Chapter 15 solutions

1. See `solutions.R`.

2. (a) The probabilities must sum to one, so $c = 1/15$.

   (b) $P(X \leq 4) = 10c = 2/3$, $P(1 < X < 5) = 8c = 8/15$

   (c) $E[X] = 1 \times (2/15) + 2 \times (3/15) + 3 \times (1/15) + 4 \times (4/15) + 5 \times (5/15) = 3.4667$

   $$\begin{aligned} \text{var}[X] = &(2/15)(1-3.4667)^2 + (3/15)(2-3.4667)^2 + (1/15)(3-3.4667)^2 + \\ &(4/15)(4-3.4667)^2 + (5/15)(5-3.4667)^2 = 2.1156 \end{aligned}$$

4. (a) $E = \{$ all subsets of $\Omega\ \} = \{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\},$
   $\{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$

   (b) Two independent events: $\{b,c\}$ and $\{c,d\}$

   Two dependent events: $\{a\}$ and $\{b\}$

   (c) $P(X=0|Y=0) = P(\{c,d\} \cap \{a,c\})/P(\{c,d\}) = P(\{c\})/P(\{c,d\}) = 0.25/0.50 = 0.50$

   which is equal to $P(X=0) = P(\{c,d\}) = 0.50$

   In the same way we can show that $P(X=x|Y=y) = P(X=x)$ and $P(X=x|Z=y) = P(X=x)$ for each value of x and y.

   (d) Summing the columns of the table in (c), we find $W(a) = 2$, $W(b) = 2$, $W(c) = 0$, $W(d) = 1$, so the pmf is

   $$f(x) = \begin{cases} 0.25 & x = 0 \\ 0.25 & x = 1 \\ 0.50 & x = 2 \end{cases}$$

   Using the same expansions as in 2(c), we find $\mathrm{E}[W] = 1.25$ and $\text{var}[W] = 0.6875$.

9. See `solutions.R`.

15. (a) See `solutions.R`.

    (b) The cdf has no steps, so X is continuous. The possible values of X are the regions where the slope of F_X is greater than zero, which is the range $x > 0$.

    (c) $P(X \geq 2) = 1 - P(X < 2) = 1 - F_X(2) = e^{-2} = 0.1353$

    $P(X \leq 2) = F_X(2) = 1 - e^{-2} = 0.8647$

    $P(X = 0) = 0$

    Note that because the probability of any individual value of X is zero, we can use F_X to evaluate both $P(X \leq x)$ and $P(X < x)$.

18. See `solutions.R`.

Chapter 16 solutions

1. The number of people who recover is a binomial random variable with $n = 9$ and $p = 0.15$. The probability that at most two people recover is $P(X \leq 2)$, which is given by the binomial cdf $F_X(2, n, p)$, and the R function `pbinom` reports this to be 0.8591. The expected number of people who recover is $E[X] = np = 1.35$.

2. The number of correct answers from guessing is a binomial random variable with $n = 10$ and $p = 0.2$. The probability of three or more correct answers is $P(X \geq 3) = 1 - P(X \leq 2)$, which `pbinom` reports to be 0.3222. The expected number of correct answers is $E[X] = np = 2$.

3. (a) `pbinom( 18, size=20, prob=0.9 )` $= 0.6083$

   (b) N_0 is a random variable representing the number of flights on which everyone gets a seat. In part (a) we found that the probability of this happening is 0.6083, so N_0 is a binomial random variable with $n = 15$ and $p = 0.6083$. It has mean $np = 9.1238$ and variance $np(1-p) = 3.5742$.

   N_1 is a random variable representing the number of flights on which exactly one person is left behind. The probability of this happening is `dbinom( 19, size=20, prob=0.9 )` $= 0.2702$. Thus N_1 is a binomial random variable with $n = 15$ and $p = 0.2702$. It has mean $np = 4.0526$ and variance $np(1 - p) = 2.9577$.

   (c) The independence assumption is not realistic because people often travel in groups, and so several "successes" and "failures" will occur together. We could incorporate this grouping into our model. We could assume that each set of 20 booked passengers is divided into some number of groups, e.g., we could model the group size using the contiguous-sequence model from chapter 15, problem 9. Then we could assume that each group either shows up on time or not, as a whole.

Chapter 17 solutions

4. (a) $P(4.92 \leq X \leq 5.08) = \Phi(5.08; 5, 0.05^2) - \Phi(4.92; 5, 0.05^2) = 0.8904$

   (b) Let μ_T and σ_T be the mean and standard deviation of the tube lengths.

   We are told that $1 - \Phi(20.9; \mu_T, \sigma_T^2) = 0.95$, so $\Phi(20.9; \mu_T, \sigma_T^2) = 0.05$. We can also write this as $\Phi((20.9 - \mu_T)/\sigma_T) = 0.05$, and thus $(20.9 - \mu_T)/\sigma_T = \Phi^{-1}(0.05)$, which `qnorm` reports to be -1.6449.

   We are also told that $1 - \Phi(21.6; \mu_T, \sigma_T^2) = 0.10$, and the same reasoning shows that $(21.6 - \mu_T)/\sigma_T = 1.2816$.

   This gives us two linear equations involving μ_T and σ_T. Solving them we find $\mu_T = 21.2935$ and $\sigma_T = 0.2392$.

   (c) The length of four batteries taken together is a normally distributed random variable X with $\mu_X = 4 \times 5.0 = 20$ and standard deviation $\sigma_X = \sqrt{4} \times 0.05 = 0.10$.

   Part (b) showed that the length of the tubes is a normally distributed random variable T with $\mu_T = 21.2935$ and $\sigma_T = 0.2392$.

   We are evaluating the probability $P(T > X + 0.75) = P(T - X > 0.75) = 1 - P(T - X \leq 0.75)$.

   $T - X$ is a normally distributed random variable with mean $\mu_{T-X} = \mu_T - \mu_X = 1.2935$ and standard deviation $\sigma_{T-X} = \sqrt{\sigma_T^2 + \sigma_X^2} = 0.2593$.

   Thus the probability is $1 - \Phi(0.75; 1.2935, 0.2593^2) = 0.9820$.