- Animal locomotion requires multi-dimensional co-ordinated rhythmic patterns that need to be ~~corrected~~ tuned so as to satisfy multiple constraints:-
  - capacity to generate forward motion with low energy
  - without falling over
  - and adapting to complex terrain.

- CPG is a neural circuit capable of producing co-ordinated patterns of activity in open loop. Very simple input signals are required to modulate these rhythmic activities
- The design of locomotion control systems of legged robots is a challenge that has been partially solved
- One of the main drawbacks of CPGs is related ~~to~~ ~~on~~ to the learning methodologies to generate the rhythmic ~~sys~~ signals. Methodology for designing CPGs to ~~parti~~ solve particular locomotor problems is yet missing. Another great challenge is the design of control systems that can exhibit the complexity and richness of animal locomotion.
- Locomotion solution methods:→
  - Trajectory based methods
    - Involves the use of kinematic and inverse kinematic models to implement a control strategy by calculation of joint angles for forward motion given a constraint
    - Although a model of the robot is available, learning of the trajectories is based on trial and error. from animal locomotion data and a stabilization criteria.
    - Stabilization criteria include center of mass, center of pressure ~~and~~ zero moment point. A gait is stable if these criteria lie within the support polygon for the robot.
    - Keeping the ZMP or COP within the support polygon is sufficient and necessary condition for stability of gait
  - Heuristic based methods
    - Virtual Model Control (Example of a heuristic based method)
    -
  - Biologically inspired methods -
    - Use of CPG and CPG like methods
    - CPGs are capable of ~~prod~~ producing periodic signal outputs from non-periodic signal inputs.
    -

- Advantage of CPG over other methods :-
  - CPGs do not require the exact knowledge of robot models, while other methods do.. COP and ZMP methods require
  - CPG are able to recover from perturbations ~~smot~~ smoothly and quickly.
  - Same CPG can work on different speeds and conditions without the need for retraining.
  - CPG control strategy is distributed ~~and~~ ~~and~~ over the entire body, while others are black box centralized methods.

- Disadvantage of CPGs -
  - No specific design strategies for controllers.

- Locomotion modelling based on CPGs
  - CPGs are neural networks of oscillatory and inhibitory neurons, often coupled together. Each neuron performs a specific function, controlling one or more ~~two~~ motor organs.
  - Locomotion patterns obtained from CPGs ~~a~~ can be modulated using a single or a set of parameters, ~~which provide for to smooth~~
  - In CPG design there are some common assumptions -
    - the non-linear oscillators are often assumed to ~~be~~ be identical
    - the stepping movements of each limb are controlled by a single oscillator
    - Inter limb co-ordination is achieved by inter-connection of oscillatory neurons.
    - The sensory inputs from lower level central nervous systems and higher level central nervous systems can modulate the activity of CPGs. This can be used to model robot gait for controlled, autonomous and even swarm robots that have a common higher level processing system. according to environmental conditions.

- Learning CPG trajectories can involve either:-
  - supervised learning methods - (when the rhythmic pattern is known) (Gradient descent, statistical learning algorithms)
  - Unsupervised learning method - (when the rhythmic pattern is not known) (Evolutionary algorithms like genetic algorithms and simulated annealing)
  - Parameters that are optimized are synatic weights and coupling weights (a lot like deep learning)

- Parameters that need to be ~~found to design the~~ established to design a CPG based controller.
  - Neuron to model each ~~rib~~ limb/joint
  - Neural Network architecture
  - Learning method.
  - Robot-network mapping ( ~~how a neuron models what~~ (deciding what part of the robot locomotion system a neuron models).
  - ~~coupling~~ - cost function
- Van der Pol oscillator is very simple could be the first neuron that can be used to model gait for the robot

- Central Pattern generators - ~~Rev~~ Review of previous uses in literature
  - CPGs can generate rhythmic signals without the need for a rhythmic input or
  - sensory feedback
  - CPGs provide ~~with~~
    - distributed control
    - ability to deal with redundancies
    - fast control loops and
    - Modulation of locomotion by ~~sig~~ simple signals, as mentioned earlier.
  - Although sensory feedback is not needed for the generation of rhythmic patterns by CPGs, sensory feedback can be used to modulate the rhythmic pattern according to ~~the~~ environmental or robot conditions.
  - Simulation of the CPG network by sensory feedback or high level control signals, can be used to modulate the locomotion patterns and even achieve gait tranistion and switching ~~based on th~~ determined by bifurcations in the CPG network due to the modulating signal.
- According to neuroscience research, the vertebrae locomotor system is organized such that the spinal CPG are responsible for producing the basic rhythmic ~~pat~~ patterns and higher level signals are responsible for modulating these patterns according to ~~enviro~~ environmental conditions.
- Such a distributed organization presents several features like :-
  - (i) ~~Time~~ Reduction in time delay in the motor control loop
  - (ii) Reduction of the dimensionality of the descending control system.
  - (iii) Reduction of the necessary bandwidth between the higher level centers and the spinal chord.
- CPG models could be
  - detailed biophysical models that can model rhythmogenesis, like Hodkin-Huxley model
  - connectionist models that model affect of network topology and synchronization on rhythmogenesis
  - Neuro-mechanical models that also model the effects of biophysial signals on rhythmogenesis.

- CPGs for Robot Locomotion
    - Major CPG models implemented for robotics applications include
        - connectionist models
        - Vector maps
        - System of coupled oscillators.
        - spiking neural network models.
    - Virtually all CPG implementations involve sets of coupled differential equations that are numerically integrated on a microprocessor
    - CPGs can also be directly implemented into hardware on a chip or with analog electronics.
    - Quadruped walking control using CPGs have been extensively explored by Hiroshi Kimura and his **colleagues.** work by Ijspeert and his colleagues can also be referred to.
    - Properties of CPGs that make them interesting for locomotion ~~appli~~ control -
        - CPGs exhibit limit cycle behaviour
        - CPGs can be coupled together for distributed implementations.
        - CPGs can be modulated with low dimensional control signals (mentioned earlier)
        - CPGs can easily integrate sensory ~~feedback~~ feedback.
        - CPGs can be plugged with different learning and optimization algorithms.
    - A locomotion problem would involve solving for -
        - rhythmogenesis
        - co-ordination between degrees of freedom
        - Control of balance ~~and modulation~~
        - modulation of speed
        - modulation of heading direction.
- Design methodologies for CPGs -
    - The following items need to be decided (some mentioned previously) to design a CPG based controller
        - **The** general architecture (type and number of oscillator neurons etc.)
        - **The type** and topology of neuronal couplings
        - The waveforms determing the trajectories of the robot limbs
        - Affect of input signal on rhythmogenesis.
        - Affect of feedback from the body on rhythmogenesis.
- The theory of dynamical systems is used to determine almost all of the above items.
- This project will mostly involve the use of unsupervised learning methods to learn the appropriate parameters for the CPG. (E.g Evolutionary optimization algorithms like the Genetic Algorithm)

- The CPG parameters that are supposed to be optimized are-
  - synaptic weights in fixed neural network architectures
  - coupling weights in a system of coupled oscillators.
  - In some cases the architecture of CPGs is also evolved.

- ~~Bio~~ Exact biological CPG models are deemed to be too computationally expensive for real time implementations. Simplified neuron models need to be defined for real time implementations.

- ~~Rulkov Map type neuron~~
  - ~~More efficient than continuous time models like HH model,~~
  - ~~Can exhibit spiking dynamics that can be used to model biological signals.~~
  -

- Mathematical models of synchrony like the Kuramoto oscillator can be used to model CPG behaviours. Such models are used to explain the effects of inter-oscillator coupling on rhythmogenesis. Even though such models are very simple, it is possible to replicate CPG characteristics
- Equations of a Kuramoto model of synchrony :-

$$\dot{\phi}_i = \omega_i + \sum_j (w_{ij} \times r_j \times \sin(\phi_j - \phi_i - \varphi_{ij}))] \quad - (1)$$

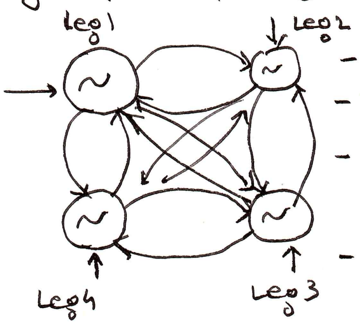
$$\ddot{r}_i = a_r \left(\frac{a_r}{4}(R_i - r_i) - \dot{r}_i\right) \quad - (2)$$

$$\ddot{x}_i = a_x \left(\frac{a_x}{4}(X_i - x_i) - \dot{x}_i\right) \quad - (3)$$

$$\theta_i = x_i + r_i \sin\phi_i \quad - (4)$$

Where,
- $\theta_i$ is the output angle of each oscillatory center
- $\phi_i$, $r_i$ and $x_i$ are the state variables that encode the variation in time of every phase, amplitude and offset
- The control parameters for each oscillator are $\omega_i$ (natural frequency), $R_i$ (target amplitude) and $X_i$ (target offset
- $a_r$ and $a_x$ are constant positive gains for Eqns (2) and (3). &
- Finally $w_{ij}$ and $\varphi_{ij}$ are respectively the coupling weights and phase biases that determine how oscillator $j$ influences oscillator $i$.

- A ~~CPG~~ simple model for quadruped gait ~~in a CPG~~ to can be




- ~~Each~~ The network is fully connected
- Each CPG controls a leg
- Need for defining how ~~second servo~~ leg servo will be controlled.
- Each CPG model will be given as input the control parameters ($\omega_i$, $R_i$, $X_i$, $w_{ij}$ & $\varphi_{ij}$)

# Analysis of the Kuramoto model

- The dynamics of the model are driven by the dominating factor of equation 1. In the absence of interactions all oscillators will oscillate with their natural frequency $\omega_i$. The second term is responsible ~~of~~ for convergence of a population of oscillators to a same phase, leading to synchronization.
For bigger values of $r_j$ and $w_{ij}$ synchronization is achieved quickly
- ~~The~~ Values of $\varphi_{ij}$ add a constant phase difference in the

## Trajectory for quadruped locomotion.

- A sprawling type quadruped robot is available for testing of designed trajectories.
-

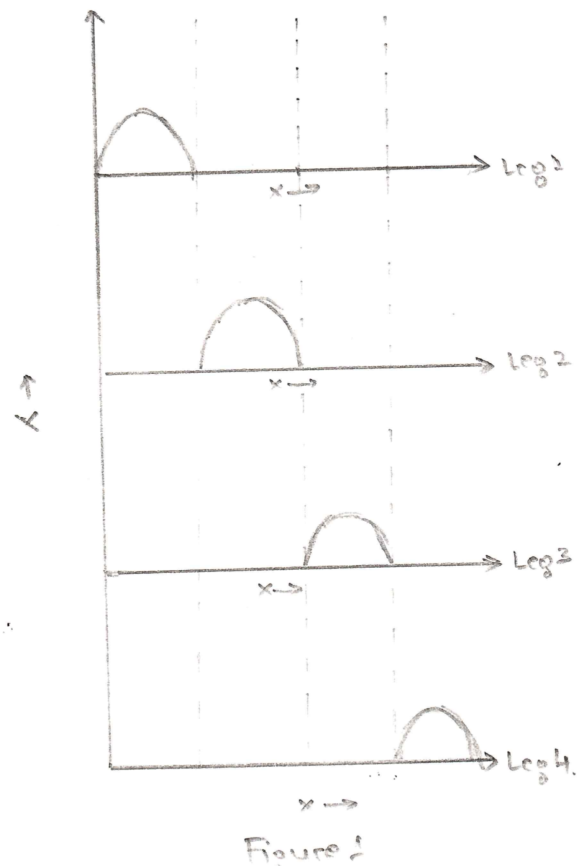



Figure 1

Figure 1 depicts the trajectory of each leg to obtain a creeping gait in the robot. The measured variable is the position of the end point of the leg

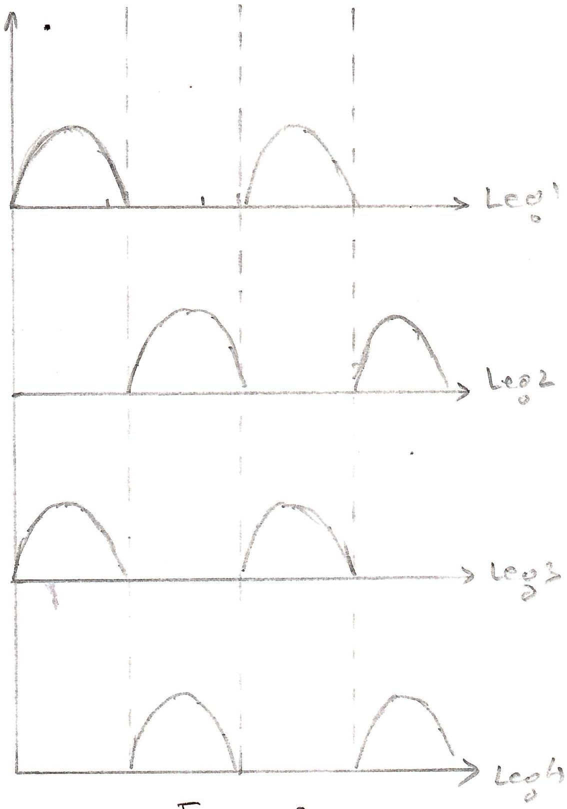



Figure 2.

Figure 2 depicts the trajectory to obtain a trot gait in the robot. The ~~measured~~ measured variable is the position of the end point of the leg.

- Refer to the CPG diagram on the previous page for ~~to le~~ leg reference.
-

- Wilson-Cowan central nervous oscillator model


Figure 3. Wilson cowan central nervous oscillator model.

A general Wilson cowan central nervous oscillator model is defined by the following equations

$$T_u \frac{du}{dt} + u = f_\mu(au - bv + S_u) \quad - \quad (5)$$

$$T_v \frac{dv}{dt} + v = f_\mu(cu - dv + S_v) \quad - \quad (6)$$

$$f_\mu(x) = \tanh(\mu x) \quad - \quad (7)$$

where

$a$ is the excitatory connection strength of neuron
$d$ is the inhibitory connection strength of neuron.
$b$ is the inhibitory connection strength from $v$ to $u$,
$c$ is the ~~inter~~ exitatory connection strength from $u$ to $v$
$S_u$ and $S_v$ are external signals
$T_u$ is the rise time constant of step input
$T_v$ is the fatigue time constant
$f_\mu(x)$ is the coupling function
$\mu$ is the gain of $f_\mu(x)$

- Implementation of the Wilson-Cowan nervous oscillator model in a CPG.
  - The CPG model on page 5 can be implemented using the Wilson-Cowan central nervous model with a few adjustments, for locomotion tasks.
  - Equations (5) through (7) will be modified as follows :-

$$T_{u_i} \frac{du_i}{dt} + u_i = f_\mu\left(au_i - bv_i + \sum_{j=1}^{n} w_{ij} u_j + \sum_{k=1}^{m} S_{ik} g_k + S_{u_i}\right) \quad - \quad (8)$$

$$T_{v_i} \frac{dv_i}{dt} + v_i = f_\mu\left(cu_i - dv_i + \sum_{j=1}^{n} w_{ij} v_j - \sum_{k=1}^{m} S_{ik} g_k + S_{v_i}\right) \quad - \quad (9)$$

$$f_\mu(x) = \tanh(\mu x) \quad - \quad (10)$$

$$Y_{out}^{i} = P(u_i - v_i) \quad - \quad (11)$$

Where,

$P$ is an amplitude limiting coefficient to adjust the output

$s_{ik}$ is the reflection information (from external feedback in the $i$th oscillator)

$g_k$ is coefficient of $s_{ik}$

$w_{ij}$ is an element from the connection matrix.

- Drive functions of the gait.
  - To be able to learn, the parameters for all the aforementioned oscillator models, the drive function to approximate the movements of the legs needs to be formulated. The drive function is, in other words, the teaching signal that the central pattern generators have to emulate to be able to successfully control the quadruped gait.
  - Drive function for the walking gait.

    ~~Representing the phase of~~

    - The following relative phases can comprise a walking gait.

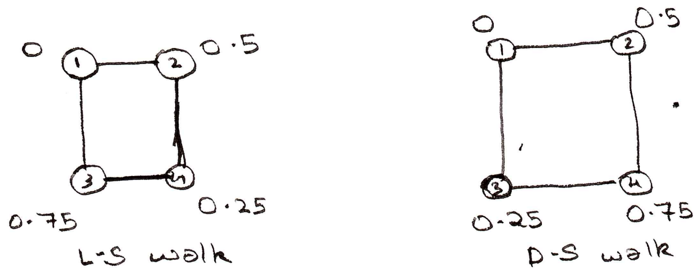



Figure 4.

Each node in figure 4 represents a leg and the corresponding value is the relative phase of the leg.

For the available robot the knee ~~has~~ joint has a predefined relationship with the hip joint movement. The following equation can be used to model this relationship:-

$$\theta_k(t) = \begin{cases} 0 & \forall\ \dot{\theta}_h < 0 \\ \mathrm{sgn}(\varphi)\,\alpha_k A_k \left[1 - \left(\dfrac{\theta_h(t)}{\alpha_h A_h}\right)^2\right] & \forall\ \dot{\theta}_h \geq 0 \end{cases}$$

$$\varphi = \begin{cases} 1 & \text{elbow joint} \\ -1 & \text{knee joint} \end{cases}$$

All joints in the robot are knee type joints.

- For hip joint

$$F_h(t) = \begin{cases} A\sin\left(\tfrac{2}{3}(2\pi f t - 1.5\pi)\right) & 0 \leq t \leq \tfrac{3}{8}T \\ A\sin\left(2(2\pi f t - 1.0\pi)\right) & \tfrac{3}{8}T \leq t \leq \tfrac{5}{8}T \\ A\cos\left(\tfrac{2}{3}(2\pi f t - 1.25\pi)\right) & \tfrac{5}{8}T \leq t \leq T \end{cases} \quad - (12)$$

where

$A$ is the amplitude of oscillation

$f$ is frequency and $T$ is time period.

$f = 1/T$

- The equation 12 varied by appropriate relative phase can be used to drive all four hip joints, as ~~the~~ all legs are equivalent for a ~~sq~~ sprawling type quadruped. For walking gait phase difference is $\frac{1}{4}T$ for each leg

- Knee drive function

$$F_k(t) = \begin{cases} 0 & 0 \le t \le \frac{3}{8}T \\ A\sin(2\pi f t - 0.75\pi)) & \frac{3}{8}T \le t \le \frac{5}{8}T \\ 0 & \frac{5}{8}T \le t \le T \end{cases} \quad - (13)$$

- The drive functions defined above can be used to learn the parameters for any of the CPG architectures previously designed by defining an appropriate objective function for an optimization algorithm.

* <u>Trajectory Generation And Modulation</u>
  - Trajectory generation and modulation is required for locomotion in dynamic and partially unknown environment.
  - A system capable of trajectory modulation would modify the parameters of smaller blocks of motor primitives, which can be modulated to perform in any given environment
  - The objective of this implementation is to design a system capable of autonomously detecting and reaching a target while avoiding obstacles in its path modulating the generated trajectory as required.
  - The modulation of the CPG parameters is encoded in high level operational locomotion parameters as the desired walking orientation, translational speed and angular velocity of the desired motion, similarly to signals derived from the brainstem of biological systems.
  - Dynamical system implemented -
    - Hopf Oscillator described by the following equations

$$\dot{x} = \alpha(\mu v - r^2)(x - O) - \omega_i z \quad - (14)$$

$$\dot{z} = \alpha(\mu v - r^2)z + - \omega_i(x - O) \quad - (15)$$

The hopf oscillator has low computational cost

where

$r = \sqrt{(x-O)^2 + z^2}$

$x$ and $z$ are the state variables

$v$ is the parameter controlling the bifurcation in the oscillator and $v \in \{-1, 1\}$

$\mu$ is ~~as~~ an oscillator parameter

$\omega$ specifies oscillation frequency

The speed of convergence to either the limit cycle (stable harmonic oscillation around $(x, z) = (O, 0)$ for $\mu v > 0$ or fixed point for $\mu v < 0$ depends on $|1/2\alpha\mu v|$

- For using the above formulation with the available robot, ~~the~~ a complex hopf oscillator can be used to compute the trajectory of the joint angles intead of computing the position of legs in the x-z plane as shown in the aforementioned formulation. Or as mentioned in the paper, of the x-z axes only x axis may be used for control of locomotion and z axis may be used for coupling of weights.
- The following is the significance of the parameters of the hopf oscillator
    - $v \in \{-1,1\}$, switches on/off the rhythmic trajectory
    - $\mu > 0$, modulates the amplitude of oscillations
    - $i \in \{-1,1\}$ modulates the direction of the trajectories
    - $\beta \in ]0,1[$ changes the walking velocity
    - $O$ sets the value of the oscillation's offset for steering
    - $\omega_{sw}$ - specifies the swing phase duration.
- Unlike the robot ~~with~~ mentioned in the paper (CPG modulation for navigation and omnidirectional quadruped locomotion), the available robot has only a single joint at the hip and another joint in the knee. Thus like the previous case, the knee joint will controlled by the oscillator in the hip joint.
    - ~~while pe~~ When the limb performs the swing phase, the knee flexes to a fixed angle $\theta_{sw}$. When performing the stance phase the knee extends to another fixed angle $\theta_{st}$.

      This motion can be generated by the following system of equations

$$\dot{y} = v \qquad -(16)$$

$$\dot{v} = -\frac{b^2}{4}(y-g) - bv \qquad -(17)$$

$$g = \frac{\theta_{st}}{e^{-az_s}+1} + \frac{\theta_{sw}}{e^{az_s}+1} \qquad -(18)$$

where the state variable $y$ is the control trajectory for the knee and follows the value of $g$,

$z_s$ is ~~the~~ the z state variable from the hip joint

- Unlike the previously considered oscillators, the neurons in this model are coupled via differential equations and not a connection matrix only..

The modified equation for the coupled oscillators is given as follows

$$\cancel{\begin{bmatrix} \dot{x}_i \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} \alpha(\mu v - r^2)(x_i - O) - \omega_i z + K_i \sum R(i,j\ \theta_i^j) \begin{Bmatrix} \frac{x_j - O_j}{r_j} \\ \frac{z_j}{r_j} \end{Bmatrix} \\ \alpha(\mu v - r^2) z - \omega_i z + 1 \end{bmatrix}}$$

$$\begin{bmatrix} \dot{x}_i \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} \alpha(\mu v - r^2)(x_i - O) - \omega_i z_i + K_i \sum R(i_j, \theta_i^j) \frac{(x_j - O_j)}{r_j} \\ \alpha(\mu v - r^2) z - \omega_i z_i + K_i \sum R(i_j, \theta_i^j) \frac{z_j}{r_j} \end{bmatrix} \quad - (16)$$

where

$K_i$ specifies the coupling strength

$R(\theta_i^j)$ is the rotation matrix, which rotates the linear terms ~~an~~ to each other

$\theta_i^j$ is the relative phase between the oscillators $i$ and $j$ to perform a certain gait.

$$\theta_i^j = (\varphi_i - \varphi_j) \cdot 2\pi \quad - (17)$$

- The final CPG would be represented by the following figure :→

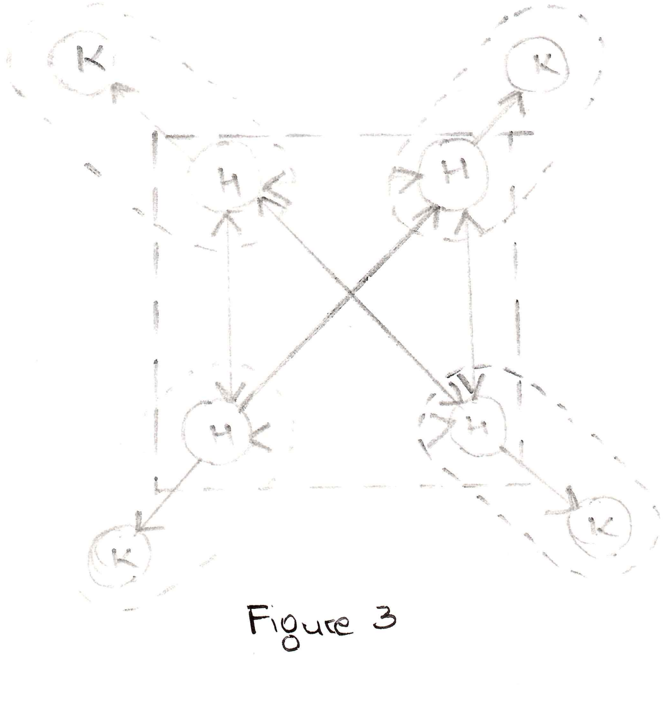



Figure 3

The four H unit neurons form the base CPG and ~~the~~ a pair of H and K neuron form a limb CPG. This architecture is very similar to the previously analyzed architecture.

- From the survey of literature performed so far, it can be concluded that the basic architecture of the CPG would be as depicted in figure 3 on the previous page, irrespective of the oscillatory neuron being chosen to model the CPG. The next step in the development of a robust controller would be the development of higher level brain functions and implementation on an arduino.

- ## CPG Modulation & Trajectory Generation

  - CPG modulation and trajectory generation constitute the higher level functions of the controller.
  - Before implementing the higher level functions design decisions as mentioned in the list below need to be made
    - What kind of modulation can the higher level functionalities perform?
    - Can ~~With~~ these ~~functionalit~~ higher level functionalities be implemented on the arduino used to control the gait or is there a need for a more powerful computation device for the implementation?
    - What kind of sensory information will the higher level functionalities be using to modulate the parameters?

- The higher level functions should be able to perform the following :->
  - Signal from higher level functions should be able to cause gait transition
  - Signal should be able to modulate the CPG parameters to be able to cause omnidirectional motion (forward, backwards, sidways, ~~rotating~~ circular)
  - Signals from the higher level functions should be able to modulate the velocity of the motion
  - Signals from the higher level functions should be able to cause ~~make~~ all the aforementioned actions given a sensory feedback from the image and ultrasonic sensor models present on the robot.
  - Signals from the higher centers are able to steer the robot