Gait Fitness Function Formulation

Design of Control System for Quadruped using Central Pattern Generators

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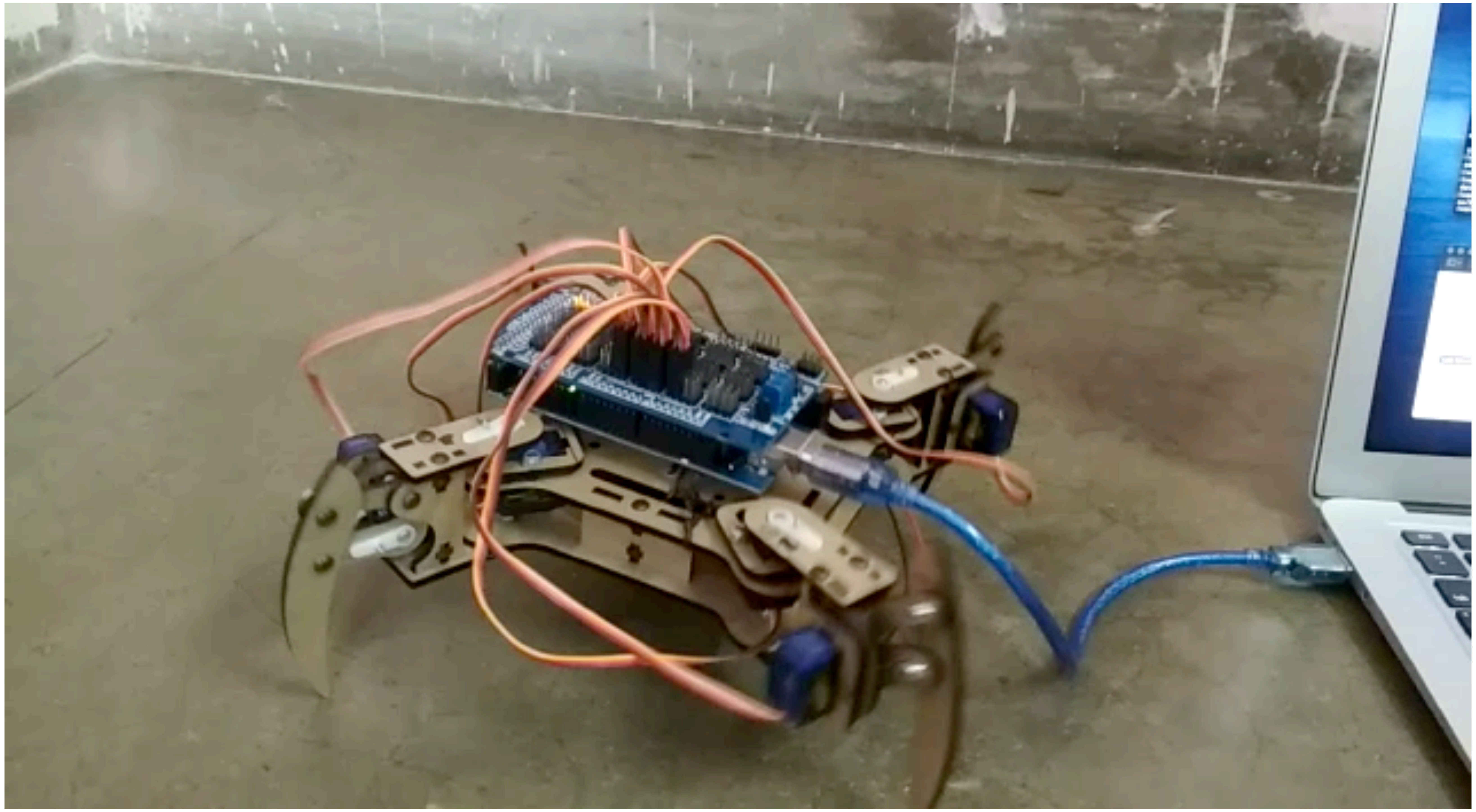
Robot walking Video

Observe the tendency of the robot to topple towards the swinging leg



Gait Pattern

Pattern in use and problems with it

- This Gait Pattern obtained from the formulation in the figure is for a creep walk of the quadruped

- The robot walks with this Gait Pattern

- But the stability of the quadruped is not maintained

- The quadruped tends to topple towards the leg in swing phase

Hip Activations:

$$\theta_h(t) = \begin{cases} \theta_h sin(\frac{(t-\frac{iT}{4})\pi}{\beta T} + \pi), & \text{if } 0 \le t \le \frac{\beta T}{2} \\ \theta_h sin(\frac{(t-\frac{iT}{4})\pi}{(1-\beta)T} + \frac{(3-4\beta)\pi}{2(1-\beta)}), & \text{if } \frac{\beta T}{2} \le t \le \frac{T(2-\beta)}{2} \\ \theta_h sin(\frac{(t-\frac{iT}{4})\pi}{\beta T} + \frac{(\beta-1)\pi}{\beta}), & \text{if } \frac{T(2-\beta)}{2} \le t \le T \end{cases}$$

For $i \in \{0,1\}$

And

$$\theta_h(t) = \begin{cases} -\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{\beta T} + \pi), & \text{if } 0 \le t \le \frac{\beta T}{2} \\ -\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{(1-\beta)T} + \frac{(3-4\beta)\pi}{2(1-\beta)}), & \text{if } \frac{\beta T}{2} \le t \le \frac{T(2-\beta)}{2} \\ -\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{\beta T} + \frac{(\beta-1)\pi}{\beta}), & \text{if } \frac{T(2-\beta)}{2} \le t \le T \end{cases}$$

For $i \in \{2,3\}$

Knee Activations:

$$\theta_k(t) = \begin{cases} \theta_k sin(\frac{t\pi}{T(1-\beta)} - \frac{\beta\pi}{2(1-\beta)}), & \text{if } \dot{\theta}_h(t) \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

For $i \in \{0,1\}$

And

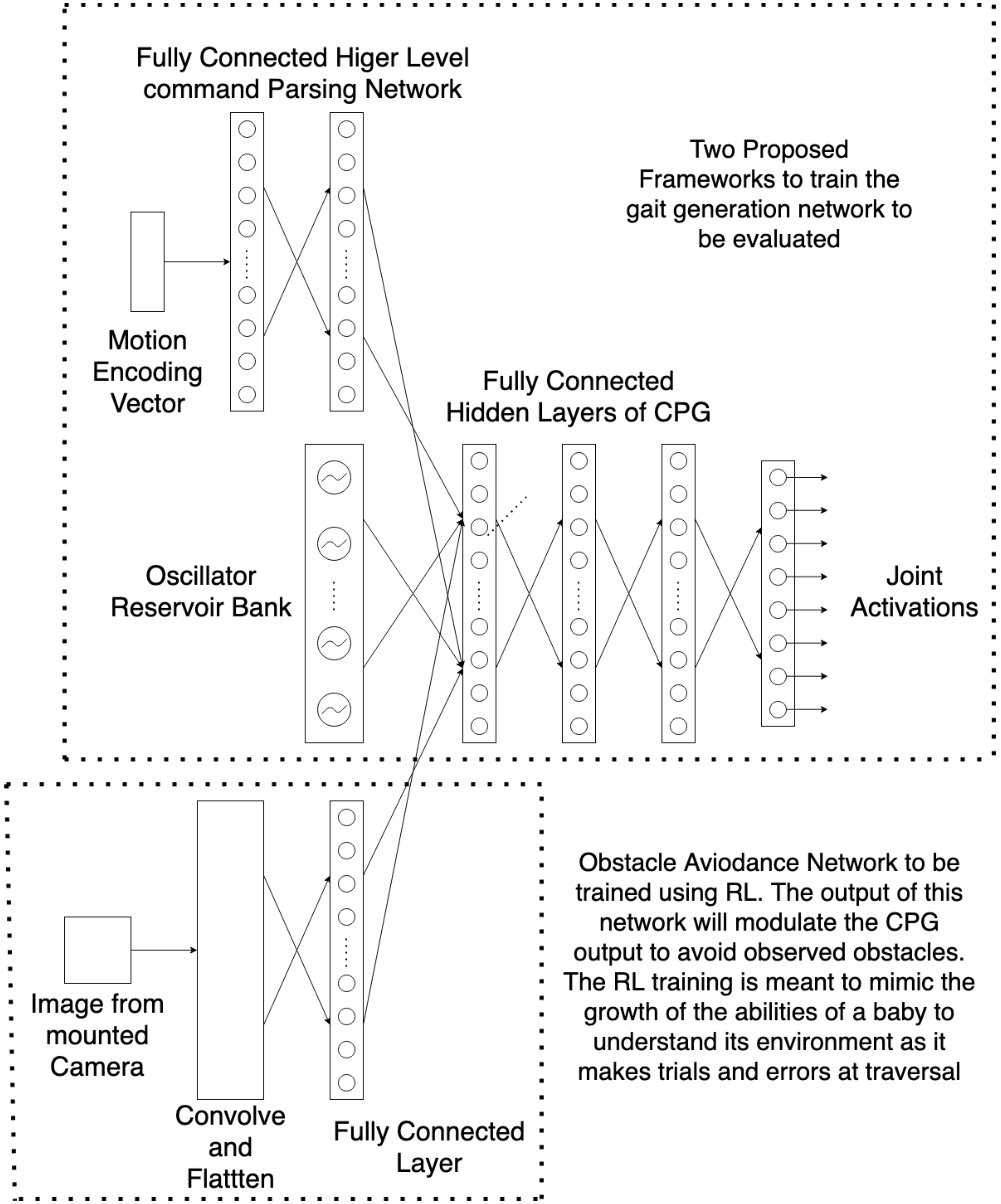
$$\theta_k(t) = \begin{cases} \theta_k sin(\frac{t\pi}{T(1-\beta)} - \frac{\beta\pi}{2(1-\beta)}), & \text{if } \dot{\theta}_h(t) \le 0 \\ 0, & \text{otherwise} \end{cases}$$

For $i \in \{2,3\}$

Architecture

Proposed solution to ensure gait stability

- Two methods Proposed to train the gait generation network
  - Train the gait generation network end to end using back props and then use RL training for improvement of gait
  - Optimise Gait Patterns using fitness function, then train the network end to end using back prop



Fitness Function

Evaluation Criteria to measure stability, speed and energy consumption

- Both the aforementioned training frameworks require a Fitness Function [1]
- Fitness Function must measure the following
  - Stability Criteria, S
  - Speed Criteria, V
  - Energy Efficiency Criteria, E
- $F = \sum_{i=t-T}^{t} S_i^{k_s} V_i^{k_v} E_i^{k_e}$ where
  - F is the Fitness Value
  - k_s, k_v and k_e are integer parameters to account for relative importance of each criteria
  - T is the gait period and t is the current time instance

Stability Criteria

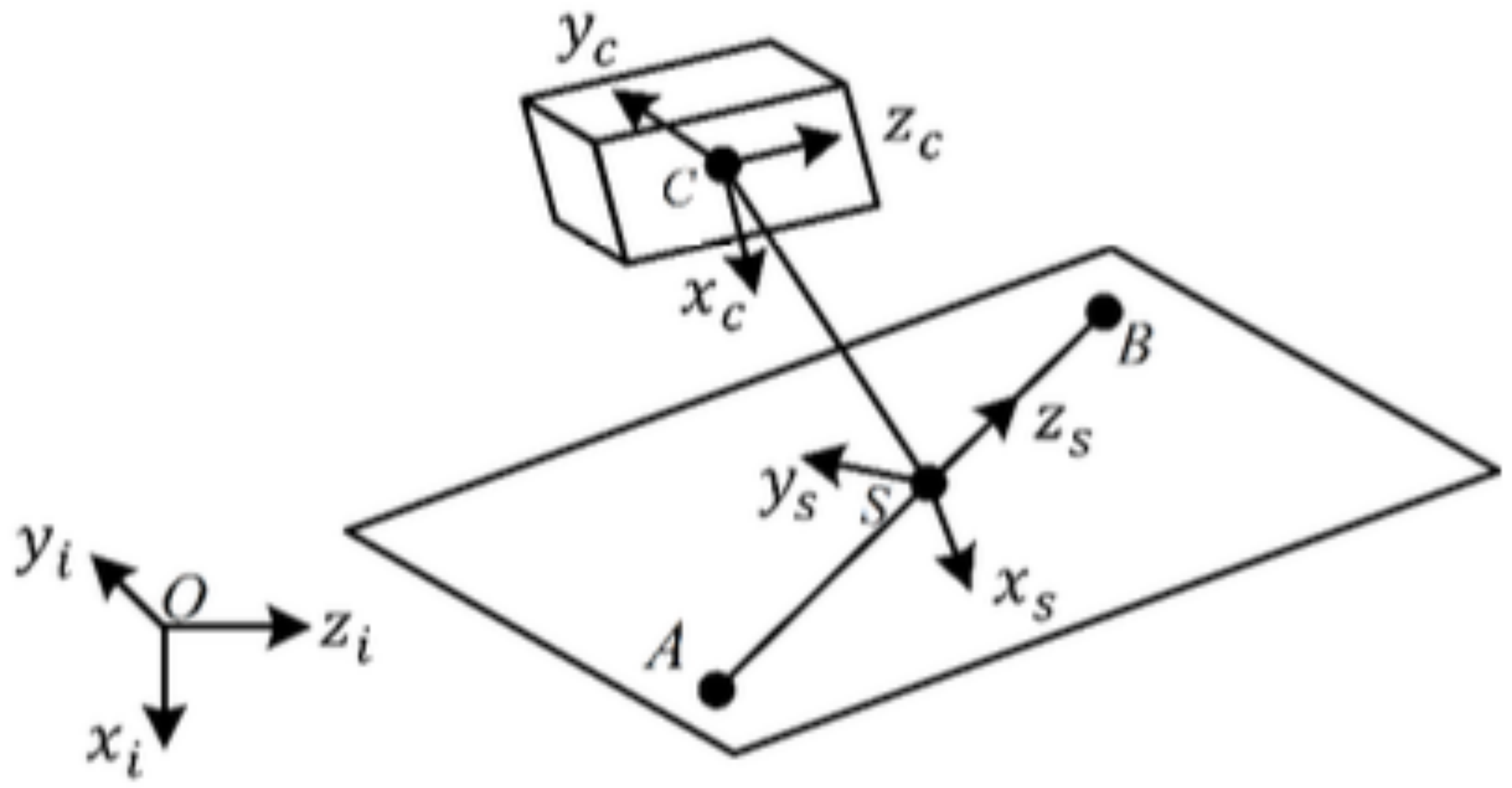
Evaluation of Dynamic and Static Stability of the Quadruped

- A Quantitative Measure of quadruped’s dynamic stability
- The Criteria must not only take into account current stability but must also **anticipate the stability at the next time** instance, given the state of motion of the quadruped, like an animal does
- Yan Jia, Xiao Luo and others [2] in their paper propose a modified ZMP based stability evaluation criteria
- The main idea is that at a certain state, the motion of the robot is considered to be stable if the **torque caused by the ground-reaction force can prevent the robot from tumbling** around any support boundary

Stability Criteria

Coordinate Systems used

- There are two coordinate systems of interest-
  - The Inertial Coordinate system
  - The Support Coordinate system
- The Support Coordinate has z-axis along the support line AB and x axis perpendicular to the current support plane



Stability Criteria

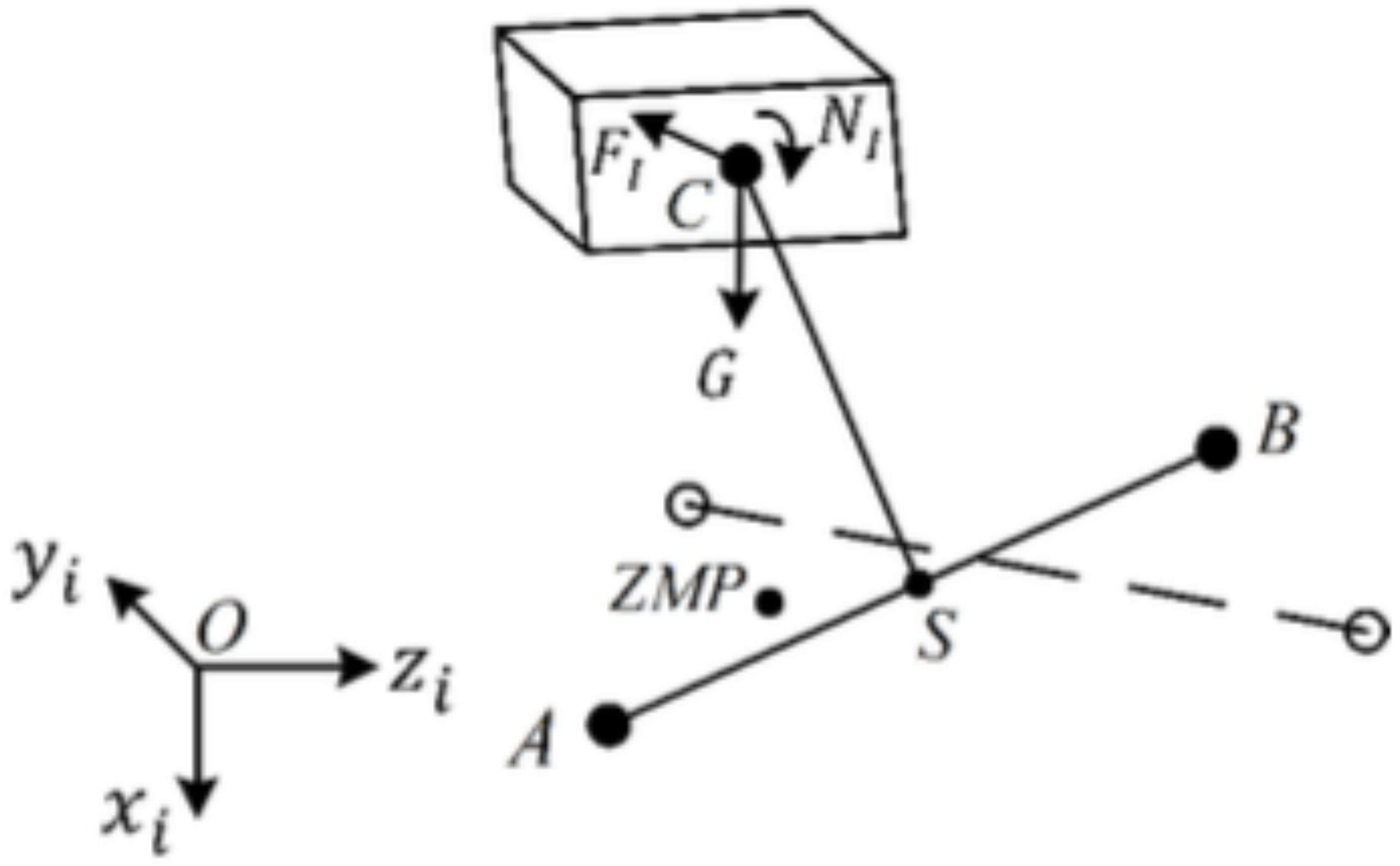
Equations for Zero Moment Point

$$-\left(z_C^i - z_{ZMP}^i\right)F_{Iy}^i + \left(y_C^i - y_{ZMP}^i\right)F_{Iz}^i + N_{Ix}^i = 0;$$

$$\left(z_C^i - z_{ZMP}^i\right)\left(F_{Ix}^i + G_x^i\right) - \left(x_C^i - x_{ZMP}^i\right)F_{Iz}^i + N_{Iy}^i = 0;$$

$$-\left(y_C^i - y_{ZMP}^i\right)\left(F_{Ix}^i + G_x^i\right) + \left(x_C^i - x_{ZMP}^i\right)F_{Iy}^i + N_{Iz}^i = 0.$$

The characteristic equations of ZMP. The inertial force and inertial moment here need to be computed by a dynamic analysis of the quadruped performed later in the report



The force schematic used to calculate the position of ZMP.

Here G is gravity, F is the inertial force on COM C, N is the inertial moment

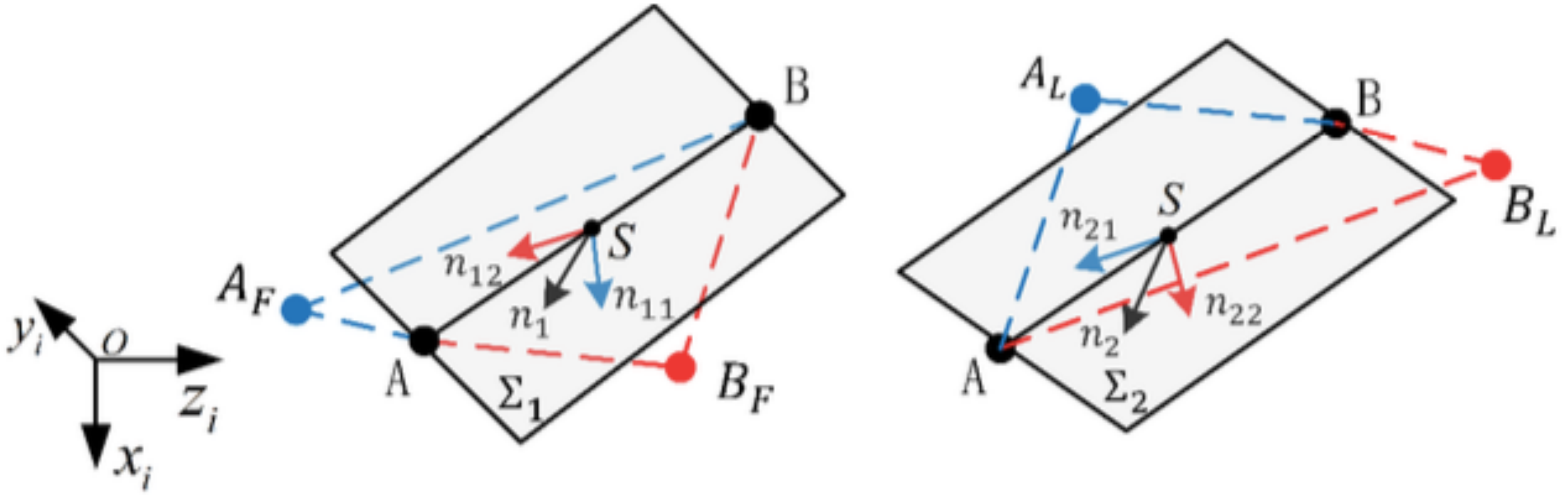
Stability Criteria

Virtual Support Plane and Modified Zero Moment Point

- The paper assumes that a robot is stable if the current or next set of support feet can provide the moment necessary to prevent the robot from tipping in any direction
- A Virtual Support Plane is proposed and it is proposed that if the modified ZMP proposed in the paper lies within this Virtual Support Plane, then the robot motion will be stable
- The Proposed Modified ZMP in the paper also includes a velocity term to account for the motion of the quadruped

Stability Criteria

Calculating the Virtual Support Plane



- S is the origin of the support coordinate system on which the ZMP will lie
- n_1 is the normal vector to the plane AA_FBB_F
- n_2 is the normal vector to the place AA_LBB_L
- x_s^i, y_s^i and z_s^i are the normal vectors to the virtual support plane in the inertial frame of reference
- T_b is the duration between two adjacent steps and *t* is the time gap between the current running time and the time point when the previous support line disappeared

$$x_s^i = \frac{\mu n_1 + (1-\mu) n_2}{\|\mu n_1 + (1-\mu) n_2\|}$$

$$\mu = -\frac{1}{T_b} t + 1 \qquad z_s^i = \frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} \qquad y_s^i = z_s^i \times x_s^i$$

$$n_{11} = \frac{\overrightarrow{AB} \times \overrightarrow{AA_F}}{\|\overrightarrow{AB} \times \overrightarrow{AA_F}\|} \qquad n_{21} = \frac{\overrightarrow{AB} \times \overrightarrow{AA_L}}{\|\overrightarrow{AB} \times \overrightarrow{AA_L}\|}$$

$$n_{12} = \frac{\overrightarrow{AB} \times \overrightarrow{BB_F}}{\|\overrightarrow{AB} \times \overrightarrow{BB_F}\|} \qquad n_{22} = \frac{\overrightarrow{AB} \times \overrightarrow{BB_L}}{\|\overrightarrow{AB} \times \overrightarrow{BB_L}\|}$$

Stability Criteria

Calculation of Modified Zero Moment Point

- Zero Moment Point is the point at which the resultant moment on a body is zero
- Modified Zero Moment Point should-
  - Assess the current stability more efficiently and accurately
  - Provide a reference to eliminate undesired velocity during motion planning
- ZMP_o can be used to compute 3 measures of dynamic stability

$$x^s_{ZMP_0} = 0;$$

$$y^s_{ZMP_0} = y^s_{ZMP} + \eta\left(v^s_{yr} - v^s_{yd}\right);$$

$$z^s_{ZMP_0} = z^s_{ZMP} + \eta(v^s_{zr} - v^s_{zd}).$$

Equations to calculate modified ZMP in support coordinate system. v^s and v^d are the actual and expected velocities in support coordinate system

$$\eta = \frac{\frac{1}{2}(L+W)}{\|v_d\|}0.1 = 0.05\frac{(L+W)}{\|v_d\|}$$

L and W are the effective length and width respectively of the quadruped

$${}^i_sR = \left[\begin{array}{ccc} x^i_s & y^i_s & z^i_s \end{array}\right]$$

Rotation Matrix for transformation from support to inertial coordinate system

Stability Criteria

Calculation of the metric S

The following three choices for S can be calculated using ZMP_o-

- Distances between ZMP_o and the boundaries of the virtual-support quadrilateral in the support plane
- Angle between the vector pointing from CoM to ZMP_o and the normal vector of the virtual support place
- Distance between ZMP_o and the support line
- The assumption in the paper that the legs have negligible weight compared to the rest of the body does not hold for the quadruped used
- There is a need for evaluation of the repercussions on the concepts introduced

Speed and Energy Criteria

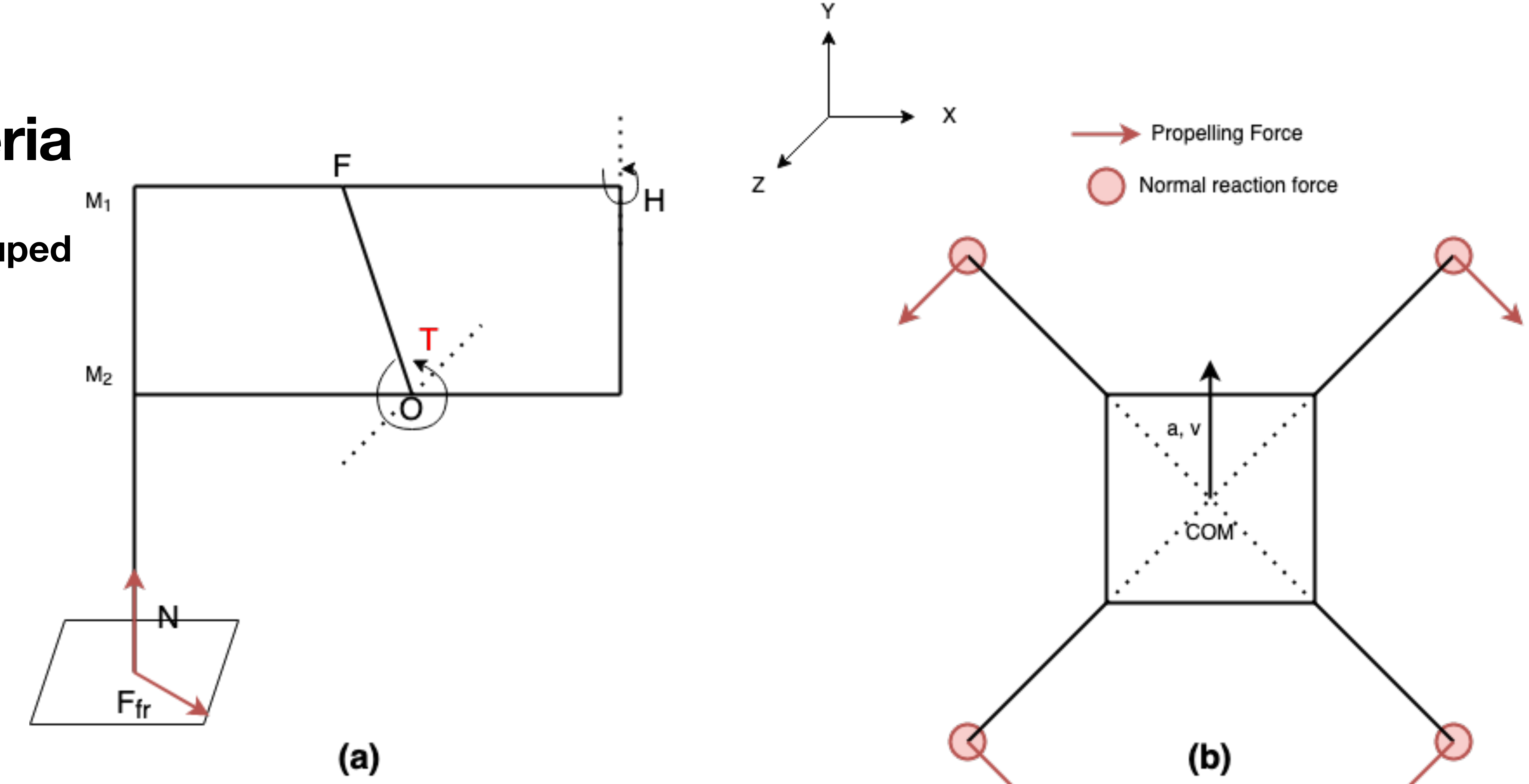
Calculation of metrics V and E

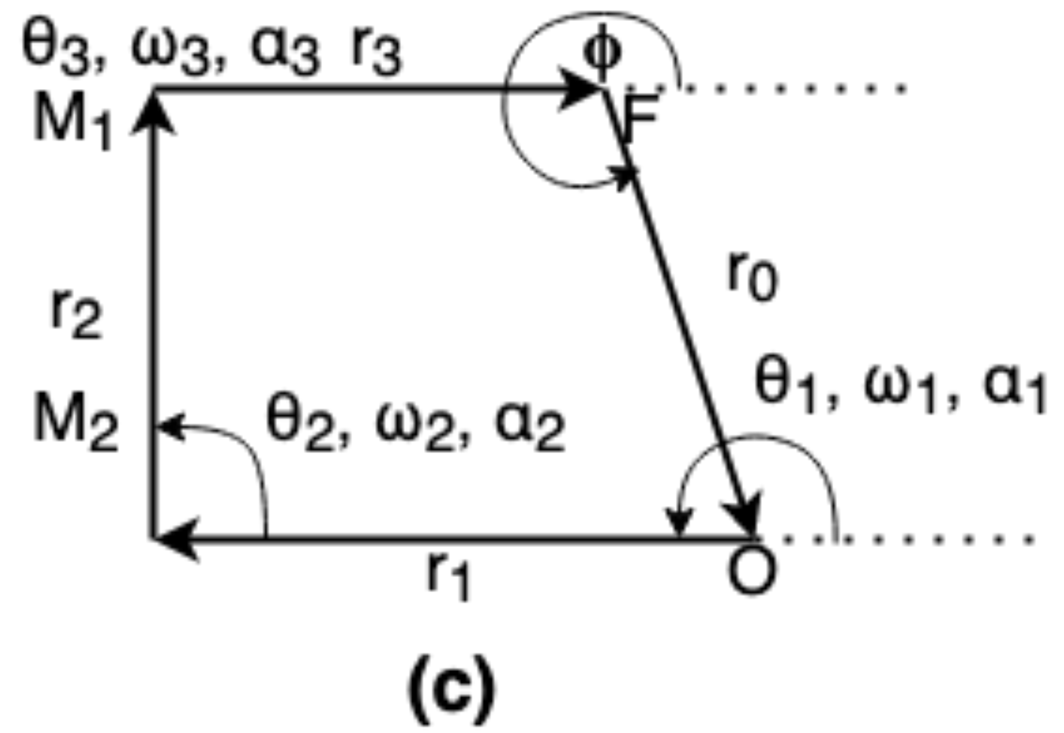
- Speed Criteria measures the discrepancy between expected and actual speed
  - $V = vT_b/d$
  - Where v is the actual speed and d is the distance travelled over period T_b
- Energy Criteria measures the energy efficiency of the quadruped
  - $E = mv^2/2T_bW$
  - where m is the mass of the quadruped, W is the power supplied to the quadruped, v is the actual velocity and T_b is the fixed period used in V

Calculating Stability Criteria

Computing ZMP for the available quadruped

- The following figure depicts the simplified model of the quadruped.
- F_{fr} is the force due to friction acting at the point of contact. This is also the propelling force on the quadruped
- N is the normal reaction force from the ground
- θ, ω and α are the state of the joints in the four bar linkage in (c)





(a) Simplified model of the leg of the quadruped

(b) Simplified model of the quadruped as seen form the top

(c) Simplified four bar linkage to model the knee of the quadruped

Calculating Stability Criteria

Relationship between knee joint activation and four bar linkage model angular positions

Of the four angular positions that in the four bar linkage model of the knee joint, the angular position at O, θ_1 and F, ϕ are known (being the controlled angle). Thus, the other angular positions must also be formulated in terms of θ_1 and ϕ. l, β_1, β_2 and δ are intermediate variables.

$$l = \sqrt{\overrightarrow{|r_0|}^2 + \overrightarrow{|r_1|}^2 - 2|r_0||r_1|cos(\theta_1 - \phi + \pi)}$$

$$\beta_1 = \arcsin(\frac{|\overrightarrow{r_1}|}{l} sin(\theta_1 - \phi + \pi))$$

$$\beta_2 = \arccos \frac{|\overrightarrow{r_2}|^2 + l^2 - |\overrightarrow{r_3}|^2}{2|\overrightarrow{r_3}|l}$$

$$\delta = \arcsin(\frac{l}{|\overrightarrow{r_3}|} sin\beta_2)$$

$$\theta_2 = \beta_2 - \beta_1 + \phi - \pi$$

$$\theta_3 = -\beta_1 - \delta + \phi - \pi$$

Calculating Stability Criteria

Linkage vector in knee joint four bar linkage model

$$\overrightarrow{r_1} = \begin{bmatrix} L_1 cos\theta_1 \\ L_1 sin\theta_1 \\ 0 \end{bmatrix} \qquad \overrightarrow{r_2} = \begin{bmatrix} L_2 cos\theta_2 \\ L_2 sin\theta_2 \\ 0 \end{bmatrix} \qquad \overrightarrow{r_3} = \begin{bmatrix} L_3 cos\theta_3 \\ L_3 sin\theta_3 \\ 0 \end{bmatrix}$$

$$\overrightarrow{r_0} = \begin{bmatrix} L_0 cos\phi \\ L_0 sin\phi \\ 0 \end{bmatrix}$$

Here $\overrightarrow{r_0}$ is fixed and ϕ, the angle between $\overrightarrow{r_0}$ and x-axis is also fixed

L_0, L_1, L_2 and L_3 are the length of the three linkages given by $\overrightarrow{r_0}$, $\overrightarrow{r_1}$, $\overrightarrow{r_2}$ and $\overrightarrow{r_3}$ respectively.

L_0, L_1, L_2 and L_3 are the length of the three linkages given by $\overrightarrow{r_0}$, $\overrightarrow{r_1}$, $\overrightarrow{r_2}$ and $\overrightarrow{r_3}$ respectively.

Calculating Stability Criteria

Kinematics model of the Knee Joint

The following equations characterise the kinematic model of the knee joint.

$$\overrightarrow{r_1} + \overrightarrow{r_2} + \overrightarrow{r_3} + \overrightarrow{r_0} = \overrightarrow{0}$$

$$\overrightarrow{v_0} + \overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3} = \overrightarrow{0}$$

$$\overrightarrow{a_0} + \overrightarrow{a_1} + \overrightarrow{a_2} + \overrightarrow{a_3} = \overrightarrow{0}$$

These aforementioned relationships arise as the four bar linkage is stationary with respect to F and O.

$$r_{1x} = L_1 cos\theta_1 \qquad r_{1x} = L_1 cos\theta_1 \qquad r_{1x} = L_1 cos\theta_1$$

$$r_{1y} = L_1 sin\theta_1 \qquad r_{1y} = L_1 sin\theta_1 \qquad r_{1y} = L_1 sin\theta_1$$

$$r_{1z} = 0 \qquad r_{1z} = 0 \qquad r_{1z} = 0$$

$$r_{0x} = L_0 cos\phi$$

$$r_{0y} = L_0 sin\phi$$

$$r_{0z} = 0$$

These relationships are shorthand for the components of the vectors $\overrightarrow{r_0}$, $\overrightarrow{r_1}$, $\overrightarrow{r_2}$ and $\overrightarrow{r_3}$

Calculating Stability Criteria

Calculating ω and α

$$\overrightarrow{\omega_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \overrightarrow{\omega_1} = \begin{bmatrix} 0 \\ 0 \\ \omega_{1z} \end{bmatrix} \qquad \overrightarrow{\omega_2} = \begin{bmatrix} 0 \\ 0 \\ \omega_{2z} \end{bmatrix} \qquad \overrightarrow{\omega_3} = \begin{bmatrix} 0 \\ 0 \\ \omega_{3z} \end{bmatrix}$$

$\overrightarrow{v_O} = 0$ The linkage $\overrightarrow{r_0}$ is fixed

$\overrightarrow{v_1} = \overrightarrow{\omega_1} \times \overrightarrow{r_1}$ Only rotation with ω_1 about origin of linkage

$\overrightarrow{v_2} = \overrightarrow{\omega_2} \times \overrightarrow{r_2}$ Only rotation with ω_2 about origin of linkage

$\overrightarrow{v_3} = \overrightarrow{\omega_3} \times \overrightarrow{r_3}$ Only rotation with ω_3 about origin of linkage

Calculating Stability Criteria

Calculating ω and α

$$\overrightarrow{v_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \overrightarrow{v_1} = \begin{bmatrix} -\omega_{1z} r_{1y} \\ \omega_{1z} r_{1x} \\ 0 \end{bmatrix} \qquad \overrightarrow{v_1} = \begin{bmatrix} -\omega_{1z} r_{1y} \\ \omega_{1z} r_{1x} \\ 0 \end{bmatrix} \qquad \overrightarrow{v_1} = \begin{bmatrix} -\omega_{1z} r_{1y} \\ \omega_{1z} r_{1x} \\ 0 \end{bmatrix}$$

Since $\overrightarrow{v_0} + \overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3} = \overrightarrow{0}$, we get the following equations.

$$0 - \omega_{1z} r_{1y} - \omega_{2z} r_{2y} - \omega_{3z} r_{3y} = 0$$

$$0 + \omega_{1z} r_{1x} + \omega_{2z} r_{2x} + \omega_{3z} r_{3x} = 0$$

Solving the above two we get ω_{2z} and ω_{3z}

$$\omega_{2z} = -\omega_{1z} \frac{r_{1y} r_{3x} - r_{3y} r_{1x}}{r_{2y} r_{3x} - r_{3y} r_{2x}}$$

$$\omega_{3z} = -\omega_{1z} \frac{r_{2y} r_{1x} - r_{1y} r_{2x}}{r_{2y} r_{3x} - r_{3y} r_{2x}}$$

Calculating Stability Criteria

Calculating ω and α

$$\overrightarrow{\alpha_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \overrightarrow{\alpha_1} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{1z} \end{bmatrix} \quad \overrightarrow{\alpha_2} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{2z} \end{bmatrix} \quad \overrightarrow{\alpha_3} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{3z} \end{bmatrix}$$

$\overrightarrow{v_O} = 0$ — The linkage $\overrightarrow{r_0}$ is fixed

$\overrightarrow{a_1} = \overrightarrow{\alpha_1} \times \overrightarrow{r_1} + \overrightarrow{\omega_1} \times \overrightarrow{\omega_1} \times \overrightarrow{r_1}$ — Only rotation with α_1 about origin of linkage

$\overrightarrow{a_2} = \overrightarrow{\alpha_2} \times \overrightarrow{r_2} + \overrightarrow{\omega_2} \times \overrightarrow{\omega_2} \times \overrightarrow{r_2}$ — Only rotation with α_2 about origin of linkage

$\overrightarrow{v_3} = \overrightarrow{\omega_3} \times \overrightarrow{r_3}$ — Only rotation with α_3 about origin of linkage

Calculating Stability Criteria

Calculating ω and α

Since $\overrightarrow{a_0} + \overrightarrow{a_1} + \overrightarrow{a_2} + \overrightarrow{a_3} = \overrightarrow{0}$, we get the following equations.

$$0 - \alpha_{1z}r_{1y} - \omega_{1z}^2 r_{1x} - \alpha_{2z}r_{2y} - \omega_{2z}^2 r_{2x} - \alpha_{3z}r_{3y} - \omega_{3z}^2 r_{3x} = 0$$

$$0 + \alpha_{1z}r_{1x} - \omega_{1z}^2 r_{1y} + \alpha_{2z}r_{2x} - \omega_{2z}^2 r_{2y} + \alpha_{3z}r_{3x} - \omega_{3z}^2 r_{3y} = 0$$

Solving the above two we get α_{2z} and α_{3z}

$$\alpha_{2z} = \frac{r_{3x}(-\alpha_{1z}r_{1y} - \omega_{1z}^2 r_{1x} - \omega_{2z}^2 r_{2x} - \omega_{3z}^2 r_{3x}) - r_{3y}(-\alpha_{1z}r_{1x} - \omega_{1z}^2 r_{1y} - \omega_{2z}^2 r_{2y} - \omega_{3z}^2 r_{3y})}{r_{2y}r_{3x} - r_{3y}r_{2x}}$$

$$\alpha_{3z} = \frac{-r_{2x}(-\alpha_{1z}r_{1y} - \omega_{1z}^2 r_{1x} - \omega_{2z}^2 r_{2x} - \omega_{3z}^2 r_{3x}) + r_{2y}(-\alpha_{1z}r_{1x} - \omega_{1z}^2 r_{1y} - \omega_{2z}^2 r_{2y} - \omega_{3z}^2 r_{3y})}{r_{2y}r_{3x} - r_{3y}r_{2x}}$$

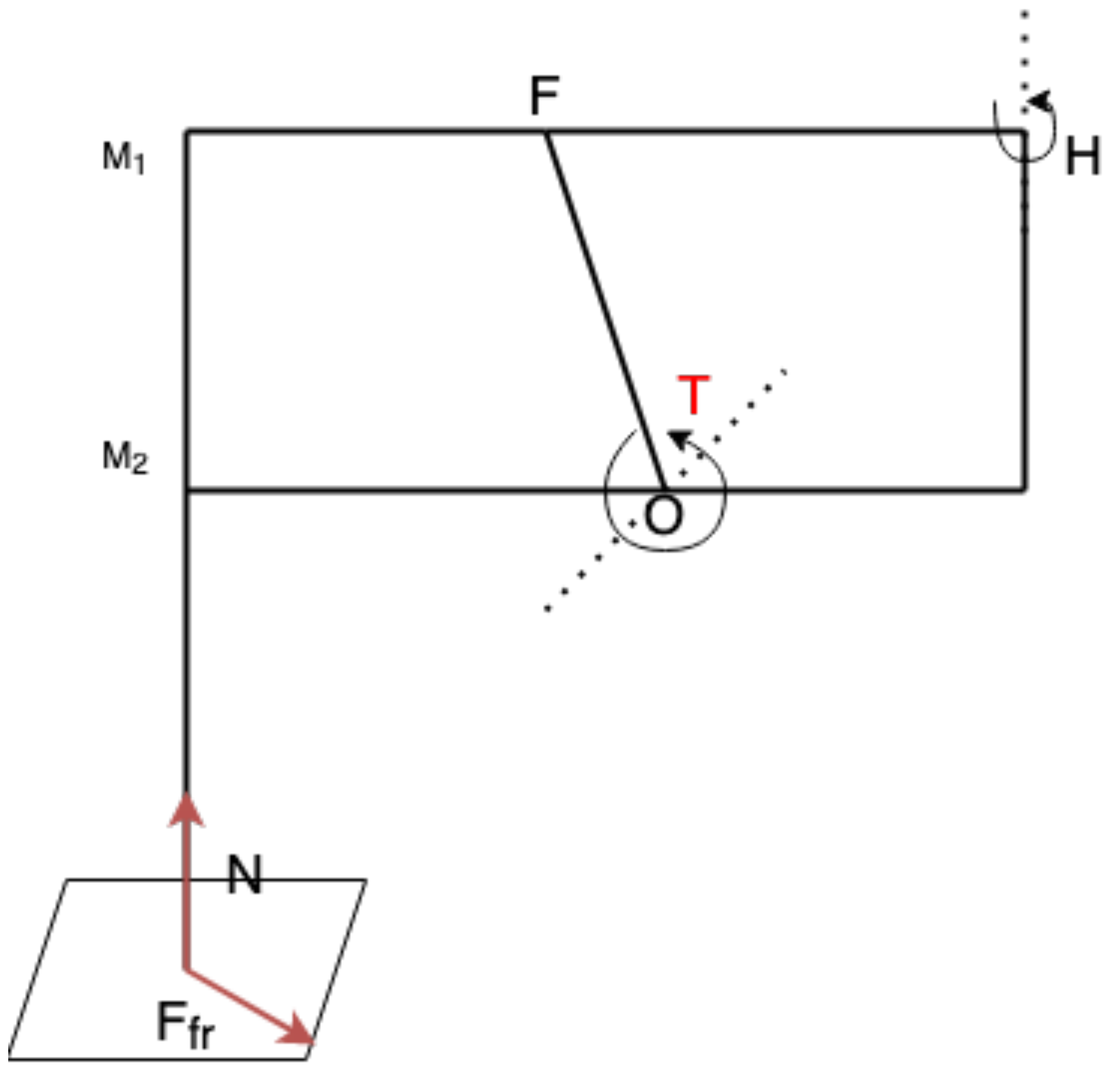
Calculating Stability Criteria

Dynamic Model of the Available Quadruped

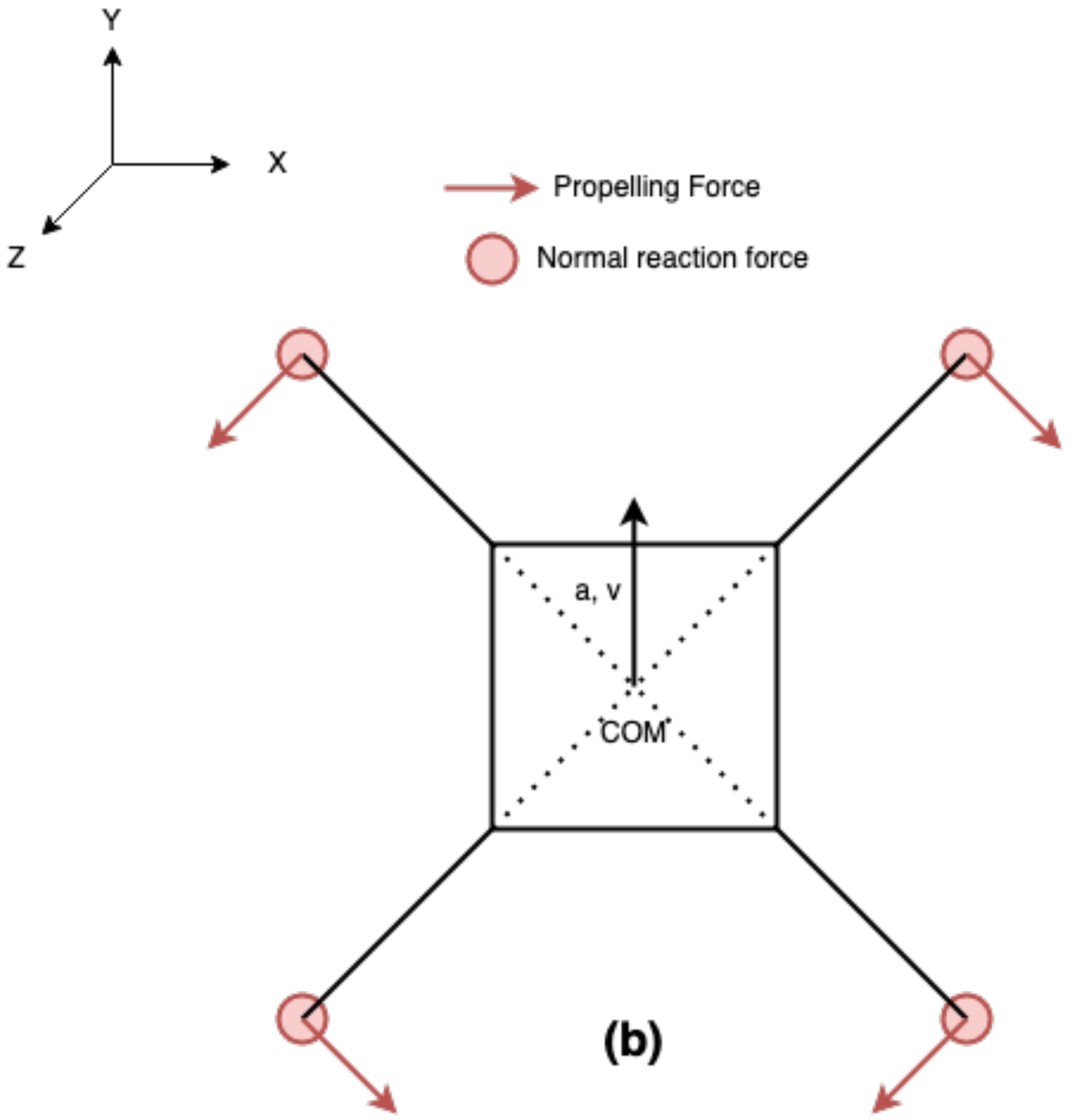
$$\sum \overrightarrow{F_{fr}} + \sum \overrightarrow{N} - m\overrightarrow{g} = m\overrightarrow{a}$$

$$\sum_i \overrightarrow{F_{fr}} \times R_i + \sum_i \overrightarrow{N} \times R_i = I_R \overrightarrow{\alpha_R}$$

The above two equations depict the dynamics of the quadruped in vector form. Rotation is possible about the y and z axes, while translation is only possible along the y and z axes.



(a)



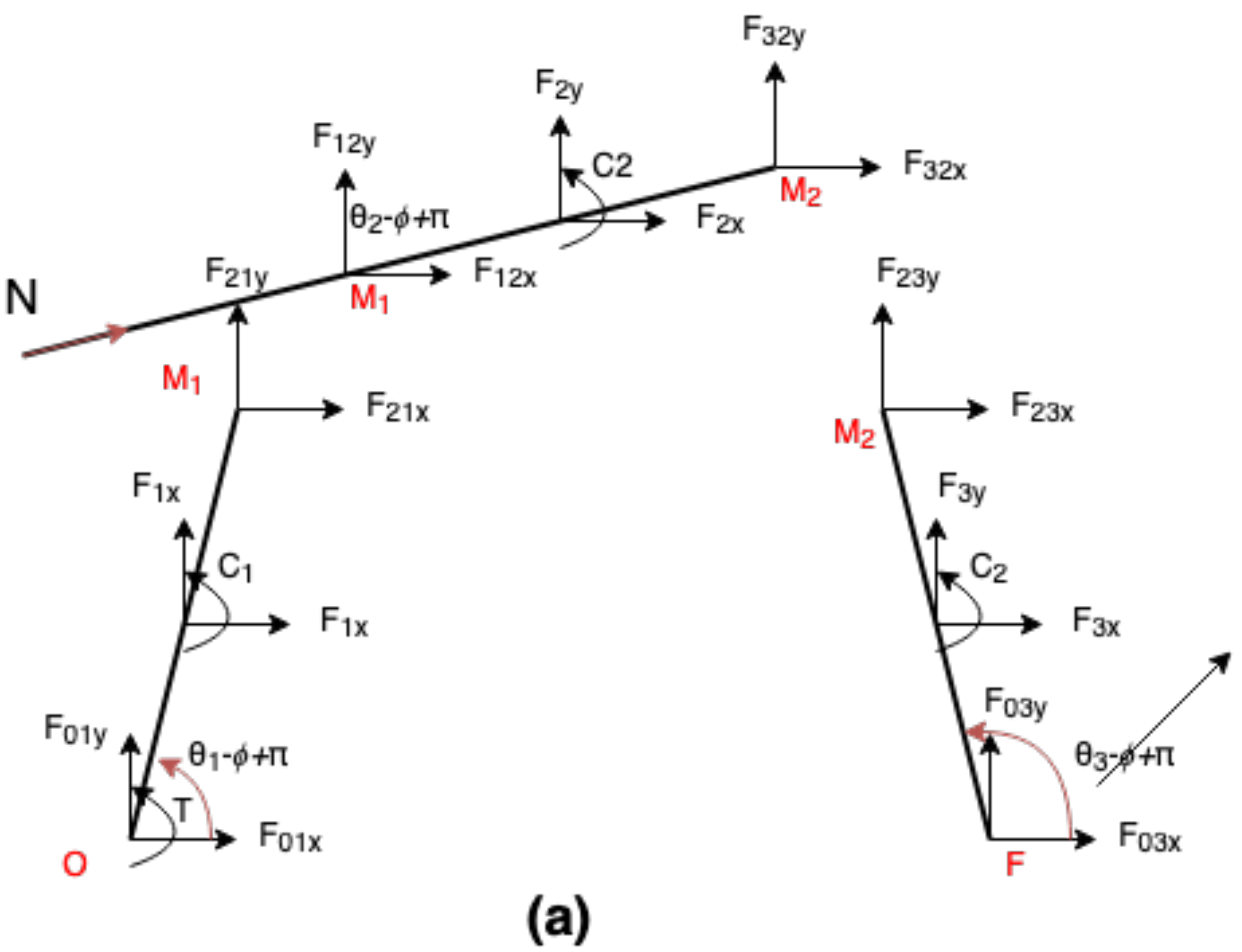
(b)

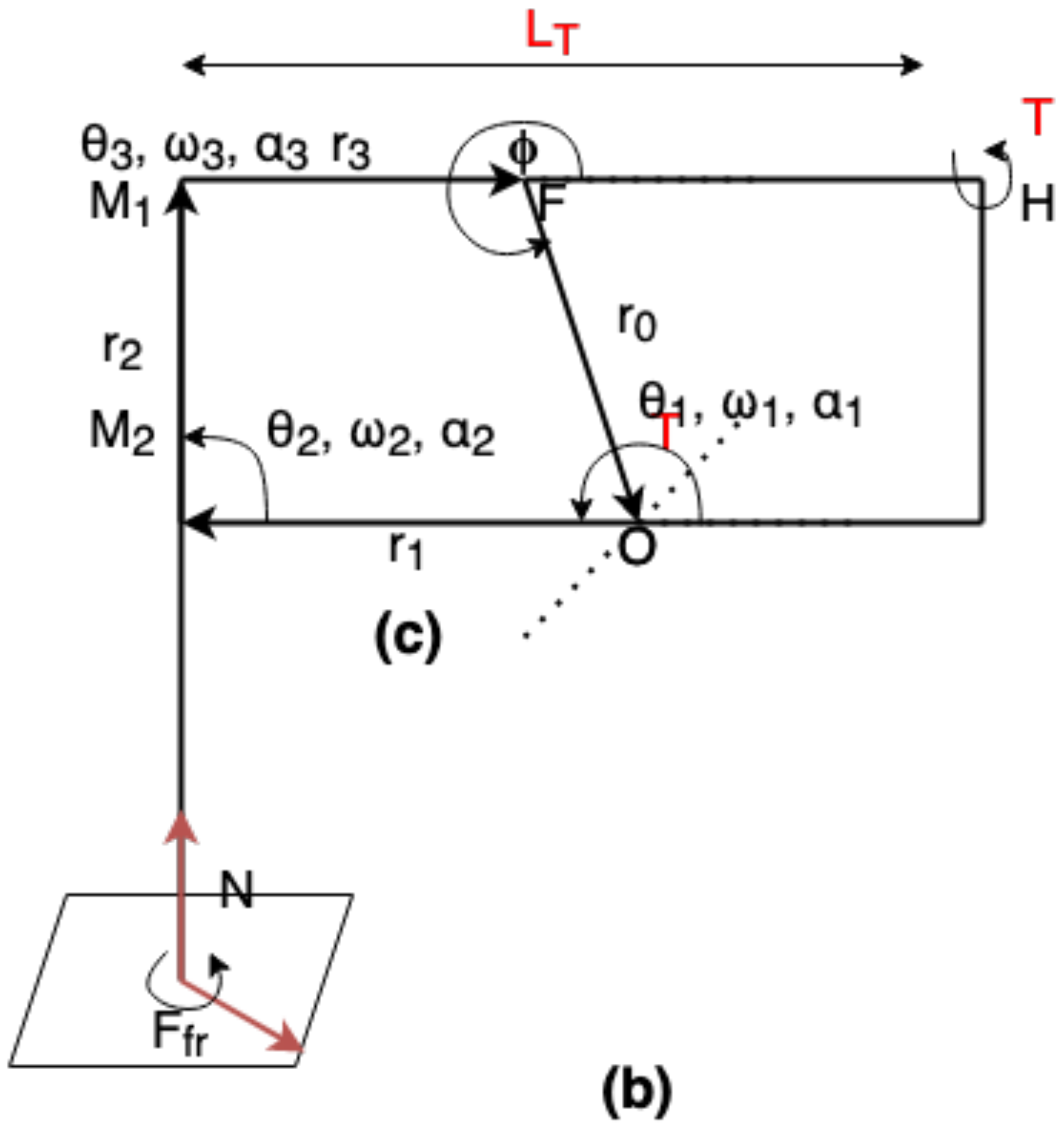
Calculating Stability Criteria

Stance Dynamics

a. Illustration of all forces acting on the four bar linkage in the knee during stance phase. The segment M1H will be rigid during stance

b. A simplified model of a leg. N and F_{fr} are normal reaction and friction forces respectively





Calculating Stability Criteria

Stance Dynamics

$$F_{01x} + F_{1x} + F_{21x} = 0$$

$$F_{01y} + F_{1y} + F_{21y} = 0$$

$$F_{12x} + F_{2x} + F_{32x} + N\cos\theta_2 - \phi + \pi = 0$$

$$F_{01y} + F_{1y} + F_{21y} + N\sin\theta_2 - \phi + \pi = 0$$

$$F_{01x} + F_{1x} + F_{21x} = 0$$

$$F_{01y} + F_{1y} + F_{21y} = 0$$

$$C_1 = 0$$

$$C_2 = 0$$

$$C_3 = 0$$

$$\overrightarrow{F_{23}} = -\overrightarrow{F_{32}}$$

$$\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$$

$$T + C_1 - F_{1x}\frac{|\overrightarrow{r_1}|}{2}\cos\theta_1 - \phi + \pi + F_{1y}\frac{|\overrightarrow{r_1}|}{2}\cos\theta_1 - \phi + \pi + F_{21y}|\overrightarrow{r_1}|\cos\theta_1 - \phi + \pi - F_{21x}|\overrightarrow{r_1}|\sin\theta_1 - \phi + \pi = 0$$

$$C_2 - F_{2x}\frac{|\overrightarrow{r_2}|}{2}\cos\theta_2 - \phi + \pi + F_{2y}\frac{|\overrightarrow{r_2}|}{2}\cos\theta_2 - \phi + \pi + F_{32y}|\overrightarrow{r_2}|\cos\theta_2 - \phi + \pi - F_{32x}|\overrightarrow{r_2}|\sin\theta_2 - \phi + \pi = 0$$

$$C_3 - F_{3x}\frac{|\overrightarrow{r_3}|}{2}\cos\theta_3 - \phi + \pi + F_{3y}\frac{|\overrightarrow{r_3}|}{2}\cos\theta_3 - \phi + \pi - F_{23y}|\overrightarrow{r_1}|\cos\theta_1 - \phi + \pi + F_{23x}|\overrightarrow{r_1}|\sin\theta_1 - \phi + \pi = 0$$

Calculating Stability Criteria

Calculating $\overrightarrow{F_{fr}}$ and $\overrightarrow{N}$

- $\overrightarrow{F_{fr}}$ and $\overrightarrow{N}$ are the normal reaction and acting at the points of contact with the ground
- To compute these forces the dynamics at the point of contact need to be considered
- For a successful walking motion, there must be no slipping at the point of contact
- Thus, we need to solve for equilibrium conditions there

$$\sum_i^k N_i - mg = 0$$

where k is the number of legs in contact with the group

$$T - F_{fr}L_T = 0$$

This is the condition of no slipping at the point of contact with the ground

$$\sum \overrightarrow{F_{fr}} = m\overrightarrow{a}$$

The sum of all frictional forces are responsible for the horizontal acceleration a of the quadruped

Calculating Stability Criteria

Calculating Inertial Force and Moment at COM

- The Inertial Force and Moment at the COM are given by the following $m\overrightarrow{a}$ and $I_R\overrightarrow{\alpha_R}$
- Both of these parameters can be calculated by solving of all the equations relating to the dynamics and kinematics of the quadruped
- Once the Inertial Force and Moment at the COM are known, the modified ZMP can computed using the method proposed in [2], which can then be used to compute the stability criteria

References

1. Masuri, Ariel; Medina, Oded; Hacohen, Shlomi; Shvalb, Nir (2020) ‘Gait and Trajectory Optimization by Self-Learning for Quadrupedal Robots with an Active Back Joint’
2. Jia, Yan; Lua, Xiao; Han, Baoling; Liang, Guanhao; Zhao, Jiaheng; Zhao, Yuting (2018) ‘Stability Criterion for Dynamic Gaits of Quadruped Robot’
3. Kramer, Oliver; Gong, Daoxiong; Yan, Jie; Zuo, Guoyu (2010) ‘A Review of Gait Optimization Based on Evolutionary Computation’