Test bank

1. Show the probability space for the roll of a fair die.

   $\Omega = \{1, 2, 3, 4, 5, 6\}$

   $E = \{$ all subsets of Ω $\}$

   $P(X) = \#X/6$, where $\#X$ is the number of outcomes in X

2. State the three axiomatic properties of a probability measure.

   (a) $P(\Omega) = 1$

   (b) $P(X) \geq 0$

   (c) $P(X \cup Y) = P(X) + P(Y)$, where X and Y are disjoint events

3. State the definition of conditional probability.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

4. State the multiplication rule for probabilities.

$$P(A \text{ and } B) = P(A)P(B|A)$$

5. State the multiplication rule for probabilities of independent events.

$$P(A \text{ and } B) = P(A)P(B)$$

6. State the addition rule for probabilities.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

7. What does it mean to say that events A and B are independent?

$$P(A|B) = P(A)$$

8. State Bayes' theorem. Name each factor.

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

   $P(X|Y)$ is the posterior probability. $P(Y|X)$ is the likelihood. $P(X)$ is the prior probability. $P(Y)$ is the marginal likelihood.

9. State the law of total probability.

   If X_1,...,X_n are mutually exclusive events whose probabilities sum to 1, then for any event A,

$$P(A) = P(A|X_1)P(X_1) + ... + P(A|X_n)P(X_n)$$

10. Write R code that creates a 1×5 matrix called `m` with elements 10, 20, 30, 40, 50.

```
m <- matrix( seq(from=10, to=50, by=10), nrow=1 )
```

11. Create an atomic vector that contains 10 real numbers evenly spaced between $-\pi$ and π. Make the vector into a 2 x 5 matrix by setting its dimension attribute directly.

```
x <- seq( -pi, pi, length=10 )
dim( x ) <- c( 2, 5 )
```

12. Suppose that in R you have a variable `m` that is a 20×5 matrix of real numbers.

   (a) Make `x` equal to the first row of `m`.

```
x <- m[1,]
```

   (b) Make `x` equal to all the elements of `m` that are less than -1 or greater than 1.

```
x <- m[ m<(-1) | m>1 ]
```

   (c) Set any elements of `m` in the third column that are less than zero to NaN.

```
m[ m[,3]<0, 3] <- NaN
```

13. Write R code for a function `sinv(x,deg)` that returns the sine of `x`, treating `x` as being in degrees if `deg` is `TRUE`, and in radians if `deg` is `FALSE`. Give `deg` a default value of `TRUE`.

```
sinv <- function(theta, deg = TRUE) {
    if( deg )
      return(sin((pi/180)*theta))
    else
      return(sin(theta))
}
```

14. Define the cumulative distribution function of a random variable X.

   a function $F_X(x)$ such that $F_X(x) = P(X \leq x)$

15. Define the probability density function of a continuous random variable X.

   a function $f_X(x)$ such that $P(u \leq X \leq v)$ equals the area under $f_X(x)$ between u and v

16. Define the probability mass function of a discrete random variable X.

   a function $f_X(x)$ such that $f_X(x) = P(X = x)$

17. Give the probability density function of a normally distributed random variable.

$$\phi(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

18. Complete the following equations for the maximum and minimum of two random variables. Assume that X and Y are independent random variables. The answers are shown at the right.

(a) $F_{\max(X,Y)}(x) =$ $\qquad F_X(x)F_Y(x)$

(b) $F_{\min(X,Y)}(x) =$ $\qquad F_X(x) + F_Y(x) - F_X(x)F_Y(x)$

19. Suppose that in a signal detection model of an observer, the decision variable has mean μ_A on signal A trials, mean μ_B on signal B trials, and standard deviation σ on both types of trials. Suppose that the observer's criterion is v.

(a) Give a definition of d' in terms of μ_A, μ_B, σ, and v.

$$d' = \frac{|\mu_A - \mu_B|}{\sigma}$$

(b) Give an equation for d' in terms of the hit rate H and the false alarm rate FA in a yes-no task.

$$d' = z(H) - z(FA)$$

(c) Give an equation for c in terms of the hit rate H and the false alarm rate FA in a yes-no task.

$$c = -0.5(z(H) + z(FA))$$

20. Write R code that reads a text data file into a data frame. The file is called data.txt, it is comma-delimited, and it has a row of column names.

```
sinv <- read.table( 'data.txt', sep=',', header=TRUE )
```

21. Write R code that creates an atomic vector with three named elements. Show how to use one of the names to access the second element.

```
x <- c( p=1, q=10, r=100 )
x['q']
```

22. Suppose you have a data frame called 'df', with columns 'ntrials' and 'ncorrect'. Write R code that uses the binomial probability mass function to find the negative log likelihood that an observer would make 'ncorrect' correct responses out of 'ntrials' attempts, if the probability of a correct response in each case is given in a vector called 'pcorrect'.

```
nll <- -sum(log( dbinom( df$ncorrect, df$ntrials, pcorrect ) ))
```

23. Write R code that uses `replicate()` to get 10,000 samples of the mean of 100 samples from the standard normal distribution.

```
replicate( 10000, mean( rnorm( 100 ) ) )
```

24. Suppose you have integer vectors `ntrials` and `ncorrect` that report the number of trials and the number of correct responses at each stimulus level in an experiment. Show R code that gets one bootstrapped resample of the number of correct trials.

```
rbinom( length( ntrials ), ntrials, ncorrect/ntrials )
```

25. According to the maximum likelihood cue combination rule, what is the variance of the optimally combined decision variable D?

    The combined variance σ_{12}^2 is given by

$$\frac{1}{\sigma_{12}^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

26. Complete the following equations. For parts (a) to (f), do not assume that X and Y are independent. The answers are shown at the right.

    (a) $E[\, aX + b\,] =$ $\quad aE[X] + b$

    (b) $E[\, X + Y\,] =$ $\quad E[X] + E[Y]$

    (c) $\mathrm{var}[X] =$ $\quad E[\,(X - E[X])^2\,]$

    (d) $\mathrm{var}[\, aX + b\,] =$ $\quad a^2\,\mathrm{var}[X]$

    (e) $\mathrm{cov}[X, Y] =$ $\quad E[\,(X - E[X])(Y - E[Y])\,]$

    (f) $\mathrm{var}[X + Y] =$ $\quad \mathrm{var}[X] + \mathrm{var}[Y] + 2\mathrm{cov}[X, Y]$

    For part (g), assume that X and Y are independent.

    (g) $\mathrm{var}[X + Y] =$ $\quad \mathrm{var}[X] + \mathrm{var}[Y]$