Network 20q HW6

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1 Probability Distribution

In this question, we need to compare the distribution of three distribution.

We could use the PDF of three functions to generate the graph.

The PDF of Pareto distribution is:

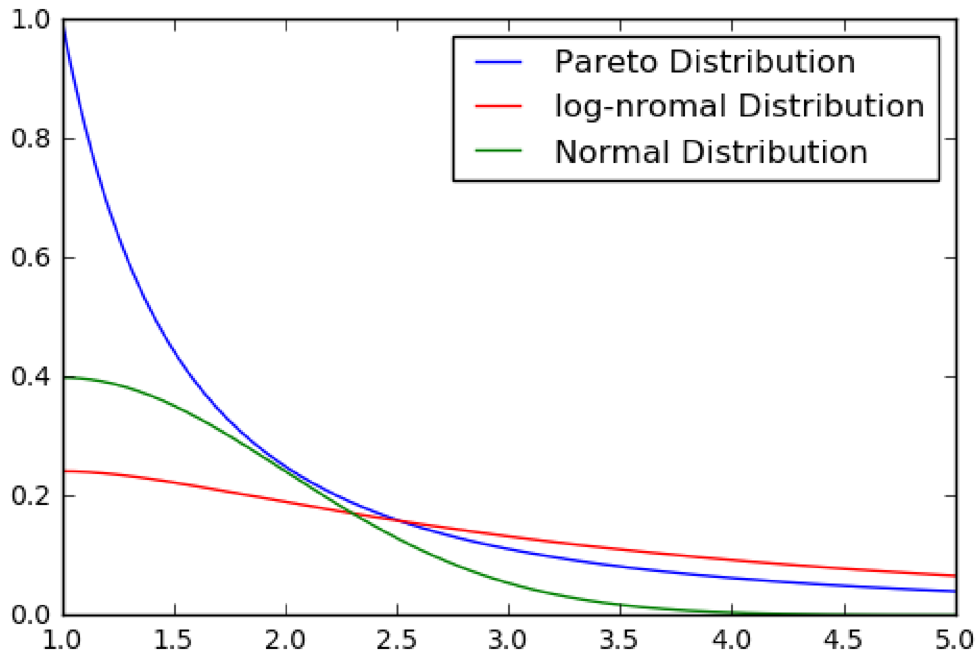
$$f(x) = \frac{\alpha * x_m^\alpha}{x^{\alpha+1}} \quad for \quad x \geq x_m \tag{1}$$

The PDF of normal distribution is:

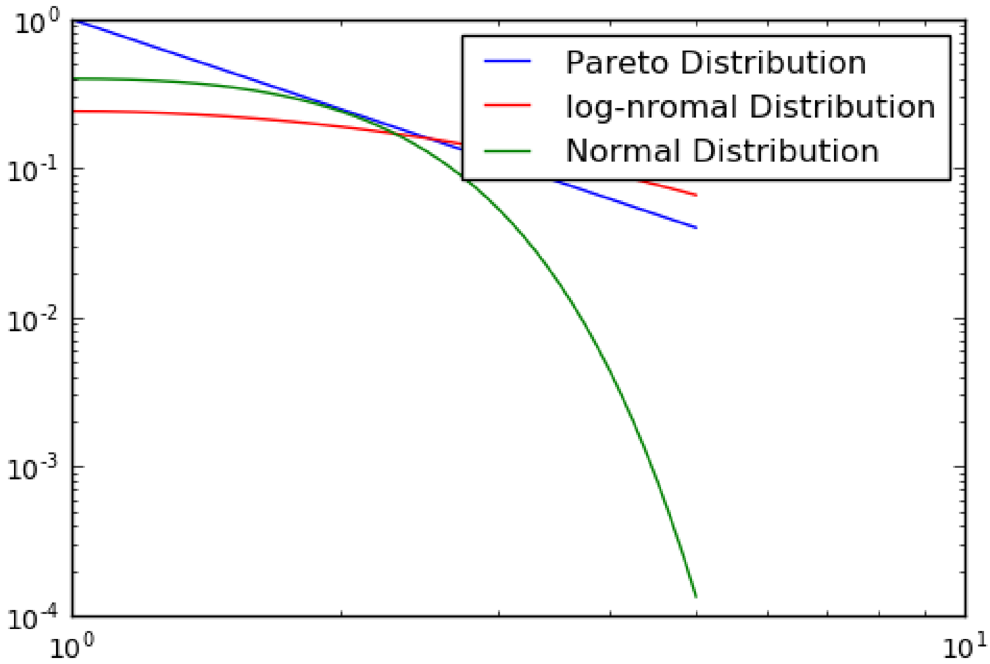
$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \tag{2}$$

The PDF of log-normal distribution is:

$$f(x) = \frac{e^{-\frac{(lnx-\mu)^2}{2\sigma^2}}}{x\sqrt{2\pi\sigma^2}} \tag{3}$$



Then we could draw the log-log plot of the three distribution function:



We could see that, the log-log plot of Pareto distribution is a straight line. We could conclude that, log-normal distribution is more similar to Pareto distribution.

2 α fairness utility function

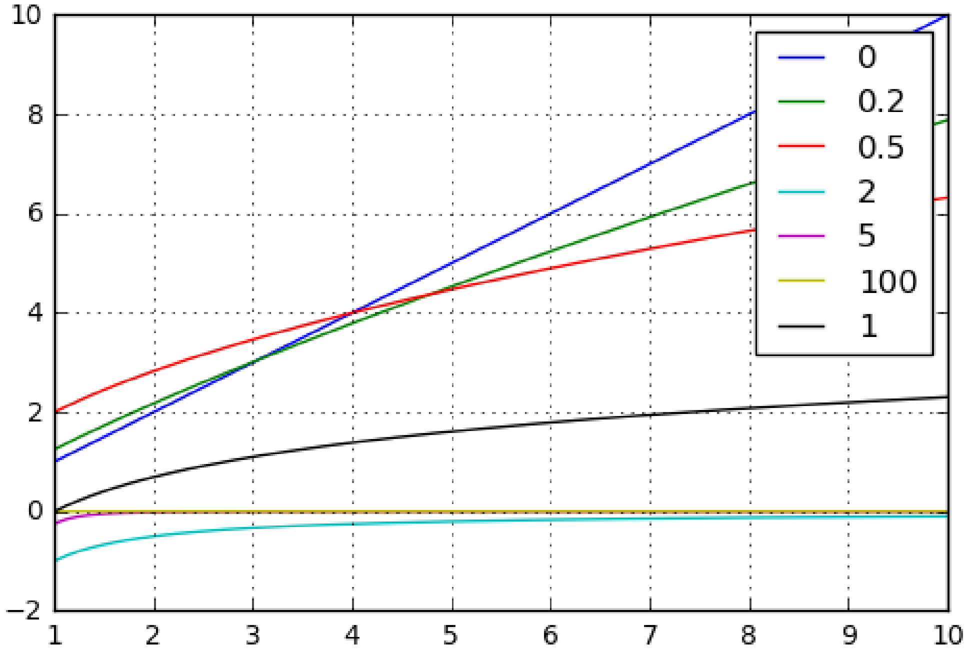
The α fairness utility function is:

$$U_\alpha(x) = \frac{x^{1-\alpha}}{1-\alpha} \quad (when \quad x \neq 1) \tag{4}$$

$$U_\alpha(x) = \log x \quad (when \quad x \neq 1) \tag{5}$$

2.1 a

Draw the function in the same graph:



From the trend of various α, we could conclude that, with the increase of α, the seems decrease, which mean the function's change seems more steady.

2.2 b

We already have the utility function:

$$U(x) = \arctan x \tag{6}$$

Then we could have:

$$U'(D(p)) = \arctan D(p) = \frac{1}{1 + D(P)^2} = p \tag{7}$$

Then demand function would be:

$$D(p) = \sqrt{\frac{1}{p} - 1} \tag{8}$$

The demand elasticity is:

$$-\frac{\partial D(p)/\partial p}{D(p)/p} = \frac{1}{2} * \frac{1}{\sqrt{\frac{1}{p} - 1}} * (-1 * \frac{1}{p^2}) = \frac{1}{2(1-p)} \tag{9}$$

2.3 c

$$U_\alpha(x) = \frac{x^{1-\alpha}}{1-\alpha} \tag{10}$$

And

$$U'_\alpha(x) = (\frac{x^{1-\alpha}}{1-\alpha})' = x^{-\alpha} = D(p)^{-\alpha} = p \tag{11}$$

So

$$D(p) = p^{-\frac{1}{\alpha}} \tag{12}$$

The demand elasticity is:

$$-\frac{\partial D(p)/\partial p}{D(p)/p} = \frac{1}{\alpha} * p^{-\frac{1}{\alpha}-1} * p * p^{\frac{1}{\alpha}} = \frac{1}{\alpha} \tag{13}$$