Topic 2: Geophysical Fluid Dynamics

MATH3261/5285 Fluids, Ocean, and Climate

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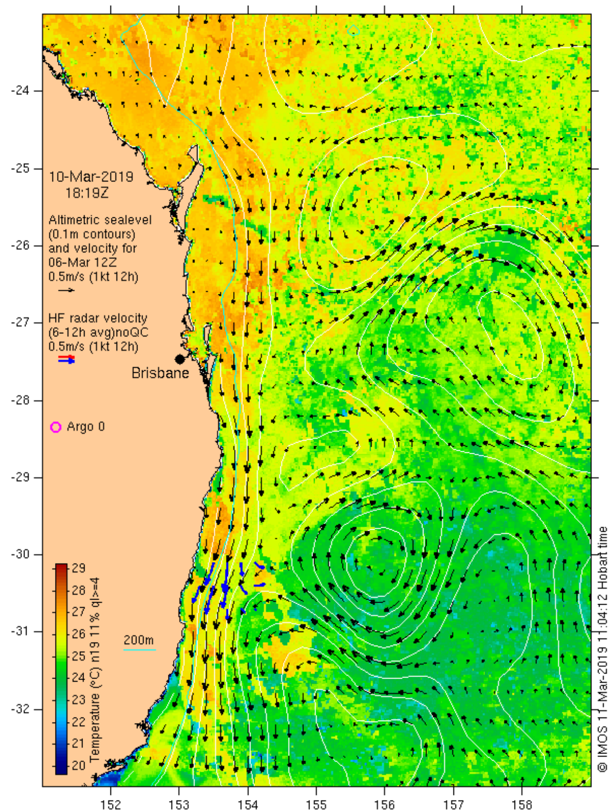
2.1 Geostrophic balance in the ocean and atmosphere

Geostrophic balance in the ocean

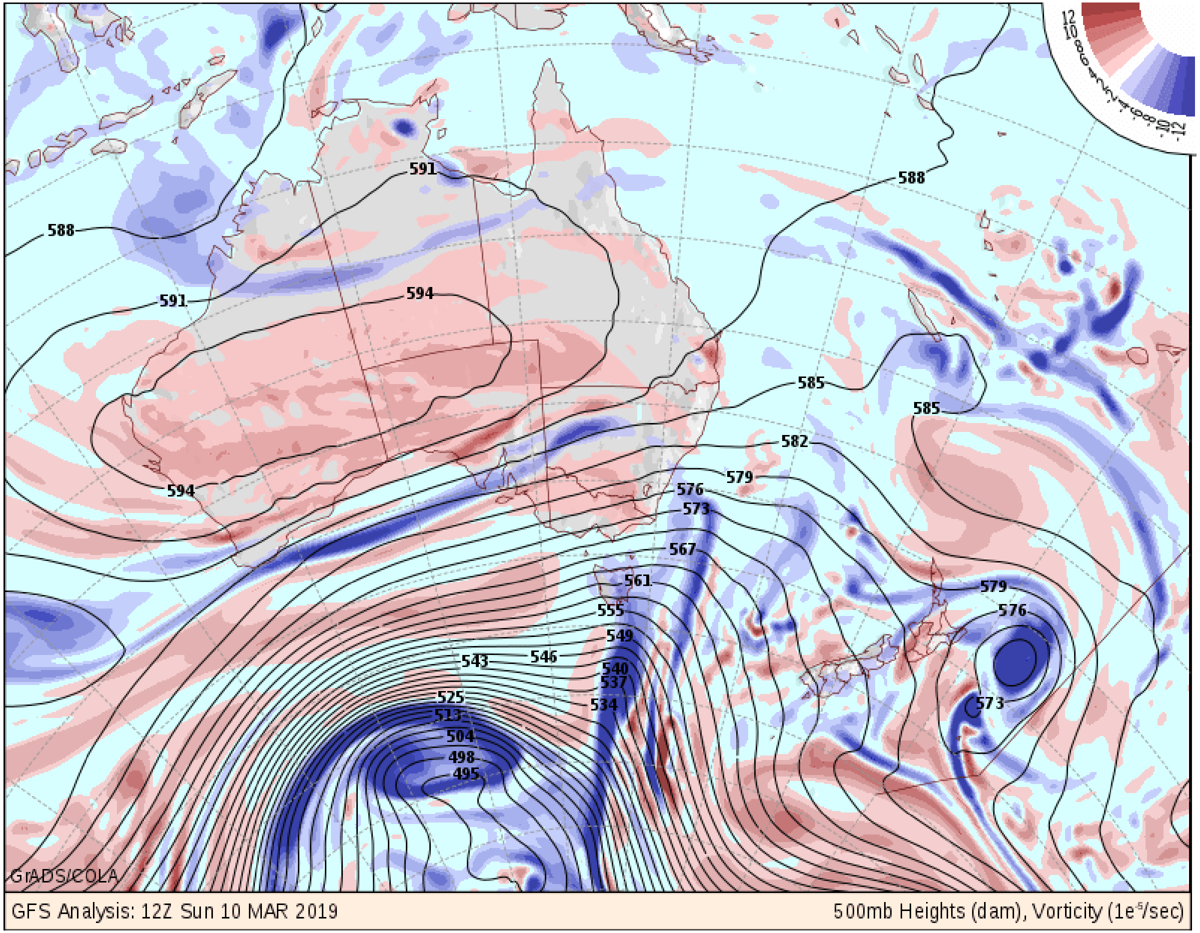
$$\boldsymbol{u} = \hat{z} \times \nabla_z \frac{g\,\eta}{f_0}, \qquad \eta = \text{sea-surface height}$$



Credit: {Integrated Marine Observing System / OceanCurrent}

Geostrophic balance in the atmosphere

$$\boldsymbol{f} \times \boldsymbol{u} = -g\nabla_p z, \qquad z = \text{geopotential height}$$



2.2 The Taylor-Proudman Theorem

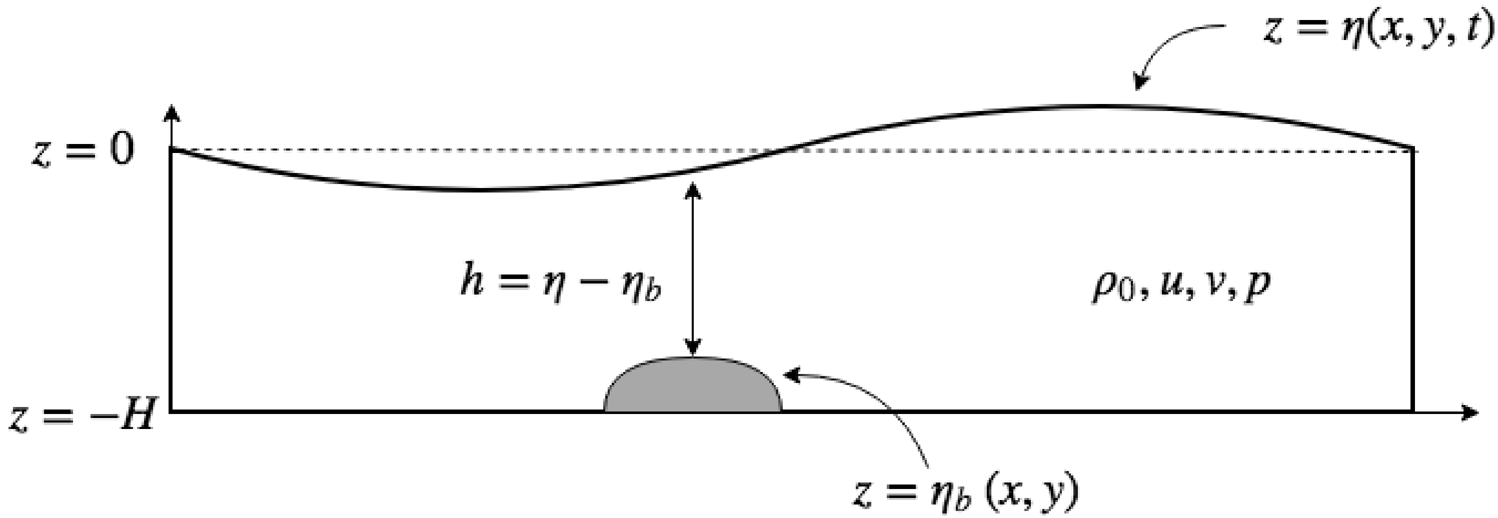
Credit: {Yous van Halder / YouTube}

2.3 The shallow water model

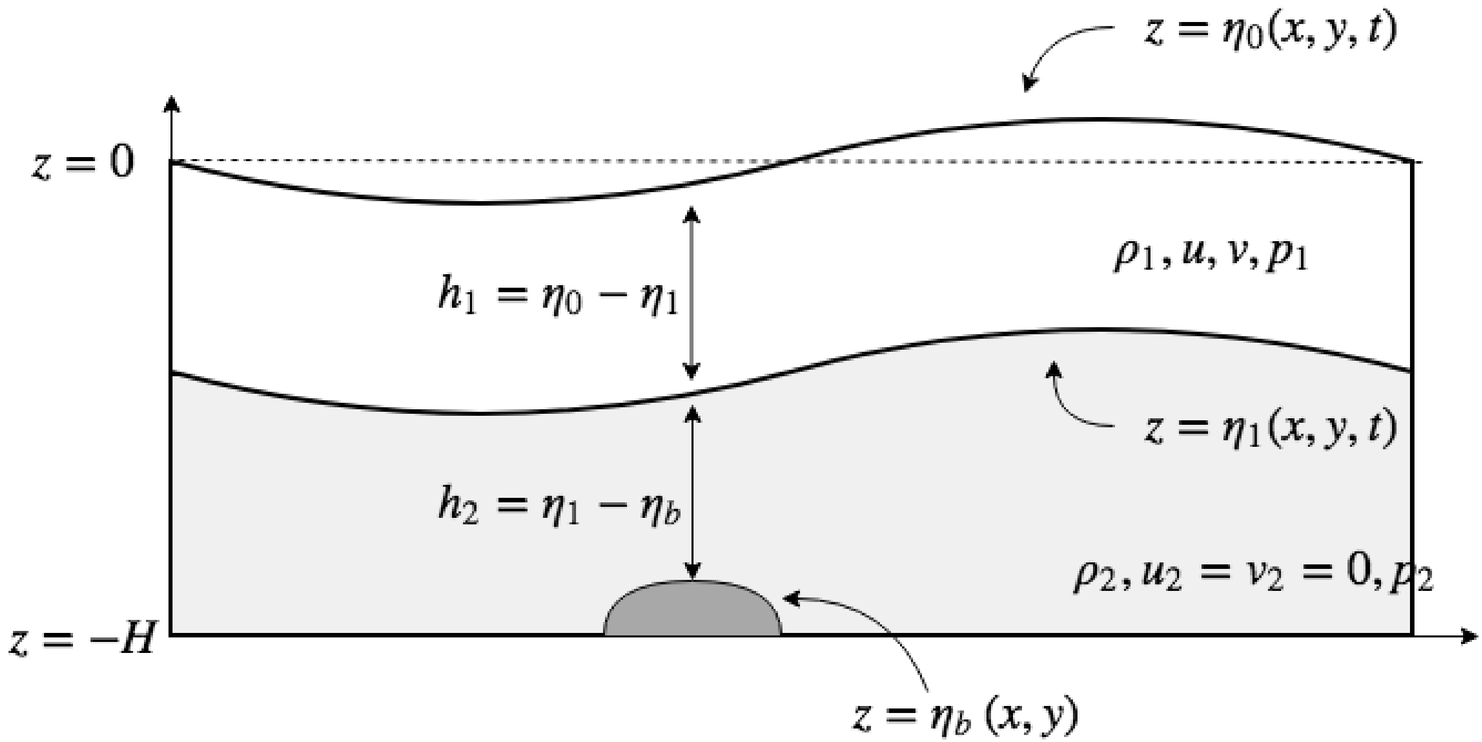
"Choke me in the shallow water before I get too deep."

Edie Brickell, *What I am*.

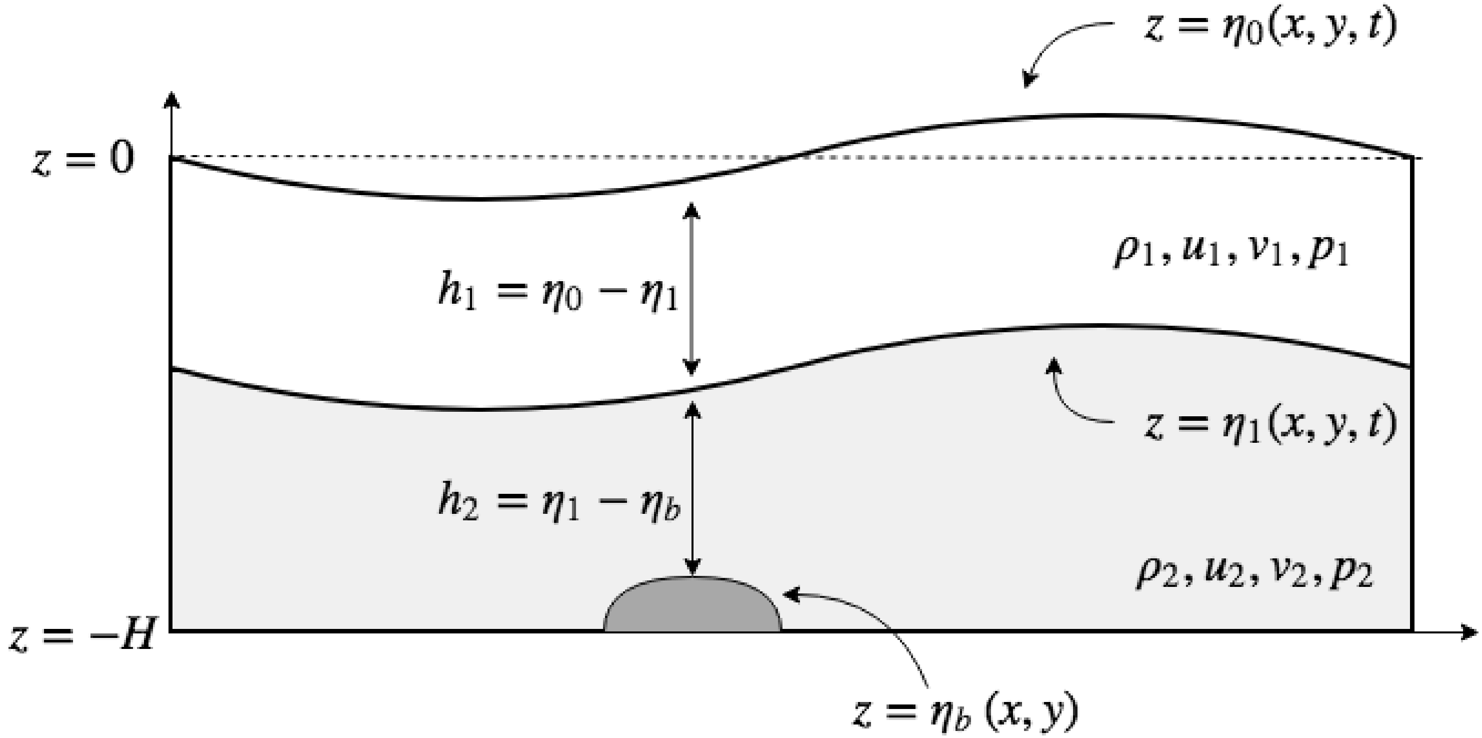
1-layer shallow water model



1.5-layer shallow water model



2-layer shallow water model



2.4 Vorticity and potential vorticity

"Donde escono quei vortici?"

("Whence come these vortices?")

Wolfgang Amadeus Mozart, *Don Giovanni, Act II*

Define 3D vorticity:

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$$

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{bmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \partial_x & \partial_y & \partial_z \\ u & v & w \end{bmatrix} = \begin{pmatrix} w_y - v_z \\ u_z - w_x \\ v_x - u_y \end{pmatrix}$$

Define circulation around a closed loop C:

$$\Gamma = \oint_C \boldsymbol{v} \cdot d\boldsymbol{r}$$

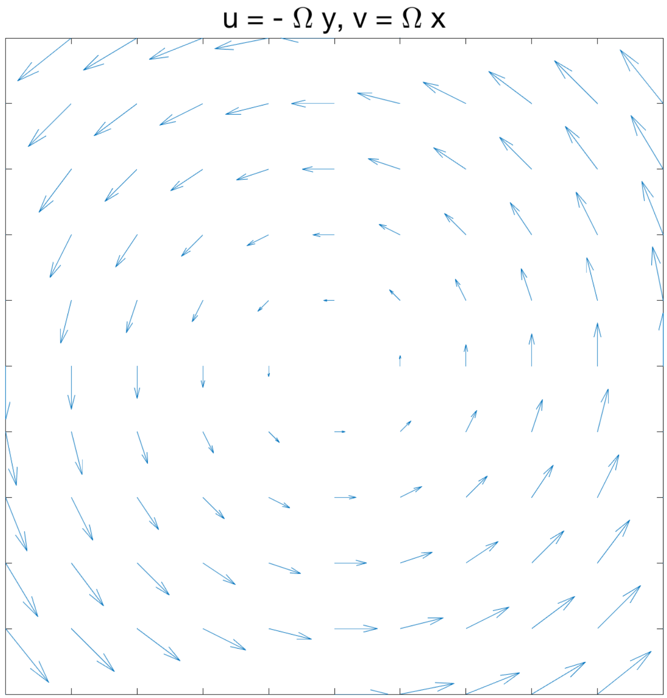
Stokes' theorem:

$$\oint_C \boldsymbol{v} \cdot d\boldsymbol{r} = \int_S \nabla \times \boldsymbol{v} \cdot d\boldsymbol{S} = \int_S \boldsymbol{\omega} \cdot d\boldsymbol{S}$$

Circulation around C = vorticity enclosed by C

Vorticity is the *circulation per area* around an infinitessimally small closed contour.

Solid body rotation with angular velocity Ω

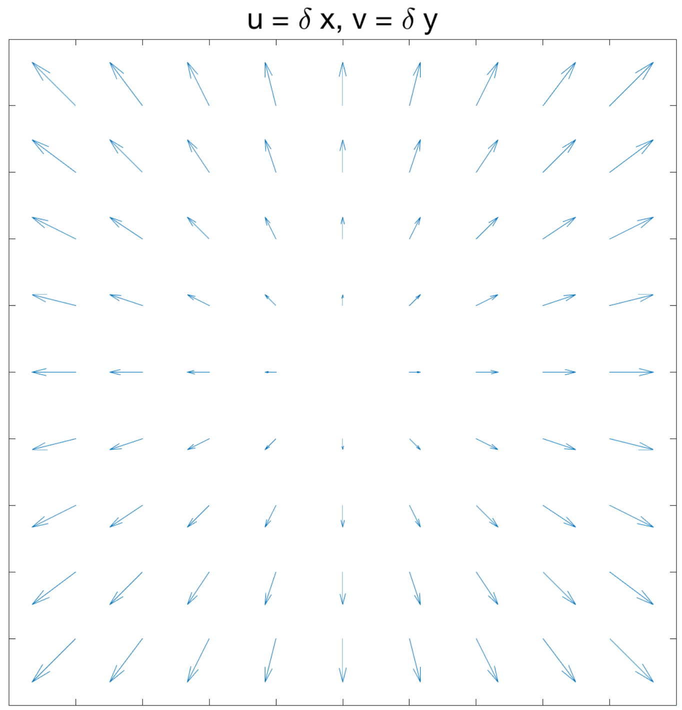


$$u = -\Omega y, v = \Omega x$$

$$\omega_3 = v_x - u_y = 2\Omega$$

Uniform vorticity: the same value at all points. Note the factor of 2!

Divergent flow

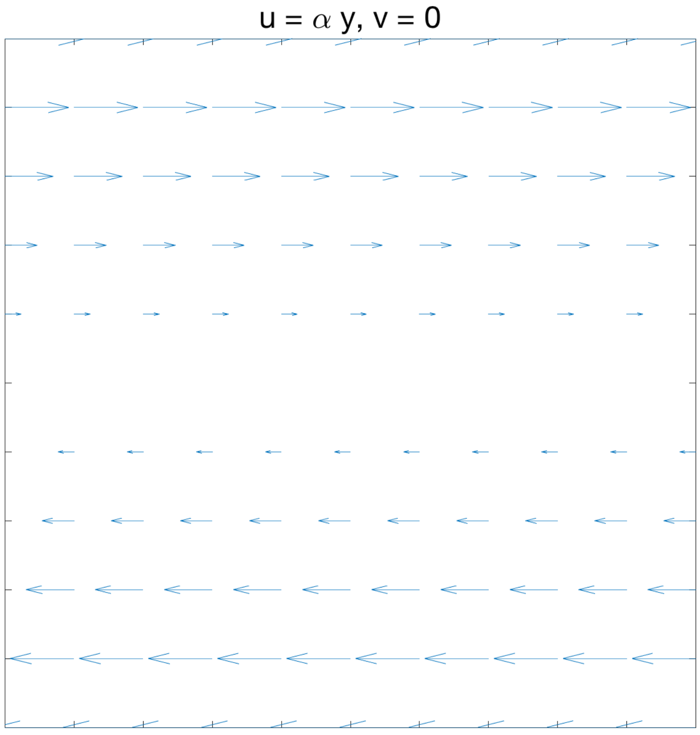


$$u = \delta x, v = \delta y$$

$$\omega_3 = v_x - u_y = 0$$

Divergent / convergent flows are irrotational (zero vorticity).

Shear flow

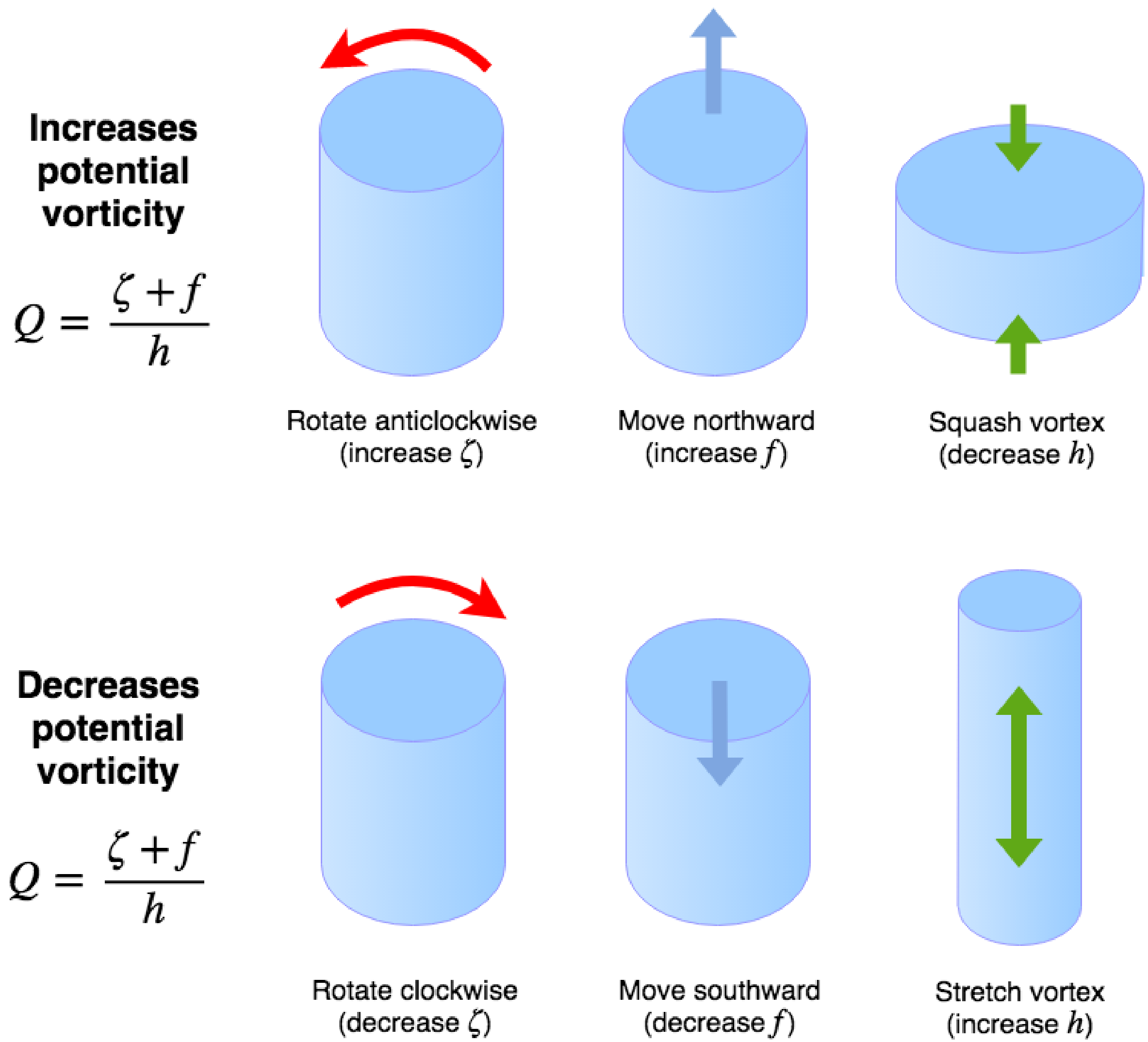


$$u = \alpha y, v = 0$$

$$\omega_3 = v_x - u_y = -\alpha$$

Even parallel flows can have vorticity!

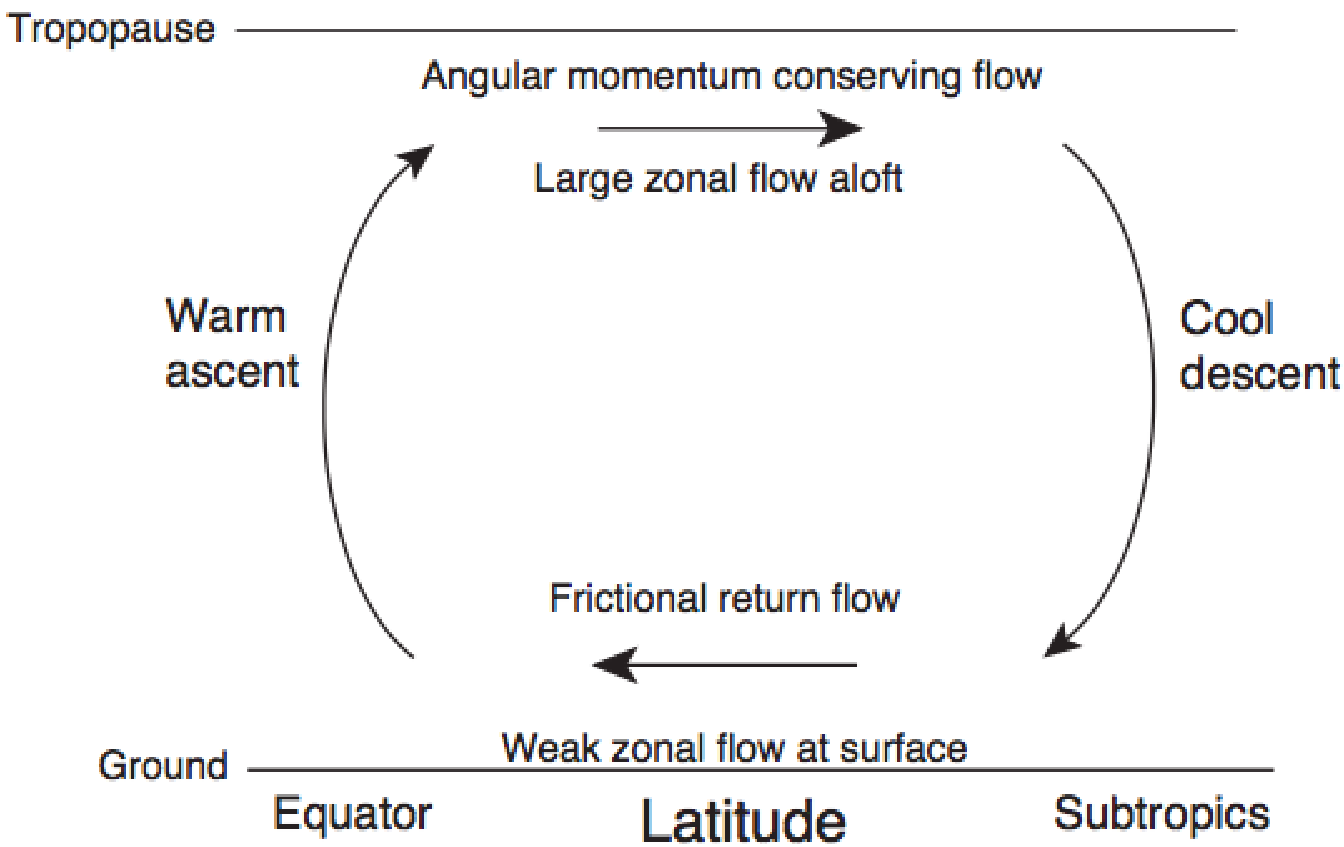
Conservation of potential vorticity



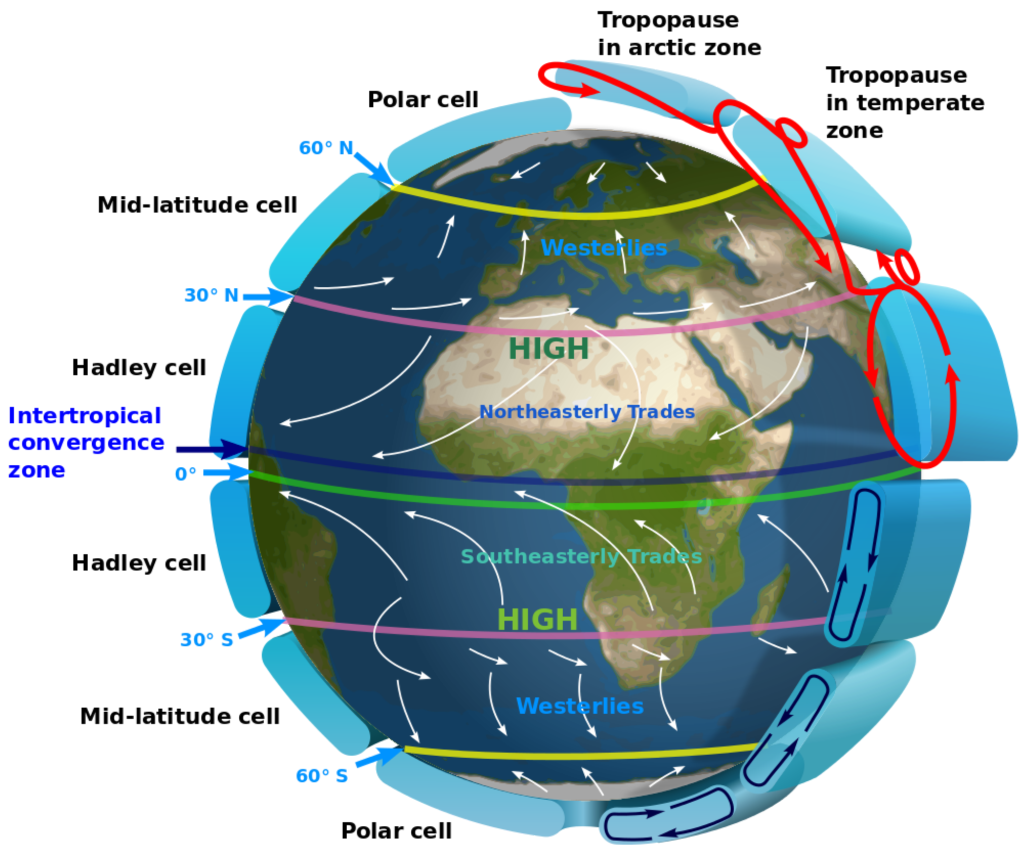
2.5 The Hadley Cell

"The answer, my friend, is blowin' in the wind."

Bob Dylan



(Credit: G. Vallis)


Tropopause
in arctic zone
Tropopause
in temperate
zone
Polar cell
60° N
Mid-latitude cell
Westerlies
30° N
HIGH
Hadley cell
Intertropical
convergence
zone
Northeasterly Trades
0°
Southeasterly Trades
Hadley cell
HIGH
30° S
Westerlies
Mid-latitude cell
60° S
Polar cell