

# Network 20q HW10

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# 1 Aloha

## 1.1 a

The probability of a successful transformation is  $P_s = N * p * (1 - p)^{N-1}$ , only successful transmission could count throughput, so:

$$S = 1 * P_s = N * p * (1 - p)^{N-1} (Mbps) \quad (1)$$

## 1.2 b

We need to calculate the differentiate of  $S(p)$ :

$$S(p)' = (N * p * (1 - p)^{N-1})' = N * (1 - p)^{N-1} - N * (1 - p)^{N-1} * p = (1 - N) * N * (1 - p)^{N-2} \quad (2)$$

Set the equation to 0, we could get  $p^* = \frac{1}{N}$

## 1.3 c

The maximum throughput is  $S = N * \frac{1}{N} * (1 - \frac{1}{N})^{N-1} = (1 - \frac{1}{N})^{N-1}$ , so  $\lim_{N \rightarrow \infty} (1 - \frac{1}{N})^{N-1}$  let  $u = N - 1$ , we could get  $\lim_{N \rightarrow \infty} (1 - \frac{1}{N})^{N-1} = \lim_{u \rightarrow \infty} (1 - \frac{1}{u+1})^u = \lim_{u \rightarrow \infty} (\frac{u}{u+1})^u = \lim_{u \rightarrow \infty} (\frac{1}{1 + \frac{1}{u}})^u = \frac{1}{e} (Mbps)$

# 2 Overhead

If we are going to send 250 bytes data, the payload percentage is:

$$\frac{250}{8 + 6 + 6 + 2 + 40 + 20 + 4 + 12 + 250} = 0.718390804 = 71.8\% \quad (3)$$