

Bisection

1) Bisection

$$f(x) = x^3 + 3x - 1 \text{ on } [0, 1]$$

Iteration 1

$$a = 0$$

$$b = 1$$

$$f(a) = -1$$

$$f(b) = 3$$

$$c = (0+1)/2 = 0.5$$

$$f(c) = 0.625$$

$$b \leftarrow c = 0.5$$

Iteration 4

$$a = 0.25$$

$$b = 0.375$$

$$f(a) = -0.234375$$

$$f(b) = 0.177734375$$

$$c = (0.25 + 0.375)/2 = 0.3125$$

$$f(c) = -0.0319824219$$

$$a \leftarrow c = 0.3125$$

Iteration 2

$$a = 0$$

$$b = 0.5$$

$$f(a) = -1$$

$$f(b) = 0.625$$

$$c = (0+0.5)/2 = 0.25$$

$$f(c) = -0.234375$$

$$a \leftarrow c = 0.25$$

Iteration 5

$$a = 0.3125$$

$$b = 0.375$$

$$f(a) = -0.0319824219$$

$$f(b) = 0.177734375$$

$$c = (0.3125 + 0.375)/2 = 0.34375$$

$$f(c) = 0.0718688965$$

$$b \leftarrow c = 0.34375$$

Iteration 3

$$a = 0.25$$

$$b = 0.5$$

$$f(a) = -0.234375$$

$$f(b) = 0.625$$

$$c = (0.25 + 0.5)/2 = 0.375$$

$$f(c) = 0.177734375$$

$$b \leftarrow c = 0.375$$

Iteration 6

$$a = 0.3125$$

$$b = 0.375$$

$$f(a) = -0.0319824219$$

$$f(b) = 0.0718688965$$

$$c = (0.3125 + 0.375)/2 = 0.34375$$

$$f(c) = 0.0197029114$$

$$b \leftarrow c = 0.34375$$

Iteration 2

$$a = 0.3125$$

$$b = 0.328125$$

$$f(a) = -0.0061984062$$

$$f(b) = 0.0197029114$$

$$c = (0.3125 + 0.328125)/2 = 0.3203125$$

$$f(c) = -0.0061984062$$

$$a \leftarrow c = 0.3203125$$

Iteration 3

$$a = 0.3203125$$

$$b = 0.328125$$

$$f(a) = -0.0061984062$$

$$f(b) = 0.0197029114$$

$$c = (0.3203125 + 0.328125)/2 = 0.3242875$$

$$f(c) = 0.0067374110$$

$$b \leftarrow c = 0.3242875$$

Iteration 4

$$a = 0.3203125$$

$$b = 0.3242875$$

$$f(a) = -0.0061984062$$

$$f(b) = 0.0067374110$$

$$c = (0.3203125 + 0.3242875)/2 = 0.32226875$$

$$f(c) = 0.0002658144$$

$$b \leftarrow c = 0.32226875$$

Iteration 10

$$a = 0.3203125$$

$$b = 0.322265625$$

$$f(a) = -0.0061984062$$

$$f(b) = 0.0002658144$$

$$c = (0.3203125 + 0.322265625)/2 = 0.3212890625$$

$$f(c) = -0.0029673151$$

$$a \leftarrow c = 0.3212890625$$

O found at $x = 0.3212890625$)

Bisection

$$g(x) = x^3 - 2 \sin x \quad \text{on } (0, 3, 2)$$

Iteration 1

$$a = 0.5$$

$$b = 2$$

$$f_a = -0.48338510772$$

$$f_b = 0.1916051463$$

$$c = (0.5 + 2)/2 = 1.25$$

$$f_c = 0.0551557613$$

$$b \leftarrow c = 1.25$$

Iteration 2

$$a = 0.5$$

$$b = 1.25$$

$$f_a = -0.48338510772$$

$$f_b = 0.0551557613$$

$$c = (0.5 + 1.25)/2 = 0.875$$

$$f_c = -0.281681295$$

$$a \leftarrow c = 0.875$$

Iteration 3

$$a = 0.875$$

$$b = 1.25$$

$$f_a = -0.281681295$$

$$f_b = 0.0551557613$$

$$c = (0.875 + 1.25)/2 = 1.0625$$

$$f_c = -0.124798647$$

$$a \leftarrow c = 1.0625$$

Iteration 4

$$a = 1.0625$$

$$b = 1.25$$

$$f_a = -0.124798647$$

$$f_b = 0.0551557613$$

$$c = (1.0625 + 1.25)/2 = 1.15625$$

$$f_c = -0.2847914008$$

$$a \leftarrow c = 1.15625$$

Iteration 5

$$a = 1.15625$$

$$b = 1.25$$

$$f_a = -0.2847914008$$

$$f_b = 0.0551557613$$

$$c = (1.15625 + 1.25)/2 = 1.203125$$

$$f_c = -0.1247986155$$

$$a \leftarrow c = 1.203125$$

Iteration 6

$$a = 1.203125$$

$$b = 1.25$$

$$f_a = -0.1247986155$$

$$f_b = 0.0551557613$$

$$c = (1.203125 + 1.25)/2 = 1.2265625$$

$$f_c = -0.0373598065$$

$$a \leftarrow c = 1.2265625$$

Bisection

Iteration 7

$$a = 1.2265625$$

$$b = 1.28$$

$$f_a = -0.037359806$$

$$f_b = 0.0551557613$$

$$c = (1.2265625 + 1.28) / 2 = 1.23828125$$

$$f_c = 0.0082880159$$

$$b \leftarrow c = 1.23828125$$

Iteration 8

$$a = 1.2265625$$

$$b = 1.23828125$$

$$f_a = -0.0373598065$$

$$f_b = 0.0082880159$$

$$c = (1.2265625 + 1.23828125) / 2 = 1.23242875$$

$$a \leftarrow c = 1.23242875$$

Iteration 9

$$a = 1.23242875$$

$$b = 1.23828125$$

$$f_a = -0.0147102162$$

$$f_b = 0.0082880159$$

$$c = (1.23242875 + 1.23828125) / 2 = 1.2353515625$$

$$f_c = -0.0032460417$$

$$a \leftarrow c = 1.2353515625$$

Iteration 10
 $a = 1.2353515625$

$$b = 1.23828125$$

$$f_a = -0.0032460417056$$

$$f_b = 0.0082380159$$

$$c = (1.2353515625 + 1.23828125) / 2 \\ = 1.23681640625$$

$$f_c = 0.0024860119$$

$$b \leftarrow c = 1.23681640625$$

No convergence

Final answer = 1.3681640625

Bisection

$h(x) = x + 10 - x \cosh(50/x)$, on $[120, 130]$

Iteration 1

$$a = 120$$

$$b = 130$$

$$f_a = -0.5667455953$$

$$f_b = 0.265496809$$

$$c = (120+130)/2 = 125$$

$$f_c = -0.1340464798$$

$$a < c = 125$$

Iteration 4

$$a = 126.25$$

$$b = 127.5$$

$$f_a = -0.031080617904$$

$$f_b = 0.0697896064$$

$$c = (126.25 + 127.5)/2 = 126.875$$

$$f_c = 0.0196123828$$

$$a < c = 126.875$$

Iteration 2

$$a = 125$$

$$b = 130$$

$$f_a = -0.1340464789$$

$$f_b = 0.265496809$$

$$c = (125+130)/2 = 127.5$$

$$f_c = 0.069789606$$

$$b < c = 127.5$$

Iteration 5

$$a = 126.25$$

$$b = 126.875$$

$$f_a = -0.0310806179$$

$$f_b = 0.0196123868$$

$$c = (126.25 + 126.875)/2 = 126.5625$$

$$f_c = -0.0056671387$$

$$a < c = 126.5625$$

Iteration 3

$$a = 125$$

$$b = 127.5$$

$$f_a = -0.1340464791$$

$$f_b = 0.0697896064$$

$$c = (125+127.5)/2 = 126.25$$

$$f_c = -0.031686414908$$

$$a < c = 126.25$$

Iteration 6

$$a = 126.5625$$

$$b = 126.875$$

$$f_a = -0.0056691387$$

$$f_b = 0.01961238868$$

$$c = (126.5625 + 126.875)/2 = 126.71875$$

$$f_c = 0.0069878655$$

$$b < c = 126.71875$$

Bisection

iteration 7

$$a = 126.5628$$

$$b = 126.71875$$

$$f_a = -0.0056691387$$

$$f_b = 0.00698780853$$

$$c = (126.5628 + 126.71875) / 2 = 126.640625$$

$$f_c = 0.0006633864$$

$$b \leftarrow c = 126.640625$$

iteration 8

$$a = 126.5628$$

$$b = 126.640625$$

$$f_a = -0.0056691387$$

$$f_b = 0.0006633864$$

$$c = (126.5628 + 126.640625) / 2 = 126.6015625$$

$$f_c = -0.00280186142$$

$$a \leftarrow c = 126.6015625$$

iteration 9

$$a = 126.6015625$$

$$b = 126.640625$$

$$f_a = -0.002801864$$

$$f_b = 0.0006633864$$

$$c = (126.6015625 + 126.640625) / 2 = 126.62109375$$

$$f_c = -0.000918984323$$

$$a \leftarrow c = 126.62109375$$

Bisection

Bisection 10

$$a = 126.62109375$$

$$b = 126.640625$$

$$f_a = -0.0009189843$$

$$f_b = 0.000663386403$$

$$c = (126.62109375 + 126.640625)/2 = 126.630859375$$

$$f_c = -0.00012773567$$

$$a \leftarrow c = 126.630859375$$

Convergence not reached

$$\text{final answer} = 126.630859375$$

Newtons Method

$$f(x) = x^3 + 2x^2 + 10x - 20 \quad x \text{ at } 2$$

$$x_{i+1} = x_i - f(x_i) / f'(x_i)$$

Iteration 1

$$x_1 = x_0 - f(x_0) / f'(x_0)$$

$$f(x_0) = 16$$

$$f'(x) = 30$$

$$x_1 = 2 - (16/30) = 1.46$$

Iteration 2

$$x_2 = x_1 - f(x_1) / f'(x_1)$$

$$f(x_1) = 2.1238518519$$

$$f'(x_1) = 22.32$$

$$x_2 = 1.46 - (2.1238518519 / 22.32) = 1.3715120138$$

Iteration 3

$$x_3 = x_2 - f(x_2) / f'(x_2)$$

$$f(x_2) = 0.0520866419$$

$$f'(x_2) = 21.1291836673$$

$$x_3 = 1.3715120138 - (0.0520866419 / 21.1291836673) \\ = 1.3688102226$$

Iteration 4

$$x_4 = x_3 - f(x_3) / f'(x_3)$$

$$f(x_3) = 4.4614466963 \times 10^{-5}$$

$$f'(x_3) = 21.0961651673$$

$$x_4 = 1.368810226 - (4.4614466963 \times 10^{-5} / 21.0961651673) \\ = 1.3688081074$$

Method de La Newton

Iteration 5

$$x_5 = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$f(A) = 2.7309710049 \times 10^{-11}$$

$$f'(A) = 21.096139339$$

$$x_5 = 1.3688081078 - \frac{(2.7309710049 \times 10^{-11})}{21.096139339}$$

Converged

0 found at $x = 1.3688081078$

Secant Method

$$x_{i+2} = x_{i+1} - (x_{i+1} - x_i) \cdot f(x_{i+1}) / (f(x_{i+1}) - f(x_i))$$

$$f(x_0) = f(2) = 16$$

$$f(x_1) = f(1) = -7$$

swap x_0 and x_1

Iteration 1

$$x_2 = 2 - (2-1) \cdot f(2) / f(2) - f(1)$$

$$f(1) = -7$$

$$f(2) = 16$$

$$x_2 = 2 - (2-1) \cdot 16 / (16 - -7)$$

$$x_2 = 1.3043782609$$

Iteration 2

$$x_3 = 1 - (1 - 1.3043782609) \cdot f(1) / f(1) - f(1.3043782609)$$

$$f(1.3043782609) = -1.3347579518$$

$$f(1) = -7$$

$$x_3 = 1 - (1 - 1.3043782609) * -7 / (-7 - -1.3347579518)$$

$$x_3 = 1.3760536204$$

Iteration 3

$$x_4 = 1.3043782609 \cdot (1.3043782609 - 1.3760536204) \times f(1.3043782609) \\ / f(1.3043782609) - f(1.3760536204)$$

$$f(1.3760536204) = 0.1831732187$$

$$f(1.3043782609) = -1.3347579518$$

$$x_4 = 1.3686219535$$

Secant method

Iteration 4

$$x_5 = x_4 - (x_4 - x_3) \cdot f(x_4) / (f(x_4) - f(x_3))$$

$$f(1.3686719535) = -0.0028722167$$

$$f(1.3760536204) = 0.1531732949$$

$$x_5 = 1.3688078 \approx 25$$

Iteration 5

$$x_6 = x_5 - (x_5 - x_4) \cdot f(x_5) / (f(x_5) - f(x_4))$$

$$f(1.3688078325) = -6.0186468751 \times 10^{-6}$$

$$f(1.3686719535) = -0.0028722167$$

$$x_6 = 1.3688081078$$

Iteration 6

$$x_7 = x_6 - (x_6 - x_5) \cdot f(x_6) / (f(x_6) - f(x_5))$$

$$f(1.3688081078) = 2.3720403619 \times 10^{-10}$$

$$f(1.3686719535) = -6.0186468751 \times 10^{-6}$$

No sign

Converges!

0 found at 1.3688081078