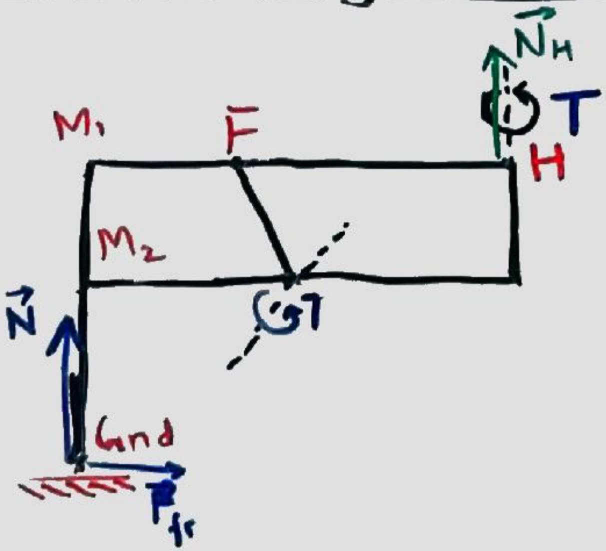
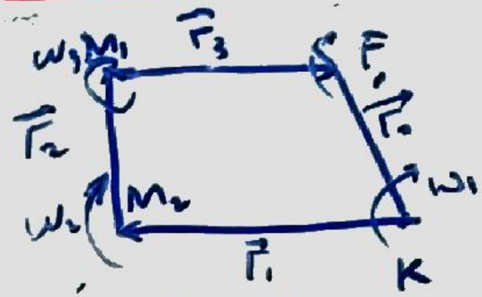
Everyday it gets easier. But you have to do it Everyday



Do not erase diagram



wrt knee

$v_1 = v_4 = 0$

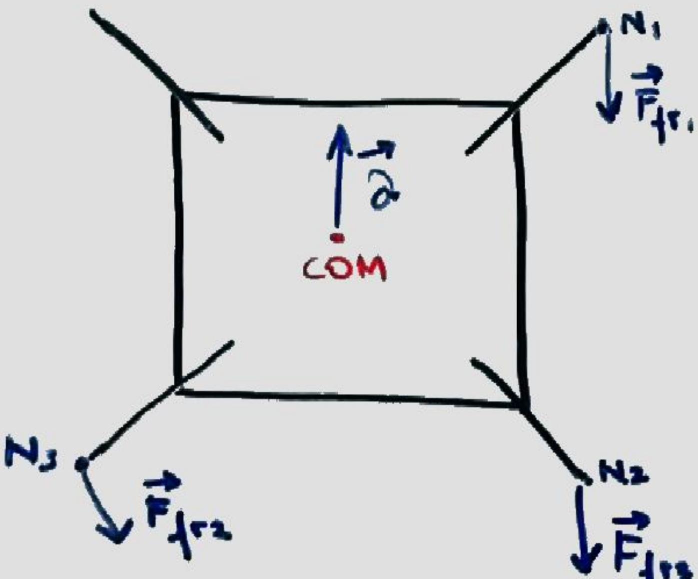
$$\vec r_0 + \vec r_1 + \vec r_2 + \vec r_3 = 0 \quad - (1)$$

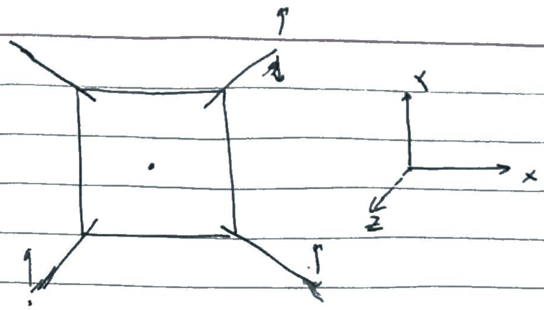
Differentiating (1)

$$\vec v_0 + \vec v_1 + \vec v_2 + \vec v_3 = 0 \quad - (2)$$

$$\vec V_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ as } \vec r_1 \text{ is fixed}$$

$$\left.\begin{aligned} \vec V_1 &= \vec r_1 \times \vec \omega_1 \\ \vec V_2 &= \vec r_2 \times \vec \omega_2 \\ \vec V_3 &= \vec r_3 \times \vec \omega_3 \end{aligned}\right\} \text{Linear velocities of links.}$$



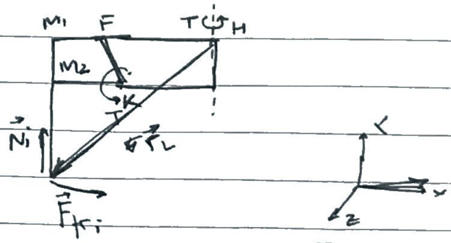


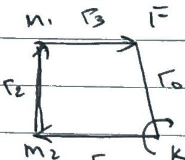
$$\sum_i \vec{F}_{fr_i} = m\vec{a}$$

$$\sum \vec{N}_i = m\vec{g}$$

$$\sum_i \vec{F}_{fr_i} \times \vec{r}_i = \vec{N}_z = I_z \vec{\alpha}_z$$

$$\sum_i \vec{N}_i \times \vec{r} = \vec{N}_x = I_x \vec{\alpha}_x$$





$$\vec{r}_0 + \vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0 \quad -(1)$$

$$\vec{V}_0 + \vec{V}_1 + \vec{V}_2 + \vec{V}_3 = 0 \quad -(2)$$

r_1 $\omega_1, \theta_1, \alpha$, (2) is obtained by differentiating (1)

$\vec{V}_0 = 0$ (K and F are fixed)

$\vec{V}_1 = \vec{\omega}_1 \times \vec{r}_1$

$\vec{V}_2 = \vec{\omega}_2 \times \vec{r}_2$

$\vec{V}_3 = \vec{\omega}_3 \times \vec{r}_3$

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \omega_{1z} \end{bmatrix} \quad \omega_2 = \begin{bmatrix} 0 \\ 0 \\ \omega_{2z} \end{bmatrix} \quad \omega_3 = \begin{bmatrix} 0 \\ 0 \\ \omega_{3z} \end{bmatrix}$$

$$r_1 = \begin{bmatrix} r_{1x} \\ r_{1y} \\ 0 \end{bmatrix} \quad r_2 = \begin{bmatrix} r_{2x} \\ r_{2y} \\ 0 \end{bmatrix} \quad r_3 = \begin{bmatrix} r_{3x} \\ r_{3y} \\ 0 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} -\omega_{1z} r_{1y} \\ \omega_{1z} r_{1x} \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} -\omega_{2z} r_{2y} \\ \omega_{2z} r_{2x} \\ 0 \end{bmatrix} \quad V_3 = \begin{bmatrix} -\omega_{3z} r_{3y} \\ \omega_{3z} r_{3x} \\ 0 \end{bmatrix}$$

since

$$\vec{V}_0 + \vec{V}_1 + \vec{V}_2 + \vec{V}_3 = 0$$

$$\Rightarrow \quad \omega_{1z} r_{1y} + \omega_{2z} r_{2y} + \omega_{3z} r_{3y} = 0$$

$$\omega_{1z} r_{1x} + \omega_{2z} r_{1x} + \omega_{3z} \omega_{3x} = 0$$

$$\Rightarrow \quad \tilde{\omega}_{2z} = -\tilde{\omega}_{1z} \frac{(r_{1y} r_{3x} - r_{3y} r_{1x})}{(r_{2y} r_{3x} - r_{3y} r_{2x})}$$

$$\tilde{\omega}_{3z} = -\tilde{\omega}_{1z} \frac{(r_{2y} r_{1x} - r_{1y} r_{2x})}{(r_{2y} r_{3x} - r_{3y} r_{2x})}$$

$$\vec{a}_0 = 0$$

$$\vec{a}_1 = \vec{\alpha}_1 \times \vec{r}_1 + \vec{\omega}_1 \times \vec{\omega}_1 \times \vec{r}_1$$

$$\vec{a}_2 = \vec{\alpha}_2 \times \vec{r}_2 + \vec{\omega}_2 \times \vec{\omega}_2 \times \vec{r}_2$$

$$\vec{a}_3 = \vec{\alpha}_3 \times \vec{r}_3 + \vec{\omega}_3 \times \vec{\omega}_3 \times \vec{r}_3$$

$$\vec{\alpha}_1 = \begin{bmatrix} 0 \\ 0 \\ \alpha_{1z} \end{bmatrix} \quad \vec{\alpha}_2 = \begin{bmatrix} 0 \\ 0 \\ \alpha_{2z} \end{bmatrix} \quad \vec{\alpha}_3 = \begin{bmatrix} 0 \\ 0 \\ \alpha_{3z} \end{bmatrix}$$

$$\vec{a}_1 = \begin{bmatrix} -\alpha_{1z} r_{1y} - \omega_{1z}^2 r_{1x} \\ \alpha_{1z} r_{1x} - \omega_{1z}^2 r_{1y} \\ 0 \end{bmatrix}$$

$$\vec{a}_0 + \vec{a}_2 + \vec{a}_3 + \vec{a}_1 = 0$$

$$\vec{a}_2 = \begin{bmatrix} -\alpha_{2z} r_{2y} - \omega_{2z}^2 r_{2x} \\ \alpha_{2z} r_{2x} - \omega_{2z}^2 r_{2y} \\ 0 \end{bmatrix}$$

$$\vec{a}_3 = \begin{bmatrix} -\alpha_{3z} r_{3y} - \omega_{3z}^2 r_{3x} \\ \alpha_{3z} r_{3x} - \omega_{3z}^2 r_{3y} \\ 0 \end{bmatrix}$$

solving for α_{2z} and α_{3z} using the above equations we get

$$\cancel{\alpha_{2z} = r_{3x}(-\alpha_{1z} r_{1y} - \omega^2_{1z} r_{1x} - \omega^2_{2z} r_{2x} - \omega^2_{3z} r_{3x}) - r_{3y}(}$$

$$\alpha_{2z} = \frac{r_{3x}(-\alpha_{1z} r_{1y} - \omega^2_{1z} r_{1x} - \omega^2_{2z} r_{2x} - \omega^2_{3z} r_{3x}) - r_{3y}(-\alpha_{1z} r_{1x} + \omega^2_{1z} r_{1y} + \omega^2_{2z} r_{2y} + \omega^2_{3z} r_{3y})}{r_{2y} r_{3x} - r_{3y} r_{2x}}$$

$$\alpha_{3z} = \frac{-r_{2x}(-\alpha_{1z} r_{1y} - \omega^2_{1z} r_{1x} - \omega^2_{2z} r_{2x} - \omega^2_{3z} r_{3x}) + r_{2y}(-\alpha_{1z} r_{1x} + \omega^2_{1z} r_{1y} + \omega^2_{2z} r_{2y} + \omega^2_{3z} r_{3y})}{r_{2y} r_{3x} - r_{3y} r_{2x}}$$

Using the above equations the kinematics can be can calculated, which will be used for calculation of dynamics

E

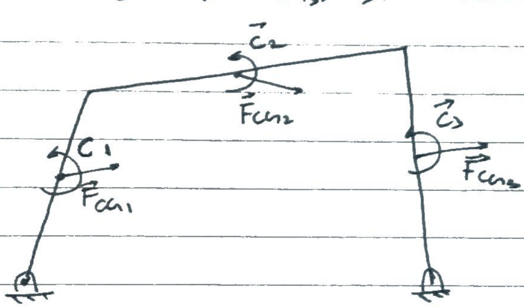
$$\vec{F}_{cg_1} = m_1 \vec{a}_{cg_1}$$
$$\vec{F}_{cg_2} = m_2 \vec{a}_{cg_2}$$
$$\vec{F}_{cg_3} = m_3 \vec{a}_{cg_3}$$

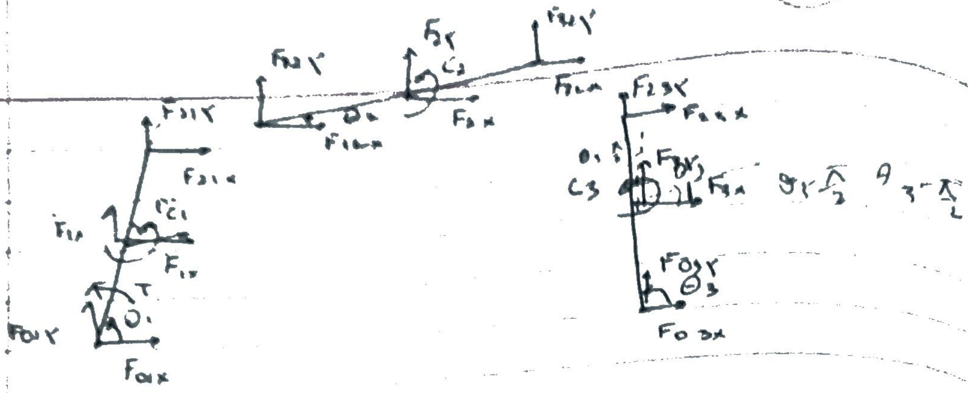
$$\vec{a}_{cg_1} = \vec{\alpha}_1 \times r_{cg_1/O_1} + \vec{\omega}_1 \times \vec{\omega}_1 \times r_{cg_1/O_1}$$
$$\vec{a}_{cg_2} = \vec{\alpha}_1 \times \vec{r}_1 + \vec{\omega}_1 \times \vec{\omega}_1 \times \vec{r}_1 + \vec{\alpha}_2 \times \vec{r}_{cg_2/B} + \vec{\omega}_2 \times \vec{\omega}_2 \times \vec{r}_{cg_2/B}$$
$$\vec{a}_{cg_3} = \vec{\alpha}_3 \times \vec{r}_{cg_3/O_3} + \vec{\omega}_3 \times \vec{\omega}_3 \times \vec{r}_{cg_3/O_3}$$

$$\vec{C}_1 \ \cancel{\vec{M}}_1 = I_{cg_1} \times \alpha_1$$
$$\vec{C}_2 \ \cancel{\vec{M}}_2 = I_{cg_2} \times \alpha_2$$
$$\vec{C}_3 \ \cancel{\vec{M}}_3 = I_{cg_3} \times \alpha_3$$

α_1

T





$F_{01x} + F_{1x} + F_{21x} = 0$

$F_{01y} + F_{1x} + F_{21y} = 0$

$F_{12x} + F_{2x} + F_{32x} = 0$

$F_{12y} + F_{2y} + F_{32y} = 0$

$F_{03x} + F_{3x} + F_{23x} = 0$

$F_{03y} + F_{3y} + F_{23y} = 0$

$\vec{F}_{21} = -\vec{F}_{12}$

$\vec{F}_{23} = -\vec{F}_{32}$

$T + C_1 + \cancel{F_1 \times r_1}\ F_{1x} r_1 \cos\theta_1$

$T + C_1 - F_{1x} r_1 \cos\theta_1 + F_{1y} r_1 \sin\theta_1 + F_{21y} \cos\theta_1 l_1 - F_{21x} \sin\theta_1 l_1 = 0$

$C_2 + F_{2y} r_2 \cos\theta_2 - F_{2x} r_2 \sin\theta_2 + F_{32y} \cos\theta_2 l_2 - F_{32x} \sin\theta_2 l_2 = 0$

$\cancel{C_3 + F_{3y} r_2}$

$C_3 \cancel{- F_{3x} \cos(\theta_3 - \frac{\pi}{2})} + F_{3y}$

$\cancel{C_3 - F_{3x} \cos}$

$C_3 - F_{3x} \sin\theta_3 r_3 \cancel{+ F_{3y} \sin\theta_3} + F_{3y} \cos\theta$

$C_3 - F_{3x} \sin\theta_3 r_3 + F_{3y} \cos\theta_3 r_3 - F_{23x} \sin\theta_3 l_3 + F_{23y} \sin\theta_3 l_3 = 0$