In [1]:

```
%matplotlib inline
```

In [2]:

```
import numpy as np
```

In [6]:

```
R = np.array([[5,0,5,4],[0,1,1,4],[4,1,2,4],[3,4,0,3],[1,5,3,0]])
```

In [7]:

```
R
```

Out[7]:

```
array([[5, 0, 5, 4],
       [0, 1, 1, 4],
       [4, 1, 2, 4],
       [3, 4, 0, 3],
       [1, 5, 3, 0]])
```

In [13]:

```
len(np.where(R!=0)[0])
```

Out[13]:

```
16
```

In [15]:

```
len(R[R!=0])
```

Out[15]:

```
16
```

In [16]:

```
np.average(R[R!=0])
```

Out[16]:

```
3.125
```

In [18]:

```
A = np.zeros((16,9))
```

In [19]:

```
A
```

Out[19]:

```
array([[ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.]])
```

In [27]:

```
np.where(R!=0)
```

Out[27]:

```
(array([0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 4, 4, 4]),
 array([0, 2, 3, 1, 2, 3, 0, 1, 2, 3, 0, 1, 3, 0, 1, 2]))
```

In [28]:

```
for row, each in enumerate(range(len(np.where(R!=0)[0]))):
    A[row][np.where(R!=0)[0][each]]  = 1
    A[row][np.where(R!=0)[1][each]+5]  = 1
```

In [29]:

```
A
```

Out[29]:

```
array([[ 1.,  0.,  0.,  0.,  0.,  1.,  0.,  0.,  0.],
       [ 1.,  0.,  0.,  0.,  0.,  0.,  0.,  1.,  0.],
       [ 1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  1.],
       [ 0.,  1.,  0.,  0.,  0.,  0.,  1.,  0.,  0.],
       [ 0.,  1.,  0.,  0.,  0.,  0.,  0.,  1.,  0.],
       [ 0.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  1.],
       [ 0.,  0.,  1.,  0.,  0.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.,  0.,  0.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.,  0.,  0.,  0.,  1.],
       [ 0.,  0.,  0.,  1.,  0.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  1.,  0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  1.,  0.,  0.,  0.,  0.,  1.],
       [ 0.,  0.,  0.,  0.,  1.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  1.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  1.,  0.,  0.,  1.,  0.]])
```

In [ ]:

```
from numpy import linalg
```

In [48]:

```
b = R[R!=0] - np.average(R[R!=0])
```

In [49]:

```
linalg.lstsq(A,b)
```

Out[49]:

```
(array([ -9.25783376e+14,  -9.25783376e+14,  -9.25783376e+14,
         -9.25783376e+14,  -9.25783376e+14,   9.25783376e+14,
          9.25783376e+14,   9.25783376e+14,   9.25783376e+14]),
 array([ 19.23685578]),
 9,
 array([  2.70236482e+00,   2.14896114e+00,   2.14896114e+00,
          2.14896114e+00,   1.92281678e+00,   1.54336192e+00,
          1.54336192e+00,   1.54336192e+00,   3.14537679e-16]))
```

In [39]:

```

```

In [38]:

```
A.shape
```

Out[38]:

```
(16, 9)
```

In [40]:

```
b.shape
```

Out[40]:

```
(16,)
```

In [44]:

```
result = np.array([  2.70236482e+00,   2.14896114e+00,   2.14896114e+00, 2.14896114e+00,
  1.92281678e+00,   1.54336192e+00, 1.54336192e+00,   1.54336192e+00,   3.14537679e-16])
```

In [45]:

```
np.average(R[R!=0])
```

Out[45]:

```
3.125
```

In [46]:

```
A.dot(result)
```

Out[46]:

```
array([ 4.24572674,  4.24572674,  2.70236482,  3.69232306,  3.69232306,
        2.14896114,  3.69232306,  3.69232306,  3.69232306,  2.14896114,
        3.69232306,  3.69232306,  2.14896114,  3.4661787 ,  3.4661787 ,
        3.4661787 ])
```

In [47]:

```
b
```

Out[47]:

```
array([ 1.875,  1.875,  0.875, -2.125, -2.125,  0.875,  0.875, -2.125,
       -1.125,  0.875, -0.125,  0.875, -0.125, -2.125,  1.875, -0.125])
```

In [50]:

```
b
```

Out[50]:

```
array([ 1.875,  1.875,  0.875, -2.125, -2.125,  0.875,  0.875, -2.125,
       -1.125,  0.875, -0.125,  0.875, -0.125, -2.125,  1.875, -0.125])
```

In [58]:

```
b.dot(linalg.pinv(A).transpose())
```

Out[58]:

```
array([ 1.52020202, -1.20707071, -0.38888889,  0.06565657,  0.06565657,
       -0.19065657, -0.00883838, -0.37247475,  0.62752525])
```

In [62]:

```
A.dot(b.dot(linalg.pinv(A).transpose())) +np.average(R[R!=0])
```

Out[62]:

```
array([ 4.45454545,  4.27272727,  5.27272727,  1.90909091,  1.54545455,
        2.54545455,  2.54545455,  2.72727273,  2.36363636,  3.36363636,
        3.        ,  3.18181818,  3.81818182,  3.        ,  3.18181818,
        2.81818182])
```

In [63]:

```
1.52020202 +  -0.19065657 + +np.average(R[R!=0])
```

Out[63]:

```
4.4545454499999995
```

In [64]:

```
1.52020202 + -0.37247475 + np.average(R[R!=0])
```

Out[64]:

```
4.2727272699999999
```

In [66]:

```
1.52020202 -0.00883838+ np.average(R[R!=0])
```

Out[66]:

```
4.6363636399999999
```

In [67]:

```
1.52020202 +  -0.00883838 + +np.average(R[R!=0])
```

Out[67]:

```
4.6363636399999999
```

In [70]:

```
linalg.pinv(A).dot(b)
```

Out[70]:

```
array([ 1.52020202, -1.20707071, -0.38888889,  0.06565657,  0.06565657,
       -0.19065657, -0.00883838, -0.37247475,  0.62752525])
```

In [72]:

```
A.dot(linalg.pinv(A).dot(b))+np.average(R[R!=0])
```

Out[72]:

```
array([ 4.45454545,  4.27272727,  5.27272727,  1.90909091,  1.54545455,
        2.54545455,  2.54545455,  2.72727273,  2.36363636,  3.36363636,
        3.        ,  3.18181818,  3.81818182,  3.        ,  3.18181818,
        2.81818182])
```

In [73]:

```
A_all = np.zeros((20,9))
for row, each in enumerate(range(len(np.where(R!=-1)[0]))):
    A_all[row][np.where(R!=-1)[0][each]]  = 1
    A_all[row][np.where(R!=-1)[1][each]+5]  = 1
```

In [74]:

```
A_all
```

Out[74]:

```
array([[ 1.,  0.,  0.,  0.,  0.,  1.,  0.,  0.,  0.],
       [ 1.,  0.,  0.,  0.,  0.,  0.,  1.,  0.,  0.],
       [ 1.,  0.,  0.,  0.,  0.,  0.,  0.,  1.,  0.],
       [ 1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  1.],
       [ 0.,  1.,  0.,  0.,  0.,  1.,  0.,  0.,  0.],
       [ 0.,  1.,  0.,  0.,  0.,  0.,  1.,  0.,  0.],
       [ 0.,  1.,  0.,  0.,  0.,  0.,  0.,  1.,  0.],
       [ 0.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  1.],
       [ 0.,  0.,  1.,  0.,  0.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.,  0.,  0.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.,  0.,  0.,  0.,  1.],
       [ 0.,  0.,  0.,  1.,  0.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  1.,  0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  1.,  0.,  0.,  0.,  1.,  0.],
       [ 0.,  0.,  0.,  1.,  0.,  0.,  0.,  0.,  1.],
       [ 0.,  0.,  0.,  0.,  1.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  1.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  1.,  0.,  0.,  1.,  0.],
       [ 0.,  0.,  0.,  0.,  1.,  0.,  0.,  0.,  1.]])
```

In [81]:

```
R_hat = np.reshape(A_all.dot(linalg.pinv(A).dot(b))  +np.average(R[R!=0]),(5,4))
```

In [ ]:

```
R_hat
```

In [145]:

```
R_hat[0,3] = 5
```

In [146]:

```
R_tilde = (R - R_hat)*((R>0)*1)
```

In [147]:

```
R_tilde
```

Out[147]:

```
array([[  5.45454545e-01,  -0.00000000e+00,   7.27272727e-01,
         -1.00000000e+00],
       [ -0.00000000e+00,  -9.09090909e-01,  -5.45454545e-01,
          1.45454545e+00],
       [  1.45454545e+00,  -1.72727273e+00,  -3.63636364e-01,
          6.36363636e-01],
       [ -8.88178420e-16,   8.18181818e-01,  -0.00000000e+00,
         -8.18181818e-01],
       [ -2.00000000e+00,   1.81818182e+00,   1.81818182e-01,
         -0.00000000e+00]])
```

In [148]:

```
R
```

Out[148]:

```
array([[5, 0, 5, 4],
       [0, 1, 1, 4],
       [4, 1, 2, 4],
       [3, 4, 0, 3],
       [1, 5, 3, 0]])
```

In [149]:

```
from scipy.spatial.distance import cosine
```

In [150]:

```
from itertools import combinations
```

In [151]:

```
d = np.zeros((4,4))
```