MODEL 1

TITLE: Integrated multi-product production scheduling and vehicle-routing problem using heterogeneous fleet with multiple trips and time windows

CLASSIFICATION: Operational integrated production and outbound distribution problem (manufacturer – customers; vehicle-routing delivery method) known as Integrated production scheduling and vehicle routing problem (PS-VRP)

RATIONALE: This research is a variant of the PS-VRP, focusing on introducing a new and more comprehensive model addressing suggested gaps in literature.

GAPS IN LITERATURE: Consolidation of real-life problem features in a single problem. Features include: Production – setup operations for multi-product production; Distribution – limited fleet size available for delivery, allowing multiple trips for each vehicle, heterogeneous fleet (varying capacity and cost rates) for delivery, inclusion of service operation (loading and unloading time), customer specified delivery time windows, and penalty for early and late deliveries.

*Literature review criteria: PS-VRP papers which minimized cost function and consider multiple products in production

**Remark. Add a definition of terms section.

PROBLEM DESCRIPTION

A manufacturer operates a single production center to produce multiple types of a product ordered from different retailers in the city. In the production facility, a single machine is used and can produce all demands at the manufacturing plant; adapting a batch production system. A set of customer orders is known in advance and must be processed and delivered within customer specified time windows. Each customer orders all types of products and the single machine can produce all types of the product following setup operations. Each type of product has its incurred processing time. Since some equipment need to be reconfigured to produce different variants, setup times and costs are assumed. At the beginning of production, the machine is setup for a product and setup time between product variants depend on the previously produced variant.

> So, the production planning concerns (1) Batching of customer orders, (2) Sequencing of the products per batch, and (3) Scheduling/sequencing of production batches.

Loading of orders in the plant is fixed. Unloading times are proportional to the work quantity (unloading rate x demand size of customer). Split delivery is not allowed which means that every customer's demand must be satisfied by one vehicle on one stop. The distribution is performed by a heterogeneous fleet with varying capacities and cost rates. Travel times and costs are accounted, and routing decisions should be made so that distribution costs are minimized. The delivery operation starts by loaded vehicles in the production plant. After delivery, the vehicles return to the production plant. Multiple trips are possible for each vehicle. In the problem, there are customer specified time windows. When the delivery arrives earlier than the earliest delivery time, the vehicle is entertained by a penalty cost is incurred for possible disruption in the customer's schedule. Similarly, when the delivery arrives later than the latest delivery time, penalty cost is incurred. Penalty cost is proportional to the delay time (penalty rate x time delay).

So, the distribution planning concerns (1) Assignment of production batches to delivery vehicles/trips, (2) Routing of customers in the vehicle trip, and (3) Recycling of vehicles.

The objective is to minimize total operating cost which includes processing cost + setup cost for production and vehicle cost + traveling cost + penalty cost for distribution.

ADDITIONAL ASSUMPTIONS:

Production is scheduled without idle time on machine. There is no preemption.

Different variants of the product are similar in size. There is no concern with unit space.

Vehicles start and end at the production plant.

No order splitting.

*A production batch consists of multiple customers. (Different batch = different customers)

*Each production batch is assigned to a vehicle trip and is composed of only one batch. (Customers in the production batch are same customers to be visited in the trip)

---MODEL FORMULATION---

INDICES

$i, j \in I = \{0,1,2,\dots,n,n+1\}$	customer indices where $i = 0, n+1$ is the manufacturing plant
$p, q \in P = \{1,2,\dots,r\}$	product type
$v \in V = \{1,2,\dots,s\}$	vehicle index specified distinctly (type and number)
$h \in H = \{1,2,\dots,g\}$	trip index
$f, f' \in B = \{1,2,\dots,t\}$	production batch name

PARAMETERS

$d_{i,p}$	demand quantity for product p of customer i
$\sigma_{p,q}$	set-up time of producing product q immediately after product p $\sigma_{0,p}$ – p is the first item in the production sequence $\sigma_{p,0}$ – p is the last item in the production sequence
ρ_p	processing time per unit of product p
$\tau_{i,j}$	travel time from customer i to customer j
$[a_i, b_i]$	time window for customer i
c_v	capacity of vehicle type v
s_i	service time (loading and unloading) at customer i
C^ρ	processing cost (per process time)
C^σ	set-up cost (per set-up time)
C^τ	travel cost (per time travelled)
C^e	penalty cost for early delivery (per unit time early)
C^l	penalty cost for late delivery (per unit time late)
F_v	fixed cost of vehicle type v used

M sufficiently large number

DEPENDENT VARIABLES

no solution is generated for this variable as this is part of no constraint in this document

s_f^p start time of production for batch f

c_f^p completion time of production for batch f

$s_{v,h}^d$ start time of delivery for the h^{th} trip of vehicle type v

$\alpha_{j,v,h}$ arrival time at customer j for the h^{th} trip of vehicle type v

$e_{j,v,h}$ time (in minutes) when vehicle arrives earlier than earliest arrival to customer j for the h^{th} trip of vehicle type v

$l_{j,v,h}$ time (in minutes) when vehicle arrives later than latest arrival from to customer j for the h^{th} trip of vehicle type v

DECISION VARIABLES (BINARY)

$x_{p,q}$ 1 if product q is produced immediately after product p
0 otherwise

$\beta_{j,f}$ 1 if customer j's order is produced in batch f
0 otherwise

for some reasons no solution is generated for this variable

δ_f 1 if batch f is active (if batch f is produced)
0 otherwise

$\gamma_{f,f'}$ 1 if batch f is scheduled before batch f'
0 otherwise

$\theta_{f,v,h}$ 1 if batch f is assigned to the h^{th} trip of vehicle type v
0 otherwise

$u_{j,v,h}$ 1 if customer j is visited on the h^{th} trip of vehicle type v
0 otherwise

$y_{i,j,v,h}$ 1 if customer j is visited immediately after customer i in h^{th} trip of vehicle type v
0 otherwise

for some reasons no solution is generated for this variable

w_v 1 if the vehicle type v is used for delivery
0 otherwise

OBJECTIVE FUNCTION

Without the highlighted part of the objective, the value is very low (80.0 for 8 variable case). With the highlighted part the value is is very high (32375.0) for the 8 variable case. The solution remains the same irrespective. Moreover, there are no model variables in this part of the objective

$$Minimize\ TOC = C^{\rho}\left(\sum_{j=1}^{n}\sum_{p=1}^{r}\rho_p \cdot d_{j,p}\right) + C^{\sigma}\left(\sum_{p=1}^{r}\sum_{q=1,p\neq q}^{r}\sigma_{p,q}\cdot x_{p,q}\right) + C^{\tau}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{s}\sum_{h=1}^{g}\tau_{i,j}\,y_{i,j,v,h}\right) + \sum_{v=1}^{s}F_v\,w_v + C^{e}\left(\sum_{j=1}^{n}\sum_{v=1}^{s}\sum_{h=1}^{g}e_{j,v,h}\right) + C^{l}\left(\sum_{j=1}^{n}\sum_{v=1}^{s}\sum_{h=1}^{g}l_{j,v,h}\right)$$

CONSTRAINTS

The following applies for all permissible indices:

Binary Variables

$$x_{p,q,f} \in \{0,1\}$$
$$\beta_{j,f} \in \{0,1\}$$
$$\delta_f \in \{0,1\}$$
$$\gamma_{f,f'} \in \{0,1\}$$
$$\theta_{f,v,h} \in \{0,1\}$$
$$u_{j,v,h} \in \{0,1\}$$
$$y_{i,j,v,h} \in \{0,1\}$$
$$w_v \in \{0,1\}$$

Non-negativity Constraints

$$s_f^p \geq 0$$
$$c_f^p \geq 0$$
$$s_{v,h}^d \geq 0$$
$$\alpha_{j,v,h} \geq 0$$
$$e_{j,v,h} \geq 0$$
$$l_{j,v,h} \geq 0$$

Others

$$s, g, t \leq n$$

Vehicle Routing Constraints

1. Each customer should be visited once and only once.

   For $j = 1,2, \ldots, n$

$$\sum_{v=1}^{s} \sum_{h=1}^{g} u_{j,v,h} = 1$$

2. Tour is denoted empty if there is no customer assigned to it. Tours with at least one customer are referred to as active tours. Processing site must be included in each active tour.

   For $j = 1,2, \ldots, n;\ \ v = 1,2, \ldots, s;\ \ h = 1,2, \ldots, g$

$$u_{0,v,h} \geq u_{j,v,h}$$
$$u_{n+1,v,h} \geq u_{j,v,h}$$

3. Total demand quantity of all the customers in the same trip should not exceed the capacity of the vehicle assigned to it.

   For $v = 1,2, \ldots, s;\ \ h = 1,2, \ldots, g$

$$\sum_{j=1}^{n} \sum_{p=1}^{r} d_{j,p} \cdot u_{j,v,h} \leq c_v$$

4. Trip must start and end at the plant. If customer j is visited in the trip, vehicle v either travels from a previous customer i or from the production center. Afterwards, the vehicle returns to the production site or delivers to another customer.

Start at the plant:

For $i, j = 1,2, \dots, n \; and \; i \neq j; \;\; v = 1,2, \dots, s; \;\; h = 1,2, \dots, g$

$$y_{0,i,v,h} + y_{0,j,v,h} + u_{i,v,h} + u_{j,v,h} \leq 3$$

Middle:

For $j = 1,2, \dots, n; \;\; v = 1,2, \dots, s; \;\; h = 1,2, \dots, g$

$$u_{j,v,h} = \sum_{i=0, i \neq j}^{n} y_{i,j,v,h}$$

$$u_{j,v,h} = \sum_{i=1, i \neq j}^{n+1} y_{j,i,v,h}$$

For $i, j = 1,2, \dots, n; \;\; v = 1,2, \dots, s; \;\; h = 1,2, \dots, g$

$$u_{i,v,h} \geq y_{i,j,v,h} + u_{j,v,h} - 1$$

$$u_{j,v,h} \geq y_{i,j,v,h} + u_{i,v,h} - 1$$

End at the plant:

For $i, j = 1,2, \dots, n \; and \; i \neq j; \;\; v = 1,2, \dots, s; \;\; h = 1,2, \dots, g$

$$y_{i,n+1,v,h} + y_{j,n+1,v,h} + u_{i,v,h} + u_{j,v,h} \leq 3$$

5. Vehicle trip is activated only if previous trip is also active constraints.

For $v = 1,2, \dots, s; \;\; h = 1,2, .., g - 1$

$$M \sum_{j=1}^{n} u_{j,v,h} \geq \sum_{j=1}^{n} u_{j,v,h+1}$$

6. The v^{th} vehicle is used if a trip is assigned to it ($u_{0,v.h} = 1$).

For $v = 1,2, \dots, s; \;\; h = 1,2, .., g$

$$u_{0,v,h} \geq \; w_v$$

Production Constraints

7. Product sequencing.

For $p, q \in \{1,2, \dots, r\}$

$$x_{p,q} \leq 1 - x_{q,p}$$

For $q \in \{1,2,\dots,r\}$

$$\sum_{p=1,p\neq q}^{r} x_{p,q} = 1$$

For $p \in \{1,2,\dots,r\}$

$$\sum_{q=1,p\neq q}^{r} x_{p,q} = 1$$

8. Total number of active trips should be the same as total number of production batches.

$$\sum_{v=1}^{s}\sum_{h=1}^{g} u_{0,v,h} = \sum_{f=1}^{t} \delta_f$$

For $f = 1,2,\dots,t-2$ (note that it was assumed t = n)

$$\delta_f \geq \delta_{f+1}$$

Note: For $f = 0$

$$\delta_f = 1$$

9. Each active trip has a corresponding production batch to deliver.

$$\sum_{f=1}^{t} \theta_{f,v,h} = u_{0,v,h}$$

For $f = 1,2,\dots,t$

$$\delta_f = \sum_{v=1}^{s}\sum_{h=1}^{g} \theta_{f,v,h}$$

10. Each customer must be assigned to one production batch.

For $j = 1,2,\dots,n$

$$\sum_{f=1}^{t} \beta_{j,f} = 1$$

11. If customer j is visited by trip vh $(u_{j,v,h} = 1)$ and batch f is assigned to trip vh $(\theta_{f,v,h} = 1)$, then customer j should be assigned to batch f.

For $j = 1,2,\dots,n;\ \ f = 1,2,\dots,t;\ \ v = 1,2,\dots,s;\ \ h = 1,2,\dots,g$

$$\beta_{j,f} + 1 \geq u_{j,v,h} + \theta_{f,v,h}$$

12. If customer j is in batch $f\ (\beta_{j,f} = 1)$ and is visited by trip vh $(u_{j,v,h} = 1)$, then batch f is assigned to trip vh.

    For $j = 1,2,\dots,n;\ \ f = 1,2,\dots,t;\ \ v = 1,2,\dots,s;\ \ h = 1,2,\dots,g$

$$\theta_{f,v,h} + 1 \geq \beta_{j,f} + u_{j,v,h}$$

13. Batch production sequencing.

    For $f, f' \in \{1,2,\dots,t\}\ and\ f \neq f'$

$$\gamma_{f,f'} \leq 1 - \gamma_{f',f}$$

    For $f' \in \{1,2,\dots,t\}$

$$\sum_{f=0, f\neq f'}^{t} \gamma_{f,f'} = \delta_{f'}$$

$$\sum_{f=1, f\neq f'}^{t} \gamma_{f',f} = \delta_{f'}$$

These condition conflicts with other conditions on gamma. The gamma variable is always 0 irrespective of parameter initialization. There is need for constraints that can replace these constraints without conflicting with 14

Time Constraints

14. Machine cannot start production for the next batch before the end of production for the current batch.

    Consider $c_f^p, \sigma_{p,q}, \rho_p$

    Note that production time (Completion time of production for batch f denoted by c_f^p):

    For $f \in \{1,2,\dots,t\}$

$$c_f^p \geq \sum_{p=1}^{r} \sum_{q=1, p\neq q}^{r} \sigma_{p,q} \cdot x_{p,q} + \sum_{j=1}^{n} \sum_{p=1}^{r} \rho_p \cdot d_{j,p} \cdot \beta_{j,f} - M(1 - \gamma_{0,f})$$

    For $f, f' \in \{1,2,\dots,t\}\ and\ f \neq f'$

$$c_{f'}^p \geq c_f^p + \sum_{p=1}^{r} \sum_{q=1, p\neq q}^{r} \sigma_{p,q} \cdot x_{p,q} + \sum_{j=1}^{n} \sum_{p=1}^{r} \rho_p \cdot d_{j,p} \cdot \beta_{j,f'} - M(1 - \gamma_{f,f'})$$

15. Arrival time to the first customer in a trip occurs after the start time of the trip from the plant plus the travel time to the first stop.

    For $j = 1,2,\dots,n;\ \ v = 1,2,\dots,s;\ \ h = 1$

$$\alpha_{j,v,h} \geq s_{v,h}^d + s_0 + \tau_{0,j} - M(2 - y_{1,j,v,h} - u_{j,v,h})$$

16. Arrival times to consecutive stops in a trip occurs after the arrival time to the previous customer of the trip plus the service time at that customer and travel time to the next.

For $i = 0,1,\dots,n;\ \ j = 1,2,\dots,n+1;\ \ v = 1,2,\dots,s;\ \ h = 1,2,..,g$

$$\alpha_{j,v,h} \geq \alpha_{i,v,h} + s_i + \tau_{ij} - M(1 - y_{i,j,v,h})$$

17. Start time of the first tour of each vehicle is greater than or equal to the completion time of the production batch assigned to it.

For $v = 1,2,\dots,s;\ \ f \in \{1,2,..,t\}, h = 1$

$$s^d_{v,h} \geq c^p_f + s_0 - M(1 - \theta_{f,v,h})$$

18. If it is not the first tour of a vehicle, start time of the trip is greater than or equal to the previous tour's arrival time at the plant or the completion time of production assigned to the trip plus service time at the plant. (How should we express this? But the concept of this constraint makes sense, right?)

For $v = 1,2,\dots,s;\ \ h = 1,2,..,g;\ \ f \in \{1,2,..,t\}$

$$s^d_{v,h+1} \geq \begin{cases} \alpha_{n+1,v,h} + s_{n+1} +, & \alpha_{n+1,v,h} \leq c^p_f \\ c^p_f + s_0 - M\left(1 - \theta_{f,v,h+1}\right), & otherwise \end{cases}$$

methods constraint19 and constraint20 in the file pulp_model.py implements 17 and 18, please confirm if the formulation is correct as the formulation given here can not be implemented in the solver

19. Time window constraints

For $j = 1,2,\dots,n;\ v = 1,2,\dots,s;\ \ h = 1,2,..,g$

$$e_{j,v,h} \geq 0$$

$$e_{j,v,h} \geq a_j - \alpha_{j,v,h}$$

$$l_{j,v,m,h} \geq 0$$

$$l_{j,v,h} \geq \alpha_{j,v,h} - b_j$$

The Problem is over-constrained. The objective converges at a high value and is not able to decrease further. To be able to solve the problem using industrial solvers, simplification and removal of constraints is required, without which arbitrary solution will be obtained

Also in the problem formulation, does each batch have to go particular vehicle? Wouldn't it be simpler to decouple the batch and vehicle constraint and just ensure that all the requirements of a particular customer are delivered by a single vehicle? This would allow the freedom of producing items in different batches. Might help simplify the problem.
The following paper implements what I want to say - https://www.researchgate.net/publication/338733345_Integrated_Production_and_Distribution_Problem_of_Perishable_Products_with_a_Minimum_Total_Order_Weighted_Delivery_Time