ED6001 Medical Image Analysis

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Question 3) The Laplacian operator is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$ for unrotated coordinates and as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x_1^2} + \frac{\partial^2 f}{\partial y_1^2}$$ for rotated co-ordinates, where

$$x = x_1 \cos\theta - y_1 \sin\theta$$
$$y = x_1 \sin\theta + y_1 \cos\theta$$ where θ is the angle of rotation

$$\Rightarrow \frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial x_1} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial x_1}$$

$$= \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

Taking the derivative of this expression again with respect x_1

$$\frac{\partial^2 f}{\partial x_1^2} = \frac{\partial^2 f}{\partial x^2}\cos^2\theta + \frac{\partial^2 f}{\partial x \partial y}\sin\theta\cdot\cos\theta + \frac{\partial^2 f}{\partial x \partial y}\cos\theta\sin\theta + \frac{\partial^2 f}{\partial y^2}\sin^2\theta$$

Similarly computing $\frac{\partial^2 f}{\partial y_1^2}$

$$\frac{\partial^2 f}{\partial y_1^2} = \frac{\partial^2 f}{\partial x^2}\sin^2\theta - \frac{\partial^2 f}{\partial x \partial y}\cos\theta\sin\theta - \frac{\partial^2 f}{\partial x \partial y}\cos\theta\sin\theta + \frac{\partial^2 f}{\partial y^2}\cos^2\theta$$

→ Summing $\frac{\partial^2 f}{\partial x_1^2}$ and $\frac{\partial^2 f}{\partial y_1^2}$

$$\frac{\partial^2 f}{\partial x_1^2} + \frac{\partial^2 f}{\partial y_1^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Thus the laplacian operator is invariant of rotation

Question 4) Unsharp masking can be defined as high boost filtering with A=1

$$f_{hb}(x,y) = \begin{cases} Af(x,y) - \nabla^2 f(x,y) & \text{if center coefficient is -ve} \\ Af(x,y) + \nabla^2 f(x,y) & \text{if center coefficient is +ve} \end{cases}$$

~~wi~~ ~~Ass.~~ Taking Laplacian with positive center coefficient.

$$f_{um}(x,y) = f(x,y) + \nabla^2 f(x,y).$$

This ~~ea~~ filter can be applied in one pass as

-1	-1	-1
-1	9	-1
-1	-1	-1

Question 5) $g(x,y) = f(x+1,y) - f(x,y)$

Taking fourier transform

$G(u,v) = F(u,v)e^{j2\pi u/M} - F(u,v)$

$G(u,v) = F(u,v)\left(e^{j2\pi u/M} - 1\right)$

$G(u,v) = H(u,v) \cdot F(u,v)$

where

$H(u,v) = \left(e^{2j\pi u/M} - 1\right)$

~~$H(u,v) =$~~

~~F~~ $H(u,v)$ increases as the value of u increases.

with for $u = 0$

$H(u,v) = 0$

and $\frac{\partial H(u,v)}{\partial u} = 2j\frac{\pi}{M} \cdot e^{2j\pi u/M}$

which is always positive, thus $H(u,v)$ will increase

as u increases

Thus $H(u,v)$ is an ~~increasing~~ High pass filter.