Test bank

1. Show the probability space for the roll of a fair die.

    $\Omega = \{1, 2, 3, 4, 5, 6\}$

    $E = \{$ all subsets of Ω $\}$

    $P(X) = \#X/6$, where $\#X$ is the number of outcomes in X

2. State the three axiomatic properties of a probability measure.

    (a) $P(\Omega) = 1$

    (b) $P(X) \geq 0$

    (c) $P(X \cup Y) = P(X) + P(Y)$, where X and Y are disjoint events

3. State the definition of conditional probability.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

4. State the multiplication rule for probabilities.

$$P(A \text{ and } B) = P(A)P(B|A)$$

5. State the multiplication rule for probabilities of independent events.

$$P(A \text{ and } B) = P(A)P(B)$$

6. State the addition rule for probabilities.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

7. What does it mean to say that events A and B are independent?

$$P(A|B) = P(A)$$

8. State Bayes' theorem. Name each factor.

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

    $P(X|Y)$ is the posterior probability. $P(Y|X)$ is the likelihood. $P(X)$ is the prior probability. $P(Y)$ is the marginal likelihood.

9. State the law of total probability.

   If X_1,...,X_n are mutually exclusive events whose probabilities sum to 1, then for any event A,

$$P(A) = P(A|X_1)P(X_1) + ... + P(A|X_n)P(X_n)$$

10. Write R code that creates a 1×5 matrix called `m` with elements 10, 20, 30, 40, 50.

```
m <- matrix( seq(from=10, to=50, by=10), nrow=1 )
```

11. Suppose that in R you have a variable `m` that is a 20×5 matrix of real numbers.

    (a) Make `x` equal to the first row of `m`.

```
x <- m[1,]
```

    (b) Make `x` equal to all the elements of `m` that are less than -1 or greater than 1.

```
x <- m[ m<(-1) | m>1 ]
```

    (c) Set any elements of `m` in the third column that are less than zero to NaN.

```
m[ m[,3]<0, 3] <- NaN
```