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# Gait Design and Comparison Study of a Quadruped Robot

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**Abstract**—This paper presents the gait design and comparison study of a quadruped robot. The main contributions of this paper are: (i) Explore the modularized system design of an 8 degree-of-freedom quadruped robot with a power management system. (ii) Propose a gait design method based on drive function. (iii) Propose another gait design method based on foot trajectory. (iv) The simulation of the walk and trot gaits with above-mentioned methods are validated in Webots. (v) The walk and trot gait are successfully implemented in the real robot. Experimental results show that the stable locomotion can be achieved with improved elliptical foot trajectory.

**Index Terms**—Quadruped robot system, gait design, gait comparison

## I. INTRODUCTION

Quadruped mammals have the excellent athletic ability such as dogs and cats. In the past decades, many scientists devoted themselves to the study of athletic ability of the four-legged robots and were eager to develop a quadruped robot with good locomotion performance. The systematic study of athletic ability of the quadruped robots started in MIT Legged Laboratory in 1980[1][2]. After that, the research of the quadruped robot became much popular and many outstanding quadruped robots have been developed in the world.

The BigDog have the exceptional mobility to traverse and autonomous navigation in the unstructured forest environments with obstacles like trees, rocks and complex ground features[3]. The HyQ is a 75kg hydraulically actuated quadruped robot and the body size is as large as the goat[4]. The ANYMal is able to carry out outdoor activities for 2h and crawl very steep stairs[5]. The StarLETH can be unmanned in 3D space at speed up to 0.7m/s, it can navigate over unperceived 5cm high obstacles and recover from unexpected external forces[6]. The ANYMal and StarLETH are torque controllable and actuated by high-torque electrical motors. The Cheetah is able to run outdoors at a relatively high speed about 6 m/s[7] and can jump the obstacles with a height of 40cm (80% of the leg length).

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The main feature among these quadruped robots is the excellent stable gait, which is the critical issue for quadruped mammals to search for food and escape from predators.

Muybridge's landmark of early photography, recorded many different gaits of mammals including walking, trotting, cantering, galloping in 1887[8], which allowed people, at a very early stage, to see clearly attitudes and gaits taken by quadruped mammals in motion.

Inspired by gaits of the quadruped mammals, many different types of methods have been proposed to design a gait for the quadruped robot's locomotion like drive function[9][10] and foot trajectory[11]. Particularly, the foot trajectory is the most critical gait design method such as elliptical trajectory[12], bezier trajectory[7], Third-Order spline interpolation trajectory[13], rectangular trajectory[13].

This paper is organized as follows. In Section II, the system structure and control frame of the robot are shown. In section III, we propose gait design methods based on drive function and foot trajectory, and compare the advantages and disadvantages between them. Thus, the improved elliptical trajectory is proposed. In section IV, the simulation and experiment results are displayed and discussed. Finally, we draw the conclusion in Section V, and the video link is given in Appendix.

## II. SYSTEM DESIGN OF QUADRUPED ROBOT

The design of quadruped robot includes following three different parts (i) The mechanical design of quadruped robot. Specifically, the Dynamixel AX-12 and Robotis Kits are applied in the robot. (ii) The electronic system design. The range of battery voltage is from 3.3v to 15v, and the power supply can last half an hour. (iii) The control frame of the system. Figure 1(b) and 2 shows the system structure of the real robot.

### A. Mechanical Design

Figure 1(a) shows the SolidWorks model of the quadruped robot. An 8-degree-of-freedom quadruped robot is designed to verify our proposed gait design methods. The size of the quadruped robot is shown in Table I. In our real prototype, the length of thigh and shank is 5.2cm and 4.1cm respectively.

Figure 1(b) illustrates the joint configuration of quadruped robot. LF, LH, RF, RH represents the left-front leg, left-hind leg, right-front leg, right-hind leg respectively. Each leg

TABLE I  
SIZE OF QUADRUPED ROBOT

Properties	Value
Body length	25.5 cm
Height	25.0 cm
Leg length	13.5 cm
L1 length	5.2 cm
L2 length	4.1 cm

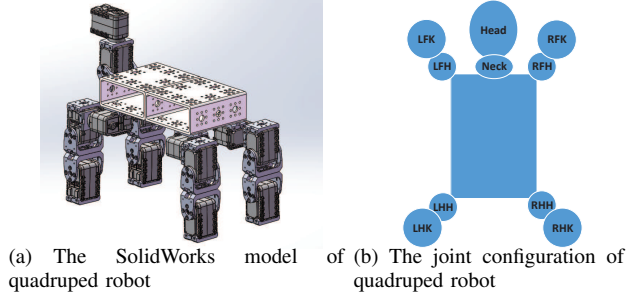


Fig. 1. The mechanical structure of the robot

consists of hip joint and knee joint in which H denotes the hip joint of the leg and K denotes the knee joint of the leg. For example, LFH and LFK means the hip joint of left-front leg and knee joint of left-front leg respectively.

### B. Battery Management System

The structure of the battery management system is shown in Figure 3. The system contains a Battery and four adjustable DC-DC convertors. Lithium Battery is the power source of the quadruped robot, and works as the input of four DC-DC convertors controlled by a mechanical switch. A1 is the

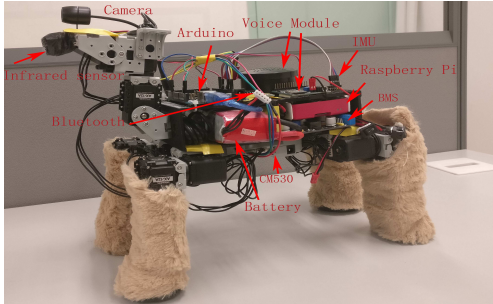


Fig. 2. The system structure of the quadruped robot

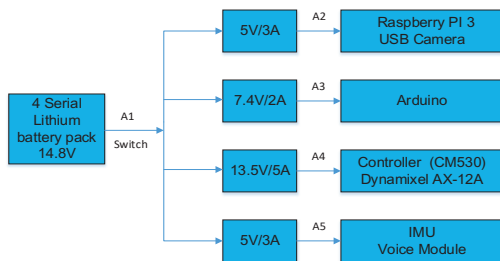


Fig. 3. The structure of the Battery Management System

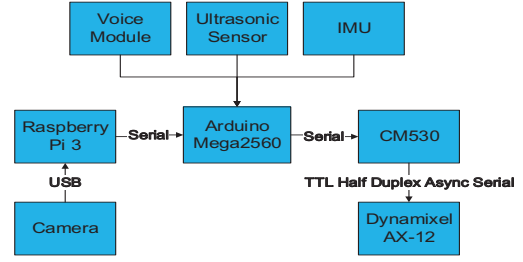


Fig. 4. System control frame of the small quadruped robot

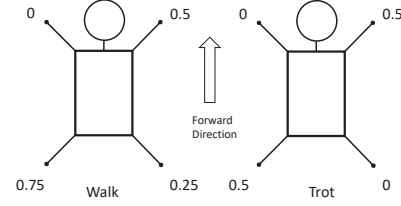


Fig. 5. Walking and Trotting phase difference

circuit that measures the battery current and voltage. A2 to A5 measures the output voltage and current of the four DC-DC convertors.

### C. Controller Frame

Figure 4 is the system control frame. As depicted in the picture, the small quadruped robot has four different kinds of sensors to sense and interact with the outside environments. The camera is mounted in Raspberry Pi 3 with USB interface. The robot has the ability of voice interaction with Voice Module. Ultrasonic Sensor is used for the obstacle detection. Through the Inertial Measure Unit(IMU), robot can acquire the information of the real-time body posture. Arduino, the main controller, gets the feedback from sensors. CM530 controls the Dynamixel AX-12 servos.

## III. GAIT DESIGN AND COMPARISON

Walk and trot are the most typical gaits of four-legged robots. In this paper, two different gait design methods are presented and applied to simulation and experiment. First, using drive function drives the angle of joints. Second, design different types of foot trajectories, like the Bezier curve, elliptic curve and improved-elliptic curve.

Figure 5[14] shows the phase difference of legs. Walk belongs to a quasi-static gait with each leg putting up and down in turn, and the motion order of four legs is LF-RH-RF-LH with fixed phase difference  $1/4T$ . Trot is dynamic gait with the diagonal legs putting up and down simultaneously at fixed phase difference  $1/2T$ .

Figure 6(a)[15] shows the single leg motion of walk gait.  $s$  represents the stride length. When the leg is in the support phase, the hip moves forward  $3/4s$ . Then, the hip continues moving forward  $1/4s$  in the swing phase, along with the toe of foot forward  $s$ . The movement mechanism of trot gait

is similar to walk gait except that the forward distance is  $s/2$  in both support phase and swing phase as Figure 6(b) illustrates.

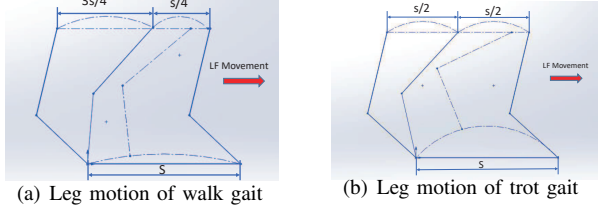


Fig. 6. The leg motion intuition

### A. Gait Design Based on Drive Function

1) *Walk gait*: According to the law of biological movement and the output wave of CPG generator, segmentation and half-wave function are used to drive the hip and knee joints on each leg respectively. In this paper, the rotation angle of the joint is defined as positive in the forward direction.

The RHH drive function of walk gait is described as equation (1):

$$F_h(t) = \begin{cases} A \sin(\frac{2}{3}(2\pi ft - 1.5\pi)) & 0 \leq t \leq 3/8T \\ A \sin(2(2\pi ft - 1.0\pi)) & 3/8T \leq t \leq 5/8T \\ A \cos(\frac{2}{3}(2\pi ft - 1.25\pi)) & 5/8T \leq t \leq T \end{cases} \quad (1)$$

In equation (1), the amplitude  $A$  is 0.4 and the frequency  $f$  is 1.5Hz. The corresponding graphics is depicted in Figure 7.

From the Figure 7, in the hip drive function curve, the phase difference between each leg is  $1/4T$ . The rising curve represents the swing phase of leg, and the falling curve represents the support phase of leg. The ratio of the rising time to the fall time is 1: 3.

When the hip drive curve is in the ascending phase, the leg is in swing phase. At this time the corresponding knee

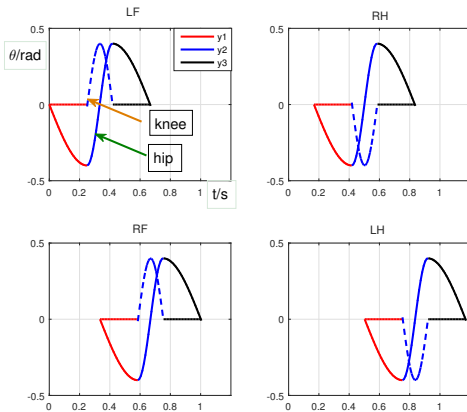


Fig. 7. Drive function curve of hip and knee joint of four legs

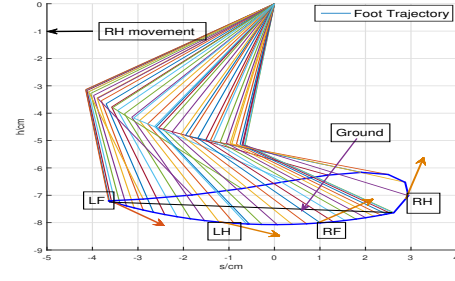


Fig. 8. The foot trajectory of walk gait based on drive function

drive function is exerted to ensure smooth motion of the leg lift action. When the hip drive curve is in the descending phase, the leg is in the support phase. At this time, knee joint remains stationary.

The RHH drive function of walk gait is described as equation 2:

$$F_k(t) = \begin{cases} 0 & 0 \leq t \leq 3/8T \\ A \sin(2(2\pi ft - 0.75\pi)) & 3/8T \leq t \leq 5/8T \\ 0 & 5/8T \leq t \leq T \end{cases} \quad (2)$$

Similarly, the amplitude  $A$  is 0.4 and the frequency  $f$  is 1.5Hz. The function graphics is depicted in Figure 7.

According to the knee drive curve, the half-wave function is used to drive the knee joint in order to achieve a forward step. The rise time and fall time are identical.

Thus, depending on the driving function of the hip joint and the knee joint, the foot trajectory can be obtained with the equation (5), as the Figure 8 shows.

2) *Trot gait*: In the trot gait, the sine and half-wave drive functions are used to drive the hip and knee joints respectively. The RHH drive function is as equation (3) shows:

$$F_h(t) = A \sin(2\pi ft - \pi/2) \quad (3)$$

The amplitude  $A$  is 0.3 and the frequency  $f$  is 2.5Hz. The phase difference between LF(RH) and RF(LH) is  $1/2T$ .

The RHH drive function is as equation (4):

$$F_k(t) = \begin{cases} A \sin(2\pi ft) & 0 \leq t \leq 0.5T \\ 0 & 0.5T \leq t \end{cases} \quad (4)$$

The amplitude  $A$  is 0.5 and the frequency  $f$  is 2.5Hz. Hip drive function curve and the knee drive function curve is depicted in Figure 9. The hip drive function of LF and RH leg is same, while the knee curve is opposite in direction, which is due to the different joint configuration of the front and hind legs. Similarly, the RF and LH drive function is same with  $1/2T$  phase delay. In the trot gait, the support and swing time of each leg is identical.

The foot trajectory of trot gait based on the driving function is depicted in Figure 10.

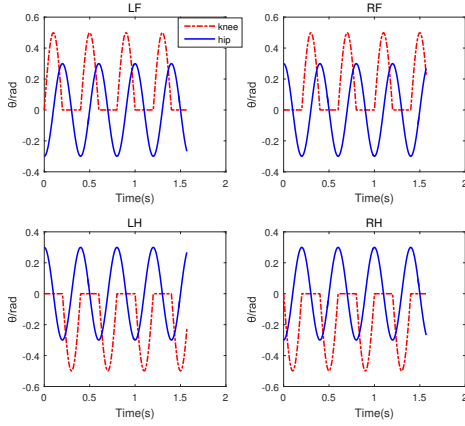


Fig. 9. Drive function curve of hip and knee joint of four legs

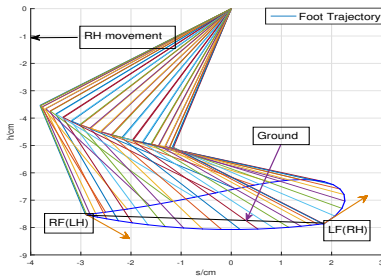


Fig. 10. The foot trajectory of trot gait based on drive function

3) *Summary*: There are the skew black lines (ground) in Figure 8 and 10, which means the height level that the toe of foot putting down and lifting the ground is different. It indicates when the swing phase is over, the toe of foot is still in the floating state(not in contact with the ground), leading to an unstable locomotion.

### B. Gait design based on Foot Trajectory

In order to avoid the flaws of the drive function, foot trajectory method becomes particularly important. Good foot trajectory can be used to suit for different environments. The most common trajectories are elliptical trajectories, trapezoidal trajectories, Bezier curves and so on. This part focus on the design of the foot trajectory. First, the inverse kinematics of LF (left front leg) can be derived by using the geometric method from Figure 11.

1) *Inverse Kinematics*: In Figure 11, the hip joint position is located in the origin of the coordinate frame.  $L_1$  and  $L_2$  represent the thigh and shank length respectively.  $(X_2, Y_2)$  is the position of toe of the foot and described as equation (5):

$$\begin{aligned} X_2 &= L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) \\ Y_2 &= -L_1 \sin \theta_1 - L_2 \sin(\theta_1 + \theta_2) \end{aligned} \quad (5)$$

The  $\theta_1$  and  $\theta_2$  are derived using geometric method as equation (6) shows:

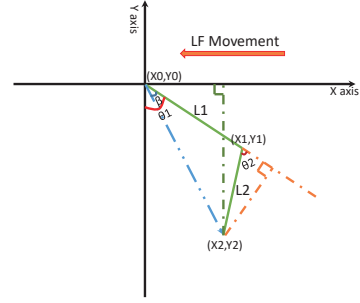


Fig. 11. Inverse kinematics solved by the geometric method

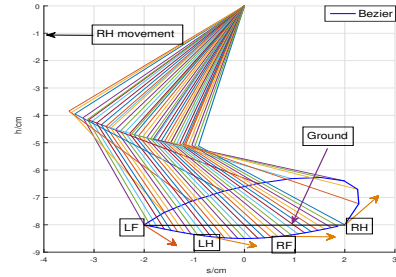


Fig. 12. The foot trajectory of walk gait using the Bezier curve

$$\begin{aligned} \theta_2 &= \cos^{-1} \left( \frac{X_2^2 + Y_2^2 - L_1^2 - L_2^2}{2L_1L_2} \right) \\ \beta &= \sin^{-1} \left( \frac{L_2 \sin \theta_2}{\sqrt{X_2^2 + Y_2^2}} \right) \\ \theta_1 &= \sin^{-1} \left( \frac{X_2}{\sqrt{X_2^2 + Y_2^2}} \right) + \beta \end{aligned} \quad (6)$$

According to the equation (6),  $(\theta_1, \theta_2)$  can be calculated with respect to the certain position  $(X_2, Y_2)$ .

2) *Foot Trajectory Design based on Bezier Curve*: The aim of designing Bezier curve is that the legs could lift faster and higher. Here, three-order bezier curve is adopted so that robot has the potential to cross the barrier.

$$B(t) = P_0(1-t)^3 + 3P_1t(1-t)^2 + 3P_2t^2(1-t) + P_3t^3, t \in [0, 1] \quad (7)$$

where  $P_0$  is the start point of the Bezier curve,  $P_3$  is the last point of the Bezier curve. Here we use  $P_0 = (-2, -8)$ ,  $P_3 = (2, -8)$ ,  $P_1 = (-3.5, -5)$ ,  $P_2 = (1, -6.5)$ .

Figure 12 shows the foot trajectory of RH leg. The curve of support phase is sine function. The time ratio of sine curve(support phase) to the Bezier curve(swing phase) is 3:1.

The foot trajectory of trot gait using Bezier curve is as Figure 13 shows. The curve of support phase is sine function as well. The time ratio of sine curve(support phase) to the Bezier curve(swing phase) is 1:1.

3) *Foot Trajectory Design based on Ellipse Curve*: The elliptic foot trajectory is efficient for the quadruped

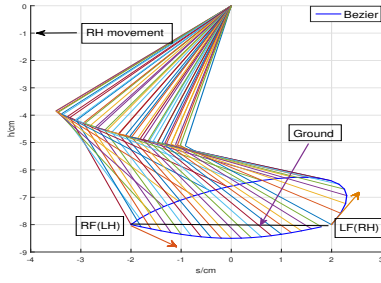


Fig. 13. The foot trajectory of trot gait using the Bezier curve

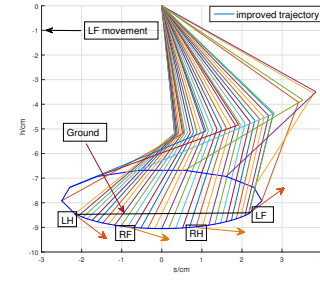


Fig. 16. Improved elliptical foot trajectory design of walk gait

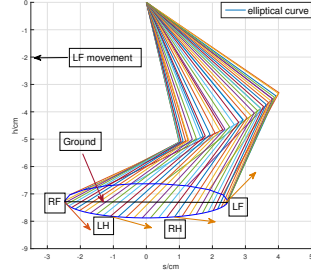


Fig. 14. Elliptical foot trajectory design of walk gait

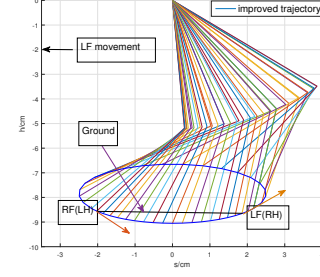


Fig. 17. Improved elliptical foot trajectory design of trot gait

robot. The locomotion is smooth and stable. The parametric equation of the ellipse are the following equation (8):

$$\begin{aligned} x &= h + a \cos(\phi) \\ y &= k + b \sin(\phi) \end{aligned} \quad (8)$$

where  $\phi$  is the phase in interval  $[0, 2\pi]$ ,  $(h, k)$  is the center of the ellipse,  $a$  and  $b$  are the size of radius of ellipse in  $x$ -axis and  $y$ -axis respectively. The value of these parameters is relative to the anatomy of the real robot [12] and is determined as equation (9):

$$\begin{aligned} b &= L_1 + (L_2/2) - \sqrt{L_1^2 + L_2^2} \\ a &= 0.95(L_1/2) \\ (h, k) &= (0, -(L_1 + (L_2/2))) \end{aligned} \quad (9)$$

The Figure 14,15 shows the elliptical foot trajectory of the walk gait and trot gait.

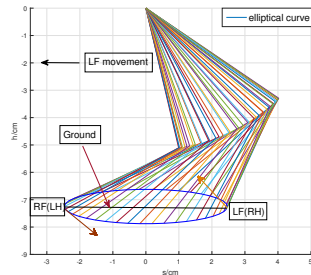


Fig. 15. Elliptical foot trajectory design of trot gait

4) *summary*: By analyzing Bezier trajectory, there is a significant impact over the ground observed by the contacting noise, leading to energy-consuming and unstable locomotion when the leg puts down on the ground. However, it's efficient to cross the short obstacle because of the abrupt lift.

Compared with Bezier curve, due to the initial leg lifting amplitude is low, it becomes hard to cross the obstacles, but the locomotion is more smooth and stable with Elliptical trajectory.

#### C. Improved foot trajectory from ellipse

In order to solve the obvious problems of the above-mentioned methods, improved foot trajectory is proposed as follows, which possess the advantages of elliptic curve and Bezier curve and avoid the disadvantages at the same time.

The parameters of improved foot trajectory are described as follows:

$$\begin{aligned} b &= 2(L_1 + (L_2/2) - \sqrt{L_1^2 + L_2^2}) \\ a &= 0.95(L_1/2) \\ (h, k) &= (0, -(L_1 + (L_2/2)) - d) \end{aligned} \quad (10)$$

In the equation (10),  $d$  represents the vertical translation of the LF (start point of foot trajectory). Here the value of  $d$  is  $b/2$ .

The Figure 16, 17 shows the improved elliptical foot trajectory of the walk gait and trot gait respectively.

In the figure, the amplitude of stride height is enough to allow the leg easily cross the obstacles. In addition, the impact between the toe of foot and the ground is obviously reduced observed by the contacting noise.



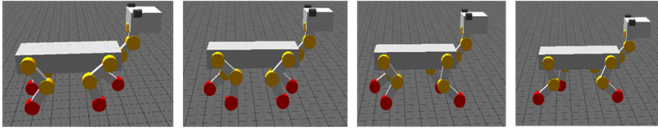


Fig. 18. Walk gait simulation on the Webots using the drive function

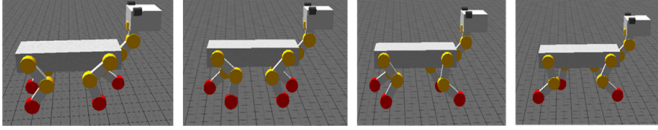


Fig. 19. Trot gait simulation on the Webots using the drive function

#### IV. EXPERIMENTS

##### A. Simulation

In this paper, a small inner-knee-elbow quadruped robot is designed with 8 degree-of-freedom, and each leg equipped with hip and knee joint servo.

The above-mentioned gait design methods have been validated in Webots. The walk and trot gait simulation based on Webots are shown in Figure 18 and 19.

##### B. Experiment

We successfully designed a Battery Management System (BMS) for servo driven quadruped robot. The advantages of the BMS are listed as follows:

- The BMS have four DC-DC Buck convertors and each convertor can supply voltage ranging from 3.3V to 15V. In addition, the real-time current and voltage can be measured.
- The BMS can be applied in various electronic system in robots.

We have carried on the corresponding experiments to all the above-mentioned gait design methods. For the improved elliptical foot trajectory, the experimental results show that quadruped robot have the stable locomotion.

Figure 20 shows the gait test of the quadruped robot with the improved elliptical method, and the robot can easily and steadily stride across a book of 1.2cm thickness.

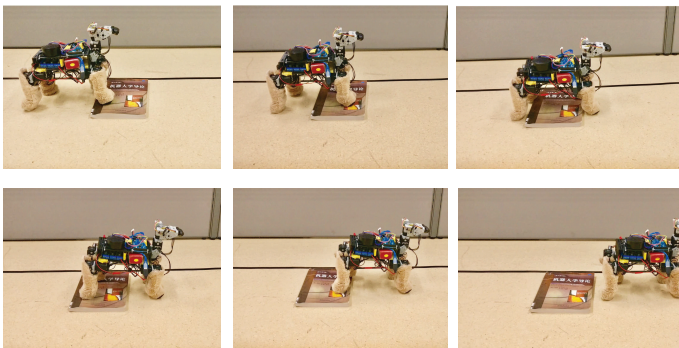


Fig. 20. Trot experiment based on improved elliptical trajectory

#### V. CONCLUSION

In this paper, we focus on the gait design and comparison study of a small quadruped robot on the flat terrain in straight direction. The drive function and foot trajectory gait design methods are proposed and have been efficiently implemented on the small quadruped robot. Furthermore, we compare the performance of the proposed gait design methods, and experimental result demonstrated that the improved elliptical trajectory shows the stable locomotion of the quadruped robot.

#### APPENDIX

The accompanying video can be accessed at the following link: <https://youtu.be/qrlXNlni0Q8>.

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