TITLE: Integrated multi-product production scheduling and vehicle-routing problem using heterogeneous fleet with multiple trips and time windows

CLASSIFICATION: Operational integrated production and outbound distribution problem (manufacturer – customers; vehicle-routing delivery method) known as Integrated production scheduling and vehicle routing problem (PS-VRP)

RATIONALE: This research is a variant of the PS-VRP, focusing on introducing a new and more comprehensive model addressing suggested gaps in literature.

GAPS IN LITERATURE: Consolidation of real-life problem features in a single problem. Features include: Production – setup operations for multi-product production; Distribution – limited fleet size available for delivery, allowing multiple trips for each vehicle, heterogeneous fleet (varying capacity and cost rates) for delivery, inclusion of service operation (loading and unloading time), customer specified delivery time windows, and penalty for early and late deliveries.

*Literature review criteria: PS-VRP papers which minimized cost function and consider multiple products in production

**Remark. Add a definition of terms section.

PROBLEM DESCRIPTION

A manufacturer operates a single production center to produce multiple types of a product ordered from different retailers in the city. In the production facility, a single machine is used and can produce all demands at the manufacturing plant; adapting a batch production system. A set of customer orders is known in advance and must be processed and delivered within customer specified time windows. Each customer orders all types of products and the single machine can produce all types of the product following setup operations. Each type of product has its incurred processing time. Since some equipment need to be reconfigured to produce different variants, setup times and costs are assumed. At the beginning of production, the machine is setup for a product and setup time between product variants depend on the previously produced variant.

> So, the production planning concerns (1) Batching of customer orders, (2) Sequencing of the products per batch, and (3) Scheduling/sequencing of production batches.

Loading of orders in the plant is fixed. Unloading times are proportional to the work quantity (unloading rate x demand size of customer). Split delivery is not allowed which means that every customer's demand must be satisfied by one vehicle on one stop. The distribution is performed by a heterogeneous fleet with varying capacities and cost rates. Travel times and costs are accounted, and routing decisions should be made so that distribution costs are minimized. The delivery operation starts by loaded vehicles in the production plant. After delivery, the vehicles return to the production plant. Multiple trips are possible for each vehicle. In the problem, there are customer specified time windows. When the delivery arrives earlier than the earliest delivery time, the vehicle is entertained by a penalty cost is incurred for possible disruption in the customer's schedule. Similarly, when the delivery arrives later than the latest delivery time, penalty cost is incurred. Penalty cost is proportional to the delay time (penalty rate x time delay).

> So, the distribution planning concerns (1) Assignment of production batches to delivery vehicles/trips, (2) Routing of customers in the vehicle trip, and (3) Recycling of vehicles.

The objective is to minimize total operating cost which includes processing cost + setup cost for production and vehicle cost + traveling cost + penalty cost for distribution.

ADDITIONAL ASSUMPTIONS:

Production is scheduled without idle time on machine. There is no preemption.

Different variants of the product are similar in size. There is no concern with unit space.

Vehicles start and end at the production plant.

No order splitting.

*A production batch consists of multiple customers. (Different batch = different customers)

*Each production batch is assigned to a vehicle trip and is composed of only one batch. (Customers in the production batch are same customers to be visited in the trip)

--MODEL FORMULATION---

INDICES

$i, j \in I = \{0,1,2, \dots, n, n+1\}$	customer indices where $i = 0, n+1$ is the manufacturing plant
$p, q \in P = \{1,2, \dots, r\}$	product type
$v \in V = \{1,2, \dots, s\}$	vehicle index specified distinctly (type and number)
$h \in H = \{1,2, \dots, g\}$	trip index
$f, f' \in B = \{1,2, \dots, t\}$	production batch name

PARAMETERS

$d_{i,p}$	demand quantity for product p of customer i
$\sigma_{p,q}$	set-up time of producing product q immediately after product p $\sigma_{0,p}$ – p is the first item in the production sequence $\sigma_{p,0}$ – p is the last item in the production sequence
ρ_p	processing time per unit of product p
$\tau_{i,j}$	travel time from customer i to customer j
$[a_i, b_i]$	time window for customer i
c_v	capacity of vehicle type v
s_i	service time (loading and unloading) at customer i
C^ρ	processing cost (per process time)
C^σ	set-up cost (per set-up time)
C^τ	travel cost (per time travelled)
C^e	penalty cost for early delivery (per unit time early)
C^l	penalty cost for late delivery (per unit time late)
F_v	fixed cost of vehicle type v used
M	sufficiently large number

DEPENDENT VARIABLES

The v and h indices are redundant in alpha, e and l. Removing them will somplify the problem as no order splitting is allowed

s_f^p	start time of production for batch f
c_f^p	completion time of production for batch f
$s_{v,h}^d$	start time of delivery for the h^{th} trip of vehicle type v
$\alpha_{j,v,h}$	arrival time at customer j for the h^{th} trip of vehicle type v
$e_{j,v,h}$	time (in minutes) when vehicle arrives earlier than earliest arrival to customer j for the h^{th} trip of vehicle type v
$l_{j,v,h}$	time (in minutes) when vehicle arrives later than latest arrival from to customer j for the h^{th} trip of vehicle type v

DECISION VARIABLES (BINARY)

$x_{p,q}$	1 if product q is produced immediately after product p 0 otherwise
$\beta_{j,f}$	1 if customer j's order is produced in batch f 0 otherwise
δ_f	1 if batch f is active (if batch f is produced) 0 otherwise
$\gamma_{f,f'}$	1 if batch f is scheduled before batch f' 0 otherwise
$\theta_{f,v,h}$	1 if batch f is assigned to the h^{th} trip of vehicle type v 0 otherwise
$u_{j,v,h}$	1 if customer j is visited on the h^{th} trip of vehicle type v 0 otherwise
$y_{i,j,v,h}$	1 if customer j is visited immediately after customer i in h^{th} trip of vehicle type v 0 otherwise
w_v	1 if the vehicle type v is used for delivery 0 otherwise

The v and h indices are redundant in y. Removing them will somplify the problem as no order splitting is allowed

OBJECTIVE FUNCTION

$$Minimize\ TOC = C^{\rho}\left(\sum_{j=1}^{n}\sum_{p=1}^{r}\rho_p \cdot d_{j,p}\right) + C^{\sigma}\left(\sum_{p=1}^{r}\sum_{q=1,p\neq q}^{r}\sigma_{p,q}\cdot x_{p,q}\right) + C^{\tau}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{s}\sum_{h=1}^{g}\tau_{i,j}\, y_{i,j,v,h}\right) + \sum_{v=1}^{s}F_v\, w_v + C^{e}\left(\sum_{j=1}^{n}\sum_{v=1}^{s}\sum_{h=1}^{g}e_{j,v,h}\right) + C^{l}\left(\sum_{j=1}^{n}\sum_{v=1}^{s}\sum_{h=1}^{g}l_{j,v,h}\right)$$

CONSTRAINTS

The following applies for all permissible indices:

Binary Variables

$$x_{p,q,f} \in \{0,1\}$$
$$\beta_{j,f} \in \{0,1\}$$
$$\delta_f \in \{0,1\}$$
$$\gamma_{f,f'} \in \{0,1\}$$

$$\theta_{f,v,h} \in \{0,1\}$$
$$u_{j,v,h} \in \{0,1\}$$
$$y_{i,j,v,h} \in \{0,1\}$$
$$w_v \in \{0,1\}$$

Non-negativity Constraints

$$s_f^p \geq 0$$
$$c_f^p \geq 0$$
$$s_{v,h}^d \geq 0$$
$$\alpha_{j,v,h} \geq 0$$
$$e_{j,v,h} \geq 0$$
$$l_{j,v,h} \geq 0$$

Others

$$s, g, t \leq n$$

Vehicle Routing Constraints

1. Each customer should be visited once and only once.

   For $j = 1,2, \ldots, n$

$$\sum_{v=1}^{s} \sum_{h=1}^{g} u_{j,v,h} = 1$$

2. Tour is denoted empty if there is no customer assigned to it. Tours with at least one customer are referred to as active tours. Processing site must be included in each active tour.

   For $j = 1,2, \ldots, n;\ \ v = 1,2, \ldots, s;\ \ h = 1,2, . . , g$ j = n+1 also has this constraint

$$u_{0,v,h} \geq u_{j,v,h}$$

3. Total demand quantity of all the customers in the same trip should not exceed the capacity of the vehicle assigned to it.

   For $v = 1,2, \ldots, s;\ \ h = 1,2, \ldots, g$

$$\sum_{j=1}^{n} \sum_{p=1}^{r} d_{j,p} \cdot u_{j,v,h} \leq c_v$$

4. Trip must start and end at the plant. If customer j is visited in the trip, vehicle v either travels from a previous customer i or from the production center. Afterwards, the vehicle returns to the production site or delivers to another customer.

   For $i, j = 1,2, \ldots, n;\ \ \ v = 1,2, \ldots, s;\ \ h = 1,2, \ldots, g$

$$y_{0,j,v,h} + y_{0,j,v,h} + u_{i,v,h} + u_{j,v,h} \leq 3$$

   For $j = 1,2, \ldots, n;\ \ \ v = 1,2, \ldots, s;\ \ h = 1,2, \ldots, g$

$$u_{j,v,h} = \sum_{i=0,i\neq j}^{n} y_{i,j,v,h}$$

$$u_{j,v,h} = \sum_{i=1,i\neq j}^{n+1} y_{j,i,v,h}$$

For $i,j = 1,2,\dots,n;\quad v = 1,2,\dots,s;\quad h = 1,2,\dots,g$

$$u_{i,v,h} \geq y_{i,j,v,h} + u_{j,v,h} - 1$$

$$u_{j,v,h} \geq y_{i,j,v,h} + u_{i,v,h} - 1$$

5. Vehicle trip is activated only if previous trip is also active constraints.

   For $v = 1,2,\dots,s;\quad h = 1,2,..,g$

   The range of h here should be upto g - 1

$$M\sum_{j=1}^{n} u_{j,v,h} \geq \sum_{j=1}^{n} u_{j,v,h+1}$$

6. The v^{th} vehicle is used if a trip is assigned to it ($u_{0,v.h} = 1$).

   For $v = 1,2,\dots,s;\quad h = 1,2,..,g$

$$u_{0,v,h} \geq w_v$$

Production Constraints

7. Product sequencing.

   For $p,q \in \{1,2,\dots,r\}$

$$\sum_{q=1,p\neq q}^{r} x_{p,q} = 1 \quad and \quad \sum_{p=1,p\neq q}^{r} x_{p,q} = 1$$

$$x_{p,q} \leq 1 - x_{q,p}$$

8. Total number of active trips should be the same as total number of production batches.

$$\sum_{v=1}^{s}\sum_{h=1}^{g} u_{0,v,h} = \sum_{f=1}^{t} \delta_f$$

   For $f = 1,2,\dots,t$

$$\delta_f \geq \delta_{f+1}$$

9. Each active trip has a corresponding production batch to deliver.

$$\sum_{f=1}^{t} \theta_{f,v,h} = u_{0,v,h}$$

These conditions (9, 11 and 12) can be removed as a batch may be split into a number of trucks leaving at the same time or within short intervals. This will also reduce the number of constraints allow the solver greater flexibility in finding appropriate solution

   For $f = 1,2,\dots,t$

$$\delta_f = \sum_{v=1}^{s} \sum_{h=1}^{g} \theta_{f,v,h}$$

10. Each customer must be assigned to one production batch.

For $j = 1,2,\dots,n$

$$\sum_{f=1}^{t} \beta_{j,f} = 1$$

11. If customer j is visited by trip vh $(u_{j,v,h} = 1)$ and batch f is assigned to trip vh $(\theta_{f,v,h} = 1)$, then customer j should be assigned to batch f.

For $j = 1,2,\dots,n;\ \ f = 1,2,\dots,t;\ \ v = 1,2,\dots,s;\ \ h = 1,2,\dots,g$

$$\beta_{j,f} + 1 \geq u_{j,v,h} + \theta_{f,v,h}$$

12. If customer j is in batch f $(\beta_{j,f} = 1)$ and is visited by trip vh $(u_{j,v,h} = 1)$, then batch f is assigned to trip vh.

For $j = 1,2,\dots,n;\ \ f = 1,2,\dots,t;\ \ v = 1,2,\dots,s;\ \ h = 1,2,\dots,g$

$$\theta_{f,v,h} + 1 \geq \beta_{j,f} + u_{j,v,h}$$

13. Batch production sequencing.

For $f' \in \{1,2,\dots,t\}$

index going out of range here

$$\sum_{f=1,f\neq f'}^{t} \gamma_{f,f'} = \delta_{f'} \quad and \quad \sum_{f=1,f\neq f'}^{t+1} \gamma_{f',f} = \delta_{f'}$$

Time Constraints

14. Machine cannot start production for the next batch before the end of production for the current batch.

Consider $s_f^p, c_f^p, \sigma_{p,q}, \rho_p$

$$c_f^p \geq \sum_{p=1}^{r} \sum_{q=1,p\neq q}^{r} \sigma_{p,q} \cdot x_{p,q} + \sum_{j=1}^{n} \sum_{p=1}^{r} \rho_p \cdot d_{j,p} \cdot \beta_{j,f} - M(1 - \gamma_{0,f})$$

$$c_{f'}^p \geq s_f^p + \sum_{p=1}^{r} \sum_{q=1,p\neq q}^{r} \sigma_{p,q} \cdot x_{p,q} + \sum_{j=1}^{n} \sum_{p=1}^{r} \rho_p \cdot d_{j,p} \cdot \beta_{j,f} - M(1 - \gamma_{f,f'})$$

15. Arrival time to the first customer in a trip occurs after the start time of the trip from the plant plus the travel time to the first stop.

For $j = 1,2,\dots,n;\ \ v = 1,2,\dots,s;\ \ h = 1,2,..,g$

$$\alpha_{j,v,h} \geq s_{v,h}^d + s_0 + \tau_{0,j} - M(1 - u_{j,v,h})$$

16. Arrival times to consecutive stops in a trip occurs after the arrival time to the previous customer of the trip plus the service time at that customer and travel time to the next.

For $i = 0,1,\dots,n;\ \ j = 1,2,\dots,n+1;\ \ v = 1,2,\dots,s;\ \ h = 1,2,..,g$

$$\alpha_{j,v,h} \geq \alpha_{i,v,h} + s_i + \tau_{ij} - M(1 - y_{i,j,v,h})$$

17. Start time of the first tour of each vehicle is greater than or equal to the completion time of the production batch assigned to it.

For $v = 1,2,\dots,s;\ \ f \in \{1,2,..,t\}$

The varying index also includes the product index as well right?

$$s^d_{v,1} \geq c^p_f + s_0 - M(1 - \theta_{f,v,1})$$

18. If it is not the first tour of a vehicle, start time of the trip is greater than or equal to the previous tour's arrival time at the plant or the completion time of production assigned to the trip plus service time at the plant.

Also AND condition is equivalent to a product and it is possible that ortools is not able to parse it properly, resulting into bad results

For $v = 1,2,\dots,s;\ \ h = 1,2,..,g;\ \ f \in \{1,2,..,t\}$

Linear solver does not support “<” or “>”

$$s^d_{v,h+1} \geq \alpha_{n+1,v,h} + s_{n+1} \qquad * \, if\ \alpha_{n+1,v,h} < c^p_f$$

AND condition can not be represented in the NumPy model. Need to reformulate this condition

$$s^d_{v,h+1} \geq c^p_f + s_0 - M\left(1 - \theta_{f,v,h+1}\right)$$

The varying index also includes the product index as well right?

19. Time window constraints

For $j = 1,2,\dots,n;\ v = 1,2,\dots,s;\ \ h = 1,2,..,g$

$$e_{j,v,h} \geq 0$$

$$e_{j,v,h} \geq a_j - \alpha_{j,v,h}$$

$$l_{j,v,m,h} \geq 0$$

$$l_{j,v,h} \geq \alpha_{j,v,h} - b_j$$