Network20q HW2

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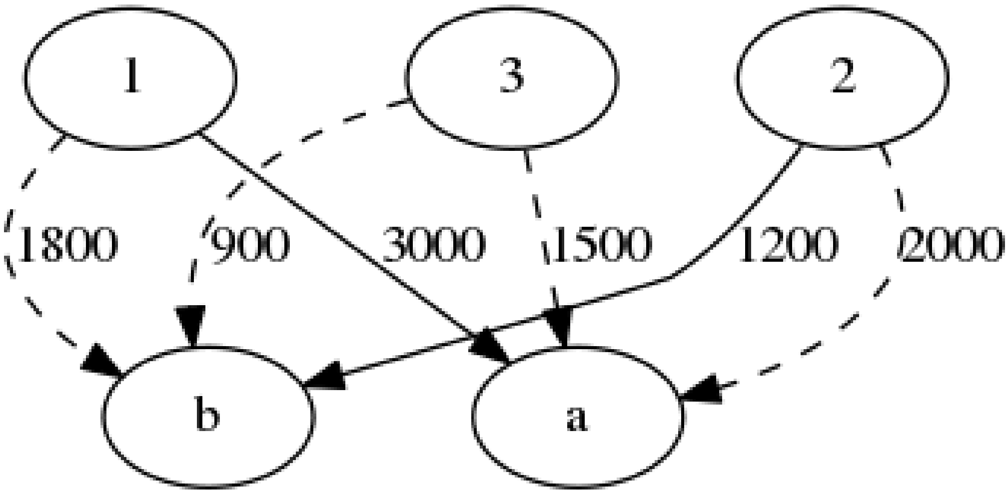
September 13, 2017

1 Ad space auction

1.1 a

In the first question, we need to draw the bipartie graph and chose the maximum matching.



The maximum matching could have the final solution: 1-A \$3000, and 2-b \$1200

1.2 b

The expected revenue vector of advertising 1,2,3 is $\begin{bmatrix}3000\\1800\end{bmatrix}$ $\begin{bmatrix}2000\\1200\end{bmatrix}$ $\begin{bmatrix}1500\\900\end{bmatrix}$ respectively. Then 1 would get A for the price of \$2000, and 2 would get B for the price of \$900. So 1 would be charged \$4 per click, and receive payoff \$2 per click. Likewise, 2 would be charged \$3 per click, and receive payoff \$1 per click.

2 Pagerank

Because node 5 do not point to another node, 5 is a dangling node. So we need to use

$$\hat{H} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1/3 & 0 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \end{bmatrix}$$

And when $\theta = 0.1$

$G = \theta \times \hat{H} + (1-\theta)\frac{1}{5} \times \vec{1}$

$$= \begin{bmatrix} 0.18 & 0.28 & 0.18 & 0.18 & 0.18 \\ 0.28 & 0.18 & 0.18 & 0.18 & 0.18 \\ 0.21333333 & 0.18 & 0.21333333 & 0.18 & 0.21333333 \\ 0.18 & 0.18 & 0.23 & 0.18 & 0.23 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

Let $\pi = [1/5, 1/5, 1/5, 1/5, 1/5]^T$

We could calculate $\pi^* = \pi \times [G]^n$, when n is big enough.

So we write a program to compare the recursive production of the multiplication of π and G, when the norm between two steps is below 1e-6, the program stops. When $\theta = 0.1$, program stops at 5th step. And the $\pi^* = [0.21117058, 0.2051142, 0.19985902, 0.18399718, 0.19985902]$

when $\theta = 0.3$

$G = \theta \times \hat{H} + (1-\theta)\frac{1}{5} \times \vec{1}$

$$= \begin{bmatrix} 0.14 & 0.44 & 0.14 & 0.14 & 0.14 \\ 0.44 & 0.14 & 0.14 & 0.14 & 0.14 \\ 0.24 & 0.14 & 0.24 & 0.14 & 0.24 \\ 0.14 & 0.14 & 0.29 & 0.14 & 0.29 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

Let $\pi = [1/5, 1/5, 1/5, 1/5, 1/5]^T$

Program stops at 9th step. And the

$\pi^* = [0.23789696, 0.22299348, 0.1937425, 0.15162455, 0.1937425]$

when $\theta = 0.5$

$G = \theta \times \hat{H} + (1-\theta)\frac{1}{5} \times \vec{1}$

$$= \begin{bmatrix} 0.1 & 0.6 & 0.1 & 0.1 & 0.1 \\ 0.6 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.26666667 & 0.1 & 0.26666667 & 0.1 & 0.26666667 \\ 0.1 & 0.1 & 0.35 & 0.1 & 0.35 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

Let $\pi = [1/5, 1/5, 1/5, 1/5, 1/5]^T$

Program stops at 16th step. And the

$\pi^* = [0.27450968, 0.25490208, 0.17647059, 0.11764706, 0.17647059]$

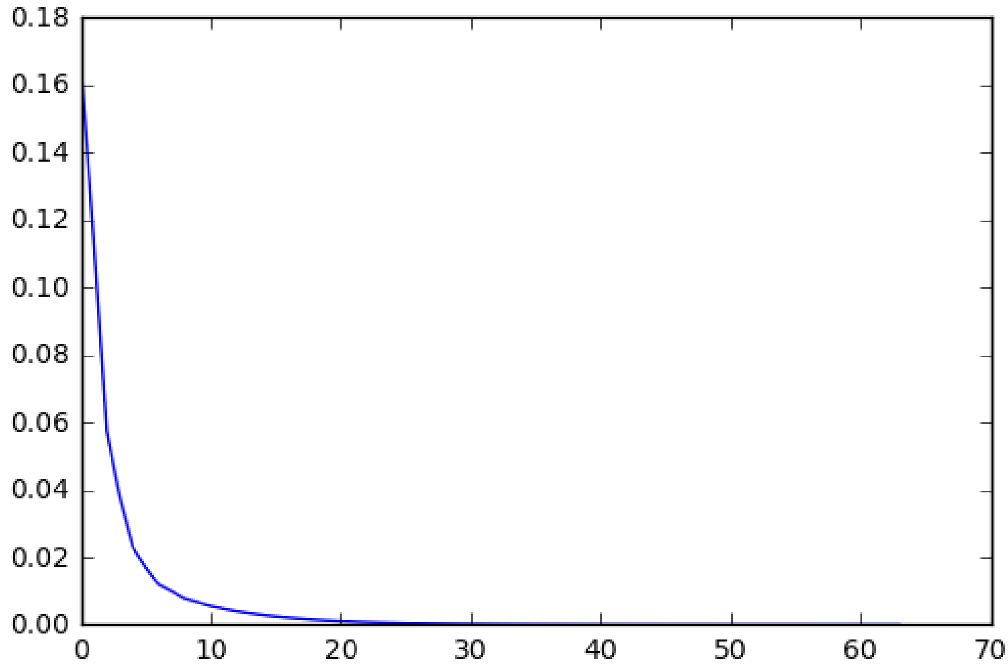
when $\theta = 0.85$

$G = \theta \times \hat{H} + (1-\theta)\frac{1}{5} \times \vec{1}$

$$= \begin{bmatrix} 0.03 & 0.88 & 0.03 & 0.03 & 0.03 \\ 0.88 & 0.03 & 0.03 & 0.03 & 0.03 \\ 0.31333333 & 0.03 & 0.31333333 & 0.03 & 0.31333333 \\ 0.03 & 0.03 & 0.455 & 0.03 & 0.455 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

Let $\pi = [1/5, 1/5, 1/5, 1/5, 1/5]^T$

When $\theta = 0.85$, program stops at 62 step. And the
$\pi^* = [0.39412997, 0.38032989, 0.09011066, 0.04531881, 0.09011066]$
The norm (distance) between two steps production is follows:



From the result, we could conlude that: first, with the increase of θ, the value of π^* could use more steps to converge, and the distribution of the value could be more imbalanced.