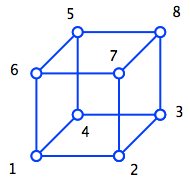
Math

1. answer: 可以这么理解，以其中一个点为起点，穿过圆心切分出一个半圆。另外两个点落在其中一个半圆中概率为 (½)\*(½), 由于三个点都可以作为起点切分出半圆，所以最后概率为3\*(½)\*(½) = ¾

2. answer:



Let *vi* denote the expected number of steps to reach the vertex numbered 8 starting at vertex numbered *i*.

*v*1=1+13(*v*2+*v*4+*v*6); *v*2=1+13(*v*1+*v*3+*v*7); *v*3=1+13(*v*2+*v*4+*v*8); *v*4=1+13(*v*1+*v*3+*v*5); *v*5=1+13(*v*4+*v*6+*v*8); *v*6=1+13(*v*1+*v*5+*v*7); *v*7=1+13(*v*6+*v*2+*v*8); *v*8=0;

Note that by symmetry you have *v*2=*v*4=*v*6 and *v*3=*v*5=*v*7.

Hence, *v*1=1+*v*2 and *v*2=1+13(*v*1+2*v*3) and *v*3=1+23*v*2.

Solving we get

*v*1*v*2=*v*4=*v*6*v*3=*v*5=*v*7=10=9=7

Hence, the expected number of steps to reach the diagonally opposite vertex is 10.

3.

answer:

f(b,r) //对剩下的b,r数量，总期望回报，包括当前的回报，和如果继续的回报

stop条件: if and only if the expected payoff of drawing more cards is less than b-r

f(0,r)=0 //black都抽到了，把最后game全部继续下去，只能持平，为0

f(b,0)=b //red都抽到了，不用继续了，回报为b

E[f(b,r)]=max(b-r, (b/(b+r))E[f(b-1,r)]+(r/(b+r))E[f(b,r-1)])

4.

answer:

if first flit is head, let H = the probability of ever obtaining an equal number of heads and tails starting from 1 head, then H satisfies

H = (1-p) + pH^2 => H = (1-p)/p for p>=1/2, and H=1 for p>=1/2;

if first flit is tail, let T = the probability of ever obtaining an equal number of heads and tails starting from 1 tail, then T satisfies

T = p + (1-p)T^2 => T = p/(1-p) for p <= 1/2, and T = 1 for p >= 1/2;

Now the overall probability of flipping either a head or a tail on the first flip and then obtaining an equal number of heads and tails is the weighted average of the above two probabilities, or

p\*H + (1-p)\*T =

p\*1 + (1-p)\*p/(1-p) for p <= 1/2 and

p\*(1-p)/p + (1-p)\*1 for p >= 1/2

This simplifies to just

2p for p <= 1/2 and

2\*(1-p) for p >= 1/2.

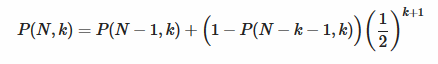
5.

no idea

6.

answer:

Denote P(N,k)P(N,k) as the probability for tossing the coin NN times, and the longest continuous heads is greater or equal than kk. Then (For N>kN>k)



Rong: P(N-1,k) //前N-1个toss没有出现连续k

(1-P(N-k-1) ) //除去后k+1，前面1->N-k-1没有连续的k个head,

(½)k+1 表示 **T**HHHH…H | **H** //总共k+1个元素