# Test bank

1. Show the probability space for the roll of a fair die.

   $\Omega = \{1, 2, 3, 4, 5, 6\}$

   $E = \{$ all subsets of $\Omega$ $\}$

   $P(X) = \#X/6$, where $\#X$ is the number of outcomes in $X$

2. State the three axiomatic properties of a probability measure.

   (a) $P(\Omega) = 1$

   (b) $P(X) \geq 0$

   (c) $P(X \cup Y) = P(X) + P(Y)$, where $X$ and $Y$ are disjoint events

3. State the definition of conditional probability.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

4. State the multiplication rule for probabilities.

$$P(A \text{ and } B) = P(A)P(B|A)$$

5. State the multiplication rule for probabilities of independent events.

$$P(A \text{ and } B) = P(A)P(B)$$

6. State the addition rule for probabilities.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

7. What does it mean to say that events A and B are independent?

$$P(A|B) = P(A)$$

8. State Bayes' theorem. Name each factor.

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

   $P(X|Y)$ is the posterior probability. $P(Y|X)$ is the likelihood. $P(X)$ is the prior probability. $P(Y)$ is the marginal likelihood.

9. State the law of total probability.

   If $X_1$,...,$X_n$ are mutually exclusive events whose probabilities sum to 1, then for any event $A$,

$$P(A) = P(A|X_1)P(X_1) + ... + P(A|X_n)P(X_n)$$

10. Write R code that creates a $1 \times 5$ matrix called `m` with elements 10, 20, 30, 40, 50.

```
m <- matrix( seq(from=10, to=50, by=10), nrow=1 )
```

11. Create an atomic vector that contains 10 real numbers evenly spaced between $-\pi$ and $\pi$. Make the vector into a 2 x 5 matrix by setting its dimension attribute directly.

```
x <- seq( -pi, pi, length=10 )
dim( x ) <- c( 2, 5 )
```

12. Suppose that in R you have a variable `m` that is a $20 \times 5$ matrix of real numbers.

    (a) Make `x` equal to the first row of `m`.

    ```
    x <- m[1,]
    ```

    (b) Make `x` equal to all the elements of `m` that are less than -1 or greater than 1.

    ```
    x <- m[ m<(-1) | m>1 ]
    ```

    (c) Set any elements of `m` in the third column that are less than zero to NaN.

    ```
    m[ m[,3]<0, 3] <- NaN
    ```

13. Write R code for a function `sinv(x,deg)` that returns the sine of `x`, treating `x` as being in degrees if `deg` is `TRUE`, and in radians if `deg` is `FALSE`. Give `deg` a default value of `TRUE`.

```
sinv <- function(theta, deg = TRUE) {
      if( deg )
        return(sin((pi/180)*theta))
      else
        return(sin(theta))
}
```

14. Define the cumulative distribution function of a random variable $X$.

    a function $F_X(x)$ such that $F_X(x) = P(X \leq x)$

15. Define the probability density function of a continuous random variable $X$.

    a function $f_X(x)$ such that $P(u \leq X \leq v)$ equals the area under $f_X(x)$ between $u$ and $v$

16. Define the probability mass function of a discrete random variable $X$.

    a function $f_X(x)$ such that $f_X(x) = P(X = x)$

17. Give the probability density function of a normally distributed random variable.

$$\phi(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

18. Complete the following equations for the maximum and minimum of two random variables. Assume that $X$ and $Y$ are independent random variables. The answers are shown at the right.

    (a) $F_{\max(X,Y)}(x) =$ $F_X(x)F_Y(x)$

    (b) $F_{\min(X,Y)}(x) =$ $F_X(x) + F_Y(x) - F_X(x)F_Y(x)$