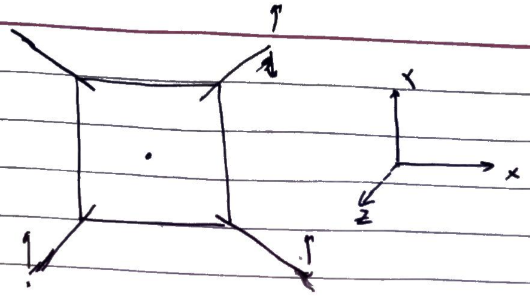


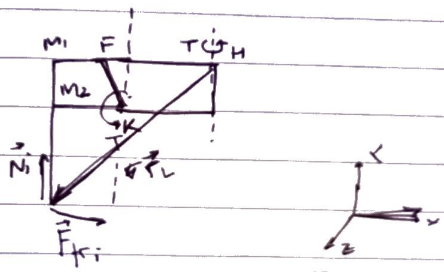
$$\sum_i \vec{F}_{fri} = m\vec{a}$$

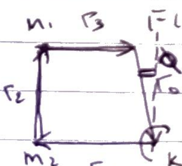
$$\sum \vec{N}_i = m\vec{g} \qquad \sum \vec{F}_{fri} + \sum N_i - m\vec{g} = \underbrace{m\vec{a}}$$

is the mechal force.

$$\sum_i \vec{F}_{fri} \times \vec{r}_i = \vec{N}_z = \boxed{I_z \vec{\alpha}_z}$$

$$\sum_i \vec{N}_i \times \vec{r} = \vec{N}_x = \boxed{I_x \vec{\alpha}_x} \rightarrow \text{Inertial moment}$$





$$\vec{r}_0 + \vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0 \quad -①$$

$$\vec{v}_0 + \vec{v}_1 + \vec{v}_2 + \vec{v}_3 = 0 \quad -②$$

② is obtained by differentiating ①

$\vec{v}_0 = 0$ (K and F are fixed)

$\vec{v}_1 = \vec{\omega}_1 \times \vec{r}_1$

$\vec{v}_2 = \vec{\omega}_2 \times \vec{r}_2$

$\vec{v}_3 = \vec{\omega}_3 \times \vec{r}_3$

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \omega_{1z} \end{bmatrix} \quad \omega_2 = \begin{bmatrix} 0 \\ 0 \\ \omega_{2z} \end{bmatrix} \quad \omega_3 = \begin{bmatrix} 0 \\ 0 \\ \omega_{3z} \end{bmatrix}$$

$$r_1 = \begin{bmatrix} r_{1x} \\ r_{1y} \\ 0 \end{bmatrix} \quad r_2 = \begin{bmatrix} r_{2x} \\ r_{2y} \\ 0 \end{bmatrix} \quad r_3 = \begin{bmatrix} r_{3x} \\ r_{3y} \\ 0 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} -\omega_{1z} r_{1y} \\ \omega_{1z} r_{1x} \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} -\omega_{2z} r_{2y} \\ \omega_{2z} r_{2x} \\ 0 \end{bmatrix} \quad V_3 = \begin{bmatrix} -\omega_{3z} r_{3y} \\ \omega_{3z} r_{3x} \\ 0 \end{bmatrix}$$

since

$$\vec{V}_0 + \vec{V}_1 + \vec{V}_2 + \vec{V}_3 = 0$$

$$\Rightarrow \quad \omega_{1z} r_{1y} + \omega_{2z} r_{2y} + \omega_{3z} r_{3y} = 0$$

$$\omega_{1z} r_{1x} + \omega_{2z} r_{1x} + \omega_{3z} \omega_{3x} = 0$$

$$\Rightarrow \quad \omega_{2z} = -\omega_{1z} \frac{(r_{1y} r_{3x} - r_{3y} r_{1x})}{(r_{2y} r_{3x} - r_{3y} r_{2x})}$$

$$\omega_{3z} = -\omega_{1z} \frac{(r_{2y} r_{1x} - r_{1y} r_{2x})}{(r_{2y} r_{3x} - r_{3y} r_{2x})}$$

$$\vec{a}_0 = 0$$

$$\vec{a}_1 = \vec{\alpha}_1 \times \vec{r}_1 + \vec{\omega}_1 \times \vec{\omega}_1 \times \vec{r}_1$$

$$\vec{a}_2 = \vec{\alpha}_2 \times \vec{r}_2 + \vec{\omega}_2 \times \vec{\omega}_2 \times \vec{r}_2$$

$$\vec{a}_3 = \vec{\alpha}_3 \times \vec{r}_3 + \vec{\omega}_3 \times \vec{\omega}_3 \times \vec{r}_3$$

$$\vec{\alpha}_1 = \begin{bmatrix} 0 \\ 0 \\ \alpha_{1z} \end{bmatrix} \quad \vec{\alpha}_2 = \begin{bmatrix} 0 \\ 0 \\ \alpha_{2z} \end{bmatrix} \quad \vec{\alpha}_3 = \begin{bmatrix} 0 \\ 0 \\ \alpha_{3z} \end{bmatrix}$$

$$\vec{a}_1 = \begin{bmatrix} -\alpha_{1z} r_{1y} - \omega_{1z}^2 r_{1x} \\ \alpha_{1z} r_{1x} - \omega_{1z}^2 r_{1y} \\ 0 \end{bmatrix}$$

$$\vec{a}_0 + \vec{a}_2 + \vec{a}_3 + \vec{a}_1 = 0$$

$$\vec{a}_2 = \begin{bmatrix} -\alpha_{2z} r_{2y} - \omega_{2z}^2 r_{2x} \\ \alpha_{2z} r_{2x} - \omega_{2z}^2 r_{2y} \\ 0 \end{bmatrix}$$

$$\vec{a}_3 = \begin{bmatrix} -\alpha_{3z} r_{3y} - \omega_{3z}^2 r_{3x} \\ \alpha_{3z} r_{3x} - \omega_{3z}^2 r_{3y} \\ 0 \end{bmatrix}$$

solving for α_{2z} and α_{3z} using the above equations we get

$$\cancel{\alpha_{2z} = r_{3x}(-\alpha_{1z} r_{1y} - \omega_{1z}^2 r_{1x} - \omega_{2z}^2 r_{2x} - \omega_{3z}^2 r_{3x}) - r_{3y}(}$$

$$\alpha_{2z} = \frac{r_{3x}(-\alpha_{1z} r_{1y} - \omega_{1z}^2 r_{1x} - \omega_{2z}^2 r_{2x} - \omega_{3z}^2 r_{3x}) - r_{3y}(-\alpha_{1z} r_{1x} + \omega_{1z}^2 r_{1y} + \omega_{2z}^2 r_{2y} + \omega_{3z}^2 r_{3y})}{r_{2y} r_{3x} - r_{3y} r_{2x}}$$

$$\alpha_{3z} = \frac{-r_{2x}(-\alpha_{1z} r_{1y} - \omega_{1z}^2 r_{1x} - \omega_{2z}^2 r_{2x} - \omega_{3z}^2 r_{3x}) + r_{2y}(-\alpha_{1z} r_{x} + \omega_{1z}^2 r_{1y} + \omega_{2z}^2 r_{2y} + \omega_{3z}^2 r_{3y})}{r_{2y} r_{3x} - r_{3y} r_{2x}}$$

Using the above equations the kinematics can be can calculated, which will be used for calculation of dynamics

E

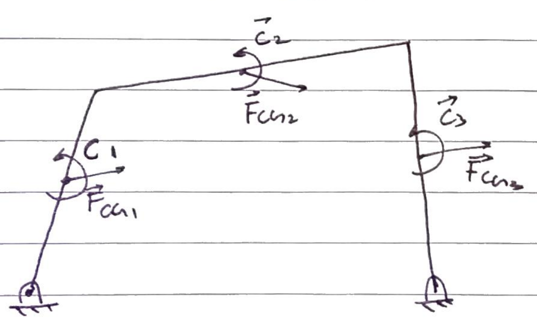
$$\vec{F}_{CG_1} = m_1 \vec{a}_{CG_1}$$
$$\vec{F}_{CG_2} = m_2 \vec{a}_{CG_2}$$
$$\vec{F}_{CG_3} = m_3 \vec{a}_{CG_3}$$

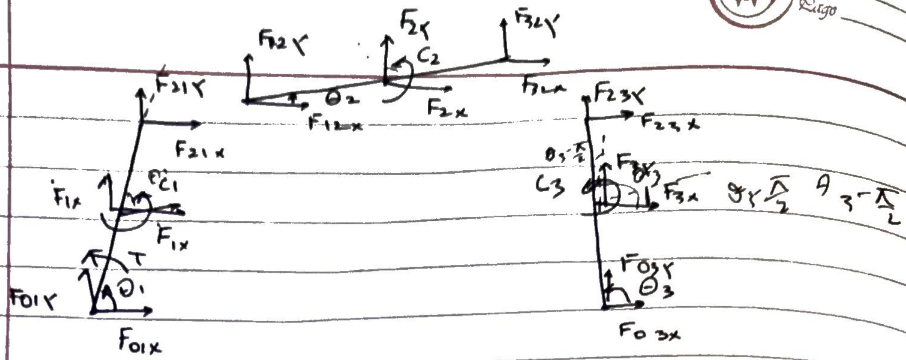
$$\vec{a}_{CG_1} = \vec{\alpha}_1 \times r_{CG_1/O_1} + \vec{\omega}_1 \times \vec{\omega}_1 \times r_{CG_1/O_1}$$
$$\vec{a}_{CG_2} = \vec{\alpha}_1 \times \vec{r}_1 + \vec{\omega}_1 \times \vec{\omega}_1 \times \vec{r}_1 + \vec{\alpha}_2 \times \vec{r}_{CG_2/B} + \vec{\omega}_2 \times \vec{\omega}_2 \times \vec{r}_{CG_2/B}$$
$$\vec{a}_{CG_3} = \vec{\alpha}_3 \times \vec{r}_{CG_3/O_3} + \vec{\omega}_3 \times \vec{\omega}_3 \times \vec{r}_{CG_3/O_3}$$

$$\vec{C}_1 \quad \cancel{\vec{M}_1} = I_{CG_1} \times \alpha_1$$
$$\vec{C}_2 \quad \cancel{\vec{M}_2} = I_{CG_2} \times \alpha_2$$
$$\vec{C}_3 \quad \cancel{\vec{M}_3} = I_{CG_3} \times \alpha_3$$

α_1

T





9×3

$F_{01x} + F_{1x} + F_{21x} = 0$

$F_{01Y} + F_{1x} + F_{21Y} = 0$

$F_{12x} + F_{2x} + F_{32x} = 0$ $\quad 9\times3$

$F_{12Y} + F_{2Y} + F_{32Y} = 0$

$F_{03x} + F_{3x} + F_{23x} = 0$

$F_{03Y} + F_{3Y} + F_{23Y} = 0$

$\vec{F}_{21} = -\vec{F}_{12}$

$\vec{F}_{23} = -\vec{F}_{32}$

$\cancel{F} \quad T + C_1 + \cancel{F_1 \times F_1} \; F_{1x} r_1 \cos\theta_1$

$T + C_1 - F_{1x} r_1 \cos\theta_1 + F_{1Y} r_1 \sin\theta_1 + F_{21Y} \cos\theta_1 l_1 - F_{21x} \sin\theta_1 l_1 = 0$

$C_2 + F_{2Y} r_2 \cos\theta_2 - F_{2x} r_2 \sin\theta_2 + F_{32Y} \cos\theta_2 l_2 - F_{32x} \sin\theta_2 l_2 = 0$

$\cancel{C_3 + F_{3Y} r_2}$

$\cancel{C_3 - F_{3x} \cos\left(\theta_3 - \frac{\pi}{2}\right)} + F_{3Y}$

$\cancel{C_3 - F_{3x} \cos}$

$C_3 - F_{3x} \sin\theta_3 r_3 \cancel{+ F_{3Y} \sin\theta_3} + F_{3Y} \cos\theta$

$C_3 - F_{3x} \sin\theta_3 r_3 + F_{3Y} \cos\theta_3 r_3 - F_{23x} \sin\theta_3 l_3 + F_{23Y} \sin\theta_3 l_3 = 0$

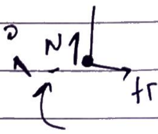
~~Problem~~ formulation

→ In case of three legged motion.

- at the point of contact

-

- We assume that there is no slipping occuring



→ Thus the force acting ~~from~~ on the leg are equally and opposite to the forces exerted ~~by~~ at the point of contact.

→ The point of contact is at a point that is an extension of the second link in the four bar linkage analogy.

→ There are two forces responsible for interaction at the point of contact
- The movement at the hip
- The movement at the knee.

→ The position of $\vec{r}_1$ is given by

$$\vec{r}_1 = -(l_1 \cos(\theta_1)\hat{i} + l_1 \sin(\theta_1)\hat{j})$$

where θ_0 is the reference position (usually $\pi/2$)

$$\vec{r}_0 = -(l_0 \sin\phi\, \hat{i} + l_0 \cos\phi\, \hat{j})$$

$$\vec{r}_3 = \ell$$

$$\overrightarrow{M_1 F} = (x - l_1 \cos(\theta_1)\hat{i}) + (y - l_1 \sin\theta_1)\hat{j}$$

$$\vec{M}_1 \vec{F}_2 = (l_2 \cos\theta_2 - l_1 \cos(\theta_1))\hat{i} + (l_2 \sin\theta_2 - l_1 \sin(\theta_1))\hat{j}$$

θ_0 θ_2 initial $= 0$

θ_1 initial $= \frac{\pi}{2}$

$\Rightarrow$ $\vec{r}_2 = l_2 \cos\theta_2 \hat{i} + l_2 \sin\theta_2 \hat{j}$

$\vec{r}_3 = l_3 \cos\theta_3 \hat{i} + l_3 \sin\theta_3 \hat{j}$

θ_3 initial $= 0^\circ$

$$l = \sqrt{|r_0|^2 + |r_1|^2 - 2|r_0||r_1|\cos\theta_1}$$

$$\phi_1 = \sin^{-1} \frac{|r_1|}{l} \cos\theta_1$$

$$\cancel{\phi_2 = -\left[\frac{|r|^2 - l^2 - |r_3|^2}{2l|r_3|}\right]} \quad \phi_2 = \cos^{-1} - \left(\frac{|r_2^2| - l^2 - |r_3|^2}{2l|r_3|}\right)$$

$$\cancel{\theta_3 = 2\pi - (\phi_1 + \phi_2)} \quad \theta_3 = 2\pi - (\phi_1 + \phi_2)$$

$$\theta_2 = \sin^{-1} \frac{||r_3| \sin\theta_3 - |r_1| \sin\theta_1|}{|r_2|}$$