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Optimal Trajectory Generation for a Biped Robot Walking a Staircase based on Genetic Algorithms

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***Abstract*— In this paper, we propose optimal trajectories of biped robots to move a staircase up and down using genetic algorithms and computed-torque controller to be dynamically stable. Firstly, in order to minimize the total energy efficiency, the Real-Coded Genetic Algorithm (RCGA)which of Operators are composed of the reproduction, crossover and mutation is used. Constraints are divided into equalities and inequalities: Equality constraints consist of position conditions at the start and end of stride period and repeatability conditions related to each joint angle and angular velocity. Next, inequality constraints include the collision avoidance conditions of swing leg at the face and edge of a stair, the knee joint conditions with respect to the singular avoidance, and the zero moment point condition with respect to the stability into the going direction. Finally, in order to approximate the walking gait, each joint angle trajectory is defined as a 4-th order polynomial of which coefficients are chromosomes. In the computer simulation used 6 degree of freedom biped robot model that consists of seven links, we analyze the energy efficiency of proposed optimal trajectories about the following cases :walking a ground, ascending stairs, and descending stairs.**

I. Introduction

Human beings walk stably under the various locomotion conditions of the environment and generate a walking pattern for minimizing energy. Therefore, many researches for the biped robot have been studied for the structure similar to human, the adaptability in various terrains and the generation of energy-efficient trajectories.

Generation of a gait trajectory for the biped robot has been proposed by many methods. Park and Kim [1] proposed the gravity-compensated inverted pendulum mode (GCIPM).

In the study to generate low energy gaits, various optimization algorithms are used and the variables of basis functions to approximate the walking gait are used for design variables generally. Choi et al. [2] proposed that optimal via-points data are found using genetic algorithm which minimizes the sum of deviation of velocities and accelerations as well as jerk. Cheng and Lin [3] proposed that the design of the controller and the gait is formulated as a parameter search problem and a genetic algorithm is applied to help the design. Park and Choi [5] proposed a method that minimizes the energy consumption by finding the optimal locations of the mass centers of the links, and the optimal trajectory of the legs. Chevallereau et al. [6] searched for the optimal stride and period to generate optimal trajectories.

Walking a staircase is one of the irregular ground conditions. Shih [7] proposed to synthesize an efficient walking pattern for a practical biped robot when ascending and descending stairs for a biped robot with 7 DOF. Jeon and Park [8] proposed to generate a trajectory to minimize the energy gait of a biped robot for ascending stairs using genetic algorithms. Hwang et al. [9] described an $O(n)$ dynamic simulation of a humanoid robot. The proposed simulation method used a virtual spring-damper contact model in order to simulate precisely a collision with friction between the foot and a floor.

In this paper, we generate a trajectory to minimize energy gait of a biped robot for walking a staircase using genetic algorithms and apply to the computed torque controller for the stable dynamic biped locomotion. In the saggital plane, a 6 degree of freedom biped robot model that consists of seven links is used. In order to minimize the total energy, the Real-Coded Genetic Algorithm (RCGA) is used. In order to approximate the walking gait, each joint angle trajectory is defined as a 4-th order polynomial of which coefficients are chromosomes. In the computer simulation, we analyze the energy efficiency of the proposed method about the following cases : walking a ground, ascending stairs, and descending stairs.

The dynamics of a biped robot is described in Section II. The constraints and formulas for walking a staircase are presented in Section III. Section IV describes computer simulation and comparisons of the energy efficiency, followed by conclusions in Section V.

II. Biped Robot Model

In the saggital plane, a 6 degree of freedom biped robot model that consists of seven links, as shown in Fig. 1, is used in the study [10][11].

The dynamic equation applied to a biped robot is derived from Lagrange's equation substituted from the kinetic and potential energies of each link. The dynamic equation can be written as:

$$M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})+G(\theta)=E\tau \qquad (1)$$

where $M(\theta)\in R^{6\times 6}$ is the inertia matrix, $C(\theta,\dot{\theta})\in R^{6\times 1}$ is the vector of coriolis and centrifugal force, $G(\theta)\in R^{6\times 1}$ is

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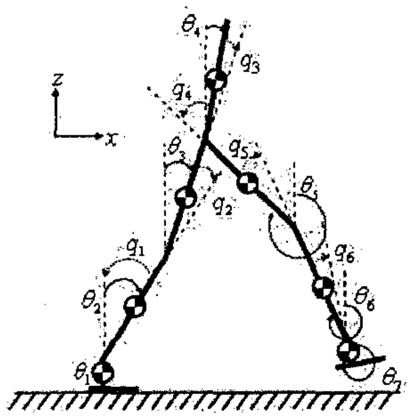

Fig. 1. 6 DOF biped robot model

the vector of gravity force, $\tau \in R^{6\times 1}$ is the vector of torque for each joint. And $E \in R^{6\times 6}$ is

$$E = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The parameters of the biped robot model are listed in Table 1.

TABLE I
PARAMETERS OF THE BIPED ROBOT

Link No.	Length (m)	Mass (kg)
Link 1	0.1	1.0
Link 2	0.4	5.0
Link 3	0.4	4.0
Link 4	0.5	6.0
Link 5	0.4	4.0
Link 6	0.4	5.0
Link 7	0.1	1.0

We assume that the mass of link is concentrated on the center of each link. We also assume that walking cycle of a biped robot is divided into two phases that repeat alternately, i.e., the single support phase and double support phase.

III. Energy-Optimized Trajectory by Genetic Algorithms

A. Genetic Algorithms

Operators of genetic algorithms are composed of the reproduction, crossover, and mutation. The reproduction increases average fitness values, the crossover exchanges chromosomes' information and the mutation prevents convergence to a local minimum/maximum [12]. Parameters of genetic algorithms are listed in Table 2.

TABLE II
PARAMETERS USED IN GENETIC ALGORITHMS

Parameters	Values
Maximum Generations	5,000
Population	30
Chromosome Length	14
Crossover Ratio	0.9
Mutation Ratio	0.02

In this study, instead of a Binary-Coded Genetic Algorithms (BCGA), we used a Real-Coded Genetic Algorithms (RCGA) because BCGA has many problems in the practical applications [13].

B. Equality Constraints

Equality constraints consist of the position conditions and the repeatability conditions.

Firstly, in order to walk a staircase for the biped robot, the tip position of swing leg must be satisfied with following the position conditions at the start and end of the stride period.

$$\text{At } t=0: \quad x_{tip}(0) = -S,\ z_{tip}(0) = -H \quad (3)$$

$$\text{At } t=t_f: \quad x_{tip}(t_f) = S,\ z_{tip}(t_f) = H \quad (4)$$

$$\text{At } t=0: \quad x_{tip}(0) = -S,\ z_{tip}(0) = H \quad (5)$$

$$\text{At } t=t_f: \quad x_{tip}(t_f) = S,\ z_{tip}(t_f) = -H \quad (6)$$

where t_f is a stride period, S is a stride and H is a stair height. Eqs. (3) and (4) are equality constraints which are related to ascending stairs and Eqs. (5) and (6) are equality constraints when descending stairs.

On the other hand, for the biped robot to have steady and repeatable walking pattern, the following the repeatability conditions should be satisfied.

$$q_i(0) = q_{7-i}(t_f) \quad (i = 1, \cdots, 6) \quad (7)$$

$$\dot{q}_i(0) = \dot{q}_{7-i}(t_f) \quad (i = 1, \cdots, 6) \quad (8)$$

where q_i is defined as the absolute coordinates based on the vertical axe as shown in Fig. 1. And, the impact at contacting the ground is ignored.

C. Inequality Constraints

Also, inequality constraints consist of the stair conditions, the knee joint conditions, and the ZMP condition.

In order to walk a staircase for a biped robot during a stride period, position of swing leg must be satisfied with following the stair conditions, Fig. 2 :

As shown in Fig. 2 (a), when the biped robot ascends stairs, the important points are the tip position of swing leg coming

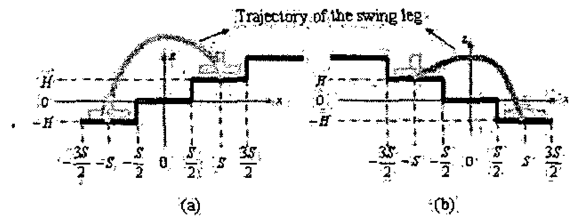

Fig. 2. (a) when ascending stairs (b) when descending stairs

up above the stairs, eqs. (9) to (11), and the toe position of swing leg to avoid colliding with the stairs, eqs. (12) to (14).

$$\text{if } -\frac{3S}{2} < x_{tip}(t) \le -\frac{S}{2} \quad \text{then } z_{tip}(t) > -H + \delta h \quad (9)$$
$$\text{if } -\frac{S}{2} < x_{tip}(t) \le \frac{S}{2} \quad \text{then } z_{tip}(t) > \delta h \quad (10)$$
$$\text{if } \frac{S}{2} < x_{tip}(t) \le \frac{3S}{2} \quad \text{then } z_{tip}(t) > H + \delta h \quad (11)$$

$$\text{if } -H < z_{toe}(t) \le 0 \quad \text{then } x_{toe}(t) < -\frac{S}{2} - \delta s \quad (12)$$
$$\text{if } 0 < z_{toe}(t) \le H \quad \text{then } x_{toe}(t) < \frac{S}{2} - \delta s \quad (13)$$
$$\text{if } H < z_{toe}(t) \le 2H \quad \text{then } x_{toe}(t) < \frac{3S}{2} - \delta s \quad (14)$$

And Fig. 2 (b) show the tip position of swing leg coming up above the stairs, eqs. (15) to (17), and the heel position of swing leg to avoid colliding with the stairs, eqs. (18) to (20), when the biped robot descends stairs.

$$\text{if } -\frac{3S}{2} < x_{tip}(t) \le -\frac{S}{2} \quad \text{then } z_{tip}(t) > H + \delta h \quad (15)$$
$$\text{if } -\frac{S}{2} < x_{tip}(t) \le \frac{S}{2} \quad \text{then } z_{tip}(t) > \delta h \quad (16)$$
$$\text{if } \frac{S}{2} < x_{tip}(t) \le \frac{3S}{2} \quad \text{then } z_{tip}(t) > -H + \delta h \quad (17)$$

$$\text{if } -H < z_{heel}(t) \le 0 \quad \text{then } x_{heel}(t) > \frac{S}{2} + \delta s \quad (18)$$
$$\text{if } 0 < z_{heel}(t) \le H \quad \text{then } x_{heel}(t) > -\frac{S}{2} + \delta s \quad (19)$$
$$\text{if } H < z_{heel}(t) \le 2H \quad \text{then } x_{heel}(t) > -\frac{3S}{2} + \delta s \quad (20)$$

where δh and δs are a safe boundary in order that it may be avoided that the swing leg collide with the stair. And time t don't include $t = 0$ and $t = t_f$.

The following inequality constraints are the knee joint conditions, eqs. (21) and (22). The knee joint condition is to have a human-like locomotion and to avoid the singular configuration.

$$\theta_2 - \theta_3 > \delta\theta \quad (21)$$

$$\theta_6 - \theta_5 > \delta\theta \quad (22)$$

where $\delta\theta$ is a safe boundary.

The final inequality constraint is related to the stability of a biped robot, that is the ZMP condition, eq. (23). Thus, when the ZMP exists within the foot print of the supporting leg, we can say that the biped robot is stable and does not fall down.

$$\|x_{ZMP}\| < \frac{\triangle}{2} \quad (23)$$

where $\triangle$ is a safety boundary with respect to the size of the robot foot.

$$x_{ZMP} = \frac{\sum_{i=1}^{6} m_i(\ddot{z}_i + g)x_i - \sum_{i=1}^{6} m_i \ddot{x}_i z_i - \sum_{i=1}^{6} I_i \ddot{\theta}_i}{\sum_{i=1}^{6} m_i(\ddot{z}_i + g)} \quad (24)$$

where $(x_i,\ z_i)$ is the position of the mass center of link i and I_i is the inertia moment of link i.

D. Optimization Methods

In order to approximate the walking gait, each joint angle is defined as a 4-th order polynomial of time t.

$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \\ t^4 \end{bmatrix} \quad (25)$$

where coefficient, $a_{i,\ j}$ $(i = 1, \cdots, 6\ ,\ j = 1, \cdots, 5)$, are design variables. Therefore, the total number of design variables is 14 except for 16 of equality constraints, Eqs. (3) to (8).

The performance index to be minimized is :

$$J = \frac{1}{2}\int_0^{t_f} p^T Q p dt \quad (26)$$

where $p = \tau\dot{q}$ describes the powers applied at each joint and $Q = diag[\omega_1\ \omega_2\ \omega_3\ \omega_4\ \omega_5\ \omega_6]^T$ is positive definite matrix and the elements of Q, $\omega_{1\sim6}$, are the weighting factor on control torque of relative driving actuators.

The inequality constraints, Eqs. (9) to (23), are as follows:

$$g_j(\alpha) \le 0 \ (j = 1, \cdots, n) \quad (27)$$

Transformation methods convert the constraint optimization problem defined in Eqs. (9) to (23) into an unconstraint problem for the transformation function :

$$F(\alpha,\ r) = J(\alpha) + P(g(\alpha),\ r) \quad (28)$$

where r is a vector of penalty parameters and P is a real valued function which of imposing the penalty is controlled by r. The form of penalty function P depends on the transformation method used. The transformation method used in this paper is the exterior penalty function method :

$$P(g(\alpha),\ r) = \sum_{j=1}^{n} r_j [g_j^+(\alpha)]^2 \qquad (29)$$

where $g_j^+(\alpha) = \max(0,\ g_j(x))$, and r_j is a scalar. The value of function $g_j^+(\alpha)$ is zero if inequality is inactive, $(g_j(\alpha) < 0)$, and it is positive if inequality is violated, i.e. $(g_j(\alpha) > 0)$.

Therefore, the flow chart of genetic algorithms is as shown in Fig. 3.

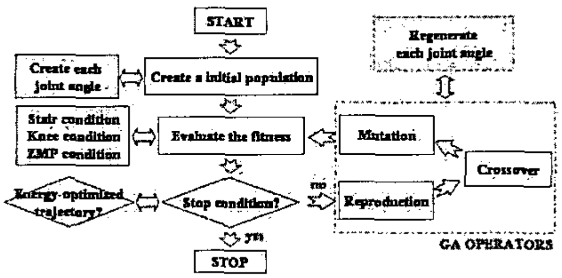

Fig. 3. Flow chart of genetic algorithms

In order to search the energy-optimized trajectory, the coefficients of 4-th order polynomials are used as design variables. Firstly, joint angles are created from the initial population. And then they are evaluated whether stair condition, knee joint condition, and ZMP condition are satisfied or dissatisfied. If an energy-optimized trajectory are searched, the searching process are stopped. If they are dissatisfied with the condition, GA operators regenerate each joint angle.

IV. Simulations

The effectiveness and performance of the energy optimized trajectories are shown in computer simulation. We used MATLAB program in order to simulate locomotion of the biped robot for walking a staircase. The parameters of genetic algorithms are as previously shown in Table 2. Genetic operators are used that reproduction is a gradient-like selection method, crossover is a modified simple crossover method, and mutation is boundary mutation method. And the parameters of model used in computer simulations are shown in Table 3 :

TABLE III
PARAMETERS USED IN SIMULATIONS

Parameters	Values
S	0.3 m
H	0.05 m
δs	0.001 m
δh	0.001 m
$\delta\theta$	0.001 rad
$\triangle$	0.19 m
Stride Period	1.0 sec

A. Ascending Stairs

Figure 4 is compared with the cost functions of inequality constraints. Figure 4 (a) is the performance index, (b) is the stair condition, (c) is the knee joint condition, and (d) is the ZMP condition. Figure 4 (a) shows how quickly the performance is improved as the generation progresses. Figures 4 (b) to (d) show that the value of inequality constraint term come to zero.

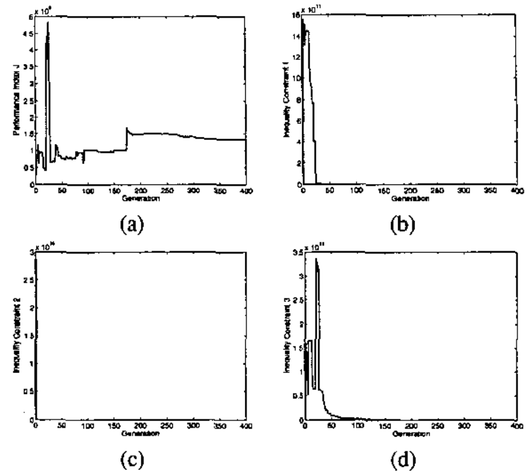

Fig. 4. Cost functions of inequality constraints when ascending a stair

Figure 5 is the torque and power on each joint with the locomotion generated by genetic algorithms. The dotted lines (torque6 and power6) in Fig. 5 show that the magnitudes of the torque and power are almost zero. This means that the ankle joint of the swing leg consumes little energy. And Figs. 6 and 7 are the locomotion of the biped robot for ascending a stair using genetic algorithms. It is similar to locomotion of a human for ascending a stair. Especially, we know that the foot moves to some backward during beginning of a stride period in order to avoid that it collide with the stair.

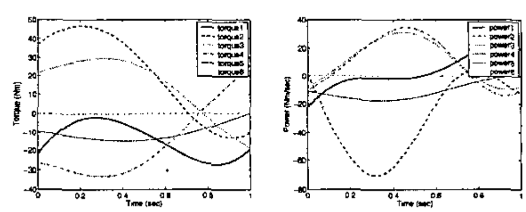

Fig. 5. Torque and power on each joint when ascending a stair

B. Descending Stairs

As ascending stairs, the cost functions of inequality constraints when descending stairs, as shown in Figs. 8 (b) to (d), converge to zero. Also Figs. 10 and 11 are similar to locomotion of a human for descending a stair.

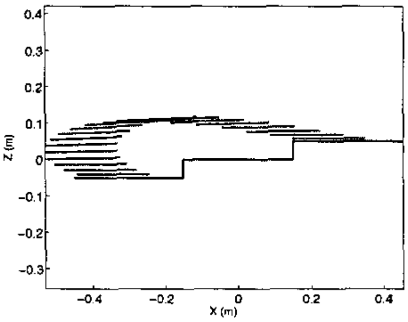

Fig. 6. Foot diagram in locomotion when ascending a stair

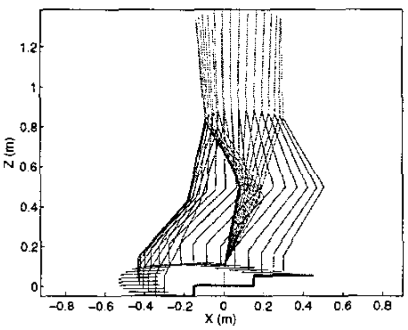

Fig. 7. Stick diagram of the biped robot when ascending a stair

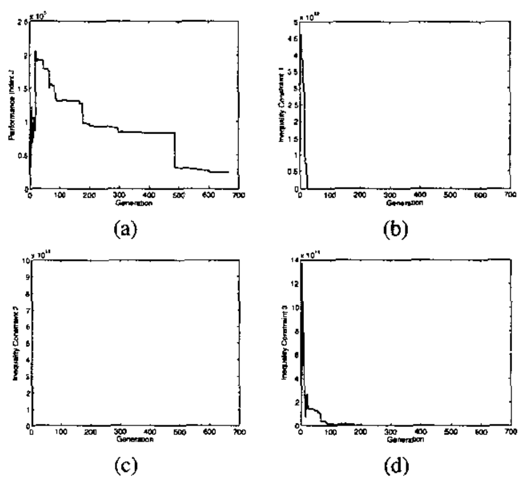

Fig. 8. Cost functions of inequality constraints when descending stairs

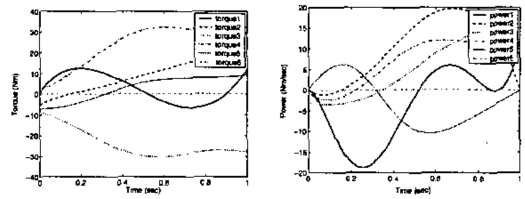

Fig. 9. Torque and power on each joint when descending a stair

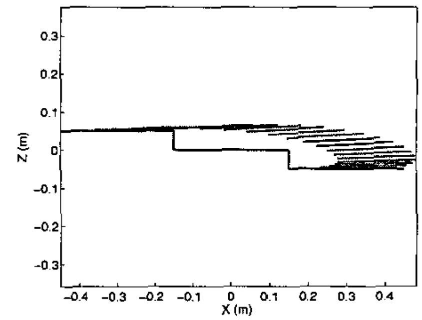

Fig. 10. Foot diagram in locomotion when descending a stair

C. Comparison of the Energy Efficiency

The computed torque controller is used for the stable dynamic locomotion of a biped robot. Angle, angular velocity, and angular acceleration on each joint which are searched by genetic algorithms are used into the desired trajectory. Figure 13 is the locomotion of the biped robot for walking a staircase. The biped robot begins to walk a ground at speed of $0.3m/s$, ascends stairs, and descends stairs.

We analyze the energy efficiency of the proposed method about the following cases : walking a ground, ascending stairs, and descending stairs.

In Fig. 12, the vertical direction is a sum of the power square on each joint. Where the dotted line denotes the power of walking a ground. The dash-dotted line denotes the power

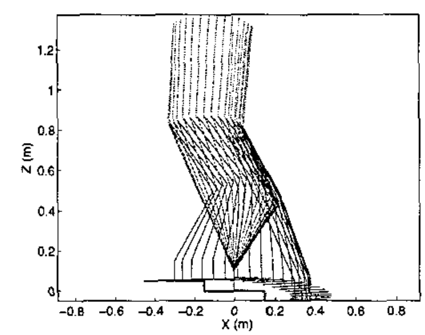

Fig. 11. Stick diagram of the biped robot when descending a stair

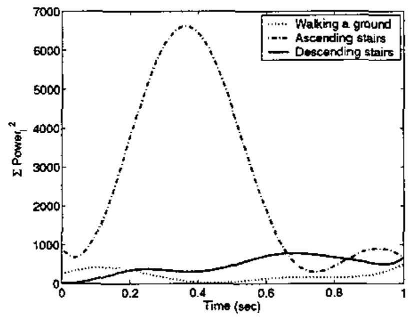

Fig. 12. Comparison of the energy efficiency

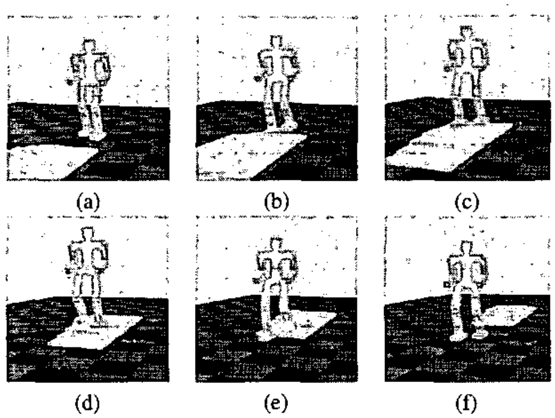

Fig. 13. Locomotion of the biped robot for walking a staircase

of ascending stairs. And the solid line denotes the power of descending stairs. As shown in Fig. 12, we know that walking a ground is the most efficient. Walking a ground is more efficient than ascending stairs, 92%, and than descending stairs, 54%. Also, descending stairs is more efficient than ascending stairs, 82%.

V. CONCLUSION

In this paper, we generate optimal trajectories for walking a staircase using genetic algorithms and apply to the computed torque controller for the stable dynamic biped locomotion. The computer simulations of walking a staircase show that the proposed method is very effective and walk stably. And this research compares the energy efficiency using the proposed method following cases : walking a ground, ascending stairs, and descending stairs. We know that walking a ground is the most efficient. In the future work, we will research for various stair height using the proposed method and consider the impact model at contacting the ground.

ACKNOWLEDGMENT

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