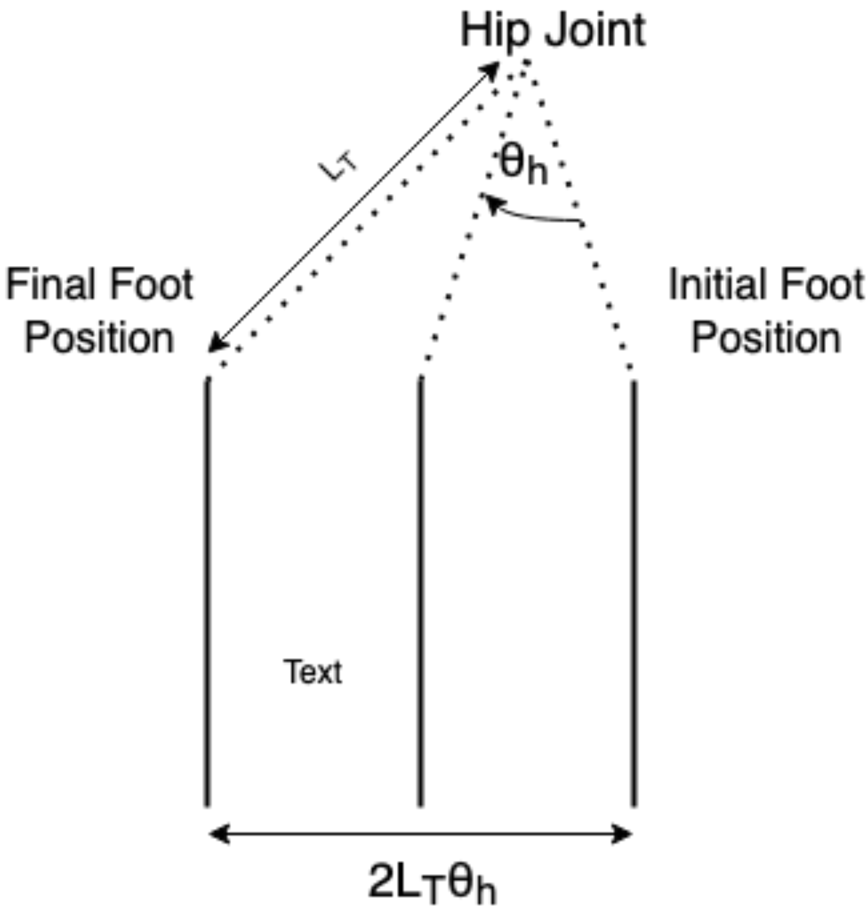
Gait Generation for Quadruped

The following figure depicts the movement of one of the legs of the quadruped with respect to its corresponding hip joint.



For a uniform swing frequency of f (gait period of T) the speed at which the foot moves ahead with respect to the hip joint can be given by the following equation,

$$v = 2L_T\theta_h f$$

Where L_T is the thigh length, θ_h is the half swing angle.

The following relationship holds true between f and T,

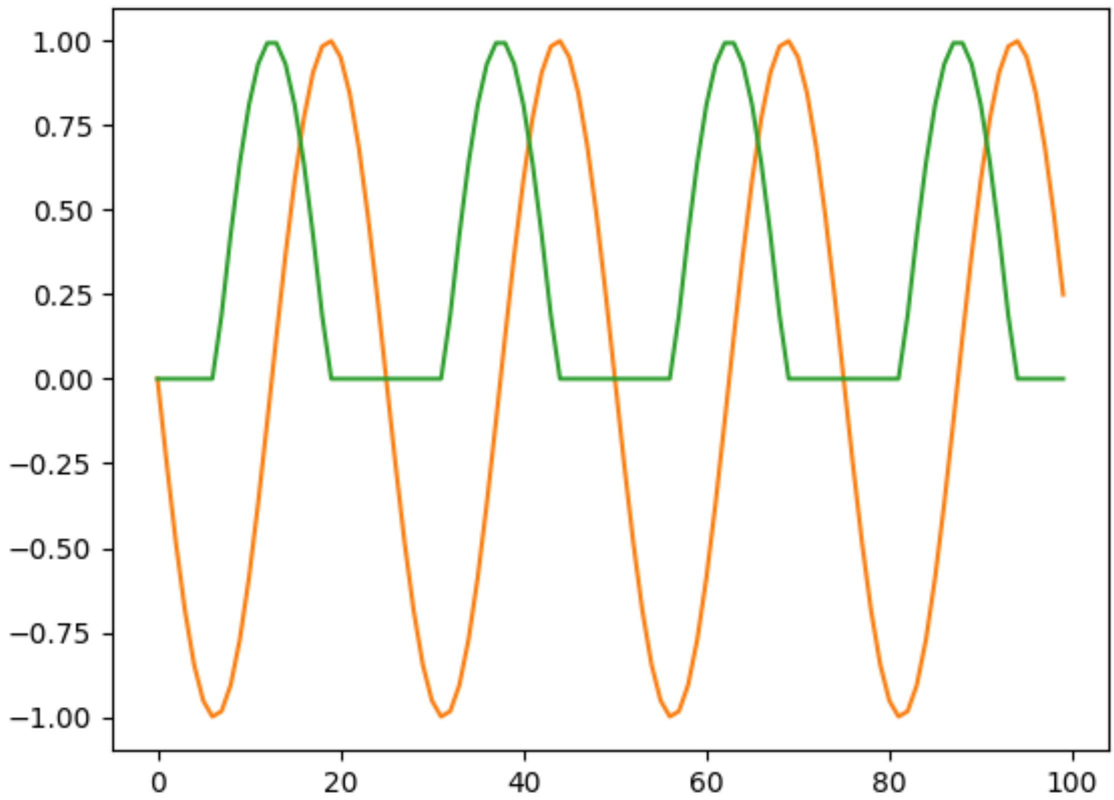
$$f = 1/T$$

For such a case, a simple sinusoidal drive function as follows may be used to represent the variation of the joint angle θ_h,

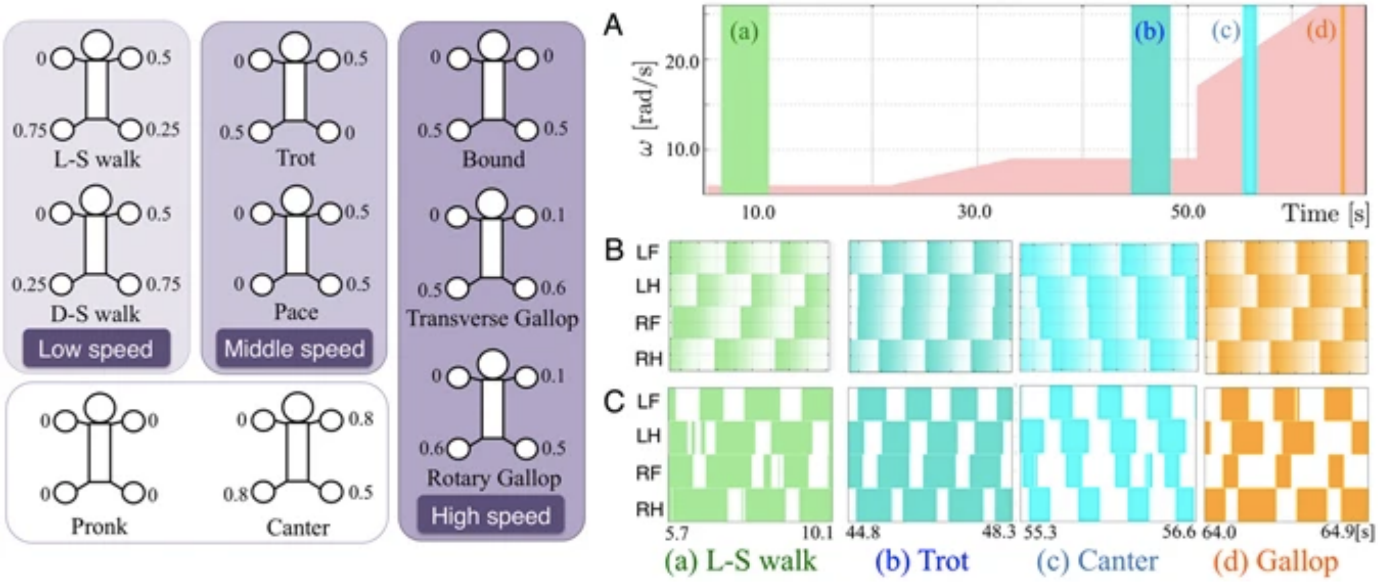
$$\theta_h(t) = \theta_h sin(ft - \pi)$$

Since the knee angle values are dependent on the hip angle value, the following equations can be used to obtain the knee angle values, given a hip angle value,

$$\theta_k(t) = \begin{cases} \theta_k sin(ft - \pi/2), & \text{if } \dot{\theta}_h(t) \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



The above image shows the gait pattern obtained for one leg from the formulations in the previous equations. The orange plot corresponds to the hip joint angle values and the green plot corresponds to knee joint angle values.



From the previous formulation and observations about quadruped motion, we can conclude the following and make modifications to the previous formulations to obtain walking gait pattern for all four legs of the quadruped:

- For a sprawling type quadruped, the drive function for motion in a straight line, will be the same periodic function separated by a phase gap.
- The amplitude of motion of the knee joint, will determine the maximum vertical clearance that the robot can perform
- The amplitude and frequency of oscillations of the hip joint will determine the speed of the quadruped.
- The offset of the hip joint from the base position will determine the heading of the quadruped
- The difference in amplitude of oscillations between different legs will determine the heading of the quadruped
- The maximum ratio between the swing period and the stance period for a walking gait must be $\frac{1}{3}$ to ensure that no two legs are in the swing phase at the same time
- Thus, given a motion in a straight line the following relations hold for speed:

$$v \propto \frac{1}{T_{sw}}$$

$$v \propto \frac{1}{T_{st}}$$

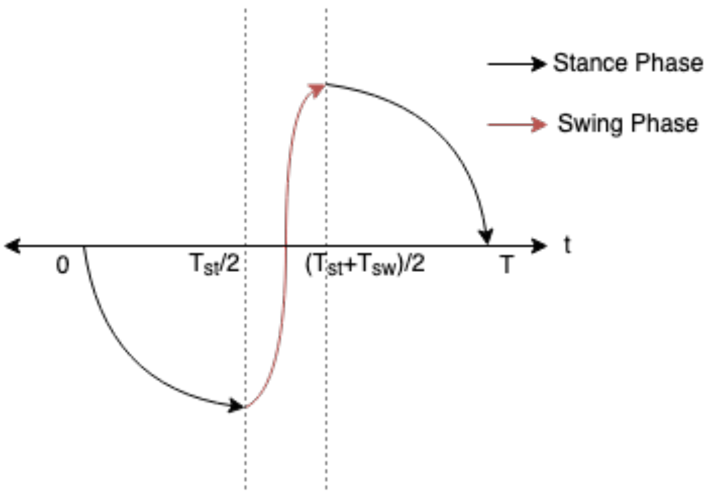
$$v \propto \theta_h$$

Thus the formulation for speed may be modified as follows:

$$v = \frac{2L_T\theta_h}{T_{st} + T_{st}}$$

Due to introduction of different stance and swing periods, the gait generation formulation will now be a piecewise function with the following parameters:

- $T = T_{sw} + T_{st}$
- $\beta = \frac{T_{st}}{T_{st} + T_{sw}}$
- i, leg index



All three pieces will be represented by sinusoidal functions. The following is the template for the piecewise function in the above figure:

$$\theta_h(t) = \begin{cases} \theta_h sin(\frac{t\pi}{T_{st}} + \varphi_1), & \text{if } 0 \le t \le \frac{T_{st}}{2} \\ \theta_h sin(\frac{t\pi}{T_{sw}} + \varphi_2), & \text{if } \frac{T_{st}}{2} \le t \le \frac{T_{st} + 2T_{sw}}{2} \\ \theta_h sin(\frac{t\pi}{T_{st}} + \varphi_3), & \text{if } \frac{T_{st} + 2T_{sw}}{2} \le t \le T_{st} + T_{sw} \end{cases}$$

φ_1 will be calculated according to the following equation:

$$\frac{t\pi}{T_{st}} + \varphi_1 = \frac{3\pi}{2}, \text{if } t = \frac{T_{st}}{2}$$

φ_2 will be calculated according to the following equation:

$$\frac{t\pi}{T_{sw}} + \varphi_2 = 2\pi, \text{if } t = \frac{T_{st} + T_{sw}}{2}$$

φ_3 will be calculated according to the following equation:

$$\frac{t\pi}{T_{st}} + \varphi_3 = \pi, \text{if } t = T_{st} + T_{sw}$$

Solving the above three equations and injecting β, i, k and T into the equations, the general gait generation equations can be given by,

$$\theta_h(t) = \begin{cases} \theta_h sin(\frac{(t - \frac{iT}{4})\pi}{\beta T} + \pi), & \text{if } 0 \le t \le \frac{\beta T}{2} \\ \theta_h sin(\frac{(t - \frac{iT}{4})\pi}{(1-\beta)T} + \frac{(3-4\beta)\pi}{2(1-\beta)}), & \text{if } \frac{\beta T}{2} \le t \le \frac{T(2-\beta)}{2} \\ \theta_h sin(\frac{(t - \frac{iT}{4})\pi}{\beta T} + \frac{(\beta - 1)\pi}{\beta}), & \text{if } \frac{T(2-\beta)}{2} \le t \le T \end{cases}$$

$$\text{where } i \in \{0,1,2,3\}$$

Similarly for the knee joint, the following formulation will give joint angle values,

$$\theta_k(t) = \begin{cases} \theta_k sin(\frac{t\pi}{T(1-\beta)} - \frac{\beta\pi}{2(1-\beta)}), & \text{if } \dot{\theta}_h(t) \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

References

1. Zhou, Liguang & Liu, Wan & Qian, Huihuan & Xu, Yangsheng. (2017). Gait Design and Comparison Study of a Quadruped Robot. 10.1109/ICInfA.2017.8078886.
2. Owaki, D., Ishiguro, A. A Quadruped Robot Exhibiting Spontaneous Gait Transitions from Walking to Trotting to Galloping. *Sci Rep* **7,** 277 (2017). https://doi.org/10.1038/s41598-017-00348-9
3. Junmin Li, Jinge Wang, Simon X. Yang, Kedong Zhou, and Huijuan Tang. (2016). Gait Planning and Stability Control of a Quadruped Robot. https://doi.org/10.1155/2016/9853070