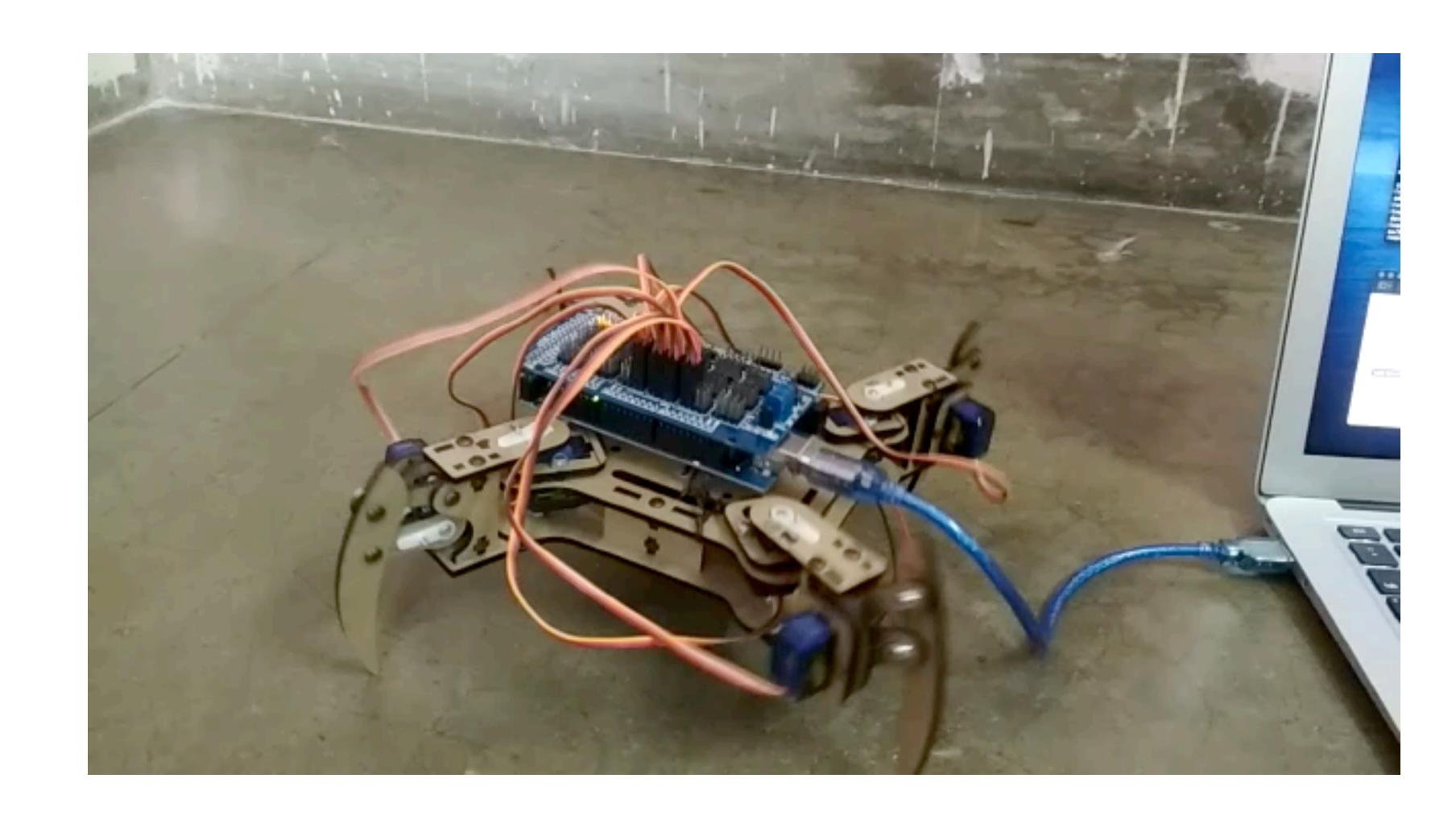
## Gait Fitness Function Formulation

Design of Control System for Quadruped using Central Pattern Generators

# Robot walking Video

Observe the tendency of the robot to topple towards the swinging leg



## Gait Pattern

#### Pattern in use and problems with it

- This Gait Pattern obtained from the formulation in the figure is for a creep walk of the quadruped
- The robot walks with this Gait Pattern
- But the stability of the quadruped is not maintained
- The quadruped tends to topple towards the leg in swing phase

#### Hip Activations:

$$\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{\beta T}+\pi), \qquad \text{if } 0 \leq t \leq \frac{\beta T}{2}$$
 
$$\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{(1-\beta)T}+\frac{(3-4\beta)\pi}{2(1-\beta)}), \quad \text{if } \frac{\beta T}{2} \leq t \leq \frac{T(2-\beta)}{2}$$
 
$$\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{\beta T}+\frac{(\beta-1)\pi}{\beta}), \quad \text{if } \frac{T(2-\beta)}{2} \leq t \leq T$$

For  $i \in \{0,1\}$ 

And

$$\theta_h(t) = \begin{cases} -\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{\beta T} + \pi), & \text{if } 0 \leq t \leq \frac{\beta T}{2} \\ -\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{(1-\beta)T} + \frac{(3-4\beta)\pi}{2(1-\beta)}), & \text{if } \frac{\beta T}{2} \leq t \leq \frac{T(2-\beta)}{2} \\ -\theta_h sin(\frac{(t-\frac{iT}{4})\pi}{\beta T} + \frac{(\beta-1)\pi}{\beta}), & \text{if } \frac{T(2-\beta)}{2} \leq t \leq T \end{cases}$$

For  $i \in \{2,3\}$ 

#### Knee Activations:

$$\theta_k(t) = \begin{cases} \theta_k sin(\frac{t\pi}{T(1-\beta)} - \frac{\beta\pi}{2(1-\beta)}), & \text{if } \dot{\theta_h}(t) \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

For  $i \in \{0,1\}$ 

And

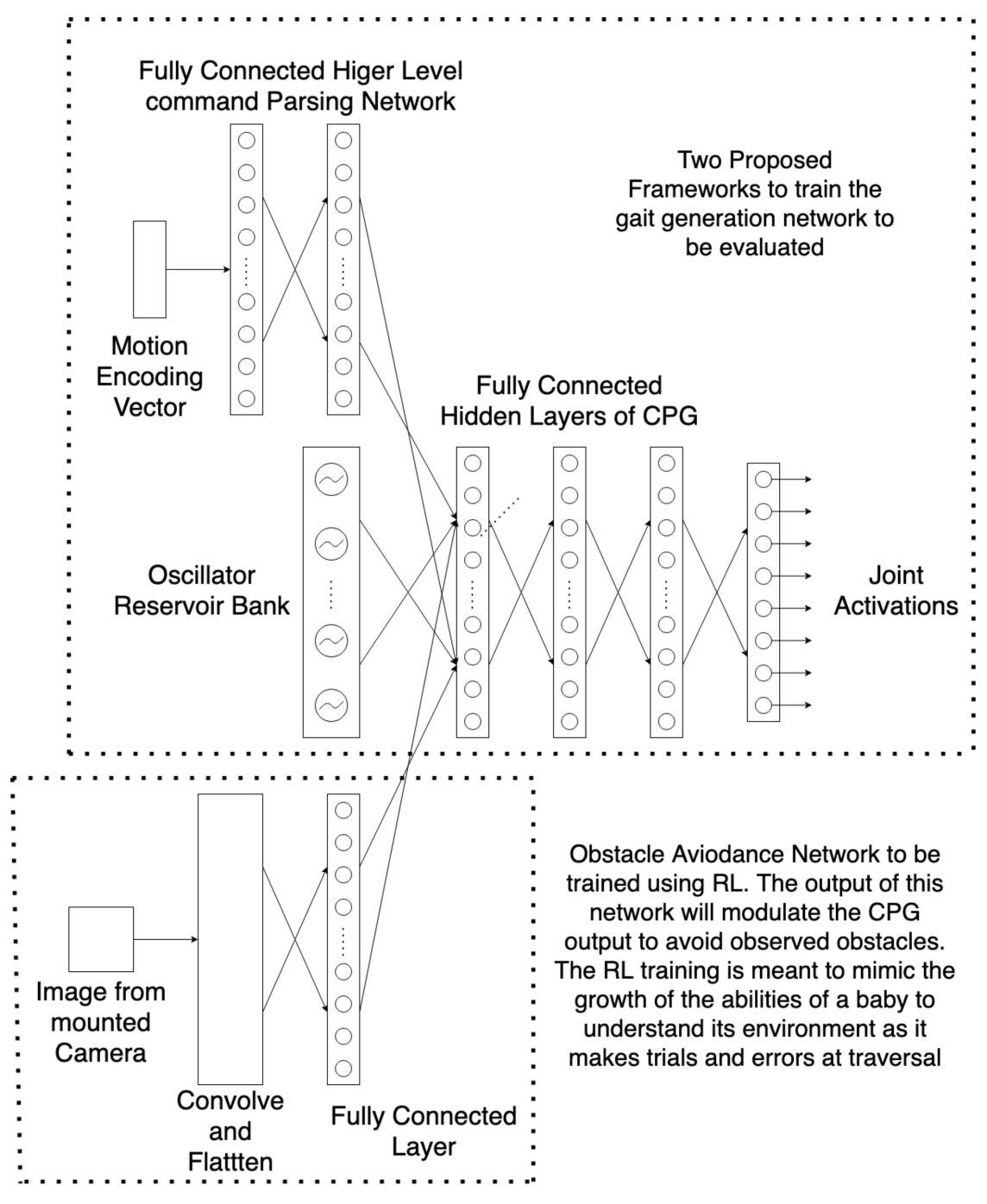
$$\theta_k(t) = \begin{cases} \theta_k sin(\frac{t\pi}{T(1-\beta)} - \frac{\beta\pi}{2(1-\beta)}), & \text{if } \dot{\theta_h}(t) \leq 0 \\ 0, & \text{otherwise} \end{cases}$$

For  $i \in \{2,3\}$ 

#### Architecture

#### Proposed solution to ensure gait stability

- Two methods Proposed to train the gait generation network
  - Train the gait generation network end to end using back props and then use RL training for improvement of gait
  - Optimise Gait Patterns using fitness function, then train the network end to end using back prop



### Fitness Function

#### Evaluation Criteria to measure stability, speed and energy consumption

- Both the aforementioned training frameworks require a Fitness Function [1]
- Fitness Function must measure the following
  - Stability Criteria, S
  - ullet Speed Criteria, V
  - Energy Efficiency Criteria, E

$$F = \sum_{i=t-T}^{t} S_i^{k_s} V_i^{k_v} E_i^{k_e} \text{ where }$$

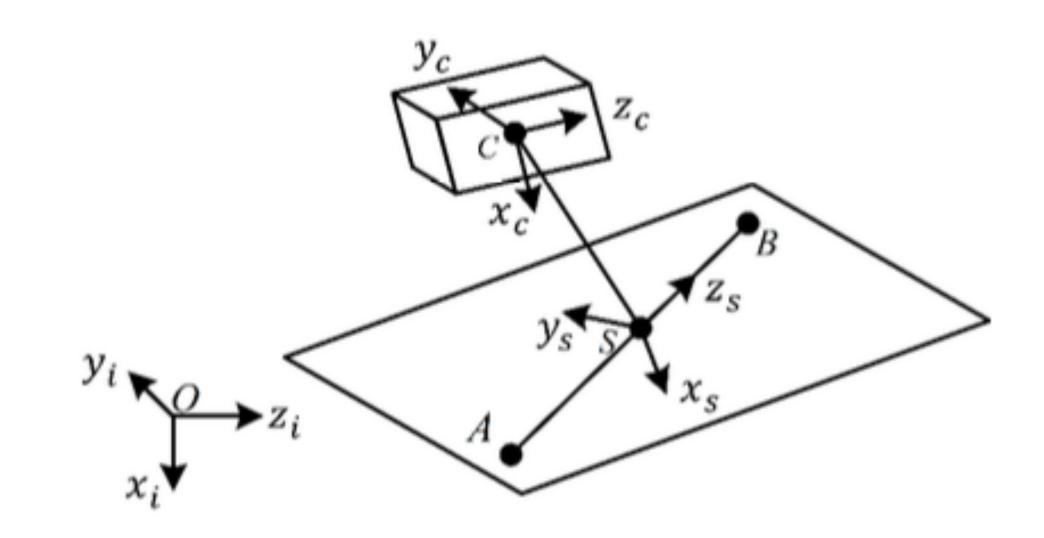
- F is the Fitness Value
- $k_s$ ,  $k_v$  and  $k_e$  are integer parameters to account for relative importance of each criteria
- T is the gait period and t is the current time instance

#### Evaluation of Dynamic and Static Stability of the Quadruped

- A Quantitative Measure of quadruped's dynamic stability
- The Criteria must not only take into account current stability but must also anticipate the stability at the next time instance, given the state of motion of the quadruped, like an animal does
- Yan Jia, Xiao Luo and others [2] in their paper propose a modified ZMP based stability evaluation criteria
- The main idea is that at a certain state, the motion of the robot is considered to be stable if the torque caused by the ground-reaction force can prevent the robot from tumbling around any support boundary

#### Coordinate Systems used

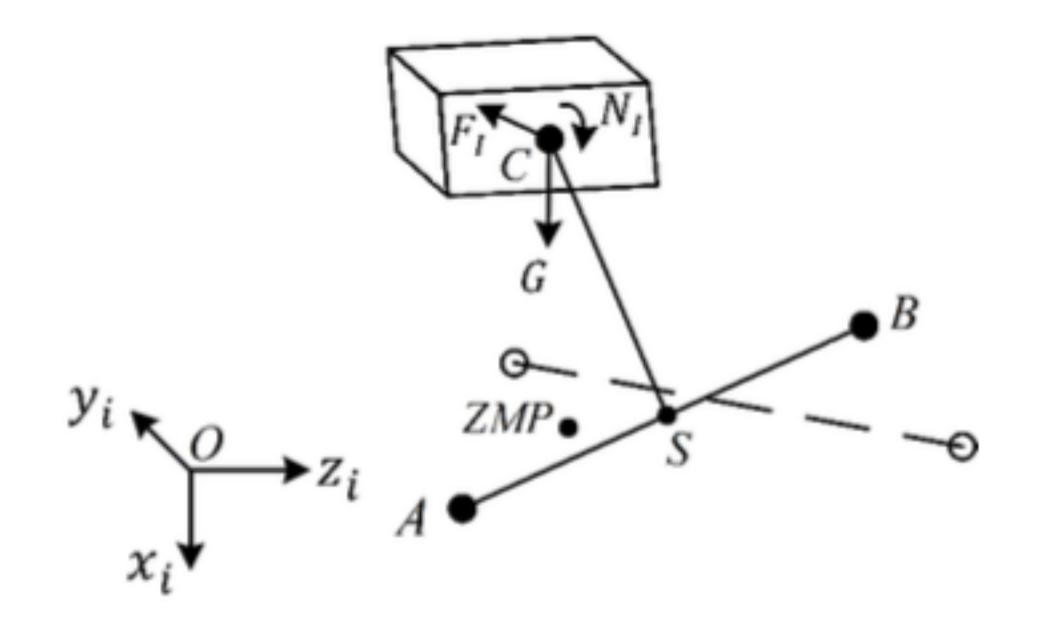
- •There are two coordinate systems of interest-
  - •The Inertial Coordinate system
  - The Support Coordinate system
- •The Support Coordinate has z-axis along the support line AB and x axis perpendicular to the current support plane



#### **Equations for Zero Moment Point**

$$\begin{split} &-\left(z_{C}^{i}-z_{ZMP}^{i}\right)F_{Iy}^{i}+\left(y_{C}^{i}-y_{ZMP}^{i}\right)F_{Iz}^{i}+N_{Ix}^{i}=0;\\ &\left(z_{C}^{i}-z_{ZMP}^{i}\right)\left(F_{Ix}^{i}+G_{x}^{i}\right)-\left(x_{C}^{i}-x_{ZMP}^{i}\right)F_{Iz}^{i}+N_{Iy}^{i}=0;\\ &-\left(y_{C}^{i}-y_{ZMP}^{i}\right)\left(F_{Ix}^{i}+G_{x}^{i}\right)+\left(x_{C}^{i}-x_{ZMP}^{i}\right)F_{Iy}^{i}+N_{Iz}^{i}=0. \end{split}$$

The characteristic equations of ZMP. The inertial force and inertial moment here need to be computed by a dynamic analysis of the quadruped performed later in the report



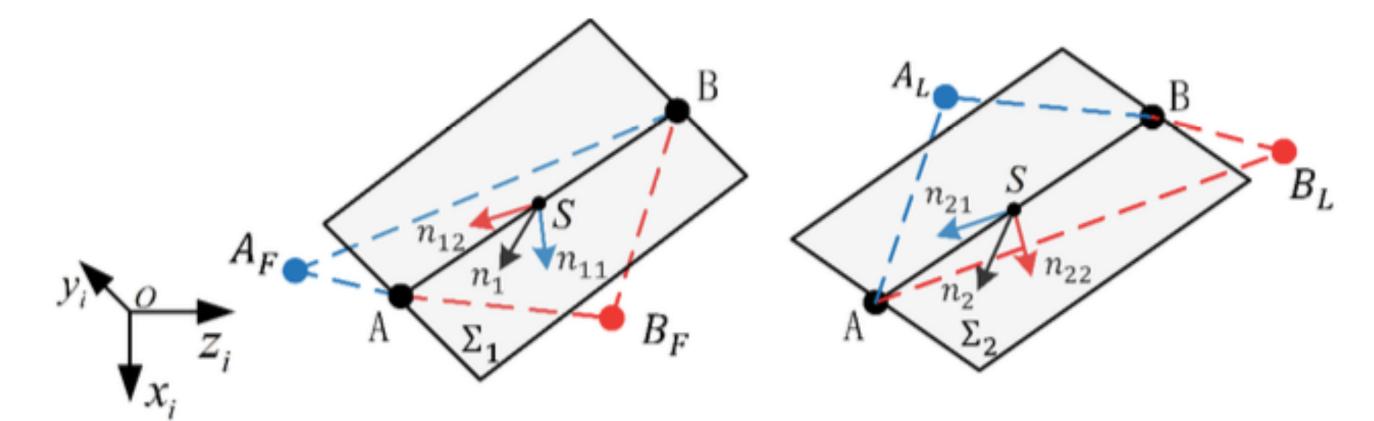
The force schematic used to calculate the position of ZMP.

Here G is gravity, F is the inertial force on COM C, N is the inertial moment

#### Virtual Support Plane and Modified Zero Moment Point

- The paper assumes that a robot is stable if the current or next set of support feet can provide the moment necessary to prevent the robot from tipping in any direction
- A Virtual Support Plane is proposed and it is proposed that if the modified ZMP proposed in the paper lies within this Virtual Support Plane, then the robot motion will be stable
- The Proposed Modified ZMP in the paper also includes a velocity term to account for the motion of the quadruped

#### Calculating the Virtual Support Plane



- S is the origin of the support coordinate system on which the ZMP will lie
- $n_1$  is the normal vector to the plane  $AA_FBB_F$
- $n_2$  is the normal vector to the place  $AA_LBB_L$
- $x_s^i$ ,  $y_s^i$  and  $z_s^i$  are the normal vectors to the virtual support plane in the inertial frame of reference
- $T_b$  is the duration between two adjacent steps and t is the time gap between the current running time and the time point when the previous support line disappeared

$$x_s^i = \frac{\mu n_1 + (1 - \mu) n_2}{\|\mu n_1 + (1 - \mu) n_2\|}$$

$$\mu = -rac{1}{T_b}t + 1$$
  $z_s^i = rac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}$   $y_s^i = z_s^i imes x_s^i$ 

$$n_{11} = \frac{\overrightarrow{AB} \times \overrightarrow{AA}_F}{\|\overrightarrow{AB} \times \overrightarrow{AA}_F\|} \qquad n_{21} = \frac{\overrightarrow{AB} \times \overrightarrow{AA}_L}{\|\overrightarrow{AB} \times \overrightarrow{AA}_L\|}$$

$$n_{12} = \frac{\overrightarrow{AB} \times \overrightarrow{BB}_F}{\|\overrightarrow{AB} \times \overrightarrow{BB}_F\|}$$
  $n_{22} = \frac{\overrightarrow{AB} \times \overrightarrow{BB}_L}{\|\overrightarrow{AB} \times \overrightarrow{BB}_L\|}$ 

#### Calculation of Modified Zero Moment Point

- Zero Moment Point is the point at which the resultant moment on a body is zero
- Modified Zero Moment Point should-
  - Assess the current stability more efficiently and accurately
  - Provide a reference to eliminate undesired velocity during motion planning
- $ZMP_o$  can be used to compute 3 measures of dynamic stability

$$x_{ZMP_0}^s=0;$$

$$y_{ZMP_0}^s = y_{ZMP}^s + \eta \left( v_{yr}^s - v_{yd}^s \right);$$
  
 $z_{ZMP_0}^s = z_{ZMP}^s + \eta \left( v_{zr}^s - v_{zd}^s \right).$ 

Equations to calculate modified ZMP in support coordinate system.  $v^s$  and  $v^d$  are the actual and expected velocities in support coordinate system

$$\eta = \frac{\frac{1}{2}(L+W)}{\|v_d\|} 0.1 = 0.05 \frac{(L+W)}{\|v_d\|}$$

L and W are the effective length and width respectively of the quadruped

$$_{s}^{i}R = \left[ \begin{array}{ccc} x_{s}^{i} & y_{s}^{i} & z_{s}^{i} \end{array} \right]$$

Rotation Matrix for transformation from support to inertial coordinate system

#### Calculation of the metric S

The following three choices for S can be calculated using  $ZMP_o$ -

- Distances between  $ZMP_{\scriptscriptstyle O}$  and the boundaries of the virtual-support quadrilateral in the support plane
- Angle between the vector pointing from CoM to  $ZMP_{o}$  and the normal vector of the virtual support place
- Distance between  $ZMP_o$  and the support line
- The assumption in the paper that the legs have negligible weight compared to the rest of the body does not hold for the quadruped used
- There is a need for evaluation of the repercussions on the concepts introduced

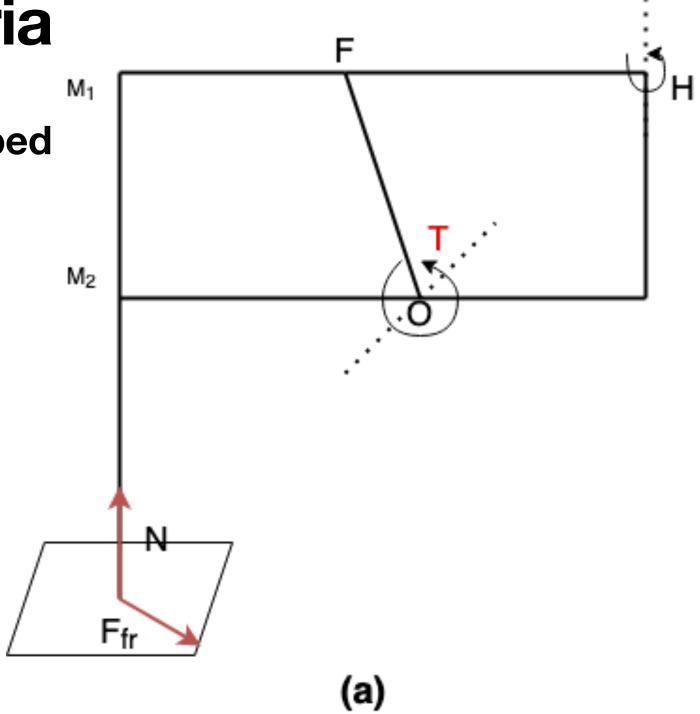
## Speed and Energy Criteria

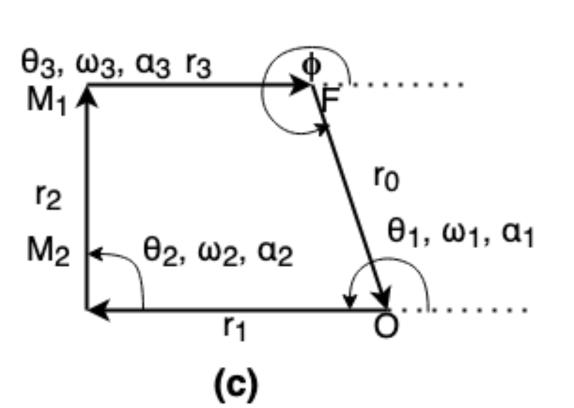
#### Calculation of metrics V and E

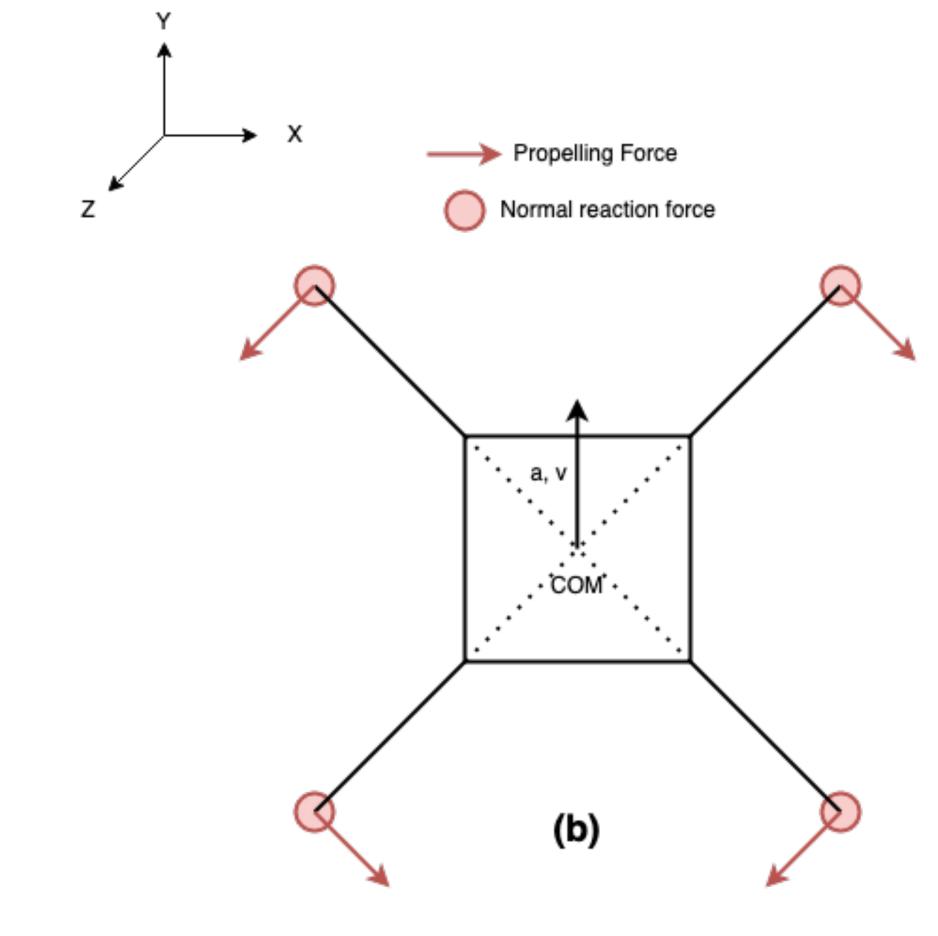
- Speed Criteria measures the discrepancy between expected and actual speed
  - $V = vT_b/d$
  - Where  $\boldsymbol{v}$  is the actual speed and d is the distance travelled over period  $T_b$
- Energy Criteria measures the energy efficiency of the quadruped
  - $E = mv^2/2T_bW$
  - where m is the mass of the quadruped, W is the power supplied to the quadruped, v is the actual velocity and  $T_b$  is the fixed period used in V

Computing ZMP for the available quadruped

- The following figure depicts the simplified model of the quadruped.
- $F_{fr}$  is the force due to friction acting at the point of contact. This is also the propelling force on the quadruped
- N is the normal reaction force from the ground
- $\theta$ ,  $\omega$  and  $\alpha$  are the state of the joints in the four bar linkage in (c)







- (a) Simplified model of the leg of the quadruped
- (b) Simplified model of the quadruped as seen form the top
- (c) Simplified four bar linkage to model the knee of the quadruped

Relationship between knee joint activation and four bar linkage model angular positions

Of the four angular positions that in the four bar linkage model of the knee joint, the angular position at O,  $\theta_1$  and F,  $\phi$  are known (being the controlled angle). Thus, the other angular positions must also be formulated in terms of  $\theta_1$  and  $\phi$ . l,  $\beta_1$ ,  $\beta_2$  and  $\delta$  are intermediate variables.

$$l = \sqrt{\overrightarrow{|r_0|}^2 + \overrightarrow{|r_1|}^2} - 2|r_0||r_1|\cos(\theta_1 - \phi + \pi)$$

$$\beta_1 = \arcsin(\frac{|\overrightarrow{r_1}|}{l}\sin(\theta_1 - \phi + \pi))$$

$$\beta_2 = \arccos\frac{|\overrightarrow{r_2}|^2 + l^2 - |\overrightarrow{r_3}|^2}{2|\overrightarrow{r_3}|l}$$

$$\beta_3 = -\beta_1 - \delta + \phi - \pi$$

#### Linkage vector in knee joint four bar linkage model

$$\overrightarrow{r_1} = \begin{bmatrix} L_1 cos\theta_1 \\ L_1 sin\theta_1 \\ 0 \end{bmatrix}$$

$$\overrightarrow{r_1} = \begin{bmatrix} L_1 cos\theta_1 \\ L_1 sin\theta_1 \\ 0 \end{bmatrix} \qquad \overrightarrow{r_2} = \begin{bmatrix} L_2 cos\theta_2 \\ L_2 sin\theta_2 \\ 0 \end{bmatrix} \qquad \overrightarrow{r_3} = \begin{bmatrix} L_3 cos\theta_3 \\ L_3 sin\theta_3 \\ 0 \end{bmatrix}$$

$$\vec{r}_3 = \begin{bmatrix} L_3 cos\theta_3 \\ L_3 sin\theta_3 \\ 0 \end{bmatrix}$$

$$\overrightarrow{r_0} = \begin{bmatrix} L_0 cos\phi \\ L_0 sin\phi \\ 0 \end{bmatrix}$$

Here  $\overrightarrow{r_0}$  is fixed and  $\phi$ , the angle between  $\overrightarrow{r_0}$  and x-axis is also fixed

 $L_0$ ,  $L_1$ ,  $L_2$  and  $L_3$  are the length of the three linkages given by  $\overrightarrow{r_0}$ ,  $\overrightarrow{r_1}$ ,  $\overrightarrow{r_2}$  and  $\overrightarrow{r_3}$  respectively.

 $L_0$ ,  $L_1$ ,  $L_2$  and  $L_3$  are the length of the three linkages given by  $\overrightarrow{r_0}$ ,  $\overrightarrow{r_1}$ ,  $\overrightarrow{r_2}$  and  $\overrightarrow{r_3}$  respectively.

#### Kinematics model of the Knee Joint

The following equations characterise the kinematic model of the knee joint.

$$\overrightarrow{r_1} + \overrightarrow{r_2} + \overrightarrow{r_3} + \overrightarrow{r_0} = \overrightarrow{0}$$

$$r_{1x} = L_1 cos\theta_1 \qquad r_{1x} = L_1 cos\theta_1$$

$$r_{1x} = L_1 sin\theta_1 \qquad r_{1y} = L_1 sin\theta_1 \qquad r_{1y} = L_1 sin\theta_1$$

$$r_{1y} = L_1 sin\theta_1 \qquad r_{1y} = L_1 sin\theta_1 \qquad r_{1z} = 0$$

$$r_{1z} = 0 \qquad r_{1z} = 0$$

$$\overrightarrow{a_0} + \overrightarrow{a_1} + \overrightarrow{a_2} + \overrightarrow{a_3} = \overrightarrow{0}$$

These aforementioned relationships arise as the four bar linkage is stationary with respect to F and O.

$$r_{0x} = L_0 cos\phi$$

$$r_{0y} = L_0 sin\phi$$

$$r_{0z} = 0$$

These relationships are shorthand for the components of the vectors  $\overrightarrow{r_0}$ ,  $\overrightarrow{r_1}$ ,  $\overrightarrow{r_2}$  and

 $r_{17} = 0$ 

#### Calculating $\omega$ and $\alpha$

$$\overrightarrow{\omega_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\overrightarrow{\omega_1} = \begin{bmatrix} 0 \\ 0 \\ \omega_{1z} \end{bmatrix}$$

$$\overrightarrow{\omega_1} = \begin{bmatrix} 0 \\ 0 \\ \omega_{1z} \end{bmatrix} \qquad \overrightarrow{\omega_2} = \begin{bmatrix} 0 \\ 0 \\ \omega_{2z} \end{bmatrix} \qquad \overrightarrow{\omega_3} = \begin{bmatrix} 0 \\ 0 \\ \omega_{3z} \end{bmatrix}$$

$$\overrightarrow{\omega_3} = \begin{bmatrix} 0 \\ 0 \\ \omega_{3z} \end{bmatrix}$$

$$\overrightarrow{v_O} = 0$$

$$\overrightarrow{v_1} = \overrightarrow{\omega_1} \times \overrightarrow{r_1}$$

$$\overrightarrow{v_2} = \overrightarrow{\omega_2} \times \overrightarrow{r_2}$$

$$\overrightarrow{v_3} = \overrightarrow{\omega_3} \times \overrightarrow{r_3}$$

The linkage  $\overrightarrow{r_0}$  is fixed

Only rotation with  $\omega_1$  about origin of linkage

Only rotation with  $\omega_2$  about origin of linkage

Only rotation with  $\omega_3$  about origin of linkage

#### Calculating $\omega$ and $\alpha$

$$\overrightarrow{v_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \overrightarrow{v_1} = \begin{bmatrix} -\omega_{1z} r_{1y} \\ \omega_{1z} r_{1x} \\ 0 \end{bmatrix}$$

$$\overrightarrow{v_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \overrightarrow{v_1} = \begin{bmatrix} -\omega_{1z}r_{1y} \\ \omega_{1z}r_{1x} \\ 0 \end{bmatrix} \qquad \overrightarrow{v_2} = \begin{bmatrix} -\omega_{1z}r_{1y} \\ \omega_{1z}r_{1x} \\ 0 \end{bmatrix} \qquad \overrightarrow{v_3} = \begin{bmatrix} -\omega_{1z}r_{1y} \\ \omega_{1z}r_{1x} \\ 0 \end{bmatrix}$$

Since  $\overrightarrow{v_0} + \overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3} = \overrightarrow{0}$ , we get the following equations.

$$0 - \omega_{1z}r_{1y} - \omega_{2z}r_{2y} - \omega_{3z}r_{3y} = 0$$

$$0 + \omega_{1z}r_{1x} + \omega_{2z}r_{2x} + \omega_{3z}r_{3x} = 0$$

Solving the above two we get 
$$\omega_{2z}$$
 and  $\omega_{3z}$ 

$$\omega_{2z} = -\omega_{1z} \frac{r_{1y}r_{3x} - r_{3y}r_{1x}}{r_{2y}r_{3x} - r_{3y}r_{2x}}$$

$$\omega_{3z} = -\omega_{1z} \frac{r_{2y}r_{1x} - r_{1y}r_{2x}}{r_{2y}r_{3x} - r_{3y}r_{2x}}$$

#### Calculating $\omega$ and $\alpha$

$$\vec{\alpha}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\overrightarrow{\alpha_1} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{1z} \end{bmatrix}$$

$$\overrightarrow{\alpha_1} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{1z} \end{bmatrix} \qquad \overrightarrow{\alpha_2} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{2z} \end{bmatrix} \qquad \overrightarrow{\alpha_3} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{3z} \end{bmatrix}$$

$$\overrightarrow{\alpha_3} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{3z} \end{bmatrix}$$

$$\overrightarrow{v_O} = 0$$

$$\overrightarrow{a_1} = \overrightarrow{\alpha_1} \times \overrightarrow{r_1} + \overrightarrow{\omega_1} \times \overrightarrow{\omega_1} \times \overrightarrow{r_1}$$

$$\overrightarrow{a_2} = \overrightarrow{\alpha_2} \times \overrightarrow{r_2} + \overrightarrow{\omega_2} \times \overrightarrow{\omega_2} \times \overrightarrow{r_2}$$

$$\overrightarrow{v_3} = \overrightarrow{\omega_3} \times \overrightarrow{r_3}$$

The linkage  $\overrightarrow{r_0}$  is fixed

Only rotation with  $\alpha_1$  about origin of linkage

Only rotation with  $\alpha_2$  about origin of linkage

Only rotation with  $\alpha_3$  about origin of linkage

#### Calculating $\omega$ and $\alpha$

Since 
$$\overrightarrow{a_0} + \overrightarrow{a_1} + \overrightarrow{a_2} + \overrightarrow{a_3} = \overrightarrow{0}$$
, we get the following equations.

$$0 - \alpha_{1z}r_{1y} - \omega_{1z}^2r_{1x} - \alpha_{2z}r_{2y} - \omega_{2z}^2r_{2x} - \alpha_{3z}r_{3y} - \omega_{3z}^2r_{3x} = 0$$

$$0 + \alpha_{1z}r_{1x} - \omega_{1z}^2r_{1y} + \alpha_{2z}r_{2x} - \omega_{2z}^2r_{2y} + \alpha_{3z}r_{3x} - \omega_{3z}^2r_{3y} = 0$$

Solving the above two we get  $\alpha_{2z}$  and  $\alpha_{3z}$ 

$$\alpha_{2z} = \frac{r_{3x}(-\alpha_{1z}r_{1y} - \omega_{1z}^2r_{1x} - \omega_{2z}^2r_{2x} - \omega_{3z}^2r_{3x}) - r_{3y}(-\alpha_{1z}r_{1x} - \omega_{1z}^2r_{1y} - \omega_{2z}^2r_{2y} - \omega_{3z}^2r_{3y})}{r_{2y}r_{3x} - r_{3y}r_{2x}}$$

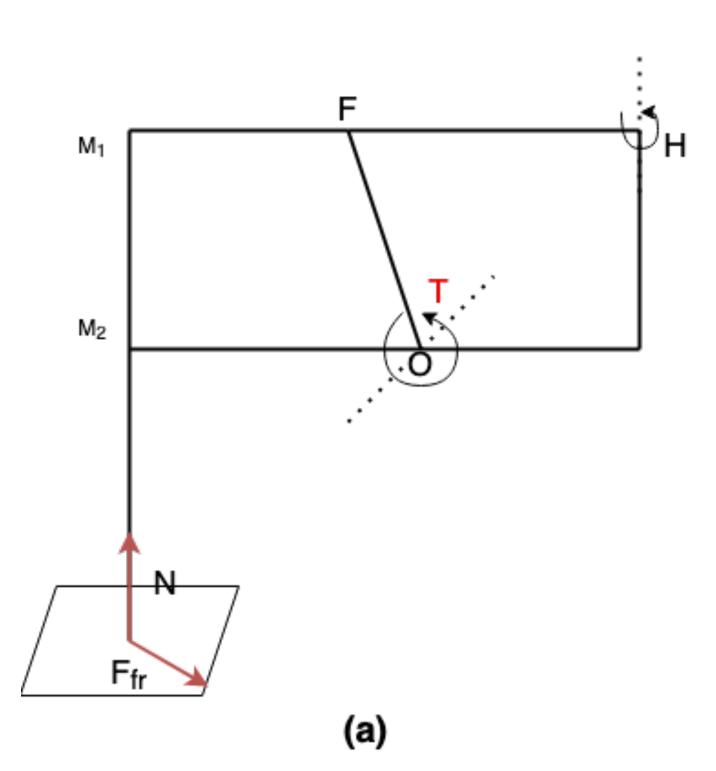
$$\alpha_{3z} = \frac{-r_{2x}(-\alpha_{1z}r_{1y} - \omega_{1z}^2r_{1x} - \omega_{2z}^2r_{2x} - \omega_{3z}^2r_{3x}) + r_{2y}(-\alpha_{1z}r_{1x} - \omega_{1z}^2r_{1y} - \omega_{2z}^2r_{2y} - \omega_{3z}^2r_{3y})}{r_{2y}r_{3x} - r_{3y}r_{2x}}$$

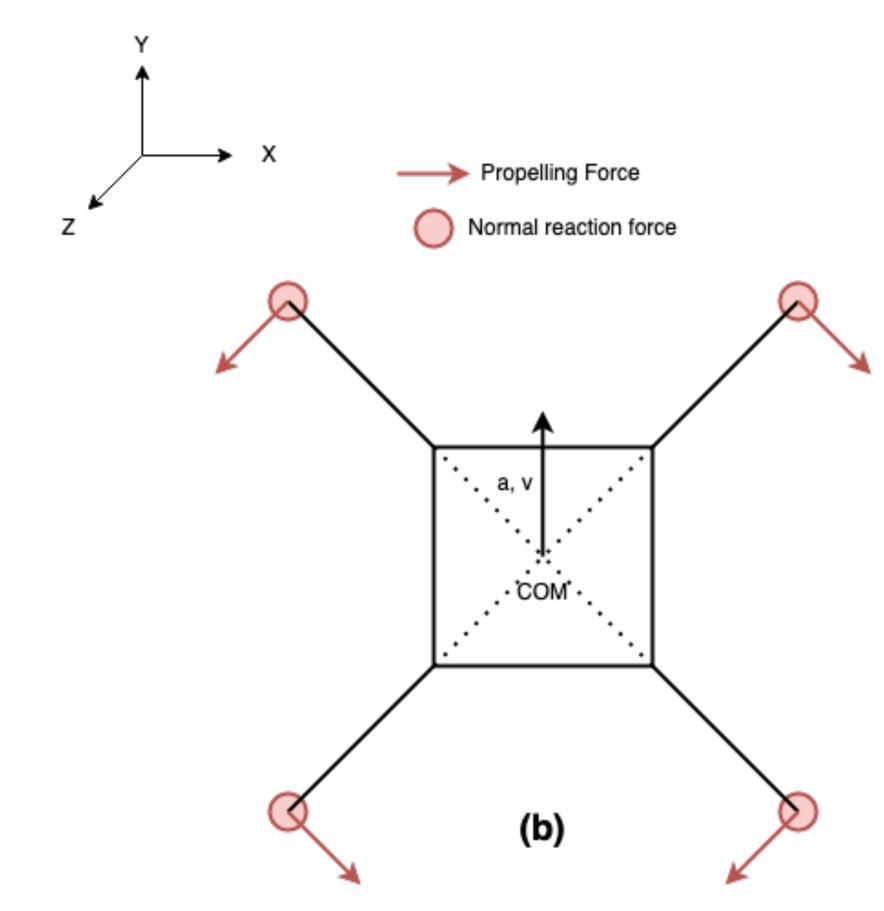
#### Dynamic Model of the Available Quadruped

$$\sum \overrightarrow{F_{fr}} + \sum \overrightarrow{N} - m\overrightarrow{g} = m\overrightarrow{a}$$

$$\sum_{i} \overrightarrow{F_{fr}} \times R_{i} + \sum_{i} \overrightarrow{N} \times R_{i} = I_{R} \overrightarrow{\alpha_{R}}$$

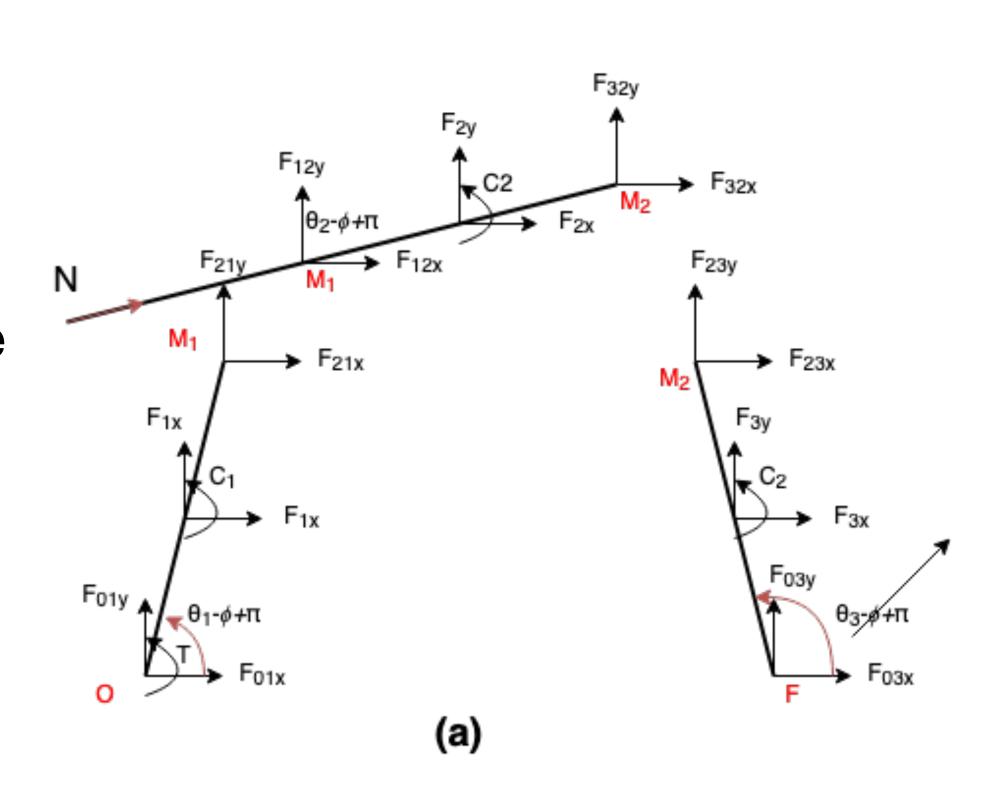
The above two equations depict the dynamics of the quadruped in vector form. Rotation is possible about the y and z axes, while translation is only possible along the y and z axes.

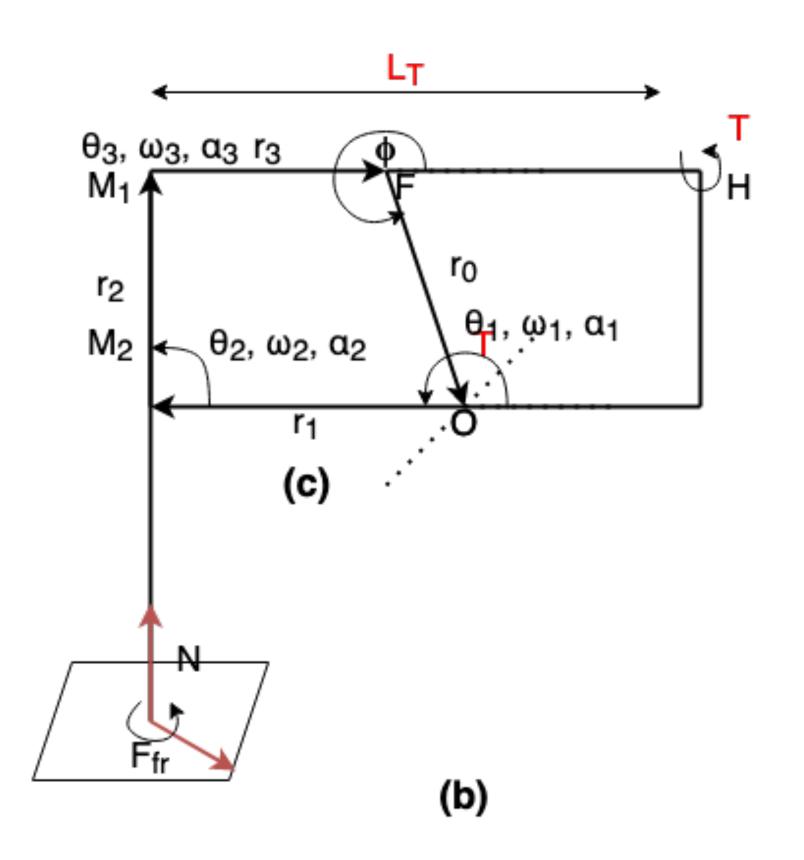




## Calculating Stability Criteria Stance Dynamics

a.Illustration of all forces acting on the four bar linkage in the knee during stance phase. The segment M1H will be rigid during stance b.A simplified model of a leg. N and  $F_{fr}$ are normal reaction and friction forces respectively





#### **Stance Dynamics**

$$F_{01x} + F_{1x} + F_{21x} = 0 \qquad F_{01y} + F_{1y} + F_{21y} = 0 \qquad \overrightarrow{F}_{23} = -\overrightarrow{F}_{32}$$

$$F_{01y} + F_{1y} + F_{21y} = 0 \qquad C_1 = 0 \qquad \overrightarrow{F}_{21} = -\overrightarrow{F}_{12}$$

$$F_{12x} + F_{2x} + F_{32x} + N\cos\theta_2 - \phi + \pi = 0 \qquad C_2 = 0$$

$$F_{12y} + F_{2y} + F_{32y} + N\sin\theta_2 - \phi + \pi = 0 \qquad C_3 = 0$$

$$F_{01x} + F_{1x} + F_{21x} = 0$$

$$T + C_1 - F_{1x} \frac{|\overrightarrow{r_1}|}{2} \cos\theta_1 - \phi + \pi + F_{1y} \frac{|\overrightarrow{r_1}|}{2} \cos\theta_1 - \phi + \pi + F_{21y} |\overrightarrow{r_1}| \cos\theta_1 - \phi + \pi - F_{21x} |\overrightarrow{r_1}| \sin\theta_1 - \phi + \pi = 0$$

$$C_2 - F_{2x} \frac{|\overrightarrow{r_2}|}{2} \cos\theta_2 - \phi + \pi + F_{2y} \frac{|\overrightarrow{r_2}|}{2} \cos\theta_2 - \phi + \pi + F_{32y} |\overrightarrow{r_2}| \cos\theta_2 - \phi + \pi - F_{32x} |\overrightarrow{r_2}| \sin\theta_2 - \phi + \pi = 0$$

$$C_3 - F_{3x} \frac{|\overrightarrow{r_3}|}{2} \cos\theta_3 - \phi + \pi + F_{3y} \frac{|\overrightarrow{r_3}|}{2} \cos\theta_3 - \phi + \pi - F_{23y} |\overrightarrow{r_1}| \cos\theta_1 - \phi + \pi + F_{23x} |\overrightarrow{r_1}| \sin\theta_1 - \phi + \pi = 0$$

Calculating  $\overrightarrow{F_{\mathit{fr}}}$  and  $\overrightarrow{N}$ 

- $\overrightarrow{F_{fr}}$  and  $\overrightarrow{N}$  are the friction and normal reaction acting at the points of contact with the ground
- •To compute these forces the dynamics at the point of contact need to be considered
- •For a successful walking motion, there must be no slipping at the point of contact
- •Thus, we need to solve for equilibrium conditions there

$$\sum_{i}^{k} N_i - mg = 0$$

where k is the number of legs in contact with the group

$$T - F_{fr}L_T = 0$$

This is the condition of no slipping at the point of contact with the ground

$$\sum \overrightarrow{F_{fr}} = m\overrightarrow{a}$$

The sum of all frictional forces are responsible for the horizontal acceleration  $\alpha$  of the quadruped

#### Calculating Inertial Force and Moment at COM

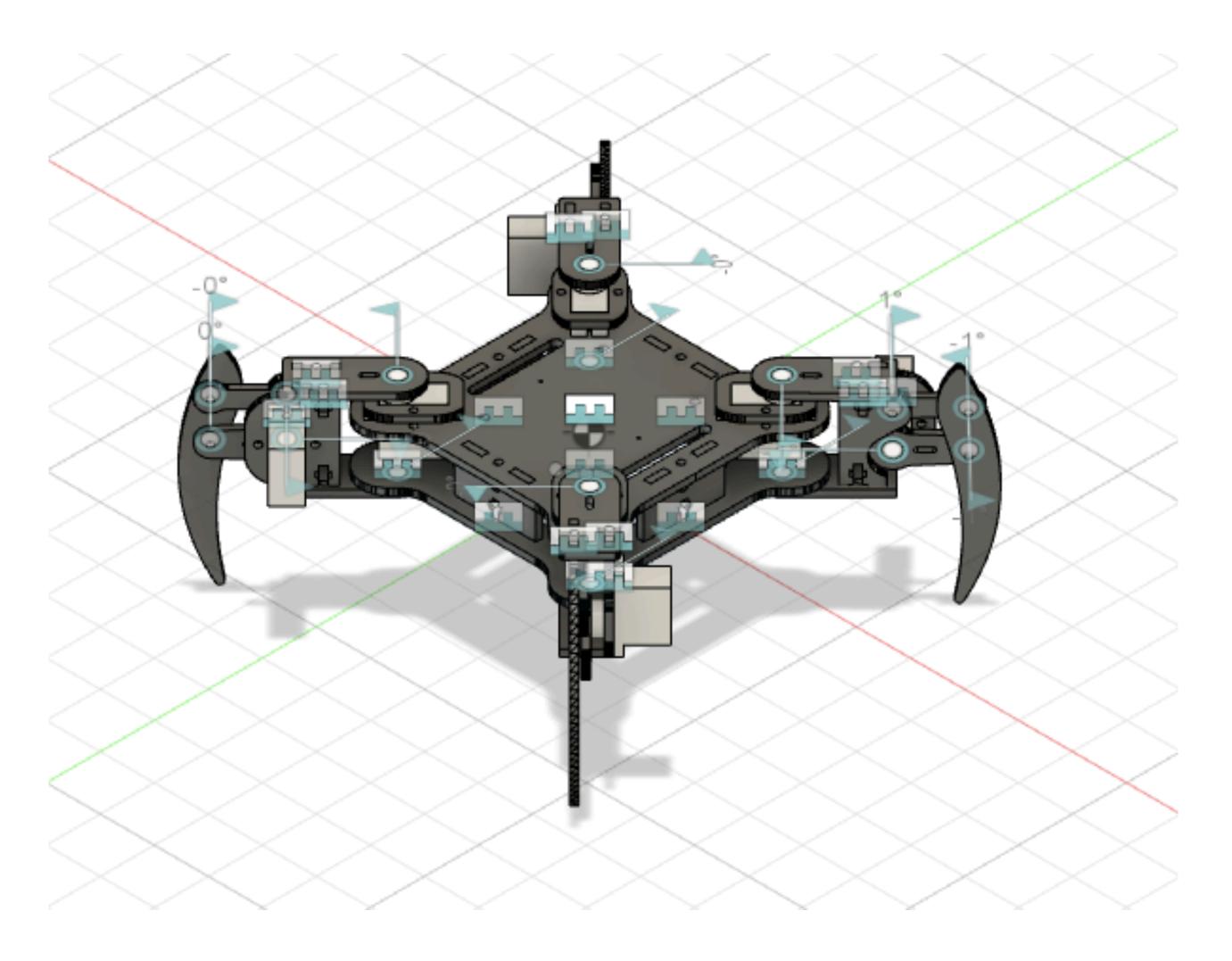
- •The Inertial Force and Moment at the COM are given by the following  $\overrightarrow{ma}$  and  $\overrightarrow{I_Rag}$
- •Both of these parameters can be calculated by solving of all the equations relating to the dynamics and kinematics of the quadruped
- •Once the Inertial Force and Moment at the COM are known, the modified ZMP can computed using the method proposed in [2], which can then be used to compute the stability criteria

#### 3D model

#### **Quadruped Assembly on Fusion 360**

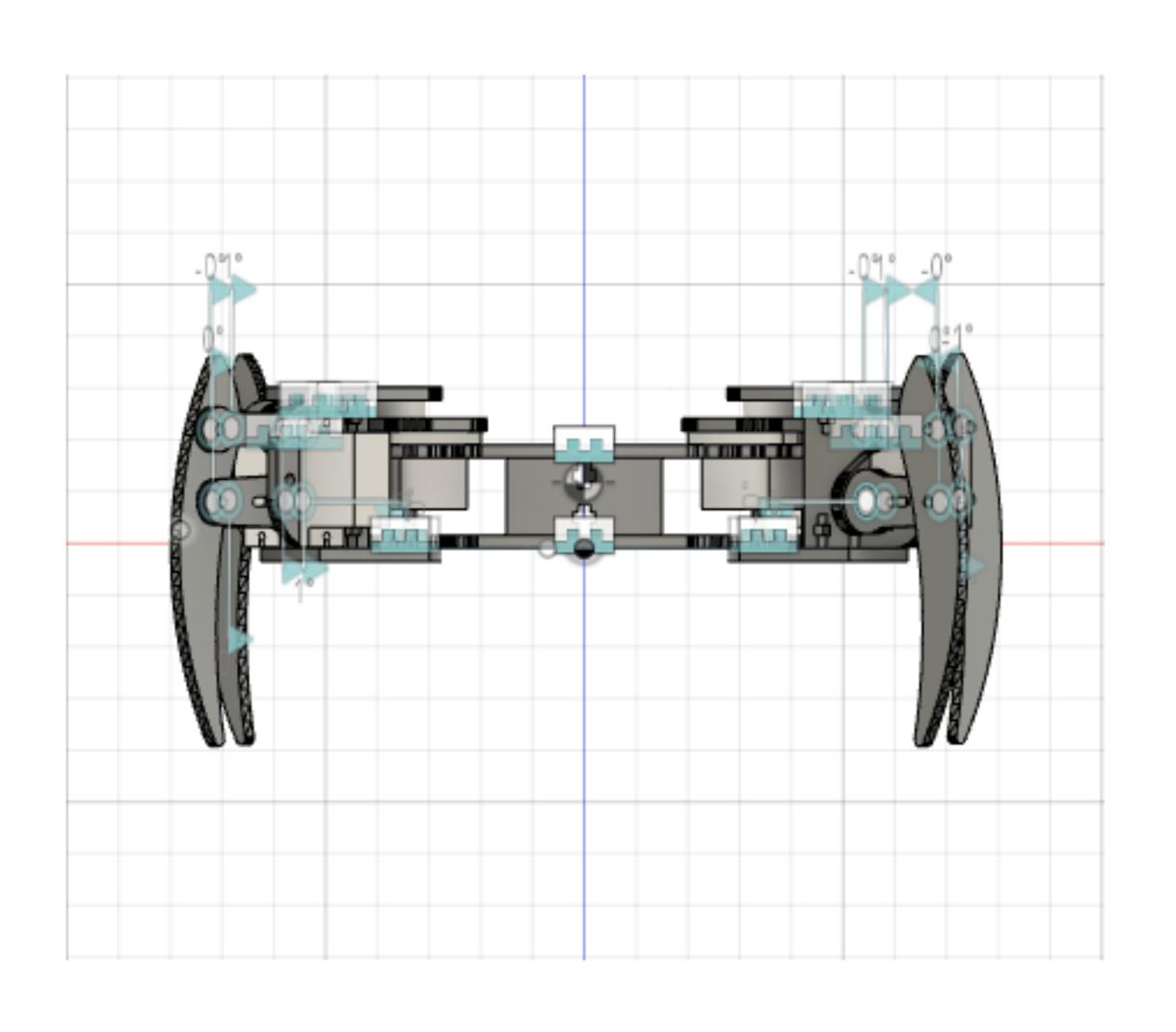
Need for 3D model-

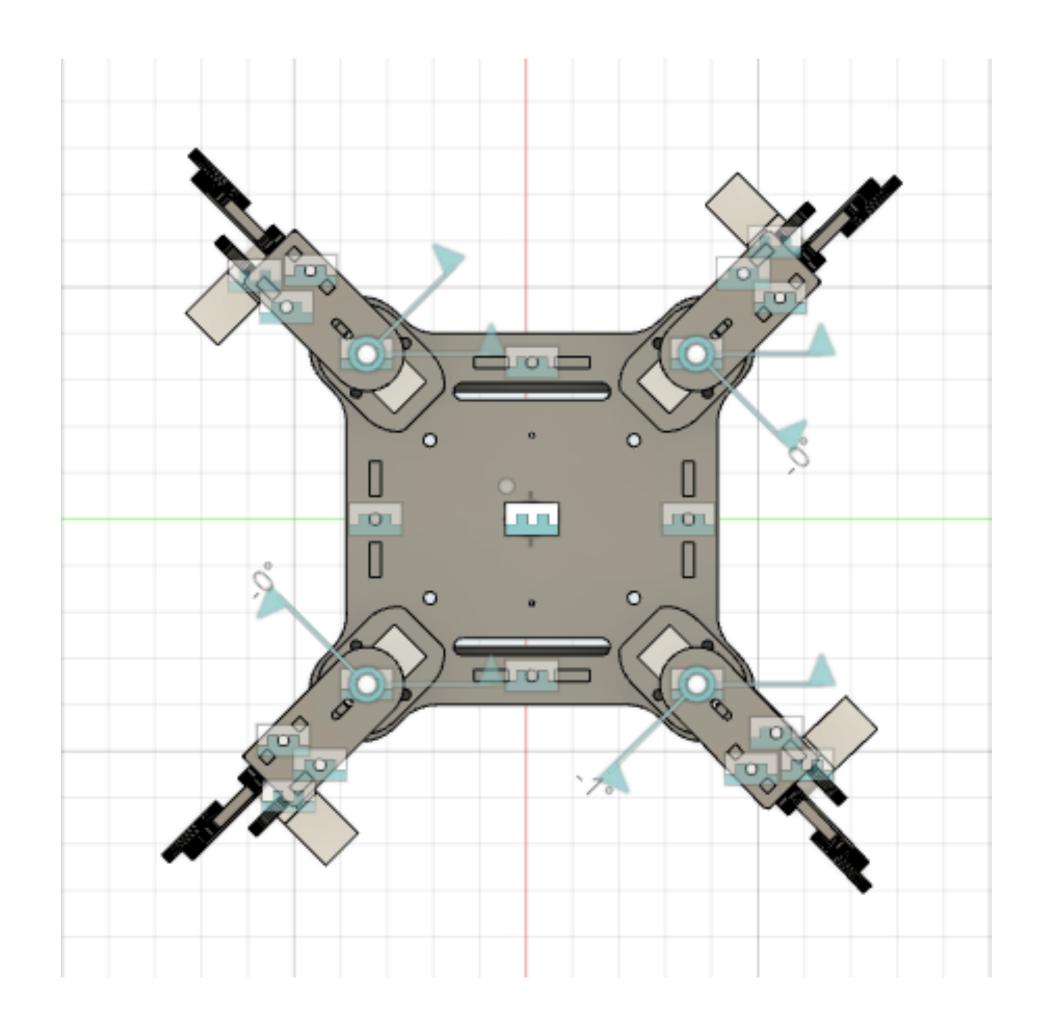
- Precise mass of quadruped components
- Precise Position of leg COM
- Moment of Inertia in Knee four bar linkage
- •Easier method for URDF model generation
- Calculating Joint motion ranges
- •Fusion 360 can also be used for ML



### 3D model

#### Quadruped Assembly on Fusion 360





## Solving Quadruped Dynamics

#### Solving a system of linear equations

- •The dynamics equations of the quadruped together form a system of 69 equations and 69 variables
- AX = B is a system of linear equations with X being the vector of the unknowns, A the matrix of coefficients and B the constants
- • $X = A^{-1}B$  gives the solution to the aforementioned system of linear equations

## Solving Quadruped Dynamics

#### Solving a system of linear equations

- ·Some simplifying assumptions were made to formulate the system of linear equations
  - A binary factor  $\gamma_i$  was introduced for each leg. If  $\gamma_i=0$  there is no contact with the ground at the leg, else there is contact
  - If  $\gamma_i = \gamma_j = 0$ , then  $N_i = N_j = 0$ .
  - If  $\gamma_i = \gamma_j = 1$ , then  $N_i = N_j \neq 0$
  - The resistive force at the hip joint is given by  $\epsilon_i$  when leg i is not in contact with the ground
  - Movement is on a flat surface with coefficient of friction  $\mu$
  - At least two legs are in contact with the ground at all points of time

## Progress and Next Steps

- •The Quadruped simulation model is complete and the module for solving dynamics and estimating modified ZMP are implemented
- •Next step is to implement the control architecture and the fitness functions in TensorFlow and integrate the architecture with gazebo
- OpenAl Gym/TensorFlow Agents to be used in conjunction with ros
  python package for implementing the Reinforcement Learning Training
  Framework
- •Before the RL pipeline can be implemented up skilling in OpenAl Gym/ TensorFlow Agents, RL and Gazebo has to be done

#### References

- Masuri, Ariel; Medina, Oded; Hacohen, Shlomi; Shvalb, Nir (2020)
   'Gait and Trajectory Optimization by Self-Learning for Quadrupedal Robots with an Active Back Joint'
- 2. Jia, Yan; Lua, Xiao; Han, Baoling; Liang, Guanhao; Zhao, Jiaheng; Zhao, Yuting (2018) 'Stability Criterion for Dynamic Gaits of Quadruped Robot'
- 3. Kramer, Oliver; Gong, Daoxiong; Yan, Jie; Zuo, Guoyu (2010) 'A Review of Gait Optimization Based on Evolutionary Computation'