

# Test bank

1. Show the probability space for the roll of a fair die.

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{ \text{all subsets of } \Omega \}$$

$$P(X) = \#X/6, \text{ where } \#X \text{ is the number of outcomes in } X$$

2. State the three axiomatic properties of a probability measure.

(a)  $P(\Omega) = 1$

(b)  $P(X) \geq 0$

(c)  $P(X \cup Y) = P(X) + P(Y)$ , where  $X$  and  $Y$  are disjoint events

3. State the definition of conditional probability.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

4. State the multiplication rule for probabilities.

$$P(A \text{ and } B) = P(A)P(B|A)$$

5. State the multiplication rule for probabilities of independent events.

$$P(A \text{ and } B) = P(A)P(B)$$

6. State the addition rule for probabilities.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

7. What does it mean to say that events A and B are independent?

$$P(A|B) = P(A)$$

8. State Bayes' theorem. Name each factor.

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

$P(X|Y)$  is the posterior probability.  $P(Y|X)$  is the likelihood.  $P(X)$  is the prior probability.  $P(Y)$  is the marginal likelihood.

9. State the law of total probability.

If  $X_1, \dots, X_n$  are mutually exclusive events whose probabilities sum to 1, then for any event  $A$ ,

$$P(A) = P(A|X_1)P(X_1) + \dots + P(A|X_n)P(X_n)$$

10. Write R code that creates a  $1 \times 5$  matrix called `m` with elements 10, 20, 30, 40, 50.

```
m <- matrix( seq(from=10, to=50, by=10), nrow=1 )
```

11. Suppose that in R you have a variable `m` that is a  $20 \times 5$  matrix of real numbers.

- (a) Make `x` equal to the first row of `m`.

```
x <- m[1,]
```

- (b) Make `x` equal to all the elements of `m` that are less than -1 or greater than 1.

```
x <- m[ m<(-1) | m>1 ]
```

- (c) Set any elements of `m` in the third column that are less than zero to NaN.

```
m[ m[,3]<0, 3] <- NaN
```

12. Write R code for a function `sinv(x,deg)` that returns the sine of `x`, treating `x` as being in degrees if `deg` is `TRUE`, and in radians if `deg` is `FALSE`. Give `deg` a default value of `TRUE`.

```
sinv <- function(theta, deg = TRUE) {  
  if( deg )  
    return(sin((pi/180)*theta))  
  else  
    return(sin(theta))  
}
```

13. Define the cumulative distribution function of a random variable  $X$ .

a function  $F_X(x)$  such that  $F_X(x) = P(X \leq x)$