# Network 20q HW3

ZHANG Xiao Research Intern in IBM CRL

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# 1 Baseline predictor

In this question, we need to implement the baseline predictor. That is

$$\hat{r_{ui}} = \bar{r} + b_u + b_i \tag{1}$$

In this question, we have $R$ matrix of 5 rows and 4 columns. So we need to solve $b_{1..5}$ and $b_{A..D}$ ,9 parameters in total.

So we need to solve:

$$A * b = c \tag{2}$$

$A$ is the indicator of valid rating, $b$ is nice parameters, and $c$ is the rating we already got sub the average value of existing rating.

So we could have:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_A \\ b_B \\ b_C \\ b_D \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \\ r_{10} \\ r_{11} \\ r_{12} \\ r_{13} \\ r_{14} \\ r_{15} \\ r_{16} \end{bmatrix} - \vec{\bar{r}} \tag{3}$$

So we could resolve this equation by:

$$
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_A \\ b_B \\ b_C \\ b_D \end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}^{-1} * [\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \\ r_{10} \\ r_{11} \\ r_{12} \\ r_{13} \\ r_{14} \\ r_{15} \\ r_{16} \end{bmatrix} - \vec{\bar{r}}] \tag{4}
$$

Use pinv to calculate the invert matrix of A, we could get the result:

$$
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_A \\ b_B \\ b_C \\ b_D \end{bmatrix} = \begin{bmatrix} 1.52020202 \\ -1.20707071 \\ -0.38888889 \\ 0.06565657 \\ 0.06565657 \\ -0.19065657 \\ -0.00883838 \\ -0.37247475 \\ 0.62752525 \end{bmatrix} \tag{5}
$$

We could finally calculate the final prediction via the following formula:

$$\vec{\hat{R}} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_A \\ b_B \\ b_C \\ b_D \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Reshape the vector we got, and we could get $\hat{R}$, that is:

$$\hat{R} = \begin{bmatrix} 4.45454545 & 4.63636364 & 4.27272727 & 5.27272727 \\ 1.72727273 & 1.90909091 & 1.54545455 & 2.54545455 \\ 2.54545455 & 2.72727273 & 2.36363636 & 3.36363636 \\ 3. & 3.18181818 & 2.81818182 & 3.81818182 \\ 3. & 3.18181818 & 2.81818182 & 3.81818182 \end{bmatrix} \tag{7}$$

But in this matrix, the rating score is between 1 and 5, we should regular the value which is larger than 5 and smaller than 1, then we could get $\hat{R}$:

$$\begin{bmatrix} 4.45454545 & 4.63636364 & 4.27272727 & 5. \\ 1.72727273 & 1.90909091 & 1.54545455 & 2.54545455 \\ 2.54545455 & 2.72727273 & 2.36363636 & 3.36363636 \\ 3. & 3.18181818 & 2.81818182 & 3.81818182 \\ 3. & 3.18181818 & 2.81818182 & 3.81818182 \end{bmatrix} \tag{8}$$

## 2 Neighborhood predictor

In this question, we should add the factor produced by similarity between items to $\hat{R}$ got from last step, first we could get $\tilde{R} =$:

$$R - \hat{R} = \begin{bmatrix} 5.45454545e-01 & -0.00000000e+00 & 7.27272727e-01 & -1.00000000e+00 \\ -0.00000000e+00 & -9.09090909e-01 & -5.45454545e-01 & 1.45454545e+00 \\ 1.45454545e+00 & -1.72727273e+00 & -3.63636364e-01 & 6.36363636e-01 \\ -8.88178420e-16 & 8.18181818e-01 & -0.00000000e+00 & -8.18181818e-01 \\ -2.00000000e+00 & 1.81818182e+00 & 1.81818182e-01 & -0.00000000e+00 \end{bmatrix} \tag{9}$$

And then we could get the similarity matrix between each validate items in every pair columns in the matrix $\tilde{R}$:

$$d = \begin{bmatrix} 0. & -0.94254269 & -0.23500697 & 0.16991415 \\ -0.94254269 & 0. & 0.80152501 & -0.81767046 \\ -0.23500697 & 0.80152501 & 0. & -0.95367866 \\ 0.16991415 & -0.81767046 & -0.95367866 & 0. \end{bmatrix} \tag{10}$$

And then we could get $R_N$, in $R_N$, the element $R_{N(u,i)}$ is extracted from the most correlated items score:

$$R_{N(u,i)} = R\hat{_{(u,i)}} + \frac{d_{(i,top1)} * R\tilde{_{(u,top1)}} + d_{(i,top2)} * R\tilde{_{(u,top2)}}}{|d_{(i,top1)}| + |d_{(i,top2)}|} \tag{11}$$

We must also consider the validation of the element in $\tilde{R}$, if the element in $R$ is not validate, the same element in $\tilde{R}$ is also invalidate, so in the computation of $R_{N(u,i)}$, we should ignore the portion of invalidate element.

Then we could get $R_N =$

$$\begin{bmatrix} 3.72727273 & 4.8088179 & 5.27272727 & 4.27272727 \\ 2.56379168 & 0.45454545 & 0.33999441 & 3.25876686 \\ 4.00058244 & 1.65279664 & 1.2291021 & 4.35673936 \\ 2.18181818 & 3.56188755 & 3.63636364 & 3. \\ 1.50839198 & 5.18181818 & 4.63636364 & 2.88100368 \end{bmatrix} \tag{12}$$

Regular the invalidate score to [1,5], we could get:

$$\begin{bmatrix} 3.72727273 & 4.8088179 & 5. & 4.27272727 \\ 2.56379168 & 1. & 1. & 3.25876686 \\ 4.00058244 & 1.65279664 & 1.2291021 & 4.35673936 \\ 2.18181818 & 3.56188755 & 3.63636364 & 3. \\ 1.50839198 & 5.18181818 & 4.63636364 & 2.88100368 \end{bmatrix} \tag{13}$$

## 3 Bayesian ranking

In this question, using naive average ranking, for product 1, $n_1 = 3$ and $r_1 = \frac{5+5+5}{3} = 5$, for product 2, $n_2 = 10$ and $r_2 = \frac{4.5+5+4+5+4+5+3+5+4.5+3.5}{10} = 4.35$. Because $r_1 > r_2$, so under naive average ranking, product 1 (Canon) is better than product 2 (HP).

Using Bayesian prediction:

$$\tilde{r_1} = \frac{N * R + n_1 * r_1}{N + n_1} = \frac{50 * 4 + 3 * 5}{50 + 3} = 4.056603773584905 \quad (14)$$

$$\tilde{r_2} = \frac{N * R + n_2 * r_2}{N + n_2} = \frac{50 * 4 + 10 * 4.35}{50 + 10} = 4.058333333333334 \quad (15)$$

$\tilde{r_2} > \tilde{r_1}$, so under Bayesian estimation, product 2 (HP)'s rating is better than product 1 (Canon)'s.