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Turning Strategy Analysis Based on Trot Gait of a Quadruped Robot

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Abstract—Accurate turning is not an easy task for the quadruped robot and previous work mainly addresses the problem with complex control strategy or systems. This paper proposes a design method for the accurate turning motion of a quadruped robot with 12 degrees of freedom (DOF). Unlike others, we have achieved precise turning motion with a relatively simple geometric approach. Once the parameters of desired movement are set, such as the turning radius and angles, the robot can realize corresponding circle locomotion. A design architecture including geometric analysis, 3D foot trajectory design and inverse kinematics is presented to solve the problem systematically. In addition, according to the different conditions, the corresponding compensation coefficient is necessarily considered in order to acquire precise values of leg joints. At last, comparison results by several experiments have identified the feasibility and accuracy of turning along a circle trajectory based on our proposed method.

Index Terms—Quadruped robot, Turning motion, Design architecture, Circle trajectory

I. INTRODUCTION

In the face of challenging terrain and heavy loads, legged robots show unique advantages over wheeled robots. According to the bionic mechanism, quadruped robot is the common object among legged robots. Early researchers focused on the gait patterns of quadruped robots [1] [2]. Turning gait is extremely important because it is the basis of the trajectory tracking. Therefore, the optimal turning strategy and precise turning control are essential for the trajectory tracking.

The turning of legged robots has always been a complex locomotion task. For quadruped robots, the easiest method is that ensuing both sides of the step has the different length [3]. Another way is to analyze the force on inside and outside legs, the details can be seen in [4]. Certainly, geometric analysis [5] is a main method without any doubt.

For instance, the synthesis and decomposition of motion and complex optimization of mathematical formulas were

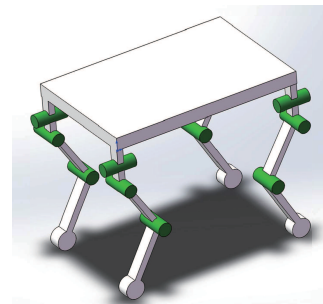


Fig. 1. Schematic model of a quadruped robot. The white boxes represent body and legs, and the green cylinders are rotate joint servos.

proposed in [6]. Despite of how clearly it described the process of movement, it's hard to understand. Furthermore, in order to obtain an accurate turning, more and more researchers have focused on precise control systems [7] [8], such as linear oscillators or non-linear oscillators [9] [10] [11]. However, to some extent, it is complex and expensive. In addition, algorithm optimization is also a relatively popular research field [12] [13]. Of course, the position of Center of Gravity (COG) and the pitch angle of the robot body are other ways to achieve the turning gait. A combination of kinematics analysis and foot trajectory design was the effective strategy [14]. Recent years, the turning on multi-legs robots or wheeled robots has became popular topic [15].

This paper proposes a design architecture for the accurate turning gait of a quadruped robot with 12 degrees of freedom. The method is easy to understand and realize, and it also has high accuracy and strong mobility. We have clearly analyzed the geometric relationship between two adjacent movement and deduced a strict mathematical formula, which is so simple that additional control can be reduced. In addition, we also have applied a 3-D foot trajectory to achieve the turning locomotion by the synthesis and decomposition of kinematics. For improving the accuracy of turning, it is no doubt that foot trajectory is the most important part among the whole design architecture. Then, inverse kinematics is introduced to obtain the rotation angles of all joints, and put the computed values into controller. The final experiment results have strongly demonstrated the feasibility of our design method. The precise execution of the turning movement is the fundamental knowledge for the trajectory tracking control

* This research is supported by State Joint Engineering Lab on Robotics and Intelligent Manufacturing ([2015]581, [2015]863), Shenzhen Engineering Lab on Robotics and Intelligent Manufacturing ([2014]1677, [2014]1722), and PF.01.000143 from the Chinese University of Hong Kong, Shenzhen, China.

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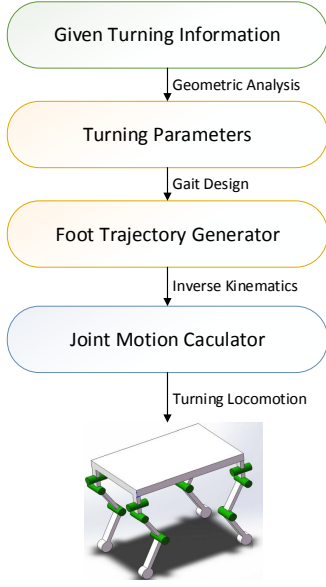


Fig. 2. A flowchart about how to design a turning gait includes three steps. First, with the desired parameters, the corresponding values of step length and central angle can be calculated. Then, design correct 3-D foot trajectory. Last, based on inverse kinematics, values of joints can be computed to achieve a great turning locomotion.

in the future.

The rest of paper are constructed as follows: Section II discusses the Robot Model and Design Architecture. Section III mainly introduces the geometric analysis, 3-D foot trajectory design and inverse kinematics which are used to acquire accurate joint values. Section IV displays the experiment results of our proposed turning strategy and the locomotion of real robot. Section V draws the conclusion about acquired results and specific analysis, as well as future works.

II. ROBOT MODEL AND DESIGN ARCHITECTURE

A. Robot Model

Given a quadruped robot model shown in Figure 1, the configuration of it adopts inner knee-elbow with 12-DOF. Each leg contains 3 actuated degrees of freedom revolute joints (marked with green colour), including side-pendulum joint (abduction/adduction), hip joint (flexion/extension) and knee joint (flexion/extension). The physical parameters information are shown in Table I. All joints rotate around the axis of the cylinder.

B. Design Architecture

Figure 2 is a flowchart of design that shows the process of obtaining turning gait. The system design architecture is described as following three layers: Perception, Calculator, Execution.

Firstly, we can give any desired turning radius and the corner angle of each step, then use geometric analysis method to calculate the step lengths of the inner and outer legs. Secondly, the required step values are used to design the corresponding 3-D foot trajectory, which is related to

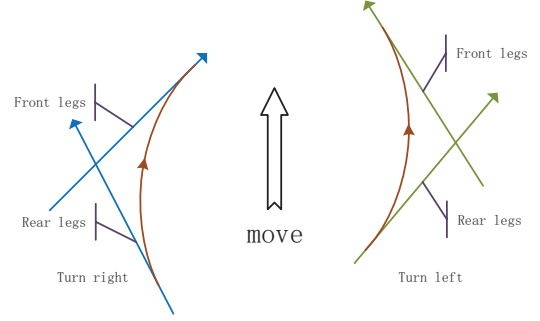


Fig. 3. The turning direction of robot motion. Blue and green vectors represent the force of legs, which is same with the differential movement of car.

the accuracy and stability of quadruped robot locomotion. After that, inverse kinematics is applied to get the three joint angle values of per leg. Finally, the corresponding joint values stored in CM530(controller) and to be tested to verify the feasibility of proposed turning strategy. More detailed analysis will be introduced in the section III.

TABLE I
PHYSICAL PARAMETERS OF QUADRUPEL ROBOT

Name	Parameters	values	Definition
Body Width	W	115 mm	The length between two front legs
Body length	L	230 mm	The length between same side legs
Pendulum Length	l_1	22 mm	The length between side-pendulum and hip joint
Thigh	l_2	53 mm	The length between hip joint and knee joint
calf	l_3	42 mm	The length between knee joint and toe foot

III. SPECIFIC ANALYSIS OF THE ARCHITECTURE

A. Geometric Analysis

Once the radius of turning is determined, corresponding parameters of the movement, including the step length and central angle, are very important. We need to analyze the turning process to acquire the relevant parameters through the geometric calculation. The circle is the track of COG, which can be artificially changed. In this paper, two adjacent movements are considered, then we can decompose the movement process into three parts (the process of 1, 2, 3 in Figure 4). These three parts represent translation and rotation locomotion, respectively.

A simple description about the turning direction can be seen in Figure 3. Turning left or right is up to the force of legs.

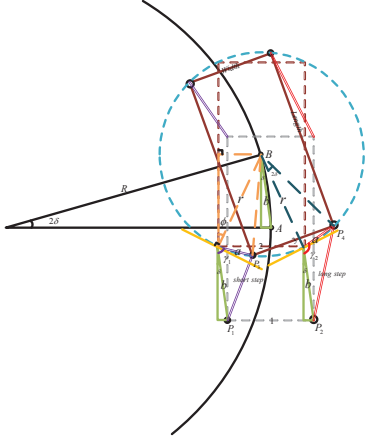


Fig. 4. The process of two adjacent movements can be divided into three parts, from 1 to 2 then 3. Through the movement decomposition, we can get the values of main parameters such as step length and center angle.

Just consider the two adjacent movement process of turning left, the detailed description is shown in Figure 4. More specific information can be obtained in [5] [12]. The process is that the COG moves from A to B while the toe of left leg moves from P_1 to P_3 and the right leg moves from P_2 to P_4 . In addition, the translation is the process of 1 to 2, and the rotation process is 2 to 3 at a same time. Through the analysis of geometric movement, the step length can be acquired. The relevant equation is written as

$$\begin{aligned} r &= \sqrt{(L/2)^2 + (W/2)^2} \\ a &= 2r \sin(\delta) \\ b &= 2R \sin(\delta) \end{aligned} \quad (1)$$

$$\begin{aligned} \phi &= \arctan(W/L) \\ \gamma_1 &= 90 - \phi \\ \gamma_2 &= 90 + \phi \end{aligned} \quad (2)$$

$$\begin{aligned} s_{step} &= \sqrt{a^2 + b^2 - 2ab \cos(\gamma_1)} \\ l_{step} &= \sqrt{a^2 + b^2 - 2ab \cos(\gamma_2)} \end{aligned} \quad (3)$$

Where the R, r represent the radius of turning movement and rotate movement, respectively. 2δ is value of central angle, and a, b are length of the secant. ϕ is angle between edge line and diagonal of the robot body. γ_1, γ_2 are the angle between a and b . s_{step}, l_{step} are length of the short step and long step. Cosine theorem is used to solve the values. By the way, the yellow line is the tangent of circle. Consider the important point in this paper is not this, so here the more specific geometric analysis process is simplified.

B. Foot Trajectory

3-D foot trajectory is essential to turning motion. We use a compound curve, which is the same method proposed in [2]. In the Figure 5, the 3-D foot trajectory is combined

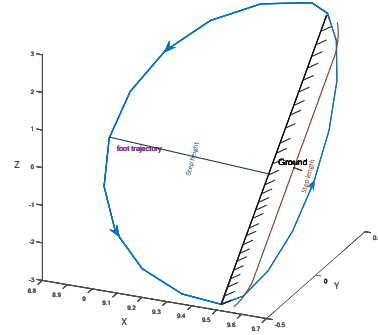


Fig. 5. The 3-D foot trajectory contains step height and length, which is a synthesis by two ellipse curves.

by two ellipse curves with different height. According to experience, the step height (h_{step}) is constant and the step length is the values of s_{step}, l_{step} (see (3)). The crucial part is the relationship between front legs and rear legs. In Figure 6, LF and RH leg has different spatial declination and step size, which is determined by the analysis of the movement of the turning. If you don't understand the move process, detailed explanation is shown in [14]. We need to compute the values of ϑ_1 and ϑ_2 . Through understanding the turning movement, the equation is

$$\begin{aligned} \xi_1 &= \frac{\arccos(b^2 + s_{step}^2 - a^2)}{2bs_{step}} + \delta \\ \xi_2 &= \frac{\arccos(b^2 + l_{step}^2 - a^2)}{2bl_{step}} - \delta \\ \vartheta_1 &= \lambda_1(\xi_1 - \delta) \\ \vartheta_2 &= \lambda_2(\xi_2 + \delta) \end{aligned} \quad (4)$$

where ξ_1, ξ_2 are the angle between body side and LH, RH leg. ϑ_1 and ϑ_2 are the space deflection angle of foot trajectory (Fig 6). Moreover, due to the phenomenon of slip, λ_1, λ_2 are introduced as the compensation coefficient, which are the crucial elements of turning motion.

Generally speaking, quadruped gaits are characterized by the time and phase of legs. Trot gait is chosen to complete turning in this paper (other gaits refer [1] [2]). The time ratio of swing and support phase is 1:1. Based on this, the follow context is about the information of swing phase and support phase, which contribute to the last foot trajectory.

1) *Swing phase*: Swing phase controls the process from the toe location, $P_1(P_2)$, to a target location, $P_3(P_4)$. Taking RH leg as an example, the parametric equations of spatial ellipse is written as

$$\begin{aligned} x &= \frac{l_{step}}{4} \cos(\psi) \cos(\vartheta_2) \\ y &= \frac{l_{step}}{4} \cos(\psi) \sin(\vartheta_2) \\ z &= h_{step} \sin(\psi) \end{aligned} \quad (5)$$

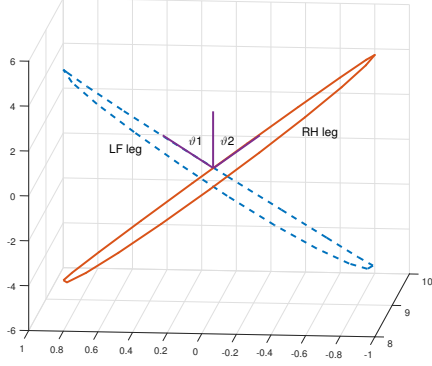


Fig. 6. The different foot trajectory design such as LF(dash line) and RH(solid line) legs are shown above. ϑ_1 , ϑ_2 represent the angles of front and rear legs.

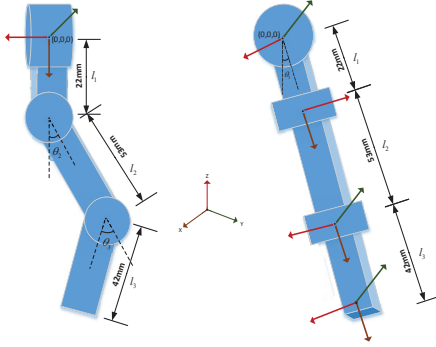


Fig. 7. The abstract joint structure of legs include all the values of leg sizes (TABLE I). Besides, the definition of θ_1 , θ_2 and θ_3 can be seen clearly in the picture.

(x, y, z) is the location point of toe at spatial coordinate system. $(\frac{l_{step}}{4}, h_{step})$ represent the lengths of the long axis and short axis of the ellipse, respectively. ψ is vertical angle and ϑ_2 is the angle in eq (4).

2) *Support phase*: The length of the duration in the support phase determines gait pattern. Support phase also influences the stability of locomotion. Thus, according to experience, the magnitude of ellipse height can not be too high. Parametric equations of support ellipse is defined as

$$\begin{aligned} x &= \frac{l_{step}}{4} \cos(\psi) \cos(\vartheta_2) \\ y &= \frac{l_{step}}{4} \cos(\psi) \sin(\vartheta_2) \\ z &= \rho h_{step} \sin(\psi) \end{aligned} \quad (6)$$

The coefficient ρ is the gain of the length of short axis, which influences the stability of turning motion.

C. Inverse Kinematics

Inverse kinematics is the essential solution to deal with the foot trajectory. As we know, the foot trajectory is known as a function of swing and support phase. Given the location of the foot at spatial coordinate system, inverse kinematics

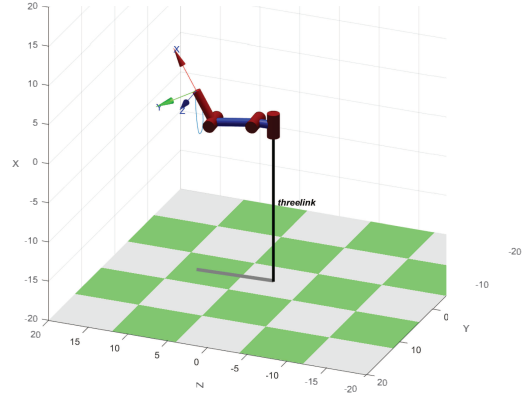


Fig. 8. The simulation movement of LF leg based on the analysis with MATLAB. It can intuitively show the movement of the left leg, and display the 3-D foot trajectory.

is often used to compute the value of all joints, which are then put into the servo controller. The simulation of LF leg can be seen in Figure 8. The definition of θ_1 , θ_2 and θ_3 is shown in an as abstract legs form in Figure 7.

The table II is the DH parameters of our designed quadruped robot.

TABLE II
D-H PARAMETERS

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\pi/2$	L_1	0	θ_2
3	0	L_2	0	θ_3
4	0	L_3	0	0

Based on our derived DH parameters, we can get the transformation of every joint and get the position of end-effector joint, which is the position of the toe of foot.

$$\begin{aligned} {}^0T_4 &= {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 = \begin{bmatrix} R_{3 \times 3} & P_{3 \times 1} \\ 0_{3 \times 1} & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & x \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & y \\ s_{23} & c_{23} & 0 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (7)$$

Here we only focus on the position of the toe of foot of

the front leg foot of the quadruped robot:

$$P_{3 \times 1} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_1 c_1 + l_2 c_1 c_2 + l_3 c_1 c_2 c_3 \\ l_1 s_1 + l_2 s_1 c_2 + l_3 s_1 c_2 c_3 \\ l_2 s_2 + l_3 s_2 c_3 \end{bmatrix} \quad (8)$$

The solution of $\theta_1, \theta_2, \theta_3$ can be derived from the equation (7) and (8), and the detailed solution process can be found in [16][17]:

$$\theta_1 = \arctan\left(\frac{y}{x}\right) \quad (9)$$

$$\theta_2 = \arctan\left(\frac{D}{\pm\sqrt{R^2 - D^2}}\right) - \arctan\left(\frac{B}{A}\right) \quad (10)$$

$$\theta_3 = \arctan\left(\frac{z - l_2 s_2}{y s_1 + x c_1 - l_1 - l_2 c_2}\right) - \theta_2 \quad (11)$$

Where D, A, B is as follows:

$$D = \frac{l_3^2 - l_2^2 - l_1^2 - z^2 + 2l_1(y s_1 + x c_1) - (y s_1 + x c_1)^2}{2l_2} \quad (12)$$

$$A = -z \quad (13)$$

$$B = l_1 - (x c_1 + y s_1) \quad (14)$$

$$R = \pm \sqrt{A^2 + B^2} \quad (15)$$

IV. EXPERIMENTS

In this section, several experiments are taken to validate our proposed method. In order to verify the accuracy and feasibility of this method, comparative experiments with different turning radius and center angle are designed. Through the comparative experiments, we can analyze some characteristics of quadruped robot when it takes the turning locomotion.

A. Experiments Parameters

For ensuring the precision of the movement, all parameters must be of sufficient accuracy. Table III shows the values used in our experiments.

TABLE III
EXPERIMENT PARAMETERS

R	2δ	s_{step}	l_{step}	ϑ_1	ϑ_2	λ_1	λ_2	h_{step}	ρ
400mm	18°	113.0mm	147.6mm	19.3°	14.7°	1.04	1.04	8mm	0.25
400mm	10°	63.0mm	82.2mm	20.6°	15.7°	1.11	1.11	8mm	0.25
500mm	18°	143.0mm	178.1mm	15.6°	12.5°	1.07	1.07	8mm	0.25
500mm	10°	79.7mm	99.2mm	15.9°	12.7°	1.09	1.09	8mm	0.25

In table III, we did four groups of contrastive experiments. The values of the parameters are adjusted through changing only one variables(radius or center angles) at a time. The

TABLE IV
EXPERIMENT RESULT

R_t	SN_t	R_r	SN_r
400 mm	20	409 mm	20
400 mm	36	406 mm	35.5
500 mm	20	515 mm	19.5
500 mm	36	505 mm	34.5

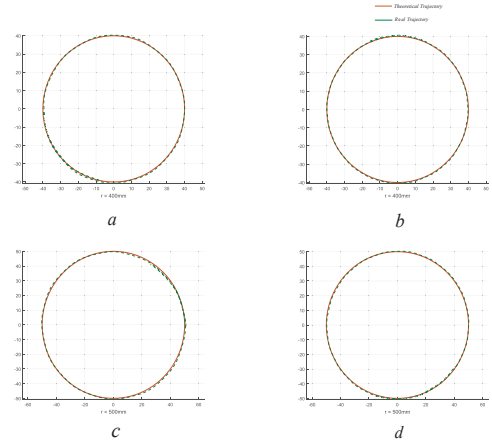


Fig. 9. Two groups of comparative experiments demonstrate the feasibility and accuracy of our proposed design method by capturing the motion of COG. The brown solid line and the green dotted line represent the theoretical trajectory and the real trajectory, respectively. a) Radius is 400 mm and center angle is 18°. b) Radius is 400 mm and center angle is 10°. c) Radius is 500 mm and center angle is 18°. d) Radius is 500 mm and center angle is 10°.

compared results of theoretical and real experiments are listed in Table IV.

Where R_t, R_r represent the radius of theoretical and real locomotion, and SN_t, SN_r are the number of steps ($SN = 360 / 2\delta$) of theoretical and real locomotion. As can be seen in table IV, the real locomotion is almost consistent with theoretical one, which clearly demonstrates the feasibility and accuracy based on our method.

B. Experiments Analysis

In Figure 9, these trajectories are obtained by capturing the motion of robot. The real trajectory matches the theoretical one very well when it takes more steps (smaller center angle). However, in table III, the compensation coefficient of group 2 or 4 is obviously larger than group 1 or 3, which suggests that the more number of steps, the larger corresponding compensation coefficient. As a result, the motion of b,d matches theoretical trajectory better than a,c.

Figure 10 shows the turning motion of a quadruped robot. The precise motion trajectory has verified feasibility and accuracy of our proposed method.

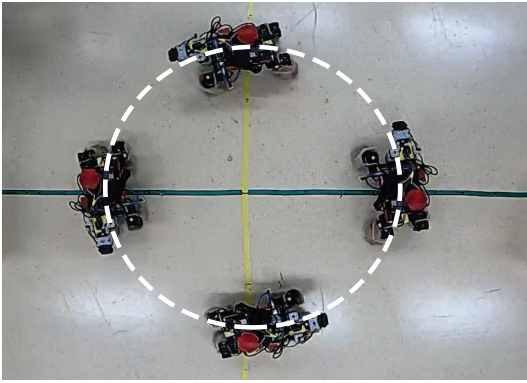


Fig. 10. The snapshots of turning experiment based on a quadruped robot. The white circle represents the real motion trajectory with a radius of 40 cm and center angle of 10° .

V. CONCLUSION AND DISCUSSION

The main contribution of this paper is that we clearly display how to achieve a desired turning gait through an efficient design architecture. The method has been applied to a quadruped robot with 12-DOF. The architecture consists of three main parts. The first part is geometric analysis about two adjacent movement for acquiring step length. Note that the step length of the inner leg is always shorter than the outer one as the robot turns left. Then, foot trajectory generator is the key point, which determines whether the turning locomotion is successful. We have applied 3-D spatial elliptic curve as foot trajectory. At last, the knowledge of inverse kinematics is applied to obtain joint values. We have realized the desired locomotion (turn along a circle track) without complex control. No doubt, the great experiment results certified the feasibility and accuracy of our method.

By comparing the experiment results, we found that the robot turns more like circle when it takes more steps. As can be seen in Fig 9, these real trajectories shown in (b), (d) matches theoretical trajectories better than in (a) (b), in terms of turning a circle. However, the disadvantage shown in table III is that the compensation coefficient of group 2 or 4 is obviously larger than group 1 or 3, which suggests that the more number of steps, the larger corresponding compensation coefficient. As a result, this method would be less accurate. Another simulation is that it can produce little unstable locomotion when the center angle is large because the larger step length will cause bigger error. Besides, the phenomenon of slip always exists, which explains the necessity of compensation coefficient.

whereas, our experiments are limited because we just consider two parameters and one gait (turning a circle). Although it doesn't combine more elements, the experiment results have proved that our proposed method is beneficial to easily generate a circle turning motion. Besides, this method is easier to understand and grasp.

Our future work will focus on realizing turning motion about arbitrary trajectories. Based on this method, any arbitrary curves are divided into many circles with different radius. An additional advantage is that we can add control

system to guarantee the perfect locomotion with little error. At last, this method should be generalized to more gait patterns, such as walk, pace etc.

APPENDIX

The accompanying video can be accessed at the following <https://youtu.be/yrHs0JC7Wn8>.

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