Integrated Multi-Product Production Scheduling and Vehicle-Routing Problem Using Heterogeneous Fleet with Multiple Trips and Time Windows

Classification: Operational integrated production and outbound distribution problem known as Integrated production scheduling and vehicle routing problem (PS-VRP)

Rationale: This research is a variant of the PS-VRP, focusing on introducing a new and more comprehensive model addressing suggested gaps in literature.

Gaps in literature: Integration of real-life problem features in single problem.

Production – setup operations for multi-product production

Distribution – heterogeneous fleet (varying capacity and cost rates), limited fleet size available for delivery, allowing multiple trips for each vehicle, inclusion of service operation (loading and unloading time), customer specified time windows, and penalty for early and later deliveries.

Problem Description

A manufacturer operates a single production center to produce multiple types of a product ordered from different retailers in the city. A single machine is used and can produce all demands, adapting a batch production system, with no provisions of shortages. The machine produces multi-variants of a product sequentially until all orders have been processed. A set of customer orders is known in advance and must be processed and delivered within customer specified time windows. Each customer orders all types of products and the single machine can produce all types of the product following setup operations. Each type of product has its incurred processing time. Since some equipment need to be reconfigured to produce different variants, setup times and costs are assumed. At the beginning of production, the machine is setup for a product and setup time/cost between product variants depend on the previously produced variant.

Loading of orders in the plant is fixed. Unloading times are proportional to the work quantity (unloading rate x demand size of customer). Idle time of orders not loaded into a vehicle are negligible and will not incur any holding time or cost. Split delivery is not allowed which means that every customer's demand must be satisfied in one delivery. The distribution is performed by a heterogeneous fleet with varying capacities and cost rates. Travel times and costs are accounted, and routing decisions should be made so that distribution costs are minimized. The delivery operation starts by loaded vehicles in the production plant. After delivery, the vehicles return to the production plant. Multiple trips are possible for each vehicle. In the problem, there are customer specified hard and soft deadlines. Hard deadline, expressed as the hard lower bound, must be met while the soft deadline or soft upper bound can be delayed. When the delivery arrives later than the soft deadline, penalty cost is incurred. Penalty cost is proportional to the tardiness at customer location (penalty rate x time tardy).

The objective is to minimize total operating cost which includes processing cost + setup cost for production and vehicle cost + traveling cost + penalty cost for distribution.

**Batching is defined as producing several customer orders in parallel (p-batching) or sequentially (s-batching) by a resource (Pinedo, 2008).*

Additional Assumptions

- Production is scheduled without idle time on machine. There is no preemption.
- Different variants of the product are similar in size. There is no concern with unit space.
- Vehicles start and end at the production plant.
- No order splitting.

Model Formulation

Table. Problem Parameters and Variables

Parameters	Description
Indices	
i, j	Customer index $1,2,\dots,n$; $0 =$ plant index (start location); $n+1 =$ plant index (final location), $0,1,2,\dots,n+1$
p	Product index $0,1,2,\dots,P$
q	Product index $0,1,2,\dots,P$
v	Vehicle index $0,1,2,\dots,V-1$
h	Vehicle tour index $0,1,2,\dots,n-1$
Parameters	

d_{ip}	Demand quantity for product p of customer i
ρ_p	Processing time of product p
σ_{pq}	Setup time of product q when product p is produced immediately before; when $p = 0$, product is first in production and when $q = 0$, product is last in production
s_0	Loading time at the manufacturing plant
s_i	Unloading time at customer location i
c_v	Capacity of vehicle v
t_{ij}	Travel time from customer i to customer j
$[a_i, b_i)$	Time window for customer i
C_{pq}^{σ}	Sequence dependent setup cost of changeover from product p to product q
C^{ρ}	Processing cost of product p
C^{v}	Fixed cost associated with each vehicle v
C^{t}	Travelling cost from customer i to customer j
C^{l}	Penalty cost for late delivery per time tardy
M	A very large number

Dependent Variables

f_p	Production finish time of product p
F	Completion time of production
k_{vh}	Delivery start time for hth trip of vehicle type v
α_i	Arrival time at node i
l_i	Tardiness at node i (time when delivery arrives later than specified soft deadline at node i)

Decision Variables

x_{pq}	q also from 0 to P? what does x[p][0] and x[p][P-1] mean respectively?	A binary variable that takes the value of 1 if product p is produced immediately before product q and no other product is processed in between, and 0 otherwise
y_{ivh}		A binary variable that takes the value of 1 if customer i is visited by the hth trip of vehicle type v, and 0 otherwise
z_{ijvh}	does z have to be 4 dimensional? I believe all the equations with z can be reduced such that z can only be 2 dimensional with only i and j	A binary variable that takes the value of 1 if customer i is immediately followed by customer j on the hth trip of vehicle type v, and 0 otherwise
w_v		A binary variable that takes the value of 1 if vehicle type v is used, and 0 otherwise

Objective Function

The objective is to minimise total operating cost which includes manufacturing cost *MC*, distribution cost *DC,* and expected penalty *PC* incurred.

$$MC = \sum_{p=0,p\neq q}^{P} \sum_{q=1}^{P} C_{pq}^{\sigma} x_{pq} + C^{\rho} \sum_{i=1}^{n} \sum_{p=1}^{P} \rho_p d_{ip} \quad (1)$$

$$DC = \sum_{v=1}^{V} C^{v} w_v + C^{t} \sum_{i=0,i\neq j}^{n} \sum_{j=1}^{n+1} \sum_{v=1}^{V} \sum_{h=1}^{n} t_{ij} z_{ijvh} \quad (2)$$

$$PC = C^{l} \sum_{i=1}^{n} l_i \quad (3)$$

$$\textbf{\textit{Minimize TOC}} = MC + DC + PC \quad (4)$$

Constraints

1. Production sequence of the products in the production run. - **Making the problem infeasible**

$$\sum_{p=0,p\neq q}^{P} x_{pq} = 1\,, \forall q = 0,1,2,\dots,P \tag{5}$$

$$\sum_{q=0,p\neq q}^{P} x_{pq} = 1\,, \forall p = 0,1,2,\dots,P \tag{6}$$

2. Each customer must be visited by one tour of one vehicle.

$$\sum_{v=0}^{V-1}\sum_{h=0}^{n-1} y_{ivh} = 1\,, \forall i = 1,2,\dots,n \tag{7}$$

3. A tour is denoted empty if there is no customer assigned to it. Tours with at least one customer are referred to as active tours. The processing site must be included in each active tour.

$$y_{0vh} \geq y_{ivh}, \forall i = 1,2,\dots,n; v = 0,1,2,\dots,\mathrm{V}-1; h = 0,1,2,\dots,\mathrm{n}-1 \tag{8}$$

$$y_{n+1vh} \geq y_{ivh}, \forall i = 1,2,\dots,n; v = 0,1,2,\dots,\mathrm{V}-1; h = 0,1,2,\dots,\mathrm{n}-1 \tag{9}$$

4. A vehicle should not be empty before an active tour. Succeeding tours for a vehicle can be active only if its previous tour is active.

$$M\sum_{i=1}^{n} y_{ivh} \geq \sum_{j=1,}^{n} y_{jvh+1}\,, \forall v = 0,1,2,\dots,\mathrm{V}-1; h = 0,1,2,\dots,n-2 \tag{10}$$

5. The total demand quantity delivered in the same tour should not exceed the capacity of the vehicle assigned to it.

$$\sum_{i=1}^{n}\sum_{p=1}^{P} d_{ip}y_{ivh} \leq c_v, \forall v = 0,1,2,\dots,V-1; h = 0,1,2,\dots,n-1 \tag{11}$$

6. Routing: If customer j is visited in the trip, vehicle v either travels from a previous customer i or from the production center. Afterwards, the vehicle returns to the production site or delivers to another customer. Tours must start and end at the manufacturing plant.

   Start and end at the plant:

$$z_{0ivh} + z_{0jvh} + y_{ivh} + y_{jvh} \leq 3 \tag{12}$$

$$z_{in+1vh} + z_{jn+1vh} + y_{ivh} + y_{jvh} \leq 3 \tag{13}$$

$$\forall i, j = 1,2, \dots, nandi \neq j; v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-1$$

Middle routing:

$$y_{j,v,h} = \sum_{i=0, i\neq j}^{n} z_{i,j,v,h}, \forall j = 1,2, \dots, n; v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-1 \quad (14)$$

$$y_{j,v,h} = \sum_{i=1, i\neq j}^{n+1} z_{j,i,v,h}, \forall j = 1,2, \dots, n; v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-1 \quad (15)$$

7. The vehicle v is used if there is a tour assigned to it.

$$y_{0,v,h} \geq w_v, \forall v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-1 \quad (16)$$

8. The start time of the hth tour of vehicle v is greater than or equal to the completion time of the production run plus the service time at the manufacturing plant.

$$k_{vh} \geq s_0 + F, \forall v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-1 \quad (17)$$

**calculation:

where the production finish time of product p, denoted by f_p, is as follows:

$$f_p = \rho_p \sum_{i=1}^{n} d_{ip}, \forall p = 0,1,2, \dots, P$$

total setup time is as follows:

$$\sum_{\substack{p=0,\\ p\neq q}}^{P} \sum_{\substack{q=0,\\ p\neq q}}^{P} \sigma_{p,q}\, x_{p,q}$$

The completion time of the entire production run is given by,

$$F = \sum_{p=0}^{P} f_p + \sum_{\substack{p=0,\\ p\neq q}}^{P} \sum_{\substack{q=0,\\ p\neq q}}^{P} \sigma_{p,q}\, x_{p,q}$$

9. If it’s not the first tour of the vehicle, the delivery start time is greater than or equal to the sum of the arrival time to the last customer on the previous tour, unloading time at the customer location, and the travel time returning to the manufacturing plant.

$$k_{vh+1} \geq \alpha_j + s_j + t_{jn+1} - M\left(1 - y_{jvh}\right) \tag{18}$$

$$\forall j = 1,2, \dots, n; v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-2$$

10. The arrival time to the first customer in each tour is greater than or equal to the sum of the start time of the tour and the travel time from the manufacturing plant to the customer location.

$$\alpha_i \geq k_{vh} + t_{0i} - M(1 - y_{ivh}) \tag{19}$$

$$\forall i = 1,2, \dots, n; v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-1$$

11. The arrival time to succeeding customers j in the tour (not the first customer) is greater than or equal to the sum of the arrival time of the previous customer i, unloading time at the customer location, and travel time between two destinations.

$$\alpha_j \geq \alpha_i + s_i + t_{ij} - M(1 - z_{ijvh}) \tag{20}$$

$$\forall i,j = 1,2, \dots, n and i \neq j; v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-1$$

12. The arrival time to each customer should be greater than or equal to the lower time window bound.

$$\alpha_i \geq a_i, \forall i = 1,2, \dots, n \tag{21}$$

13. The tardiness or delay time to each customer is greater than equal to its arrival time and upper time window bound.

$$l_i \geq \alpha_i - b_i, \forall i = 1,2, \dots, n \tag{22}$$

$$l_i \geq 0, \forall i = 1,2, \dots, n \tag{23}$$

14. Binary and Non-negativity constraints (**included in variable definitions**).

$$x_{pq} \in \{0,1\}, y_{ivh} \in \{0,1\}, z_{ijvh} \in \{0,1\}, f_p \geq 0, F \geq 0, k_{vh} \geq 0, \alpha_i \geq 0 \tag{24}$$

$$\forall i,j = 1,2, \dots, n and i \neq j; p,q = 0,1,2, \dots, P-1 and p \neq q; v = 0,1,2, \dots, V-1; h = 0,1,2, \dots, n-1$$