60$)	3269	0	30,538
Exhaustive nonminimality ($h = 5$ and 6)	6	3263	0

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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0
Certify hyperbolicity $(r = 1)$	1,072,874	0	873,908
Exhaustive nonminimality $(h = 2)$	698,650	374,224	0
Special planes (non-hyperbolicity)	33,807	664,843	0
Certify hyperbolicity $(r = 60)$	3269	0	30,538
Exhaustive nonminimality $(h = 5 \text{ and } 6)$	6	3263	0
Certify hyperbolicity $(r = 1000)$	0	0	6

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Step	Candidates	Discarded	Eligible
Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0
Certify hyperbolicity ($r = 1$)	1,072,874	0	873,908
Exhaustive nonminimality ($h = 2$)	698,650	374,224	0
Special planes (non-hyperbolicity)	33,807	664,843	0
Certify hyperbolicity ($r = 60$)	3269	0	30,538
Exhaustive nonminimality ($h = 5$ and 6)	6	3263	0
Certify hyperbolicity ($r = 1000$)	0	0	6
Total	8,373,308	7,468,856	904,452

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Deduplication: Group by canonical triangulations computed with
SnapPea/SnapPy

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Deduplication: Group by canonical triangulations computed with
SnapPea/SnapPy (float accuracy issue might occur)

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Deduplication: Group by canonical triangulations computed with
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Burton's method of seperation

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Deduplication: Group by canonical triangulations computed with
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Burton's method of seperation

- By algebraic invariants

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Deduplication: Group by canonical triangulations computed with
SnapPea/SnapPy (float accuracy issue might occur)

Burton's method of seperation

- By algebraic invariants
  - $\{\text{orientability}, H_1 = \pi_1^{ab}, S_2^{ab}(\pi_1), \ldots, S_{11}^{ab}(\pi_1)\}$

The complete census of orientable cusped hyperbolic 3-manifolds, up to 10 tetrahedra

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Deduplication: Group by canonical triangulations computed with SnapPea/SnapPy (float accuracy issue might occur)

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Deduplication: Group by canonical triangulations computed with
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Burton's method of seperation

- By algebraic invariants
    - $\{\text{orientability}, H_1 = \pi_1^{ab}, S_2^{ab}(\pi_1), \ldots, S_{11}^{ab}(\pi_1)\}$

$S_i^{ab}(G)$: the multi-set of abelianisations of index i subgroups of G,
one subgroup for each conjugacy class.

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Deduplication: Group by canonical triangulations computed with
SnapPea/SnapPy (float accuracy issue might occur)

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Our method of seperation

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Deduplication: Group by canonical triangulations computed with
SnapPea/SnapPy (float accuracy issue might occur)

Burton's method of seperation

- By algebraic invariants
  - $\{\text{orientability}, H_1 = \pi_1^{\text{ab}}, S_2^{\text{ab}}(\pi_1), \dots, S_{11}^{\text{ab}}(\pi_1)\}$

$S_i^{\text{ab}}(G)$: the multi-set of abelianisations of index i subgroups of G,
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Our method of seperation

- By verified computations (implemented in SnapPy by M. Goerner)

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Deduplication: Group by canonical triangulations computed with
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Burton's method of seperation

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$S_i^{\text{ab}}(G)$: the multi-set of abelianisations of index i subgroups of G,
one subgroup for each conjugacy class.

Our method of seperation

- By verified computations (implemented in SnapPy by M. Goerner)

With verified computation, the canonical triangulation produced is
a complete invariant of cusped hyperbolic 3-manifolds.

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The price of verified computation:

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The price of verified computation:

- Need to do algebra on the number field of shapes.

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The price of verified computation:

- Need to do algebra on the number field of shapes.

BoGwang Jeon, 2014

$$\deg\{\text{shape field}\} \propto \frac{1}{\min\{\text{length of geodesics}\}}$$

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The price of verified computation:

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$$\deg\{\text{shape field}\} \propto \frac{1}{\min\{\text{length of geodesics}\}}$$

The verified computation of $16/608{,}918$ of orientable eligible triangulations ended up OOM (with 60GB RAM).

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The verified computation of $16/608{,}918$ of orientable eligible triangulations ended up OOM (with 60GB RAM).

- Using (volume$[:60]$, H_1), the 16 triangulations were separated from other triangulations (≤ 10 tet.) and into 6 groups;

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- Using (volume[:60], H_1), the 16 triangulations were separated from other triangulations (≤ 10 tet.) and into 6 groups;
- Within each group: all reprsent the same manifold via unverified canonical triangulations (which do preserve topology).

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- Within each group: all reprsent the same manifold via unverified canonical triangulations (which do preserve topology).

L. 2025

There are precisely 150,730 orientable cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

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Census of exceptional Dehn fillings

Exceptional Dehn filling

A Dehn filling on a cusped hyperbolic 3-manifold is *exceptional* if the resulting manifold is not hyperbolic.

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Census of exceptional Dehn fillings

Exceptional Dehn filling

A Dehn filling on a cusped hyperbolic 3-manifold is *exceptional* if the resulting manifold is not hyperbolic.

6-Theorem (Agol, 2000)

Let M be a 1-cusped hyperbolic 3-manifold. If a slope (p, q) has length larger than 6, then $M(p, q)$ is hyperbolic.

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Census of exceptional Dehn fillings

Exceptional Dehn filling

A Dehn filling on a cusped hyperbolic 3-manifold is *exceptional* if the resulting manifold is not hyperbolic.

6-Theorem (Agol, 2000)

Let M be a 1-cusped hyperbolic 3-manifold. If a slope (p, q) has length larger than 6, then $M(p, q)$ is hyperbolic.

Corollary

There are only finitely many exceptional Dehn fillings on a 1-cusped hyperbolic 3-manifold.

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Dunfield, 2019

There are precisely 205,822 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

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Dunfield, 2019

There are precisely 205,822 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

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L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0

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L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	444,549	355,898	0

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L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	444,549	355,898	0
Combinatorial recognition	12,280	4647	427,622

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L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	444,549	355,898	0
Combinatorial recognition	12,280	4647	427,622
Essential S^2 or torus	4	0	12,276

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L., 2025

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Candidate generation	800,447	0	0
Certify hyperbolicity	444,549	355,898	0
Combinatorial recognition	12,280	4647	427,622
Essential S^2 or torus	4	0	12,276
Identified to be m135(1,3)	0	4	0

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L., 2025

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Candidate generation	800,447	0	0
Certify hyperbolicity	444,549	355,898	0
Combinatorial recognition	12,280	4647	427,622
Essential S^2 or torus	4	0	12,276
Identified to be m135(1,3)	0	4	0
Total	800,447	360,549	439,898

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L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimal of 10 tetrahedra.

Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	444,549	355,898	0
Combinatorial recognition	12,280	4647	427,622
Essential S^2 or torus	4	0	12,276
Identified to be m135(1,3)	0	4	0
Total	800,447	360,549	439,898

(In progress) Classification of each filling & Verify conjectures

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A conjecture on Thurston norm:

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A conjecture on Thurston norm:

Conjecture (Dunfield-Friedl-Jackson, 2024)

For any cusped hyperbolic 3-manifold M with $b_1(M) = 1$,

$$x(M) = \frac{1}{2} \deg \tau_{2,\rho}(M)$$

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A conjecture on Thurston norm:

Conjecture (Dunfield-Friedl-Jackson, 2024)

For any cusped hyperbolic 3-manifold M with $b_1(M) = 1$,

$$x(M) = \frac{1}{2} \deg \tau_{2,\rho}(M)$$

- In the 10-tet census: 143,919 manifolds with $b_1(M) = 1$

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A conjecture on Thurston norm:

Conjecture (Dunfield-Friedl-Jackson, 2024)

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- In the 10-tet census: 143,919 manifolds with $b_1(M) = 1$
- Successfully computed $x(M)$ for all but 21 of them
- Equality holds for all the computed ones

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- Integral: 10,729

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- Integral: 10,729
- Rational: 1,899,926

The complete census of orientable cusped hyperbolic 3-manifolds, up to 10 tetrahedra

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They can be used to test the *L*-space conjecture

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The one cusped manifold with closed totally geodesic surfaces

The complete census of orientable cusped hyperbolic 3-manifolds, up to 10 tetrahedra

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The one cusped manifold with closed totally geodesic surfaces

$o10\_14425$: kLwLLAQkccgghihijjjdegwtplldjo_aBBaaBBa

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The only one in the orientable cusped census up to 10-tet.

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The only one in the orientable cusped census up to 10-tet.
In particular, it is the smallest among all.

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The 10-tetrahedra census is available for installation on GitHub.

Figure: QR code to the GitHub repository

Alternatively:

shana-y-li.github.io $\to$ Research $\to$ Projects $\to$ snappy_10_tets