Advanced Nonlinear Control of Robot Manipulators

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Advanced Nonlinear Control of Robot Manipulators

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1. Introduction

In the development of modern robot manipulators, it is required that the robot controller has the capability to overcome unmodeled dynamics, variable payloads, friction torques, torque disturbances, parameter variations, measurement noises which can be often presented in the practical environment.

The objective of this chapter is to provide the reader with an overview on advanced nonlinear control techniques of a rigid robot manipulator. In nonlinear control field, a common strategy is called model based control, which can be derived from the mathematical model of the system. However, in case of robot manipulator, it is weakened by inaccuracies present in the robot model, where the performance of the control algorithm is not guaranteed. As mentioned above, these inaccuracies can be defined as parametric uncertainties, unmodeled dynamics, and unknown external disturbances. To overcome the uncertainties' drawback, robust nonlinear control can be a solution. The goal of robust control is to maintain performance in terms of stability, tracking error, or other specifications despite inaccuracies present in the system.

In this chapter we present two nonlinear model based control strategies: the feedback linearization control and a nonlinear model predictive control for rigid robot manipulator. We first consider the dynamic of the robot manipulator driven by the Euler-Lagrange equations. Based on this general representation, we are able to derive equations of the nonlinear controller for both strategies. Then, a robustness study is carried out through compensation of the system inaccuracies. Two methods are used; the first one is based on the theory of guaranteed stability of uncertain systems, while the second is figured out using the nonlinear control law.

The computation of the nonlinear model based control assumes that all state variables are available. In case of robot manipulators, it implies the presence of additional sensors in each joint such as velocity measurements. They are often obtained by means of tachometers, which are perturbed by noise, or moreover, velocity measuring equipment is frequently omitted due to the savings in cost, volume, and weight. Model-based observers are considered very well adapted for state estimation and allow, in most cases, a stability proof and a methodology to tune the observer gains, which guarantee a stable closed loop system.

In this chapter, nonlinear observer is discussed for state variables estimation. It is a powerful tool to handle nonlinear and uncertain systems, which is the case of the robot manipulator.
Finally, the coupling between the nonlinear model based control and the state observer is discussed and the global stability of the closed loop system is proven theoretically via Lyapunov stability theory.

2. Robot modeling

In this chapter, the nonlinear control laws will be developed for rigid robot manipulators. Therefore, the design and control of such robots require mathematical model of the process. The dynamic of n-link rigid robot manipulator is driven by the Euler-Lagrange equations as

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{u} \tag{1}$$

where $\mathbf{q}(t) \in \Re^n$ is the vector of the angular joint positions, which are the generalized coordinates and assumed available by measurement. $\mathbf{u}(t) \in \Re^n$ is the vector of the driving torques, which are the control inputs. $\mathbf{D}(\mathbf{q}) \in \Re^{n \times n}$, $\mathbf{D}(\mathbf{q}) = \mathbf{D}(\mathbf{q})^T > 0$ is the link inertia matrix. $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} \in \Re^n$ is the vector of the Coriolis and centripetal torques. $\mathbf{G}(\mathbf{q}) \in \Re^n$ is the vector of gravitational torques. The outputs to be controlled are the joint angles in the robot. For more detail about robot modeling, the reader can refer to (Spong et al., 2006; Kozłowski, 2004).
The practical implementation of the control law for robot manipulators requires consideration of various sources of uncertainties such as modeling errors, unknown loads, and computation errors. In order to get the real values of the system elements, the uncertainties of the system, error or mismatch represented by $\Delta(.)$, are added to the computed or nominal values represented by $(.)_0$. Therefore, the matrices are rewritten as

$$\begin{cases} \mathbf{D}(\mathbf{q}) = \mathbf{D}_0(\mathbf{q}) + \Delta\mathbf{D} \\ \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}_0(\mathbf{q},\dot{\mathbf{q}}) + \Delta\mathbf{C} \\ \mathbf{G}(\mathbf{q}) = \mathbf{G}_0(\mathbf{q}) + \Delta\mathbf{G} \end{cases} \tag{2}$$

Moreover, the frictions $\mathbf{F_r}(t) \in \Re^n$, considered as unmodeled quantities, and the external disturbances $\mathbf{b}(t) \in \Re^n$ are added to the robot model (1), which becomes

$$\left(\mathbf{D}_0(\mathbf{q}) + \Delta\mathbf{D}\right)\ddot{\mathbf{q}} + \left(\mathbf{C}_0(\mathbf{q},\dot{\mathbf{q}}) + \Delta\mathbf{C}\right)\dot{\mathbf{q}} + \mathbf{G}_0(\mathbf{q}) + \Delta\mathbf{G} + \mathbf{F_r} = \mathbf{u} + \mathbf{b} \tag{3}$$

Then, after simplification, the model dynamic of the robot is given by

$$\mathbf{D}_0(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_0(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}_0(\mathbf{q}) = \mathbf{u} + \eta(\ddot{\mathbf{q}},\dot{\mathbf{q}},\mathbf{q},\mathbf{b}) \tag{4}$$

η is called uncertainty, which is defined by

$$\eta = -\left\{\Delta\mathbf{D}\,\ddot{\mathbf{q}} + \Delta\mathbf{C}\,\dot{\mathbf{q}} + \Delta\mathbf{G} + \mathbf{F_r} - \mathbf{b}\right\} \tag{5}$$

It includes unmodeled quantities, parametric uncertainties, and external disturbances.
In a state space form, the nonlinear system of the robot model (4) can be written as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} + \mathbf{g}(\mathbf{x})\eta \tag{6}$$

where, the state vector $\mathbf{x} = [\mathbf{x}_1 \quad \mathbf{x}_2]^T = [\mathbf{q} \quad \dot{\mathbf{q}}]^T$, and the vector functions $\mathbf{f}$: $\Re^n \rightarrow \Re^{2n}$ and $\mathbf{g}$:$\Re^n \rightarrow \Re^{2n}$ are vector fields and defined as follows

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{x}_2 \\ -\mathbf{D}_0(\mathbf{x}_1)^{-1}\left(\mathbf{C}_0(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}_0(\mathbf{x}_1)\right) \end{bmatrix}, \quad \mathbf{g}(\mathbf{x}) = \begin{bmatrix} 0_{n\times n} \\ \mathbf{D}_0(\mathbf{x}_1)^{-1} \end{bmatrix} \tag{7}$$

The output vector of angular positions to be controlled is

$$\mathbf{y} = \mathbf{h}(\mathbf{x}) = C\,\mathbf{x} \tag{8}$$

where $C = [I_{n\times n} \quad 0_{n\times n}]$, and I: identity matrix. The functions $\mathbf{f}(\mathbf{x})$, $\mathbf{g}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$: $\Re^n \rightarrow \Re^n$ are assumed to be continuously differentiable a sufficient number of time.
First, the development of the control laws will be carried out for the undisturbed system where the uncertainties are not included in the analysis. Then, a robust control is studied through a compensation of the uncertainties by estimation.

3. Nonlinear model based control of robot manipulators

3.1 Feedback linearization control

Feedback linearization is one of the most important strategies for nonlinear control design. There are two general types of linearization: input-state linearization and input-output linearization. Necessary and sufficient conditions have been established for each type of linearization. For a given nonlinear system, these conditions can be checked to determine if the system is linearizable (Corriou, 2004; Nijmeijer & Van der Schaft, 1990; Isidori, 1985; Isidori & Ruberti, 1984).
In this chapter, we will study the feedback linearization, based on input-output linearization, of a rigid robot manipulator. The idea is to differentiate the output $\mathbf{y}$, using Lie derivatives, to obtain an expression where the input $\mathbf{u}$ appears explicitly. The number of times of differentiation is called relative degree.
Definition 1: The Lie derivative of function $h_j(\mathbf{x})$ along a vector field $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}) \ldots f_n(\mathbf{x}))$ is given by

$$L_{\mathbf{f}} h_j(\mathbf{x}) = \sum_{i=1}^{n} \frac{\partial h_j}{\partial x_i} f_i(\mathbf{x}) \tag{9}$$

Iteratively, we have

$$\begin{cases} L_{\mathbf{f}}^0 h_j = h_j \\ L_{\mathbf{f}}^i h_j = L_f(L_{\mathbf{f}}^{i-1} h_j) \end{cases} \tag{10}$$

and, along another vector field $\mathbf{g}(\mathbf{x}) = (g_1(\mathbf{x}) \ldots g_n(\mathbf{x}))$,

$$L_{g_i} L_{\mathbf{f}} h_j = \frac{\partial L_{\mathbf{f}} h_j}{\partial x_i} g_i(\mathbf{x}) \tag{11}$$

Definition 2: The system is said to have a relative degree r if

$$L_{g_i} L_{\mathbf{f}}^{k-1} h_j(\mathbf{x}) = 0 \text{ and } L_{g_i} L_{\mathbf{f}}^{r-1} h_j(\mathbf{x}) \neq 0;\ k = 1,\ldots, r-1 \tag{12}$$

Then, r is the number of differentiation times to appear the input $\mathbf{u}$ in the expression of $\mathbf{y}$. Differentiating the output, using Lie derivatives and the nominal robot state model (6) (without uncertainties), we have

$$\begin{aligned} \dot{\mathbf{y}} &= L_{\mathbf{f}} h_1(\mathbf{x}) = \mathbf{x}_2 \\ \ddot{\mathbf{y}} &= L_{\mathbf{f}}^2 h_1(\mathbf{x}) + L_{\mathbf{g}_2} L_{\mathbf{f}} h_1(\mathbf{x})\mathbf{u} = -\mathbf{D}_0(\mathbf{x}_1)^{-1}\big(\mathbf{C}_0(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}_0(\mathbf{x}_1)\big) + \mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u} \end{aligned} \tag{13}$$

where, $\mathbf{y} = \mathbf{x}_1 = \mathbf{q}$, and the relative degree for input $\mathbf{u}$ is $r = 2$.
The principle of linearization control law is to get a linear system, where the output is influenced by an external input $\mathbf{v}$ only through a chain of two integrators as

$$\begin{aligned} \ddot{\mathbf{y}} &= \mathbf{v} \\ &= -\mathbf{D}_0(\mathbf{x}_1)^{-1}\big(\mathbf{C}_0(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}_0(\mathbf{x}_1)\big) + \mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u} \end{aligned} \tag{14}$$

Then, the control law is carried out as

$$\mathbf{u} = \mathbf{D}_0(\mathbf{x}_1)\Big(\mathbf{v} + \mathbf{D}_0(\mathbf{x}_1)^{-1}\big(\mathbf{C}_0(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}_0(\mathbf{x}_1)\big)\Big) \tag{15}$$

It is possible to realize a pole-placement by imposing $\mathbf{v}$ as

$$\mathbf{v} = \ddot{\mathbf{y}}_{\mathbf{r}} - K_2(\dot{\mathbf{y}} - \dot{\mathbf{y}}_{\mathbf{r}}) - K_1(\mathbf{y} - \mathbf{y}_{\mathbf{r}}) \tag{16}$$

where, $K_1 = diag(k_{1i})$, $K_2 = diag(k_{2i})$, $i = 1, \ldots, n$
Applying the control law (15) with the external input (16), the tracking error $\mathbf{e}_{\mathbf{y}}(t) = \mathbf{y} - \mathbf{y}_{\mathbf{r}}$ satisfies the second linear equation

$$\ddot{\mathbf{e}}_{\mathbf{y}}(t) + K_2 \dot{\mathbf{e}}_{\mathbf{y}}(t) + K_1 \mathbf{e}_{\mathbf{y}}(t) = 0 \tag{17}$$

and, hence, the error dynamics are determined by the choice of K_2 and K_1, so that the characteristic equation is Hurwitz.

3.2 Nonlinear model based predictive control

Model based predictive control (MPC) is considered an effective control method handling with constraints, nonlinear processes and disturbances. This control strategy requires an optimization method to solve for the control trajectory over a future time horizon based on a dynamic model of the process (Bordon & Camacho, 1998; Hedjar & Boucher, 2005; Hedjar, et al., 2002; Klančar & Škrjanc, 2007; Vivas & Mosquera, 2005).

The objective of the nonlinear model based predictive controller is to carry out a control law $\mathbf{u}(t)$ in order to track the desired output trajectory $\mathbf{y_r}$ at the next time $(t+\tau)$ through minimization of a general form of the cost function defined as

$$\Im = f\left(\mathbf{e_y}(t+\tau), \mathbf{x}, \mathbf{u}\right) \tag{18}$$

where $\mathbf{e}_\mathbf{y}(t+\tau)$ is a predicted error, $\mathbf{y}(t+\tau)$ is a τ-step ahead prediction of the output (angular positions) and $\tau > 0$ is a prediction horizon .

In order to minimize the cost function (18), it is needed to define a prediction model for the behavior of the output in the moving time frame. As the robot model (6) is known, a mathematical tool based on Taylor series expansion can be used to develop the prediction model.

By definition, the Taylor series expansion is carried out using Lie derivatives and given by

$$y_i(t+\tau) = h_i(\mathbf{x}) + \tau\, L_\mathbf{f} h_i(\mathbf{x}) + \frac{\tau^2}{2!} L_\mathbf{f}^2 h_i(\mathbf{x}) + \ldots + \frac{\tau^{r_i}}{r_i!} L_\mathbf{f}^{r_i} h_i(\mathbf{x}) + \frac{\tau^{r_i}}{r_i!} L_\mathbf{g} L_\mathbf{f}^{(r_i-1)} h_i(\mathbf{x})\mathbf{u}(t) \tag{19}$$

where r_i is the relative degree.

Based on this expansion, the prediction model for robot model is expressed by

$$\mathbf{y}(t+\tau) = \mathbf{y}(t) + \tau\, \dot{\mathbf{y}}(t) + \frac{\tau^2}{2!} \ddot{\mathbf{y}}(t) \tag{20}$$

Using the output differentiations (13), we have

$$\mathbf{Y}(t) = \begin{bmatrix} \mathbf{y}(t) \\ \dot{\mathbf{y}}(t) \\ \ddot{\mathbf{y}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ -\mathbf{D}_0(\mathbf{x}_1)^{-1}\left(\mathbf{C}_0(\mathbf{x}_1, \mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}_0(\mathbf{x}_1)\right) \end{bmatrix} + \begin{bmatrix} 0_{n\times 1} \\ 0_{n\times 1} \\ \mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u}(t) \end{bmatrix} \tag{21}$$

Then, the prediction model (20) is rewritten as

$$\mathbf{y}(t+\tau) = \mathbf{T}(\tau)\mathbf{Y}(t) \tag{22}$$

where,

$$\mathbf{T}(\tau) = [I_{n\times n} \quad \tau * I_{n\times n} \quad (\tau^2/2) * I_{n\times n}]$$

A similar analysis can be used to carry out the predicted reference trajectory $\mathbf{y_r}$

$$\mathbf{y_r}(t+\tau) = \mathbf{T}(\tau)\mathbf{Y_r}(t) \tag{23}$$

where,

$$\mathbf{Y_r}(t) = [\mathbf{y_r} \quad \dot{\mathbf{y}}_\mathbf{r} \quad \ddot{\mathbf{y}}_\mathbf{r}]^T$$

It is assumed that the information about the derivatives of the reference $\mathbf{y_r}$ is available.
The predicted error is given by

$$\mathbf{e_y}(t+\tau) = \mathbf{y}(t+\tau) - \mathbf{y_r}(t+\tau) = \mathbf{T}(\tau)\big(\mathbf{Y}(t) - \mathbf{Y_r}(t)\big) \tag{24}$$

The optimal control law can be carried out through minimization of a cost function with respect to the control input. In this work, two approaches to define the cost function will be studied:

1. A cost function based only on the tracking error. The goals are to show that the use of this cost function type allows realizing a pole-placement similar to feedback linearization, and designing an uncertainty estimator (section 5. 2) based on the control law derived from this specific cost function.
2. A general form of cost function where the tracking error and the control signal are defined over a future horizon.

3.2.1 Cost function based on the tracking error

The cost function is defined as a quadratic form of the tracking error over a future horizon

$$\Im = \frac{1}{2}\int_0^{\tau_r} \big(\mathbf{y}(t+\tau) - \mathbf{y_r}(t+\tau)\big)^T \big(\mathbf{y}(t+\tau) - \mathbf{y_r}(t+\tau)\big)\, d\tau \tag{25}$$

The control weighting term is not included in the cost function. However, the control effort can be achieved by adjusting τ_r (Chan et al., 1999; Merabet & Gu, 2008).
Using the prediction model of error (24), the cost function (25) can be simplified as

$$\begin{aligned}\Im &= \frac{1}{2}\int_0^{\tau_r} \mathbf{e_y}(t+\tau)^T \mathbf{e_y}(t+\tau)\, d\tau \\ &= \frac{1}{2}\int_0^{\tau_r} \big(\mathbf{T}(\tau)\big(\mathbf{Y}(t) - \mathbf{Y_r}(t)\big)\big)^T \big(\mathbf{T}(\tau)\big(\mathbf{Y}(t) - \mathbf{Y_r}(t)\big)\big)\, d\tau \\ &= \frac{1}{2}\big(\mathbf{Y}(t) - \mathbf{Y_r}(t)\big)^T \mathbf{\Pi}\big(\mathbf{Y}(t) - \mathbf{Y_r}(t)\big)\end{aligned} \tag{26}$$

where

$$\mathbf{\Pi} = \int_0^{\tau_r} \mathbf{T}(\tau)^T \mathbf{T}(\tau) d\tau = \left[\begin{array}{cc|c} \tau_r * I_{n\times n} & (\tau_r^2/2) * I_{n\times n} & (\tau_r^3/6) * I_{n\times n} \\ (\tau_r^2/2) * I_{n\times n} & (\tau_r^3/3) * I_{n\times n} & (\tau_r^4/8) * I_{n\times n} \\ \hline (\tau_r^3/6) * I_{n\times n} & (\tau_r^4/8) * I_{n\times n} & (\tau_r^5/20) * I_{n\times n} \end{array}\right] = \begin{bmatrix} \mathbf{\Pi_1} & \mathbf{\Pi_2} \\ \mathbf{\Pi_2^T} & \mathbf{\Pi_3} \end{bmatrix}$$

The necessary and sufficient condition for cost function minimization is

$$\frac{\partial \Im}{\partial \mathbf{u}} = 0 \tag{27}$$

Using equations (21) and (26), the condition (27) can be rewritten as

$$\left(\frac{\partial\left(\mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u}(t)\right)}{\partial \mathbf{u}(t)}\right)^T [\mathbf{\Pi}_2^T \quad \mathbf{\Pi_3}]\left(\mathbf{M}(t) - \mathbf{Y_r}(t)\right) + \left(\frac{\partial\left(\mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u}(t)\right)}{\partial \mathbf{u}(t)}\right)^T \mathbf{\Pi_3}\mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u}(t) = 0 \tag{28}$$

Therefore, the optimal control is

$$\mathbf{u}(t) = -\mathbf{D}_0(\mathbf{x}_1)\left\{[\mathbf{\Pi}_3^{-1}\mathbf{\Pi}_2^T \quad I_{n\times n}]\left(\mathbf{M}(t) - \mathbf{Y_r}(t)\right)\right\} \tag{29}$$

where,

$$\mathbf{M}(t) = \begin{bmatrix} \mathbf{y}(t) \\ \dot{\mathbf{y}}(t) \\ -\mathbf{D}_0(\mathbf{x}_1)^{-1}\left(\mathbf{C}_0(\mathbf{x}_1, \mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}_0(\mathbf{x}_1)\right) \end{bmatrix}$$

$$[\mathbf{\Pi}_3^{-1}\mathbf{\Pi}_2^T \quad I_{n\times n}] = [(10/(3\tau_r^2)) * I_{n\times n} \quad (5/(2\tau_r)) * I_{n\times n} \quad I_{n\times n}]$$

Finally, the control law (29) becomes

$$\mathbf{u}(t) = -\mathbf{D}_0(\mathbf{x}_1)\left\{K_1(\mathbf{y}-\mathbf{y_r}) + K_2(\dot{\mathbf{y}}-\dot{\mathbf{y}}_\mathbf{r}) - \mathbf{D}_0(\mathbf{x}_1)^{-1}\left(\mathbf{C}_0(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}_0(\mathbf{x}_1)\right) - \ddot{\mathbf{y}}_\mathbf{r}\right\} \tag{30}$$

with, $K_1 = (10/(3\tau_r^2))* I_{n\times n}$,
$K_2 = (5/(2\tau_r))* I_{n\times n}$

From the form of the control law (30) and compared with the linearizing control law (15), it can be noticed that they are similar and allow realizing a pole-placement to have a linear dynamic of the tracking error of the closed loop system.

3.2.2 General form of the cost function

The cost function is defined as quadratic forms of the tracking error and weighting control over a future horizon

$$\Im = \frac{1}{2}\int_0^{\tau_r} \mathbf{e}_\mathrm{y}(t+\tau)^T \boldsymbol{Q}\, \mathbf{e}_\mathrm{y}(t+\tau) d\tau + \frac{1}{2}\int_0^{\tau_u} \mathbf{u}(t+\tau)^T \boldsymbol{R}\, \mathbf{u}(t+\tau) d\tau \tag{31}$$

where, $\boldsymbol{Q}\in\Re^{n\times n}$ is a positive semi-definite matrix and $\boldsymbol{R}\in\Re^{n\times n}$ is a positive definite matrix, τ_r and τ_u are respectively the observation horizon of the tracking error and the control horizon. We assume that the control signal is constant over the control horizon ($\mathbf{u}(t+\tau) = \mathbf{u}(t)$).
Using the same analysis, as in section 3.2.1, the cost function (31) can be rewritten as

$$\Im = \frac{1}{2}\left(\mathbf{Y}(t) - \mathbf{Y_r}(t)\right)^T \mathbf{\Pi}\left(\mathbf{Y}(t) - \mathbf{Y_r}(t)\right) + \frac{1}{2}\boldsymbol{R}\,\tau_u \mathbf{u}(t)^T \mathbf{u}(t) \tag{32}$$

where the new matrix $\mathbf{\Pi}$ is defined by

$$\mathbf{\Pi} = \int_0^{\tau_r} \mathbf{T}(\tau)^T \boldsymbol{Q}\mathbf{T}(\tau)\, d\tau = \begin{bmatrix} \mathbf{\Pi_1} & \mathbf{\Pi_2} \\ \mathbf{\Pi_2}^T & \mathbf{\Pi_3} \end{bmatrix}$$

The necessary and sufficient condition (27) becomes

$$\begin{gathered}\left(\frac{\partial\left(\mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u}(t)\right)}{\partial \mathbf{u}(t)}\right)^T [\mathbf{\Pi}_2^T \quad \mathbf{\Pi_3}]\left(\mathbf{M}(t) - \mathbf{Y_r}(t)\right) + \\ \left(\frac{\partial\left(\mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u}(t)\right)}{\partial \mathbf{u}(t)}\right)^T \mathbf{\Pi_3}\mathbf{D}_0(\mathbf{x}_1)^{-1}\mathbf{u}(t) + \boldsymbol{R}\,\tau_u \mathbf{u}(t) = 0 \\ \mathbf{D}_0(\mathbf{x}_1)^{-1^T}[\mathbf{\Pi}_2^T \quad \mathbf{\Pi_3}]\left(\mathbf{M}(t) - \mathbf{Y_r}(t)\right) + \left(\mathbf{D}_0(\mathbf{x}_1)^{-1^T}\mathbf{\Pi_3}\mathbf{D}_0(\mathbf{x}_1)^{-1} + \boldsymbol{R}\,\tau_u\right)\mathbf{u}(t) = 0\end{gathered} \tag{33}$$

Then, the optimal control law is given by

$$\mathbf{u}(t) = -\left(\mathbf{D}_0(\mathbf{x}_1)^{-1^T}\mathbf{\Pi_3}\mathbf{D}_0(\mathbf{x}_1)^{-1} + \boldsymbol{R}\,\tau_u\right)^{-1}\mathbf{D}_0(\mathbf{x}_1)^{-1^T}[\mathbf{\Pi}_2^T \quad \mathbf{\Pi_3}]\left(\mathbf{M}(t) - \mathbf{Y_r}(t)\right) \tag{34}$$

4. Robust control based on uncertainties compensation

Robust control is considered among the high qualified methods in motion control. The goal of robust control is to maintain performance in terms of stability, tracking error, or other specifications despite inaccuracies present in the system. The robust motion control problem can be solved by designing an estimator to compensate the system uncertainties such as unknown external disturbances, unmodeled quantities and mismatched model (Spong et al., 2006; Kozłowski, 2004; Corriou, 2004; Feuer & Goodwin, 1989; Chen et al., 2000; Curk & Jezernik, 2001; Merabet & Gu, 2008; Curk & Jezernik, 2001).
The uncertainties compensation analysis will be developed for the linearizing control laws (15) and (30).
Using the Lie derivative analysis in (13) about the uncertainties, it can be verified that the relative degree for η is r = 2. Then, the control law becomes

$$\mathbf{u}(t) = -\mathbf{D}_0(\mathbf{x}_1)\left\{K_1(\mathbf{y}-\mathbf{y_r}) + K_2(\dot{\mathbf{y}}-\dot{\mathbf{y}}_\mathbf{r}) - \mathbf{D}_0(\mathbf{x}_1)^{-1}\left(\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}(\mathbf{x}_1)\right) - \ddot{\mathbf{y}}_\mathbf{r}\right\} - \eta(t) \tag{35}$$

Usually the uncertainties η are unknown. Therefore, estimation is required to compute the control law and compensate their effects, and the robust control law is given by

$$\mathbf{u}(t) = -\mathbf{D}_0(\mathbf{x}_1)\left\{K_1(\mathbf{y}-\mathbf{y}_\mathbf{r}) + K_2(\dot{\mathbf{y}}-\dot{\mathbf{y}}_\mathbf{r}) - \mathbf{D}_0(\mathbf{x}_1)^{-1}\left(\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}(\mathbf{x}_1)\right) - \ddot{\mathbf{y}}_\mathbf{r}\right\} - \eta_{est}(t) \tag{36}$$

There are several approaches to treat the robust control problem. In this chapter two methods will be discussed to design the uncertainties estimator; the first one is based on the theory of guaranteed stability of uncertain systems, while the second one is based on the model control law.

4.1 Estimator based on the theory of guaranteed stability of uncertain systems

In this section we will detail the so-called theory of guaranteed stability of uncertain systems, which is based on Lyapunov's second method (Spong et al., 2006).
Substituting the control input (36) in the robot model differentiation (13) plus uncertainties η, the tracking error dynamic of the closed loop system is given by

$$\ddot{\mathbf{e}}_\mathrm{y}(t) + K_2\dot{\mathbf{e}}_\mathrm{y}(t) + K_1\mathbf{e}_\mathrm{y}(t) = \mathbf{D}_0^{-1}(\mathbf{x}_1)\mathbf{e}_\eta(t) \tag{37}$$

where, $\mathbf{e}_\eta(t) = \eta(t) - \eta_{est}(t)$
In terms of tracking error, the state space model of the dynamic system (37) is given by

$$\dot{\overline{\mathbf{e}}} = A_1\overline{\mathbf{e}} + B\mathbf{D}_0^{-1}\mathbf{e}_\eta \tag{38}$$

where

$$\overline{\mathbf{e}} = \begin{bmatrix}\mathbf{e}_\mathrm{y}\\ \dot{\mathbf{e}}_\mathrm{y}\end{bmatrix},\ A_1 = \begin{bmatrix}0_{n\times n} & I_{n\times n}\\ -K_1 & -K_2\end{bmatrix},\ B = \begin{bmatrix}0_{n\times n}\\ I_{n\times n}\end{bmatrix}$$

Since $\{K_1, K_2\}$>0, the matrix A_1 is Hurwitz. Thus, for any symmetric positive define matrix Q, there exists a symmetric positive defined matrix P satisfying the Lyapunov equation

$$A_1^T P + PA_1 = -Q \tag{39}$$

Let define the Lyapunov function candidate

$$V = \overline{\mathbf{e}}^T P\overline{\mathbf{e}} + \mathbf{e}_\eta^T\Gamma\mathbf{e}_\eta \tag{40}$$

where Γ is a positive definite symmetric matrix.
Using equations (38) and (39), the time derivative of V is given by

$$\dot{V} = -\overline{\mathbf{e}}^T Q\overline{\mathbf{e}} + 2\mathbf{e}_\eta^T\{(\mathbf{D}_0^{-1})^T B^T P\overline{\mathbf{e}} + \Gamma\dot{\mathbf{e}}_\eta\} \tag{41}$$

If we define

$$\dot{\mathbf{e}}_\eta = -\Gamma^{-1}(\mathbf{D}_0^{-1})^T B^T P\overline{\mathbf{e}} \tag{42}$$

Since there is no information about uncertainties variations, it can be assumed that $\dot{\eta}$ (t) = 0 (Chan et al., 1999). This assumption does not necessarily mean a constant variable, but that the changing rate in every sampling interval should be slow.
From (42), the dynamics of the uncertainties estimation is given by

$$\dot{\eta}_{est} = \Gamma^{-1}(\mathbf{D}_0^{-1})^T B^T P\overline{\mathbf{e}} \tag{43}$$

Using the definition (42), it follows that the Lyapunov function V satisfies $\dot{V} < 0$ along solution trajectories of equation (6) because

$$\dot{V} = -\overline{\mathbf{e}}^T Q\overline{\mathbf{e}} \tag{44}$$

This guarantees that $\overline{\mathbf{e}}$ (t) and $\mathbf{e}_\eta(t)$, and therefore $\eta_{est}(t)$, are bounded.
The uncertainties estimation equation (42) can also be written as

$$\eta_{est}(t) = \int \Gamma^{-1}(\mathbf{D}_0^{-1})^T B^T P\overline{\mathbf{e}}(t)dt \tag{45}$$

4.2 Estimator based on the model control law

From the model dynamic of the robot (4), an estimator for uncertainties can be defined as

$$\begin{aligned}\dot{\eta}_{est} &= L\mathbf{D}_0^{-1}(\mathbf{x}_1)(\eta - \eta_{est}) \\ &= -L\mathbf{D}_0^{-1}(\mathbf{x}_1)\eta_{est} + L\left(\ddot{\mathbf{y}} + \mathbf{D}_0^{-1}(\mathbf{x}_1)\mathbf{C}_0(\mathbf{x}_1,\mathbf{x}_2)\dot{\mathbf{y}} + \mathbf{D}_0^{-1}(\mathbf{x}_1)\mathbf{G}_0(\mathbf{x}_1) - \mathbf{D}_0^{-1}(\mathbf{x}_1)\mathbf{u}\right)\end{aligned} \tag{46}$$

where, $L = \ell * I_{n\times n} \in \Re^{n\times n}$ is a matrix gain, and ℓ is a positive constant (Chen et al., 2000; Chen et al., 1999; Feng et al., 2002).
From the equation (40) and with the assumption $\dot{\eta}$ (t) = 0, the dynamic of the uncertainty estimator is given by

$$\dot{\mathbf{e}}_\eta + L\mathbf{D}_0^{-1}\mathbf{e}_\eta = 0 \tag{47}$$

Since L> 0 and $\mathbf{D}_0$>0, it can be easily verified that the tracking error of the estimation converge to zero.
Substituting the control law (30) in the observer equation (46), the dynamic of the uncertainties estimation is given by

$$\dot{\eta}_{est}(t) = L\left(\ddot{\mathbf{e}}_{\mathrm{y}}(t) + K_2\dot{\mathbf{e}}_{\mathrm{y}}(t) + K_1\mathbf{e}_{\mathrm{y}}(t)\right) \tag{48}$$

Integrating the equation (48), the uncertainties estimation is defined by

$$\eta_{est}(t) = L\left(\dot{\mathbf{e}}_{\mathrm{y}}(t) + K_2\mathbf{e}_{\mathrm{y}}(t) + K_1\int \mathbf{e}_{\mathrm{y}}(t)\, dt\right) \tag{49}$$

The advantage of the uncertainties estimator (49) compared to (45) is that it contains an integral action, which allows achieving zero steady state error for constant reference inputs and disturbances (Corriou, 2004; Cavallo et al., 1999; Feuer, & Goodwin 1989).

5. Nonlinear observer based state estimation

The computation of a model control law, such as linearization control and model predictive control, requires angular position and velocity measurements. In the practical robotic systems all the generalized coordinates can be precisely measured by the encoder for each joint, but the velocity measurements obtained through the tachometers are easily perturbed by noises. To overcome these physical constraints, a nonlinear observer can be used for state estimation (Kozłowski, 2004; Rodriguez-Angeles & Nijmeijer, 2004; Heredia & Yu, 2000).
The state space model of rigid robot (6), (7) can be reorganized as

$$\begin{cases} \dot{\mathbf{x}}_1 = \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 = f(\mathbf{x}_1, \mathbf{x}_2) + g(\mathbf{x}_1)\mathbf{u} + g(\mathbf{x}_1)\eta \\ \mathbf{y} = \mathbf{x}_1 \end{cases} \tag{50}$$

where, $\mathbf{x}_1 = \mathbf{q}$; $\mathbf{x}_2 = \dot{\mathbf{q}}$, $\mathbf{y}$ is the measurable position vector.

$$\begin{cases} f(\mathbf{x}_1, \mathbf{x}_2) = -\mathbf{D}_0(\mathbf{x}_1)^{-1}\left(\mathbf{C}_0(\mathbf{x}_1, \mathbf{x}_2)\mathbf{x}_2 + \mathbf{G}_0(\mathbf{x}_1)\right) \\ g(\mathbf{x}_1) = \mathbf{D}_0(\mathbf{x}_1)^{-1} \end{cases} \tag{51}$$

The nonlinear state observer based on high gain for the system (50) can be designed, to estimate angular positions and velocities, as

$$\begin{cases} \dot{\hat{\mathbf{x}}}_1 = \hat{\mathbf{x}}_2 + H_1(\mathbf{y} - \hat{\mathbf{x}}_1) \\ \dot{\hat{\mathbf{x}}}_2 = f(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) + g(\hat{\mathbf{x}}_1)\mathbf{u} + g(\hat{\mathbf{x}}_1)\hat{\eta}_{est} + H_2(\mathbf{y} - \hat{\mathbf{x}}_1) \end{cases} \tag{52}$$

where, $\hat{\mathbf{x}}_i$ (i = 1, 2) are the estimated states; $\hat{\eta}_{est}$ is the estimated uncertainty carried out from (49) with estimated states.
The estimated nonlinear functions f(.) and g(.) are given by:

$$\begin{cases} f(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) = -\mathbf{D}_0(\hat{\mathbf{x}}_1)^{-1}\left(\mathbf{C}_0(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2)\hat{\mathbf{x}}_2 + \mathbf{G}_0(\hat{\mathbf{x}}_1)\right) \\ g(\hat{\mathbf{x}}_1) = \mathbf{D}_0(\hat{\mathbf{x}}_1)^{-1} \end{cases} \tag{53}$$

From (50) and (52), the observer error dynamic is given, in matrix form, by

$$\dot{\tilde{\mathbf{e}}}(t) = H\tilde{\mathbf{e}}(t) + W\delta(t) \tag{54}$$

where,

$$\tilde{\mathbf{e}} = \begin{bmatrix} \tilde{\mathbf{e}}_y \\ \dot{\tilde{\mathbf{e}}}_y \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 - \hat{\mathbf{x}}_1 \\ \mathbf{x}_2 - \hat{\mathbf{x}}_2 \end{bmatrix}, H = \begin{bmatrix} -H_1 & I_{n\times n} \\ -H_2 & 0_{n\times n} \end{bmatrix}, W = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \end{bmatrix},$$

$H_1 = h_1 * I_{n\times n}$, $H_2 = h_2 * I_{n\times n}$, and h_1, h_2 are positive constants.

$\delta(.)$ is the disturbance term in the state observer. It is given by

$$\delta(.) = f(.) - f(\hat{.}) + \big(g(\hat{.}) - g(\hat{.})\big)\mathbf{u} + g(.)\eta - g(\hat{.})\hat{\eta}_{est} \tag{55}$$

The observer gain H is chosen to be a Hurwitz matrix in order to guarantee the convergence. In the presence of δ, the observer gains are adjusted as

$$h_1 = \frac{\gamma_1}{\varepsilon},\ h_2 = \frac{\gamma_2}{\varepsilon^2} \tag{56}$$

where, $0 < \varepsilon \ll 1$, and γ_1, γ_2 are positive constants.
This adjustment allows making the transfer function from δ to the error small so that the estimation error is not sensitive to the modeling error (Wang & Gao, 2003; Khalil, 1999; Heredia & Yu, 2000).

6. Global stability of the closed loop system

This section aims to discuss the global convergence of the tracking error for the closed loop system. The theory of stability, based on Lyapunov method, is used to prove the global stability of the robot system controlled by the robust estimated nonlinear control law.
The propriety of boundedness of the model elements of the robot are given from (Spong et al., 2006).

- Since $\mathbf{D}_0(\mathbf{q}) > 0$, it can be assumed that $\underline{\mathbf{D}} \leq \left\|\mathbf{D}_0(\mathbf{q})^{-1}\right\| \leq \overline{\mathbf{D}}$, where $\underline{\mathbf{D}}$, $\overline{\mathbf{D}}$ are positive constants.
- The matrix $\mathbf{C}_0(\mathbf{q},\dot{\mathbf{q}})$ is linear on $\dot{\mathbf{q}}(t)$ and bounded on $\mathbf{q}(t)$. Therefore, $\left\|\mathbf{C}_0(\mathbf{q},\dot{\mathbf{q}})\right\| \leq \alpha_1\left\|\dot{\mathbf{q}}\right\|; \alpha_1 \in \Re^+$.
- The vector $\mathbf{G}_0(\mathbf{q})$ satisfies $\left\|\mathbf{G}_0(\mathbf{q})\right\| \leq \alpha_2$; $\alpha_2 \in \Re^+$.
- All variations $\Delta(.)$ are bounded.
- The signals $\mathbf{q}_\mathbf{r}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r}$ are bounded, such as $\left\|\mathbf{q}_\mathbf{r}(t)\right\| \leq r_0$, $\left\|\dot{\mathbf{q}}_\mathbf{r}(t)\right\| \leq r_1$ and $\left\|\ddot{\mathbf{q}}_\mathbf{r}(t)\right\| \leq r_2$.
- The disturbance term $\delta(.)$ is smaller than the state observer error. Thus, $\left\|\delta(t)\right\| \leq \gamma\left\|\tilde{\mathbf{e}}(t)\right\|$.
- The vector function $f(\mathbf{x}_1, \mathbf{x}_2)$ is Lipschitz with respect to $\mathbf{x}_2$. Thus, there exists $\kappa > 0$ such that
  $\left\|f(\mathbf{x}_1,\mathbf{x}_2) - f(\mathbf{x}_1,\dot{\mathbf{q}}_{ref})\right\| \leq \kappa\left\|\mathbf{x}_2 - \dot{\mathbf{q}}_{ref}\right\| = \kappa\left\|\mathbf{e}_2\right\|;$
  $\forall(\mathbf{x}_1,\mathbf{x}_2) \in \Re^n \times \Re^n$

Integrating the state observer in the control loop, the control law is carried out with the state estimation (Rodriguez-Angeles & Nijmeijer, 2004). Based on state observer (52), the model control law (36) becomes

$$\mathbf{u}(t) = -\mathbf{D}_0(\hat{\mathbf{x}}_1)\left\{K_1\hat{\mathbf{e}}_{\mathrm{y}} + K_2\dot{\hat{\mathbf{e}}}_{\mathrm{y}} - \mathbf{D}_0(\hat{\mathbf{x}}_1)^{-1}\left(\mathbf{C}(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2)\hat{\mathbf{x}}_2 + \mathbf{G}(\hat{\mathbf{x}}_1)\right) - \ddot{\mathbf{y}}_{1\mathbf{r}}\right\} - \hat{\eta}_{est}(t) \tag{57}$$

where, $\hat{\mathbf{e}}_{\mathrm{y}} = \hat{\mathbf{y}} - \mathbf{y}_{\mathbf{r}}$

The disturbance estimator (49) is carried out with the state estimation, which is expressed by

$$\hat{\eta}_{est}(t) = L\left(\dot{\hat{\mathbf{e}}}_{\mathrm{y}}(t) + K_2\hat{\mathbf{e}}_{\mathrm{y}}(t) + K_1\int \hat{\mathbf{e}}_{\mathrm{y}}(t)\, dt\right) \tag{58}$$

Substituting the control law (57) in the equation (13) with estimated states from (52), we have the dynamic of the tracking error as

$$\ddot{\hat{\mathbf{e}}}_{\mathrm{y}}(t) + K_2\dot{\hat{\mathbf{e}}}_{\mathrm{y}}(t) + K_1\hat{\mathbf{e}}_{\mathrm{y}}(t) = H_2 C\widetilde{\mathbf{e}}(t) \tag{59}$$

Using the state space form, the tracking error system (59) can be written as

$$\dot{\hat{\mathbf{e}}}(t) = A\hat{\mathbf{e}}(t) + B\widetilde{\mathbf{e}}(t) \tag{60}$$

where,

$$\hat{\mathbf{e}} = \begin{bmatrix} \hat{\mathbf{e}}_{\mathrm{y}} \\ \dot{\hat{\mathbf{e}}}_{\mathrm{y}} \end{bmatrix}, A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -K_1 & -K_2 \end{bmatrix}, \ B = \begin{bmatrix} H_2 & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix}$$

Using the estimated states of the robot model (52), the uncertainty estimator (46) can be re-designed as

$$\dot{\hat{\eta}}_{est} = -L\mathbf{D}_0^{-1}(\hat{\mathbf{x}}_1)\eta_{est} + L\left(\ddot{\hat{\mathbf{y}}} + \mathbf{D}_0^{-1}(\hat{\mathbf{x}}_1)\mathbf{C}_0(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2)\dot{\hat{\mathbf{y}}} + \mathbf{D}_0^{-1}(\hat{\mathbf{x}}_1)\mathbf{G}_0(\hat{\mathbf{x}}_1) - \mathbf{D}_0^{-1}(\hat{\mathbf{x}}_1)\mathbf{u}\right) \tag{61}$$

where, $\hat{\eta}_{est}$ is the uncertainty estimation based on estimated states.

So, the new error dynamic of the uncertainty estimator, based on estimation state model (52), is given by

$$\dot{\hat{\mathbf{e}}}_{\eta}(t) = -LH_2\widetilde{\mathbf{e}}(t) \tag{62}$$

where, $\hat{\mathbf{e}}_{\eta} = \eta - \hat{\eta}_{est}$ is uncertainty error

From (62), it can be noticed that the convergence of the uncertainty estimator is related to the convergence of the state observer.

Under the state space form, the tracking error of the global system (robot + state observer + controller) can be carried out using error models (54) and (60)

$$\dot{\mathbf{e}}(t) = \mathbf{A}\mathbf{e}(t) + \mathbf{B}\delta(t) \tag{63}$$

where,

$$\mathbf{e} = \begin{bmatrix} \hat{\mathbf{e}} \\ \tilde{\mathbf{e}} \end{bmatrix}, \mathbf{A} = \begin{bmatrix} A & B \\ 0_{2n\times 2n} & H \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0_{n\times 2n} & W \end{bmatrix}$$

By an appropriate choice of control parameters K_i (i=1,…,n) and state observer gain H, it can be ensured that the matrix **A** is Hurwitz. Therefore, for any symmetric positive define matrix **Q**, there exists a symmetric positive defined matrix **P** satisfying the Lyapunov equation

$$\mathbf{A}^T\mathbf{P} + \mathbf{PA} = -\mathbf{Q} \tag{64}$$

Let define the Lyapunov function candidate

$$V = \mathbf{e}^T\mathbf{Pe} \tag{65}$$

Its derivative is given by

$$\dot{V} = -\mathbf{e}^T\mathbf{Qe} + 2\mathbf{e}^T\mathbf{PB}\,\delta \tag{66}$$

Using the relationship

$$\lambda_{\min}(\mathbf{Q})\|\mathbf{e}\|^2 \le \mathbf{e}^T\mathbf{Qe} \le \lambda_{\max}(\mathbf{Q})\|\mathbf{e}\|^2 \tag{67}$$

where $\lambda_{\min}(\mathbf{Q})$, $\lambda_{\max}(\mathbf{Q})$ denote the minimum and the maximum eigenvalues, respectively, of the matrix **Q**.
We have

$$\dot{V} \le -\lambda_{\min}(\mathbf{Q})\|\mathbf{e}\|^2 + 2\|\delta\|\,\|\mathbf{B}\|\,\|\mathbf{P}\|\,\|\mathbf{e}\| \tag{68}$$

Using the last propriety of boundedness, we have

$$\begin{aligned} \dot{V} &\le -\lambda_{\min}(\mathbf{Q})\|\mathbf{e}\|^2 + 2\gamma\lambda_{\max}(\mathbf{P})\,\|\mathbf{e}\|^2 \\ &\le -\left(\lambda_{\min}(\mathbf{Q}) - 2\gamma\lambda_{\max}(\mathbf{P})\right)\|\mathbf{e}\|^2 \end{aligned} \tag{69}$$

The condition, $\dot{V}$ is definite negative, is held when $\gamma < \dfrac{\lambda_{\min}(\mathbf{Q})}{2\lambda_{\max}(\mathbf{P})}$. Therefore, by LaSalle's invariance theorem, the origin is asymptotically stable. The global asymptotic stability of the estimated closed loop system with uncertainties is guaranteed.

7. Simulation results and discussion

We consider the two-link rigid robot manipulator to illustrate the performances of the nonlinear model predictive controller (36) with uncertainties compensation expressed by the observers (45) and (49) respectively (Merabet & Gu, 2008). The structure of the robot system driven by nonlinear model based control law is shown in figure 1.

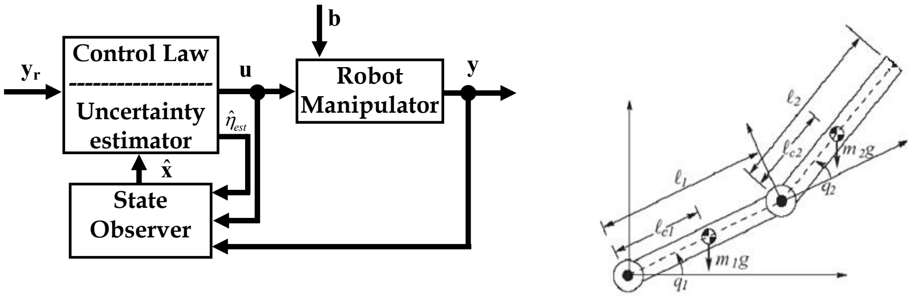

Fig. 1. Nonlinear model based control for two link rigid robot manipulator

The elements of the two-link robot model are given by

$$D_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos q_2) + I_1 + I_2;$$
$$D_{12} = D_{21} = m_2(l_{c2}^2 + l_1 l_{c2}\cos q_2) + I_2;$$
$$D_{22} = m_2 l_{c2}^2 + I_2$$
$$C_{11} = -(m_2 l_1 l_{c2}\sin q_2)\dot{q}_2;$$
$$C_{12} = -(\dot{q}_1 + \dot{q}_2)m_2 l_1 l_{c2}\sin q_2;$$
$$C_{21} = (m_2 l_1 l_{c2}\sin q_2)\dot{q}_1; C_{22} = 0$$
$$G_1 = (m_1 l_{c1} + m_2 l_1)g\cos q_1 + m_2 l_{c2} g\cos(q_1 + q_2);$$
$$G_2 = m_2 l_{c2} g\cos(q_1 + q_2)$$

For i = 1, 2, q_i denotes the joint angle; m_i denotes the mass of link i; l_i denotes the length of link i; l_{ci} denotes the distance from the previous joint to the center of mass of link i; and I_i denotes the moment of inertia of link i (Spong et al., 2006).
The nominal values of robot parameters are:

Link 1: m_1 = 10 kg, l_1 = 1 m, l_{c1} = 0.5 m, I_1 = 10/12 kg-m^2.
Link 2: m_2 = 5 kg, l_2 = 1 m, l_{c2} = 0.5 m, I_2 = 5/12 kg-m^2.

The model is simulated with a sample time of 10^{-4}s and the initial values of angular positions and velocities are $\hat{\mathbf{x}}$ = [0.1 rad 0 rad/s]^T for the state observer, and for the robot model $\mathbf{x}$ (0) = [0 rad 0 rad/s]^T. The parameters of the controller, uncertainties observers and state observer are chosen by trial and error in order to achieve accurate performances.
First, the tracking performance of robot system, driven by the nonlinear model predictive control law (36), is tested without the uncertainties observer. The robot system is affected by external disturbance **b**, which has the value 10 in the time interval [0.5s 4s]. The disturbance term is included in the robot model and the information about it is not taken into account when carrying out the control law. The value of prediction time is τ_r = 10^{-3}s. The state observer gain is taken as H_1=H_2= [10^4 0; 0 10^8]. Figure 2 shows the result for angular positions and tracking errors. It can be seen that small tracking errors, for both joints, are successfully achieved. However, steady errors occur in the system responses. The present

situation can be explained by the fact that the control law has no information about the external disturbances in order to compensate their effects. Figure 3 illustrates the induced control torque applied to robot manipulator. Note that the control torque lie inside the saturation limits. From figure 4, we can observe that the estimation errors are good although the presence of steady errors in the responses. As shown in the equation of state observer (52), the information about uncertainties is needed to have an accurate performance.

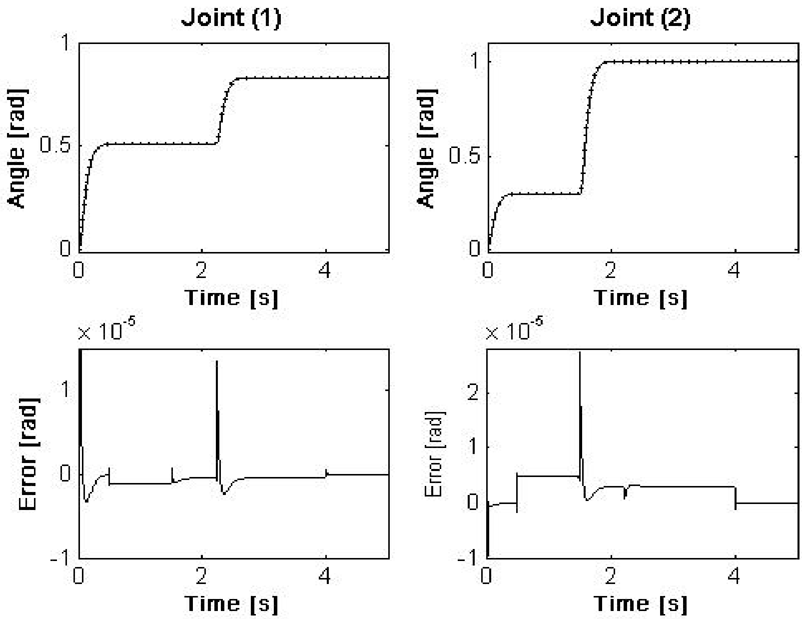

Fig. 2. Angular positions and tracking errors of distributed system without uncertainties compensator. ……. reference, ———estimate

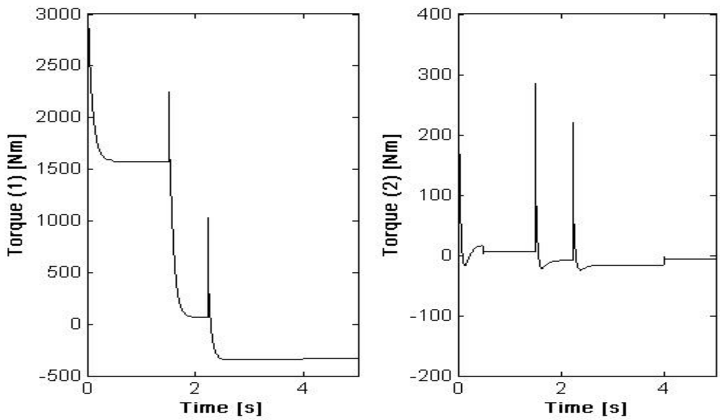

Fig. 3. Induced torque control produced from the nonlinear model predictive controller

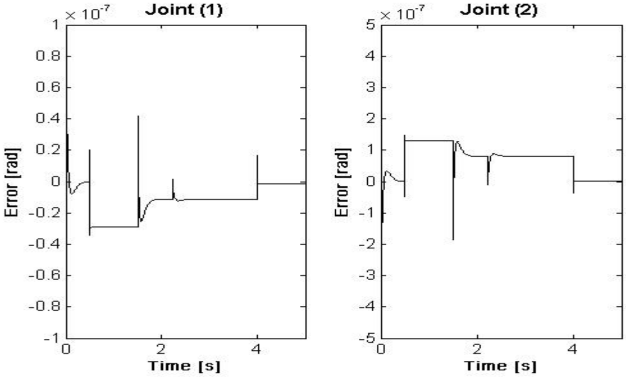

Fig. 4. Error estimation of the nonlinear state observer (controller without compensation)

Then, the uncertainties observers (45) and (49) are applied to the control law (36) respectively. The matrix P has the value [10^6 0; 0 10^6] and $\Gamma = I_{n\times n}$ for the observer (45), and L=[10^2 0; 0 10^2] for the observer (49). Figure 5 illustrates the angular positions and tracking errors of the system with uncertainties compensators. The steady error is vanished completely with the compensator (49), which means that the disturbance is well rejected. However, with the compensator (45), the steady error is only reduced compared with the results in figure (2).The elimination of steady errors by the compensator (49) can be explained by the presence of the integral action. It is known in control theory that an integral action achieves zero steady state error for constant reference inputs and disturbances. The same observation can be noticed in the result of state estimation errors shown in figure 6, where the uncertainties, carried out by the compensator (49), are included in state observer.

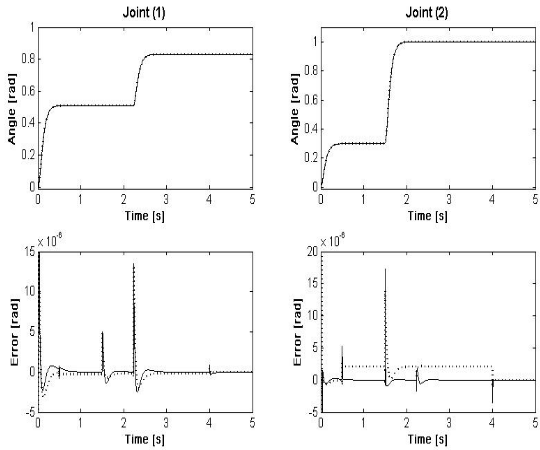

Fig. 5. Angular positions and tracking errors of distributed system with uncertainties compensator. ….. reference, —·— estimate with compensator (45) , —— estimate with compensator (49) ….. tracking error with compensator (45), —— tracking error with compensator (49)

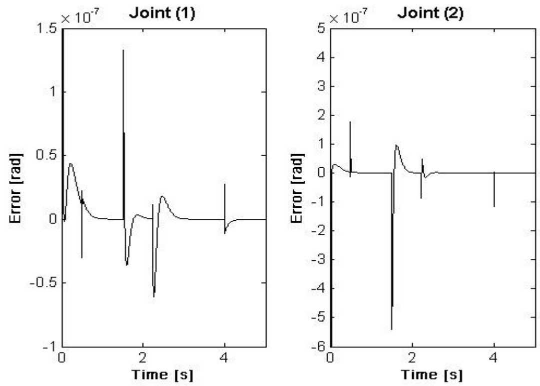

Fig. 6. Error estimation of the nonlinear state observer (controller with compensation)

In case of mismatched model, an unknown load carried by the robot is regarded as part of the second link, then the parameters m_2, l_{c2}, I_2 will change, $m_2+\Delta m_2$, $l_{c2}+\Delta l_{c2}$, $I_2+\Delta I_2$, respectively. The variations values are $\Delta m_2 = 1.5$, $\Delta l_{c2} = 0.125$, $\Delta I_2 = 1/12$. Also, the friction (Coulomb and viscous friction) given by $\mathbf{F_r}(\mathbf{x}_2)=\mathbf{F_c}\, sign(\mathbf{x}_2)+\mathbf{F_v}\mathbf{x}_2$, with values $\mathbf{F_c}=\mathbf{F_v} = diag(5, 5)$, are added to the robot model. The same parameters values of the controller, disturbance observer (45) and state observer are used as declared above. However, the gain of compensator (49) is decreased L = [70 0; 0 70]. As shown in figure 7, in case of the compensator (49), the errors occur in transient response, for this raison the gain is decreased, then reach zero. In case of the compensator (45), the errors in transient response are smaller than in the first case, but they do not reach zero like the other observer.

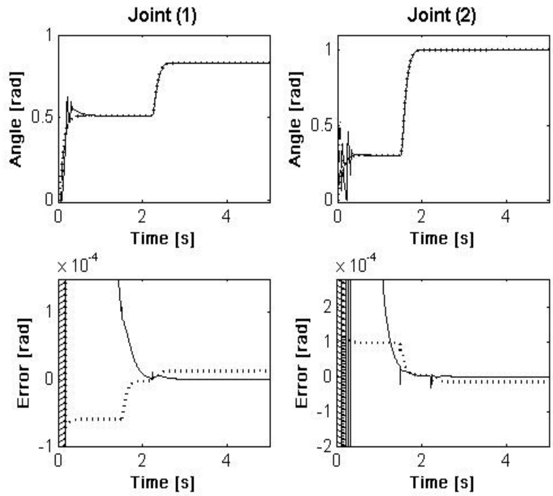

Fig. 7. Angular positions and tracking errors of mismatched model with uncertainties compensator. ….. reference, –·–·– estimate with compensator (45) , ——— estimate with compensator (49). ….. tracking error with compensator (45), ——— tracking error with compensator (49)

The results show that the tracking performance is successfully achieved and the effect of external disturbance is well rejected with the compensator (49). Concerning the unmodeled quantities and parametric uncertainties, the nonlinear model predictive controller, combined with uncertainties observer, deals well with their variations. It can be mentioned also that the state estimation, given by the nonlinear observer, is accurate for the tracking performance. The accuracy of the estimated nonlinear model predictive control combined with the compensator (49) is justified by the presence of the integral action, which eliminates steady state error.

8. Conclusions and future work

This chapter has presented some methods of advanced nonlinear control for robot systems. However, to cover all issues related to nonlinear control in detail will demand more than a chapter. The study has focused on model based control where a model dynamic of the process is needed to carry out the control law.

Two nonlinear control approaches have been detailed in this work. A feedback linearization control based on input-output linearization has been developed using differential-geometric methods for nonlinear systems. Then, a model based predictive control has been discussed for a nonlinear control design to robot manipulators. The predictive control law minimizes a cost function for the control trajectory over a future time horizon. The control solution has been analytically derived, with no need of an online optimization, which enables fast real-time implementation.

Because of the uncertainties present in the system, a robustness strategy has been studied to enhance the tracking response of the system. Two methods have been investigated to deal with system uncertainties. One method is based on the theory of guaranteed stability of uncertain systems, which results to an observer taking information from the system tracking errors. The other one is an observer derived from the nonlinear model control law. It contains an integral action on system tracking errors. This type of control strategy is robust with respect to modeling errors, very effective in disturbance rejection, and gives no steady error caused by either parameters uncertainties or external disturbances.

The development of these control strategies is related to the dynamic model of the process. In case of missing information about the system states, a version of control law based on state has been carried out with the quantities, angular positions and velocities, issued from a nonlinear state estimator. It has been shown that the tracking performance is achieved successfully when the uncertainties are well compensated.

The issue of global stability of the closed loop system has been proved analytically via Lyapunov stability theory.

The nonlinear control laws developed in this chapter are based on a dynamic model of the process. However, it is well known that mathematical representation of a dynamic model does not refer accurately to the reality. This is why it is very important to add to the control strategy a robustness analysis in order to compensate the uncertainties present in the dynamic model. As an alternative of this approach, intelligent control based on the process behavior can be considered as a solution for tracking motion of robot manipulators. Intelligent control achieves automation via the emulation of behavioral intelligence such as biological intelligence (e.g., the use of neural networks and genetics for control); the use of human's knowledge to design a smart control methodology (fuzzy control). This research area is very wide and the issues of modeling, mathematical stability, convergence and robustness analysis for learning systems must be investigated to design an accurate controller.

9. References

Bordon, C. & Camacho, EF. (1998). A generalized predictive controller for a wide class of industrial processes. *IEEE Transactions on Control Systems Technology 6*(3), pp. 372-387.

Cavallo, A.; De Maria, G. & Nistri, P. (1999). Robust control design with integral action and limited rate control. *IEEE Transactions on Automatic Control 44*(8), pp. 1569-1572.

Chen, W-H.; Balance, DJ.; Gawthrop, PJ. & O'Reilly, J. (2000). A nonlinear disturbance observer for robotic Manipulators. *IEEE Transactions on Industrial Electronics 47* (4), pp. 932-938.

Chen, W-H.; Balance, DJ.; Gawthrop, PJ.; Gribble JJ. & O'Reilly, J. (1999). Nonlinear PID predictive controller. *IEE Proceedings Control Theory application 146*(6), pp. 603-611.

Corriou, JP. (2004). *Process Control. Theory and Applications*. Springer, London, UK.

Curk, B. & Jezernik, K. (2001). Sliding mode control with perturbation estimation: Application on DD robot mechanism. *Robotica 19*(10), pp. 641-648.

Feng, W.; O'Reilly, J. & Balance, DJ. (2002). MIMO nonlinear PID predictive controller. *IEE Proceedings Control Theory application 149*(3), pp. 203-208.

Feuer, A. & Goodwin, GC. (1989). Integral Action in Robust Adaptive Control. *IEEE Transactions on Automatic Control 34*(10), pp. 1082-1085.

Hedjar, R. & Boucher, P. (2005). Nonlinear receding-horizon control of rigid link robot manipulators. *International Journal of Advanced Robotic Systems 2*(1), pp. 015-024.

Hedjar, R.; Toumi, R.; Boucher, P. & Dumur, D. (2002). Feedback nonlinear predictive control of rigid link robot manipulator. *Proceedings of the American Control Conference*, Anchorage AK, pp. 3594-3599.

Heredia, JA. & Yu, W. (2000). A high-gain observer-based PD control for robot manipulator. *Proceedings of the American Control Conference*, Chicago, Illinois, USA, pp. 2518-2522.

Isidori, A. (1985). *Nonlinear Control Systems: An Introduction*. Springer-Verlag, Berlin, New York.

Isidori, A., & Ruberti, A. (1984). On the synthesis of linear input output responses for nonlinear systems. Systems and Control Letters, 4(1), pp. 17-22.

Khalil, HK. (1999). High-gain observers in nonlinear feedback control. New Directions in nonlinear observer design. *Lecture Notes in Control and Information Sciences 24*(4), pp. 249-268.

Klančar, G. & Škrjanc, I. (2007). Tracking-error model-based predictive control for mobile robots in real time. *Robotics and Autonomous Systems 55*, pp. 460-469.

Kozłowski, K. (2004), *Robot motion and control. Recent developments*. Springer, London, UK.

Merabet, A. & Gu, J. (2008). Robust nonlinear predictive control based on state estimation for robot manipulator. *International Journal of Applied Mathematics and Mechanics*, Vol. 5, No. 1, 48-64.

Merabet, A. & Gu, J. (2008). Estimated feedback linearization controller with disturbance compensator for robotic applications. *The Mediterranean Journal of Measurement and Control*, Vol. 4, No. 3, 101-110

Nijmeijer, H., & A. J. van der Schaft. (1990). Nonlinear Dynamic Control Systems. Springer-Verlag, New York, 1990.

Richalet, J. (1993). Industrial Applications of Model Based Predictive Control. *Automatica 29*(5), pp. 1251-1274.

Rodriguez-Angeles, A. & Nijmeijer, H. (2004). Synchronizing Tracking Control for Flexible Joint Robots via Estimated State Feedback. *ASME Journal of Dynamic Systems, Measurement and Control 126*, pp. 162-172.

Spong, MW.; Hutchinson, S. & Vidyasagar, M. (2006). *Robot modeling and control*. John Wiley & Sons, USA.

Vivas, A. & Mosquera, V. (2005). Predictive functional control of a PUMA robot. *ICGST, ACSE 05 Conference*, Cairo, Egypt, pp. 372-387.

Wang, W. & Gao, Z. (2003). A comparison study of advanced state observer design techniques. *Proceedings of the American Control Conference*, Denver, Colorado, USA, pp. 4754-4759.