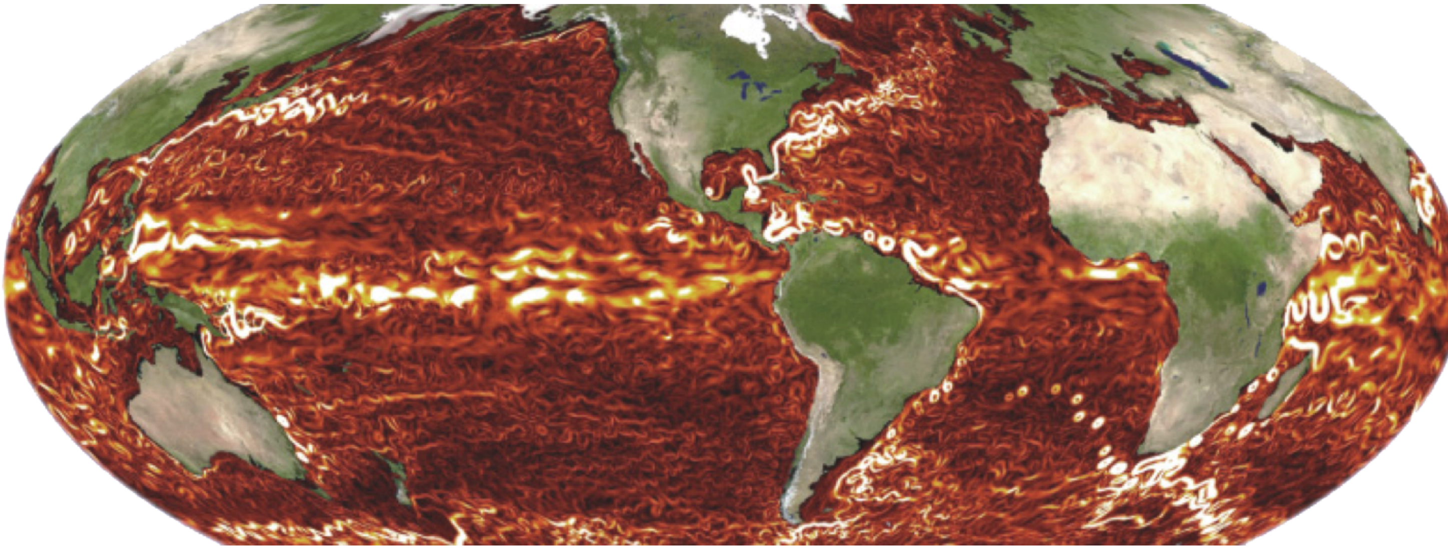

Topic 5: Ocean circulation

MATH3261/5285: Fluids, Oceans, and Climate

{Shane Keating}, School of Mathematics and Statistics

UNSW Sydney, Term 1, 2019

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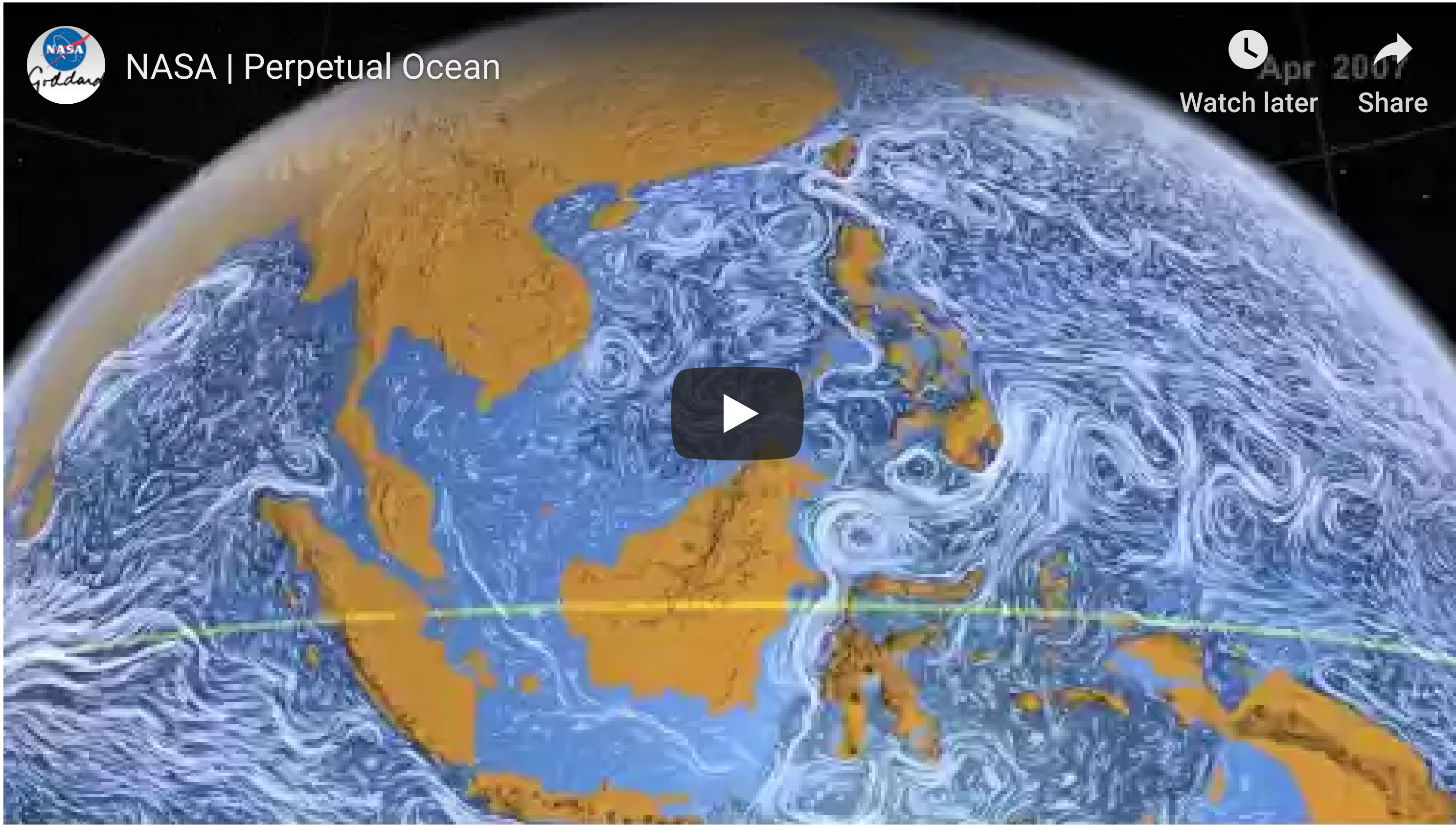
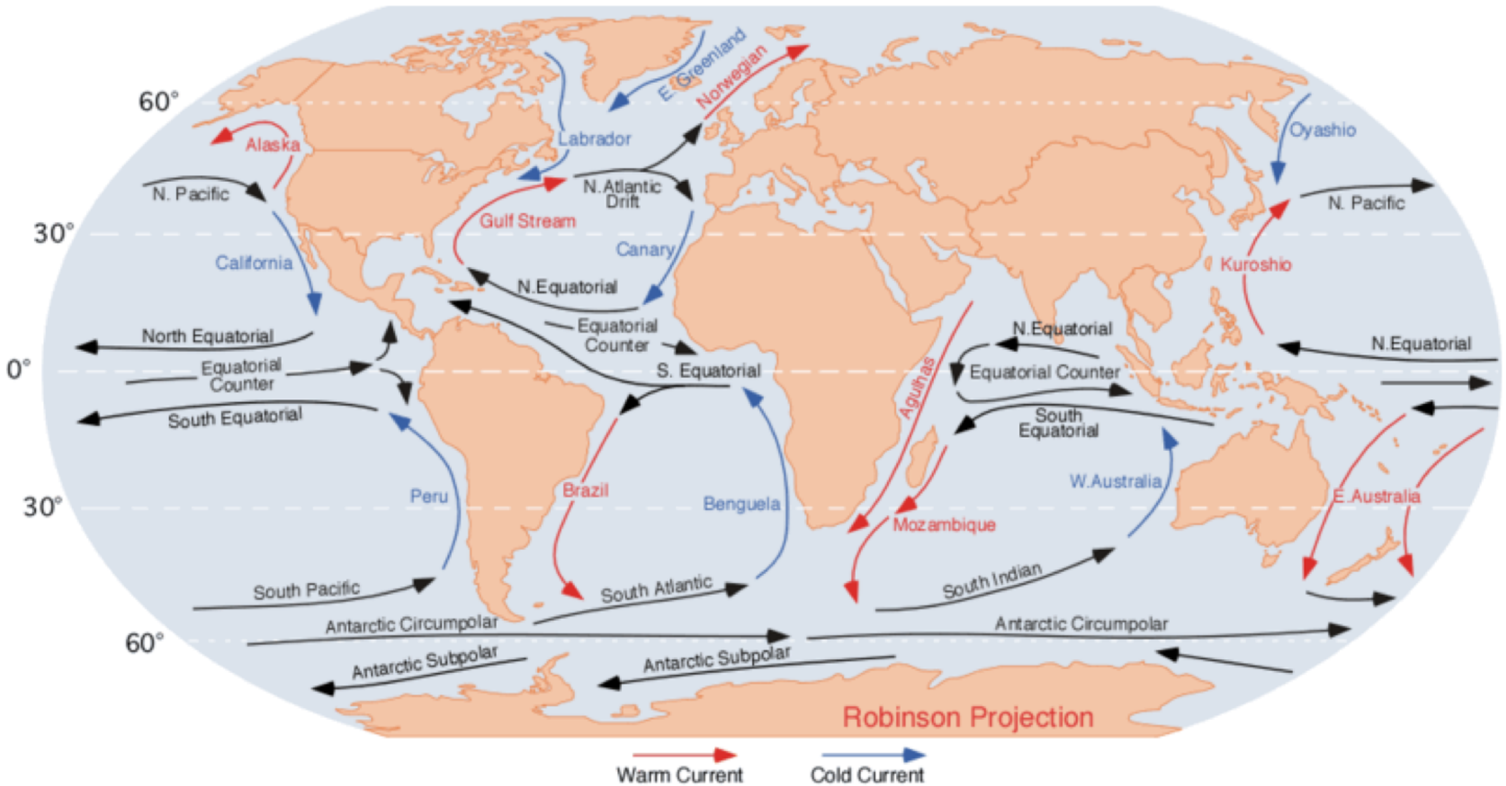
5.1 A brief overview of the ocean circulation

> "Look at the crowds of water-gazers there."
>
> Herman Melville, *Moby Dick*



{NASA | Perpetual Ocean}

Major surface ocean currents



Baroclinic instability

Baro *(pressure)* + ***cline*** *(slope)* : instability arising from sloped pressure contours

> "It is the instability that gives rise to the large-scale and mesoscale motion in the atmopshere and the ocean - it produces atmopsheric weather systems, for example, and so is, perhaps, the form of hydrodynamic instability that most affects the human condition."
>
> G. J. Vallis, *Atmospheric and Oceanic Fluid Dynamics*

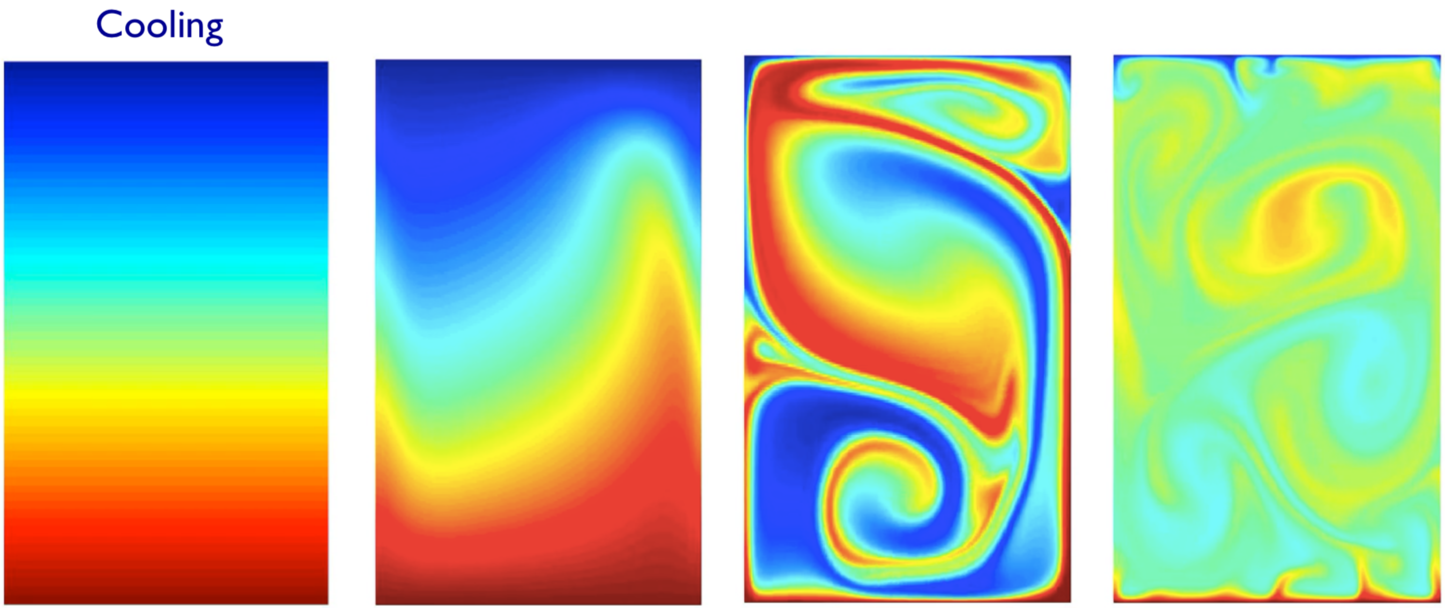
Baroclinic instability

- generates midlatitude ocean eddies and weather systems
- converts gravitational potential energy (stored in sloping density surfaces) into kinetic energy
- transports heat poleward modifying climate and global energy budget

Convective instability

- At rest with cold/dense fluid over warm/light fluid
- Rest state is in unstable equilibrium

Convective instability

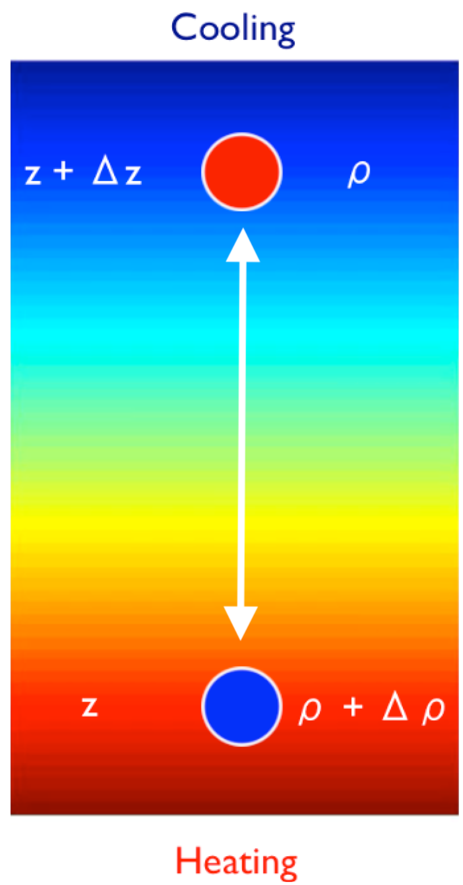


- At rest with cold/dense fluid over warm/light fluid
- Rest state is in unstable equilibrium

Convective instability

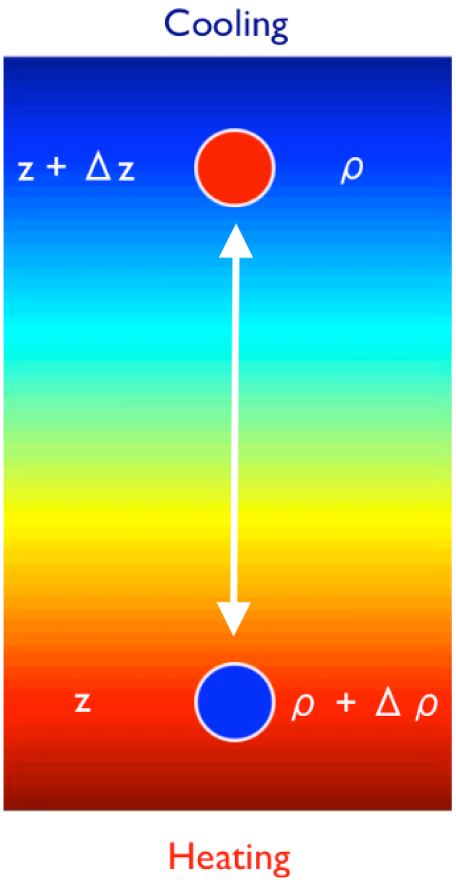
- Initial potential energy: $PE_1 = g\ \rho\ z + g\ \ (\rho + \Delta\rho)\,(z + \Delta z)$

Convective instability



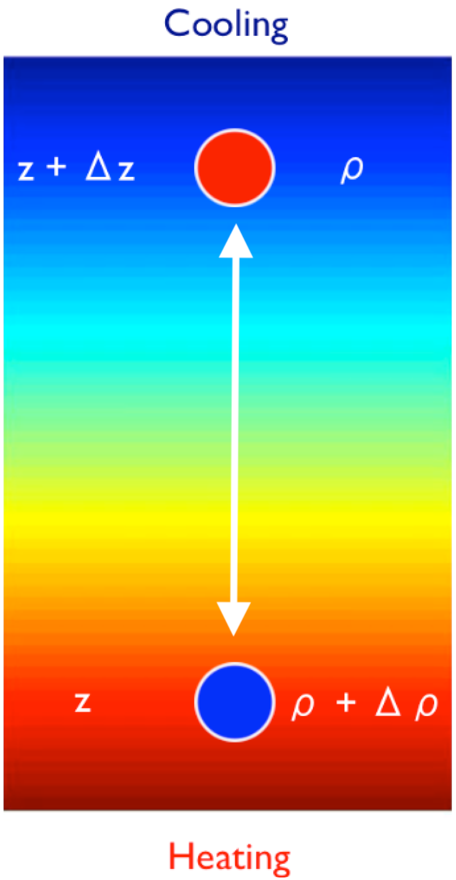
- Final potential energy: $PE_2 = g\ (\rho + \Delta\rho)\ z + g\ \rho\ (z + \Delta z)$

Convective instability



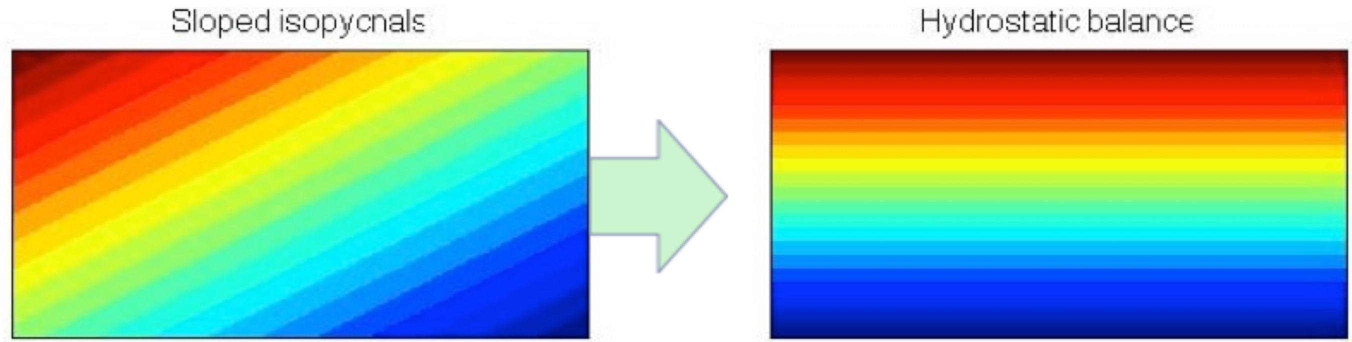
- Final potential energy: $PE_2 = g\ (\rho + \Delta\rho)\ z + g\ \rho\ (z + \Delta z)$
- If $\Delta\rho > 0$ then total PE decreases

Convective instability



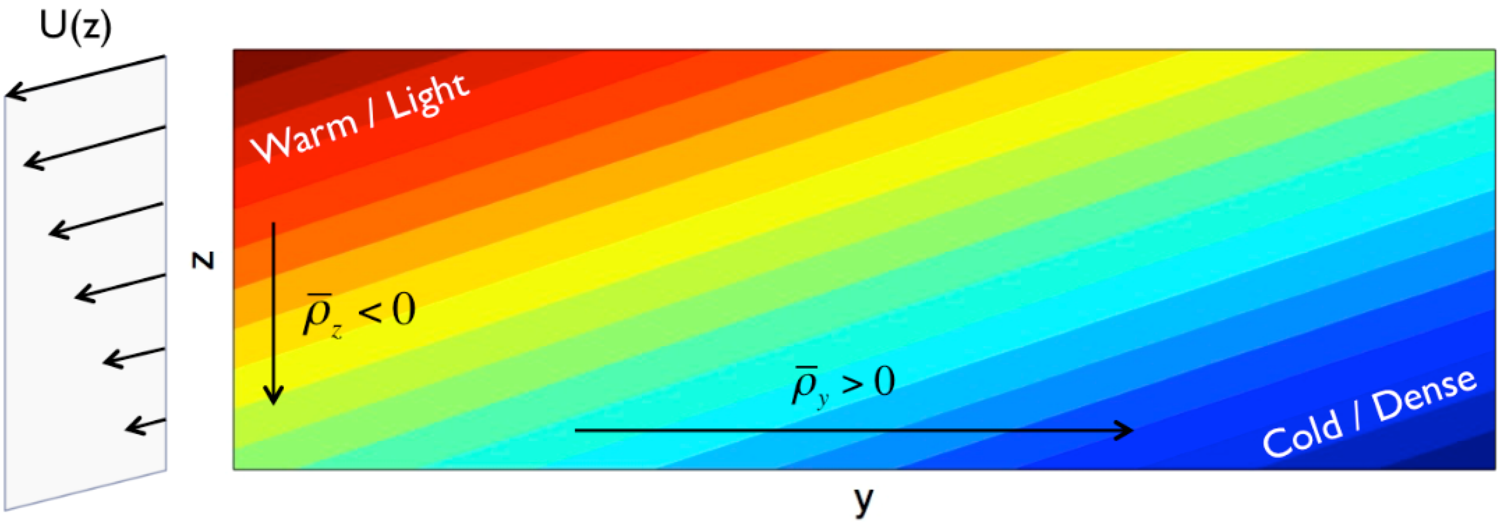
- Final potential energy: $PE_2 = g\ (\rho + \Delta\rho)\ z + g\ \rho\ (z + \Delta z)$
- If $\Delta\rho > 0$ then total PE decreases
- Kinetic energy increases (instability)

Available potential energy



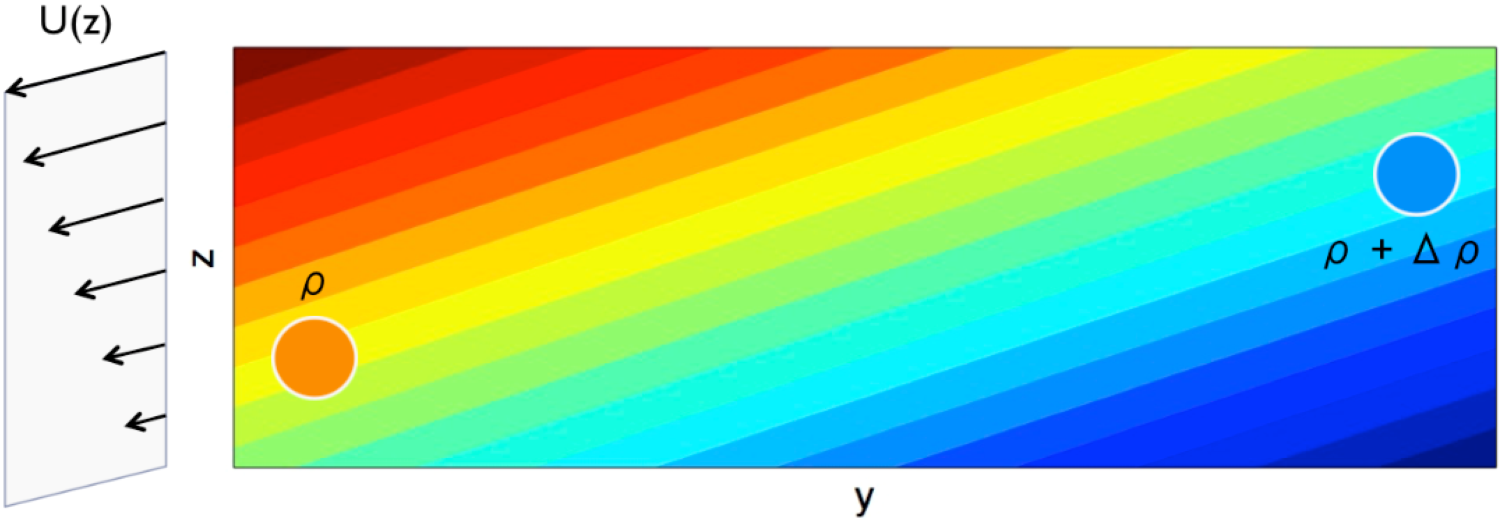
- For a statified fluid with horizontal density surfaces, any adiabatic rearrangement of water parcels *increases* potential energy
- Potential energy cannot be converted into kinetic energy
- Define **Available Potential Energy (APE)** as the PE available for conversion to KE
- Sloping density surfaces in thermal wind balance have APE!

Available potential energy



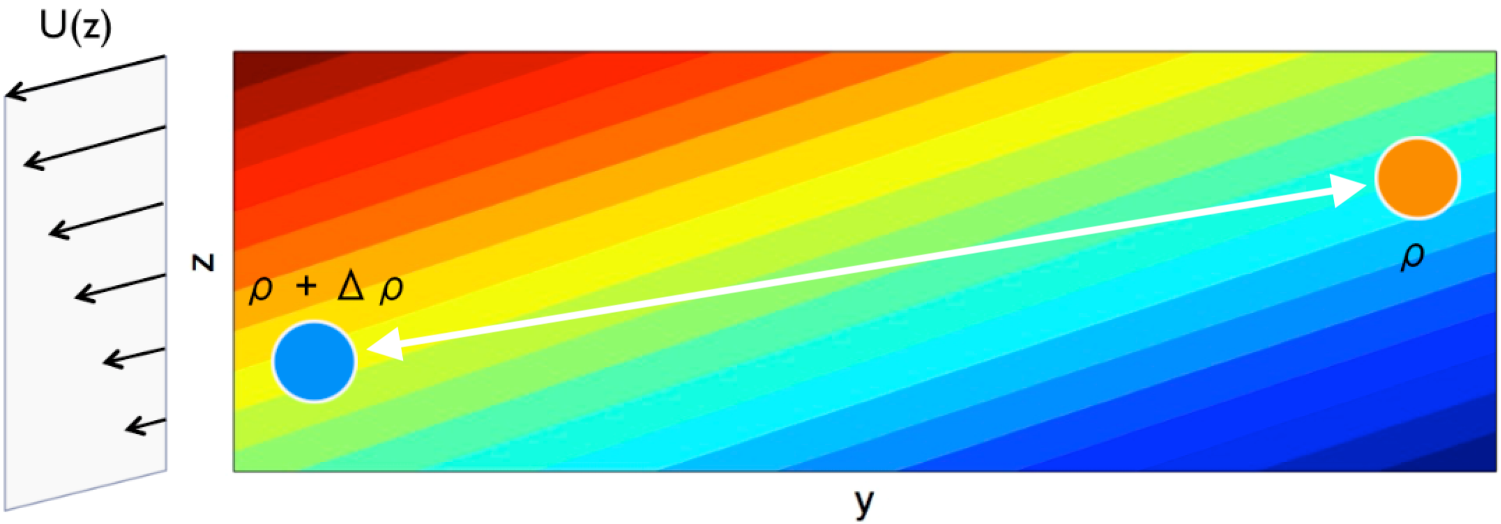
- Rest state with sloping isopycnals: $\overline{\rho}(y,z) = \overline{\rho}_y \ y + \overline{\rho_z} \ z$
- Equilibrium state in thermal wind balance: $f_0 U_z = g/\rho_0 \ \overline{\rho}_y$

Available potential energy



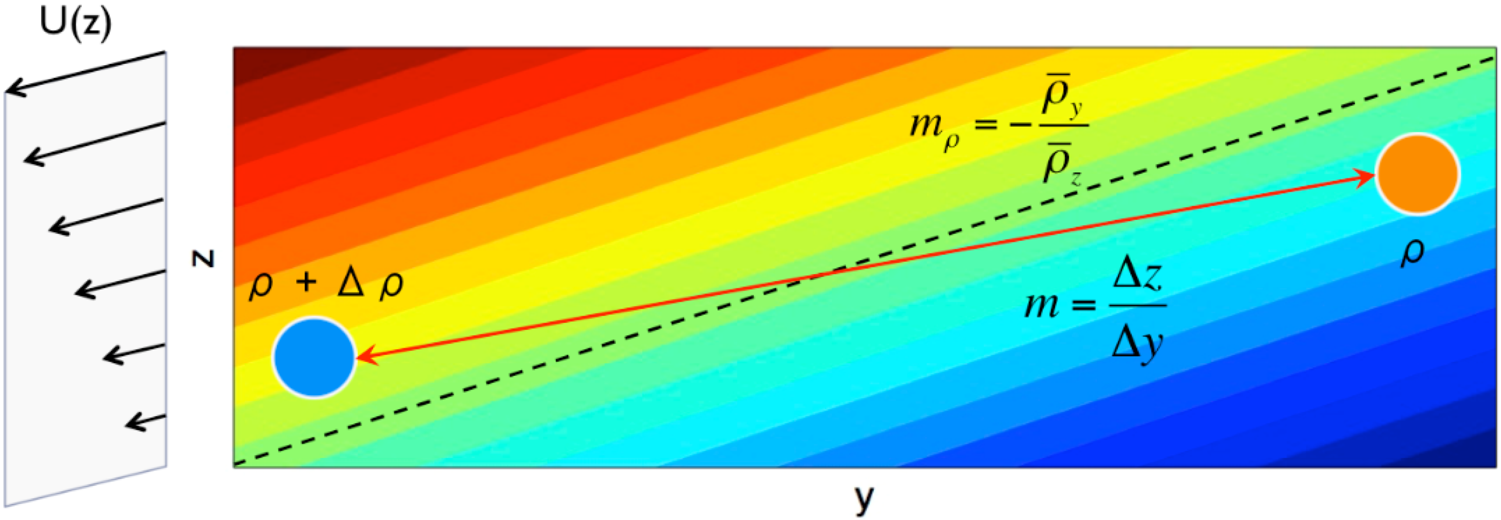
- Rest state with sloping isopycnals: $\overline{\rho}(y,z) = \overline{\rho}_y\ y + \overline{\rho_z}\ z$
- Equilibrium state in thermal wind balance: $f_0 U_z = g/\rho_0\ \overline{\rho}_y$
- Density difference: $\Delta\rho = \overline{\rho}_y\ \Delta y + \overline{\rho}_z\ \Delta z$

Available potential energy



- Rest state with sloping isopycnals: $\overline{\rho}(y,z) = \overline{\rho}_y\ y + \overline{\rho_z}\ z$
- Equilibrium state in thermal wind balance: $f_0 U_z = g/\rho_0\ \overline{\rho}_y$
- Density difference: $\Delta\rho = \overline{\rho}_y\ \Delta y + \overline{\rho}_z\ \Delta z$

Available potential energy

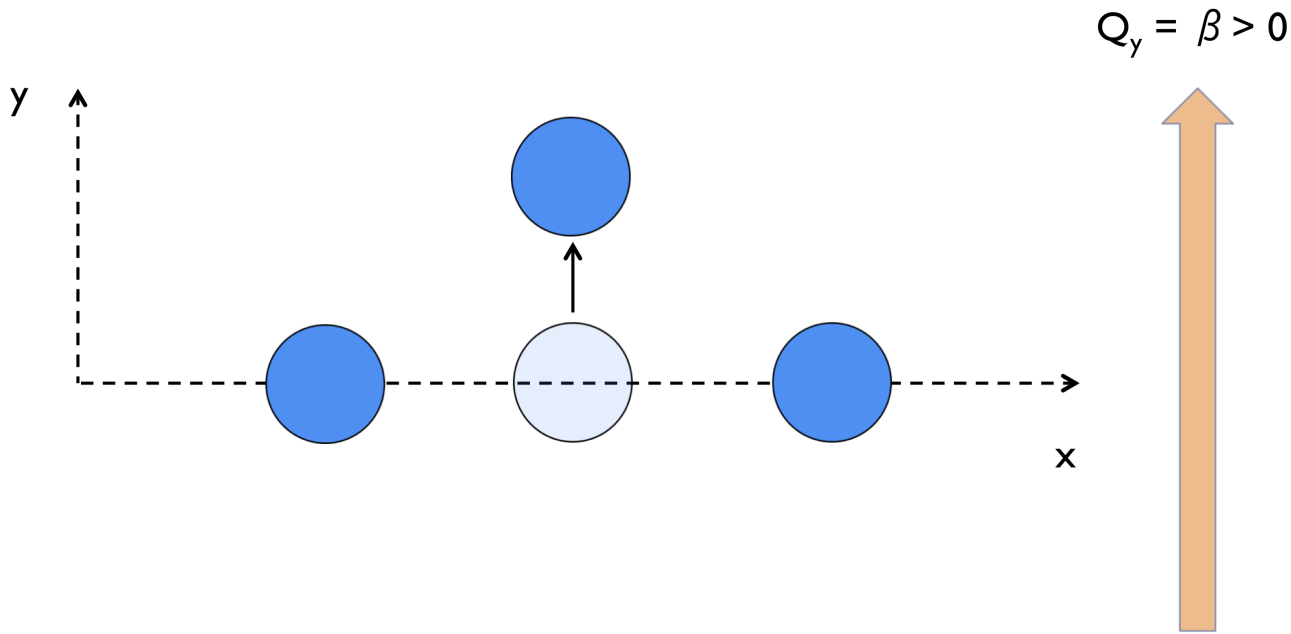


- Rest state with sloping isopycnals: $\overline{\rho}(y,z) = \overline{\rho}_y\ y + \overline{\rho_z}\ z$
- Equilibrium state in thermal wind balance: $f_0 U_z = g/\rho_0\ \overline{\rho}_y$
- Density difference: $\Delta\rho = \overline{\rho}_y\ \Delta y + \overline{\rho}_z\ \Delta z$
- PE difference:

$$\Delta PE = -g\ \Delta\rho\ \Delta z = -g\overline{\rho}_z\ \Delta z\ \Delta y\ (m - m\rho)$$

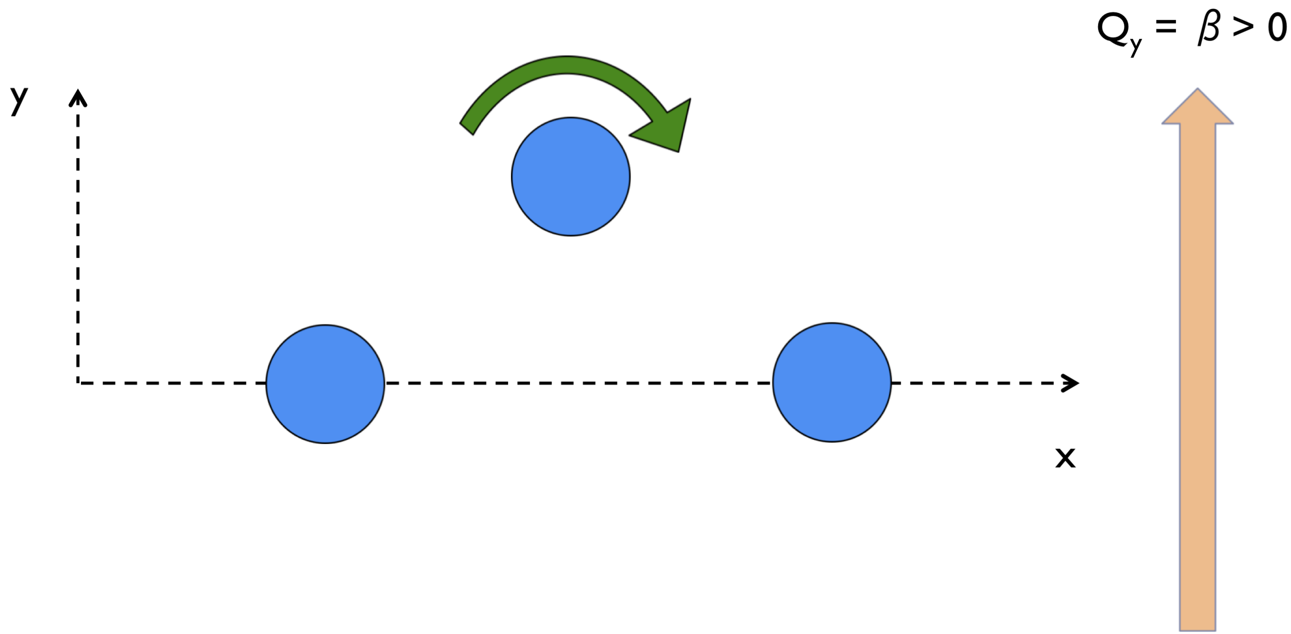
- If $m < m_\rho$ then total PE decreases and KE increases

Mechanism: Barotropic Rossby wave



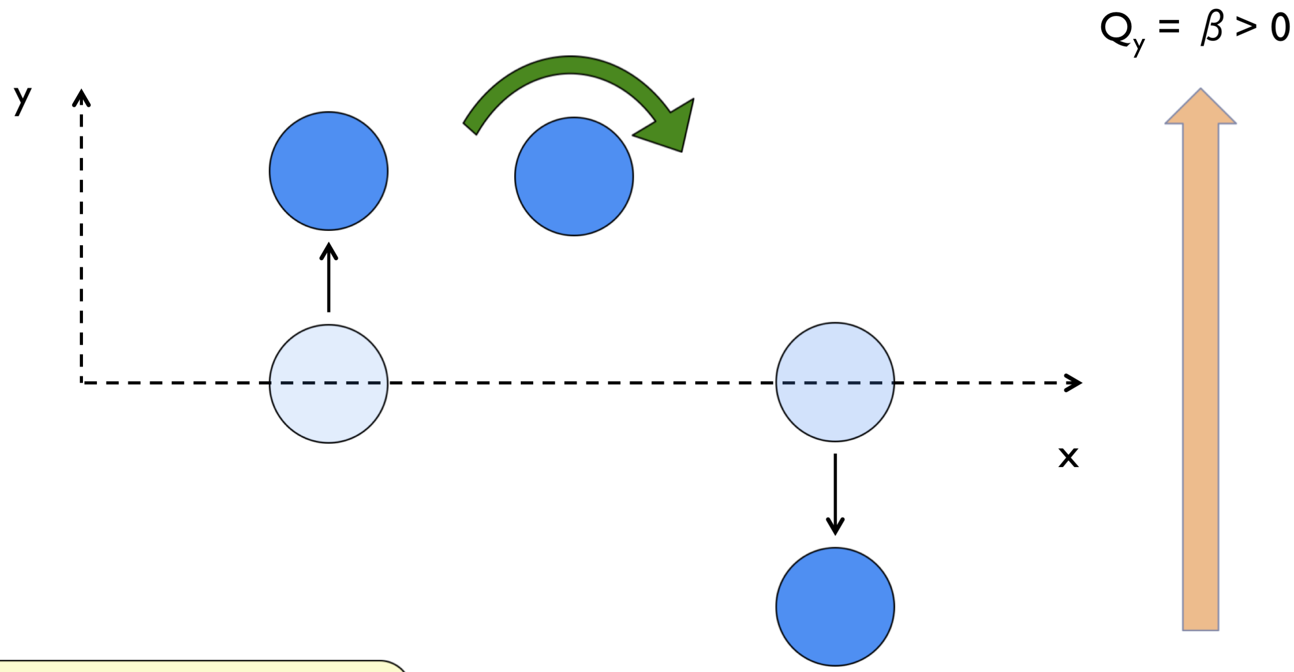
1. Perturb a fluid parcel northward into region of higher planetary vorticity

Mechanism: Barotropic Rossby wave



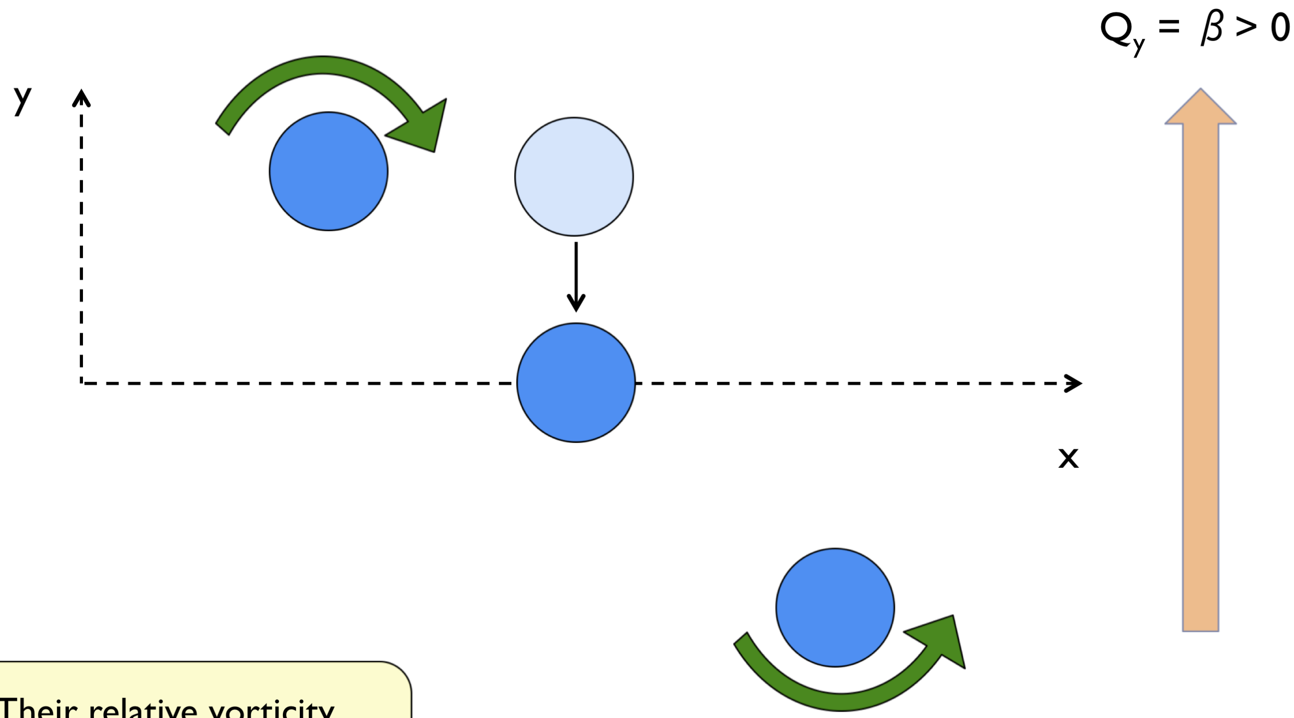
2. To conserve total PV, the parcel develops negative relative vorticity

Mechanism: Barotropic Rossby wave



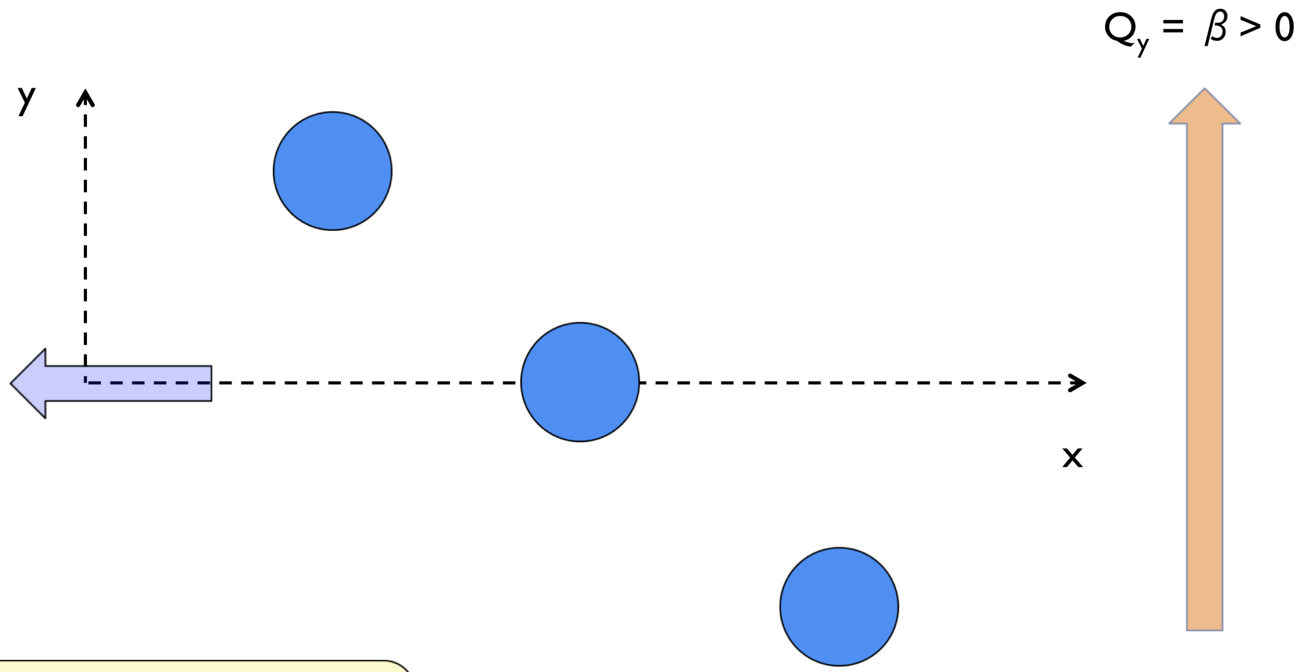
3. This relative vorticity perturbs neighboring fluid parcel north and south

Mechanism: Barotropic Rossby wave



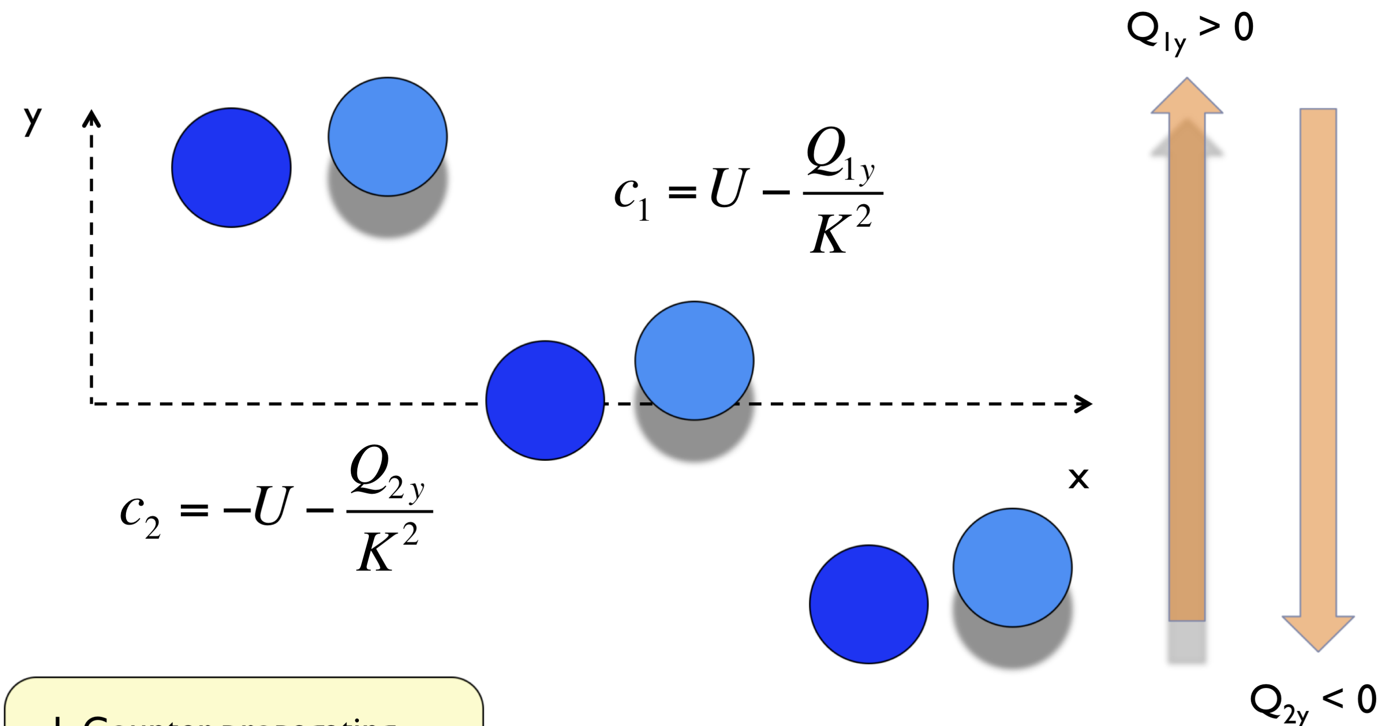
4. Their relative vorticity restores the first parcel to its original position

Mechanism: Barotropic Rossby wave



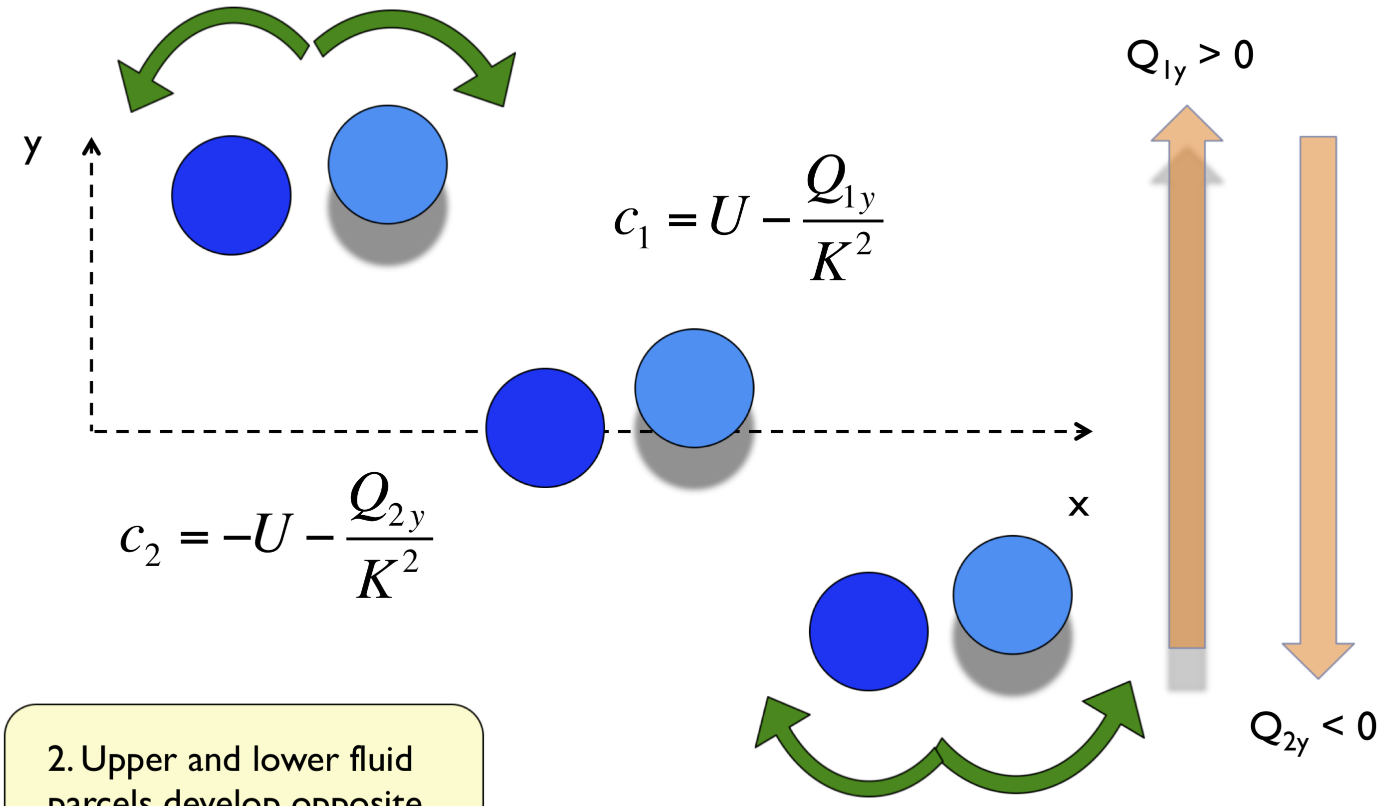
5. The perturbation propagates westward as a Rossby wave

Mechanism: Baroclinic instability



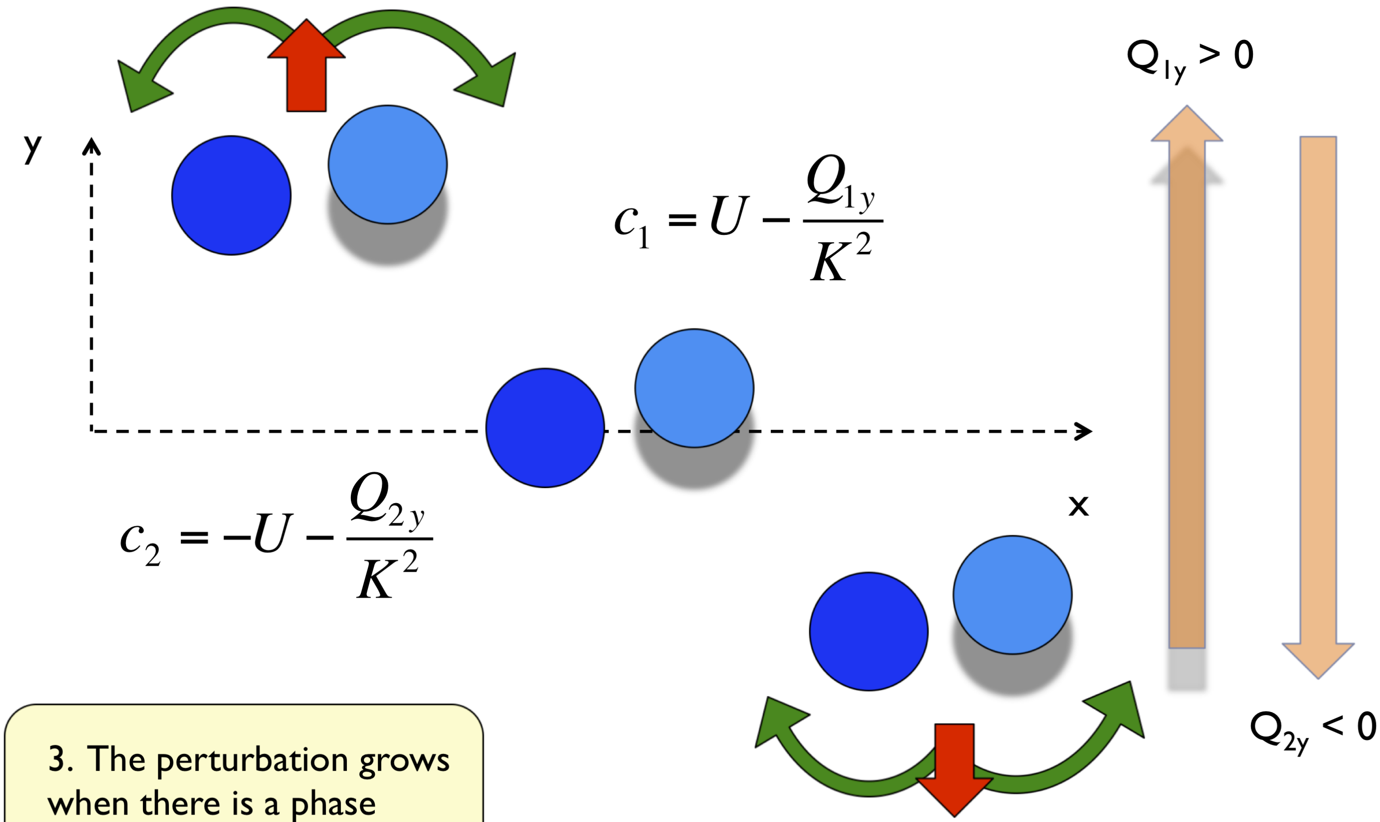
1. Counter-propagating Rossby waves with equal phase speed

Mechanism: Baroclinic instability

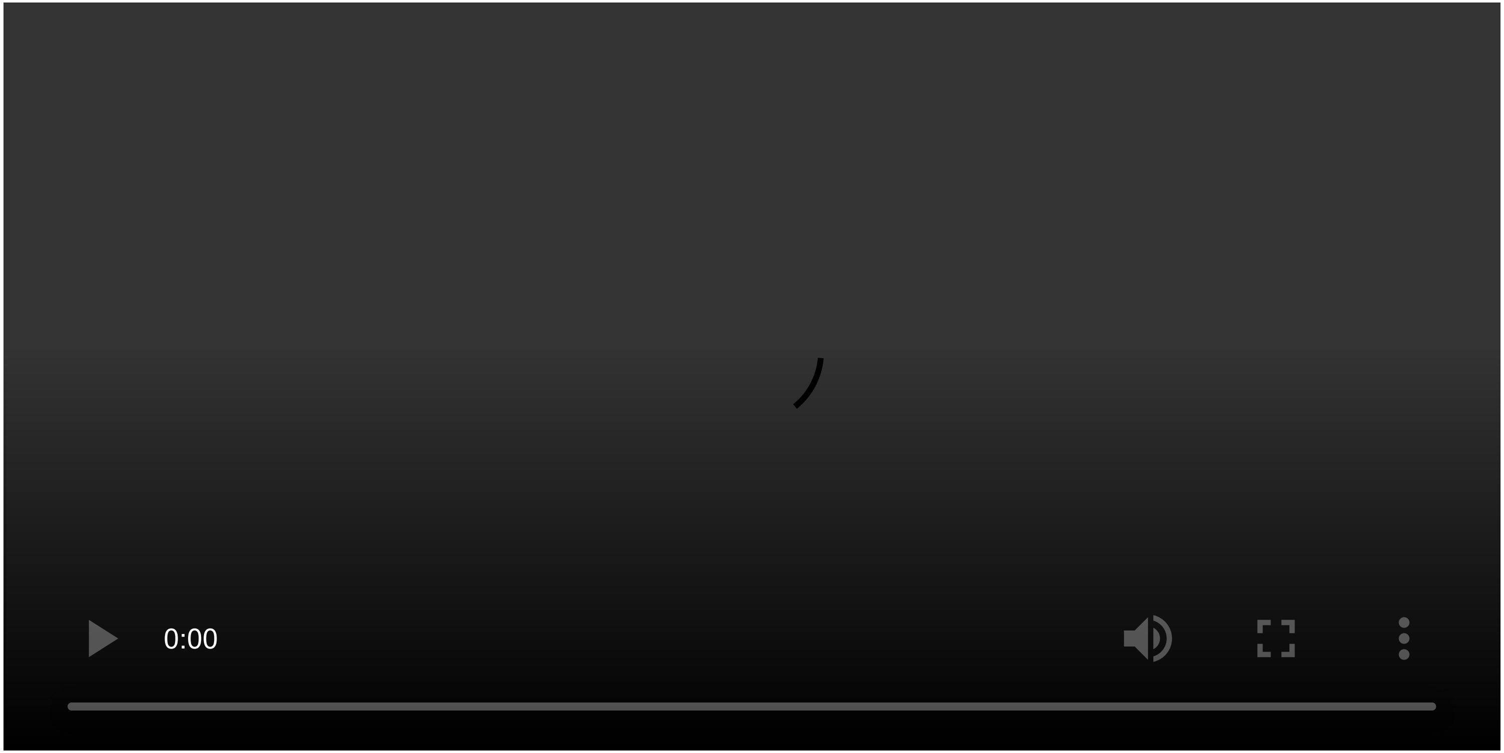


2. Upper and lower fluid parcels develop opposite relative vorticities

Mechanism: Baroclinic instability



3. The perturbation grows when there is a phase difference of π/2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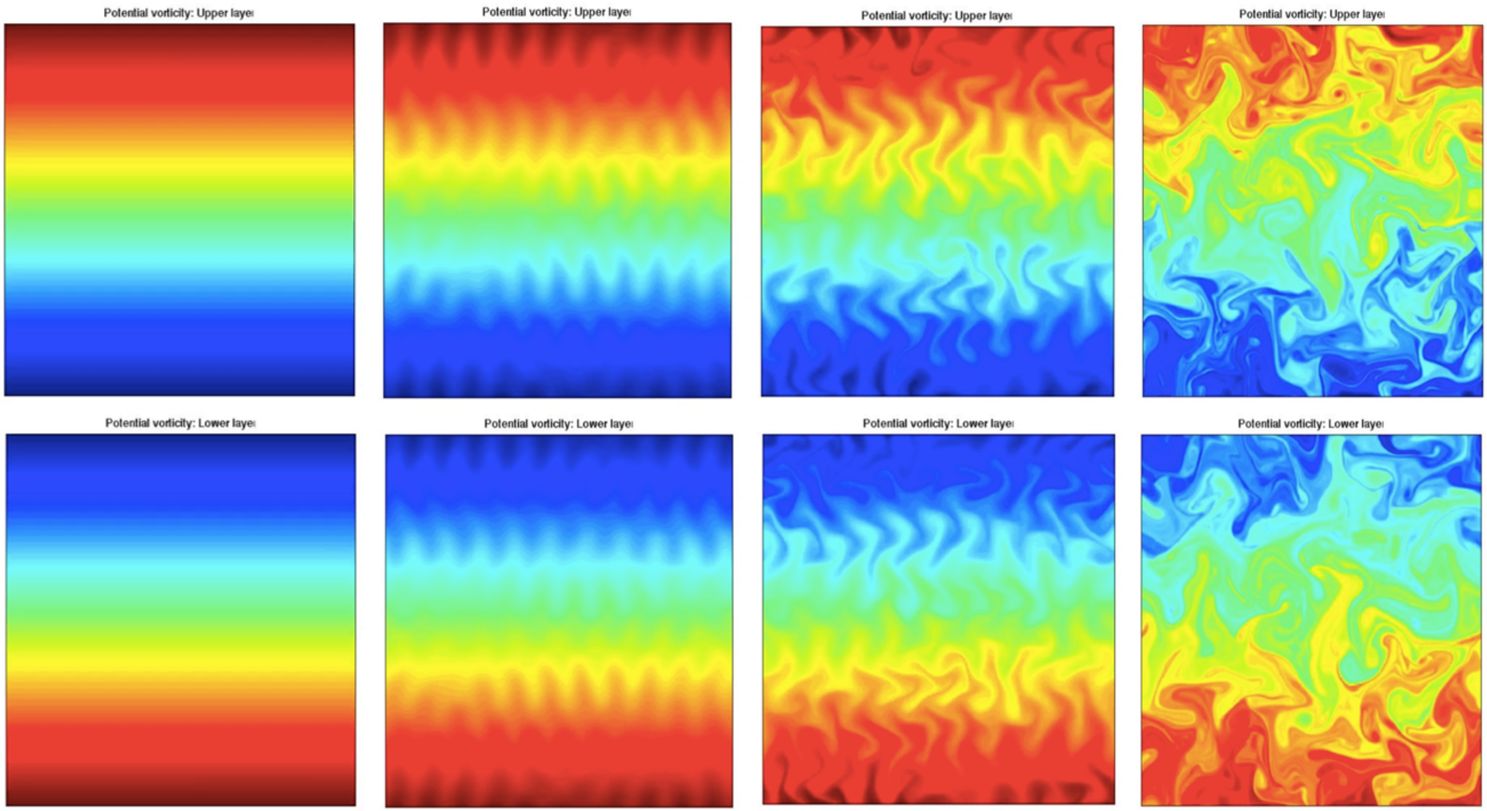
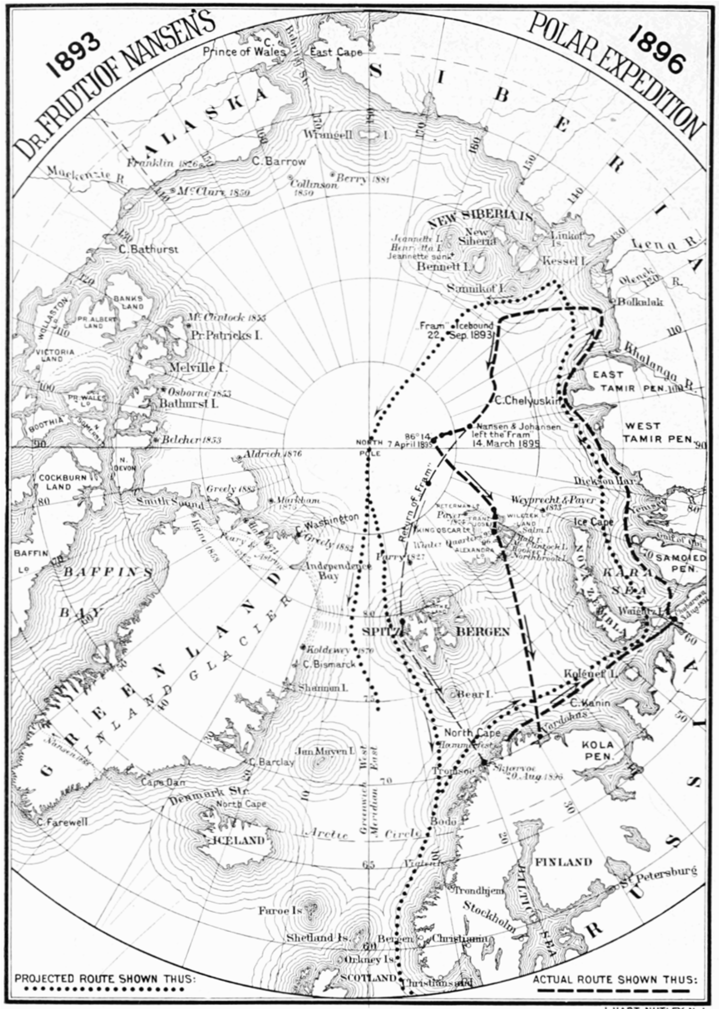
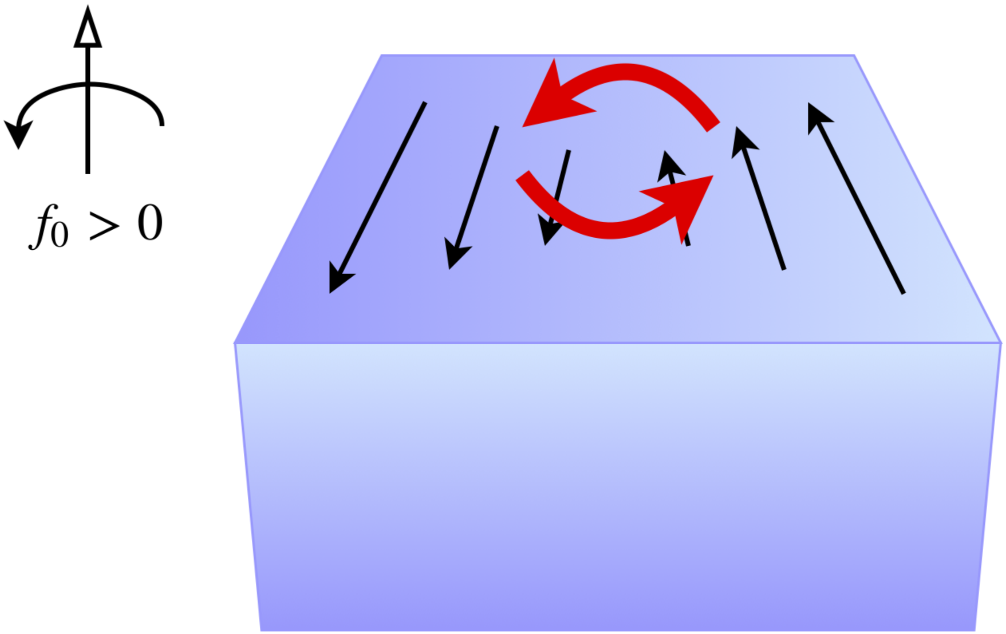

Potential vorticity: Upper layer
Potential vorticity: Upper layer
Potential vorticity: Upper layer
Potential vorticity: Upper layer
Potential vorticity: Lower layer
Potential vorticity: Lower layer
Potential vorticity: Lower layer
Potential vorticity: Lower layer

5.2 The Ekman layer

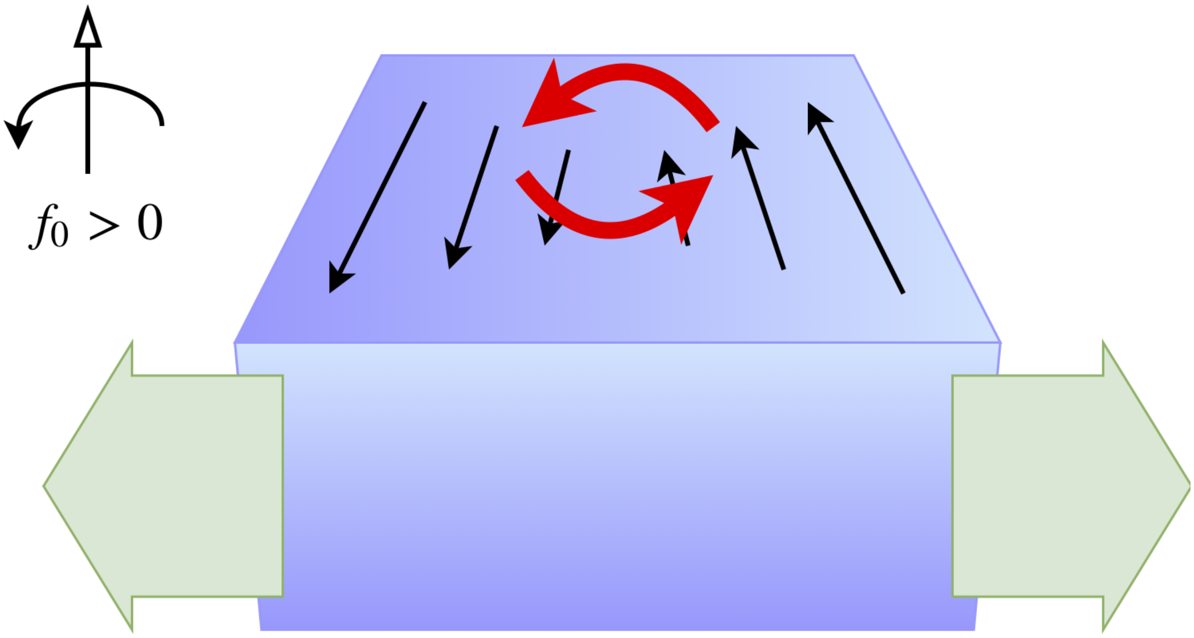
"On studying the observations of wind and ice-drift taken during the drift of the *Fram*, Fridtjof Nansen found that the drift produced by a given wind did not, according to the general opinion, follow the wind's direction but deviated 20°-40° to the right."

Vagn Walfrid Ekman, *On the influence of the Earth's rotation on ocean currents* (1905).

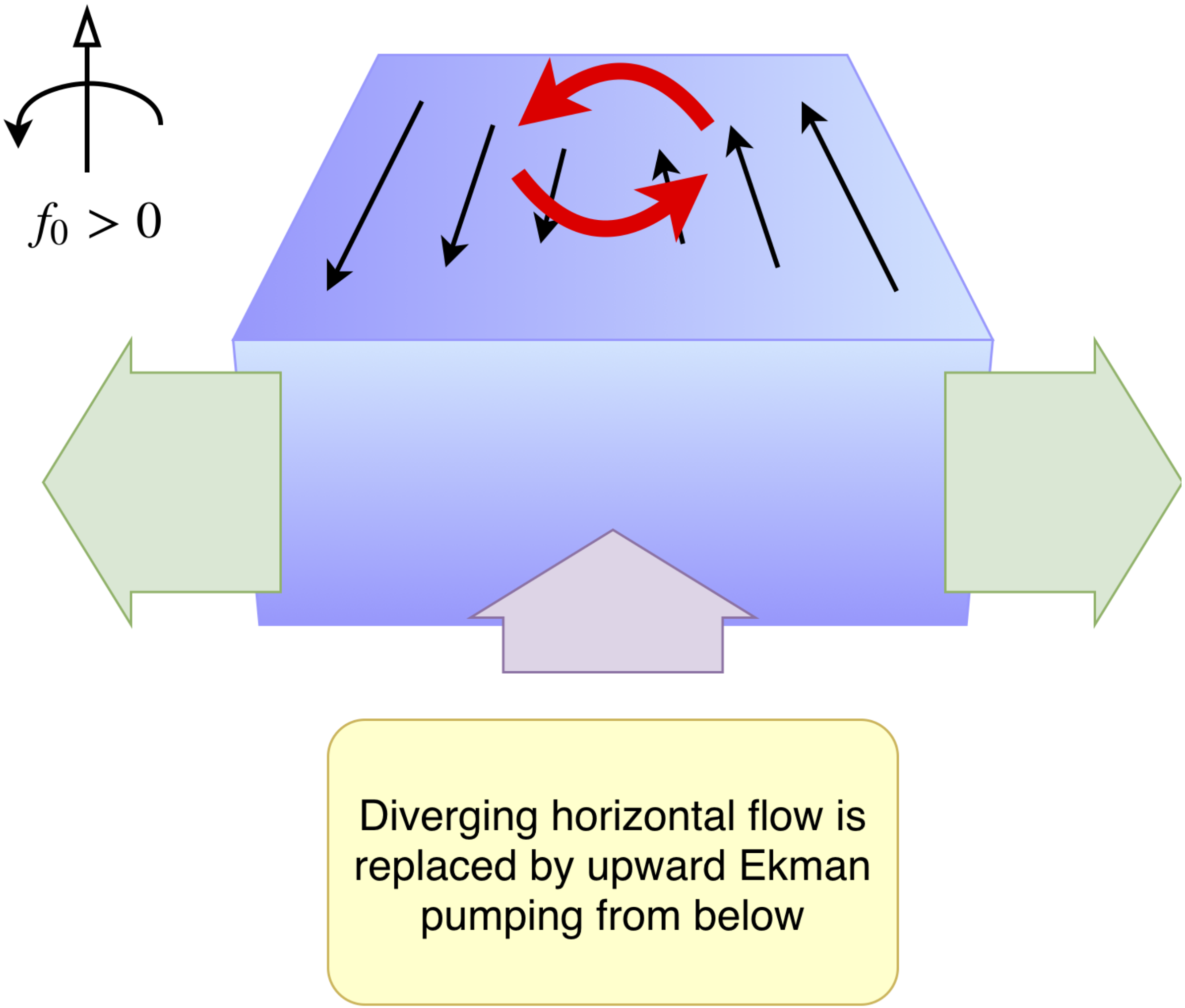

1893 Dr. FRIDTJOF NANSEN'S POLAR EXPEDITION 1896
PROJECTED ROUTE SHOWN THUS:
ACTUAL ROUTE SHOWN THUS:
J. HART, NUTLEY N.J.



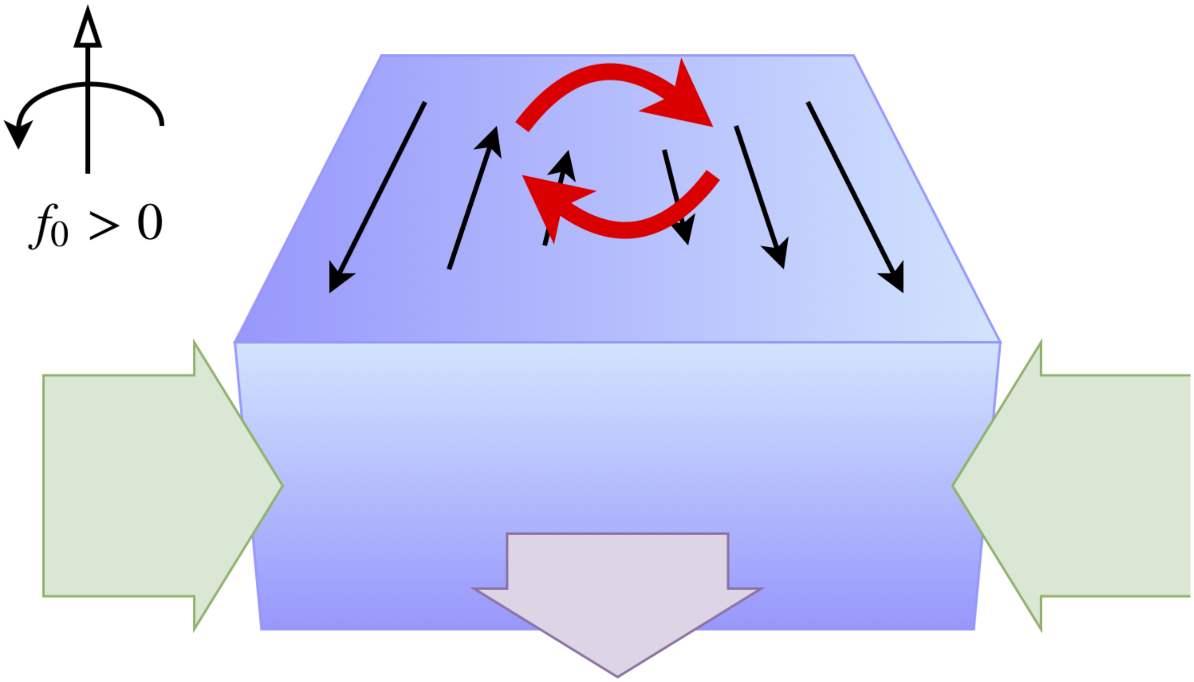
Wind stress at the surface has a positive curl.



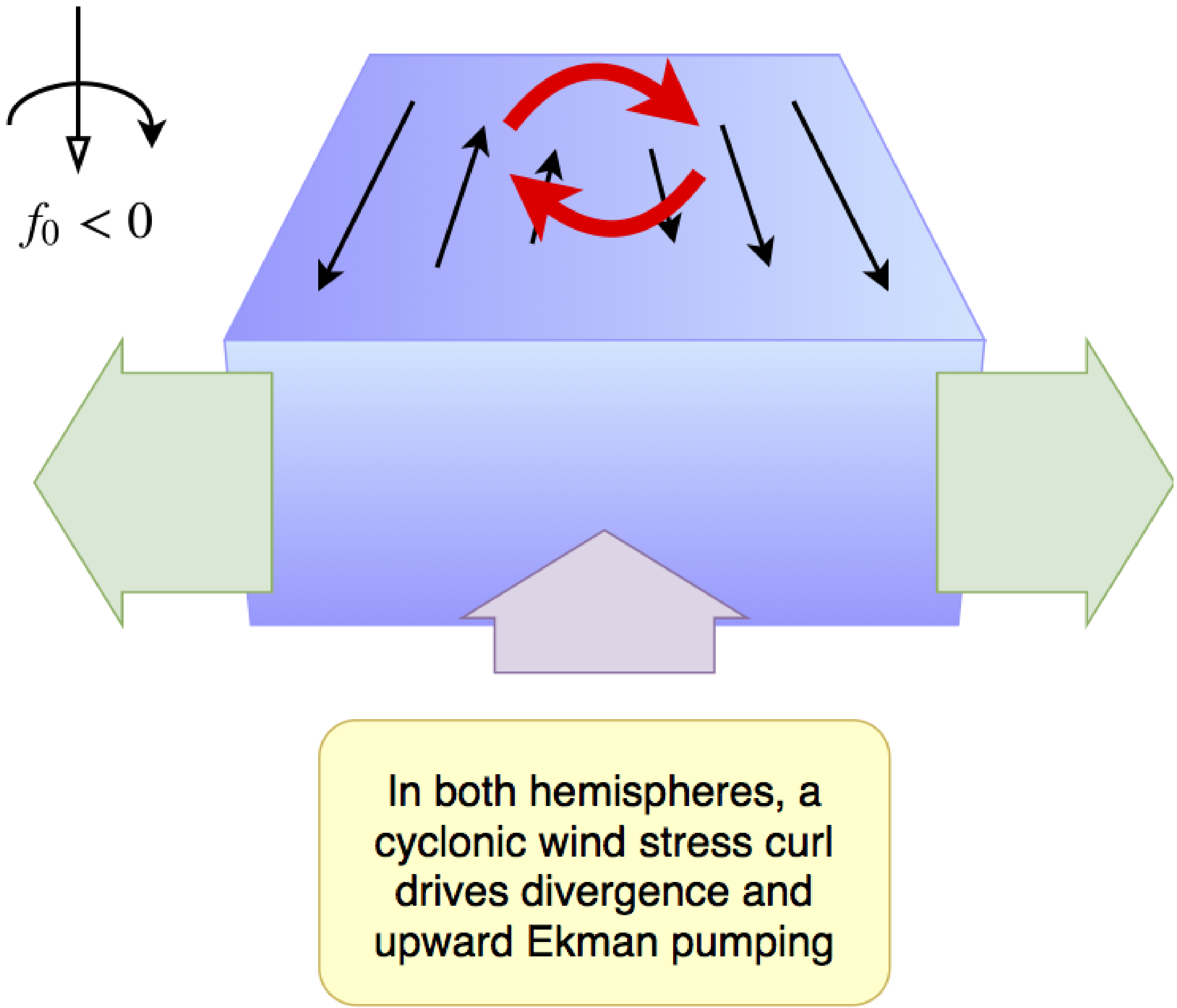
In the northern hemisphere, a positive wind-stress curl drives divergent Ekman flow

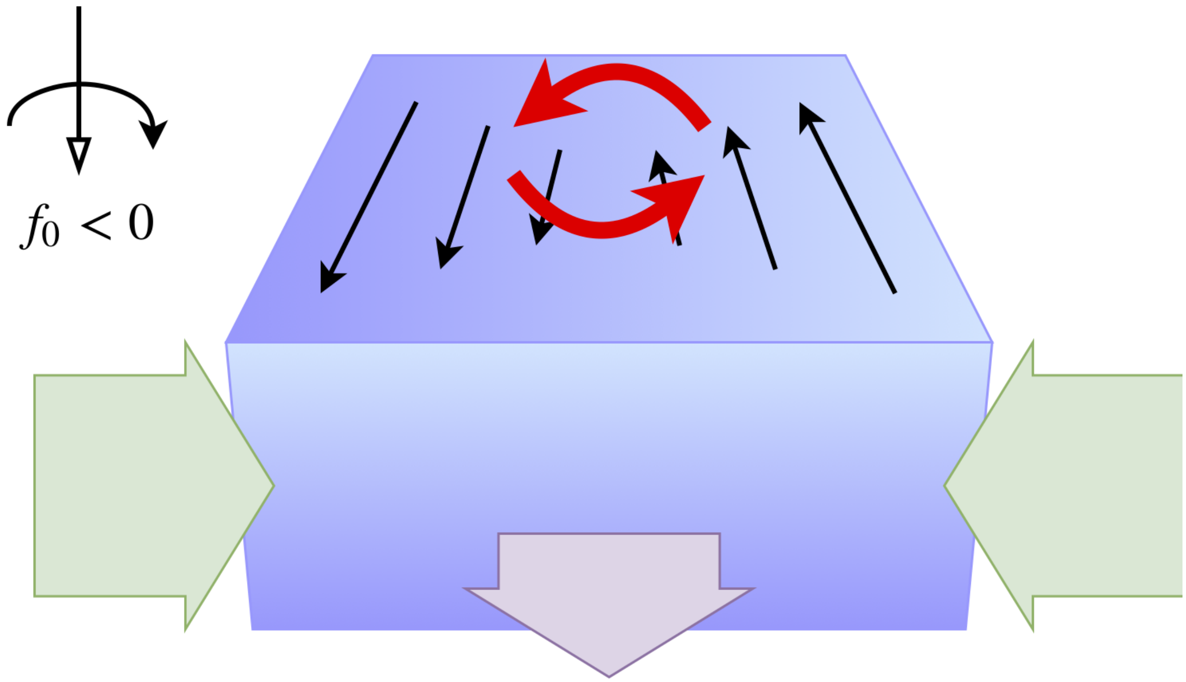


$f_0 > 0$
Diverging horizontal flow is replaced by upward Ekman pumping from below



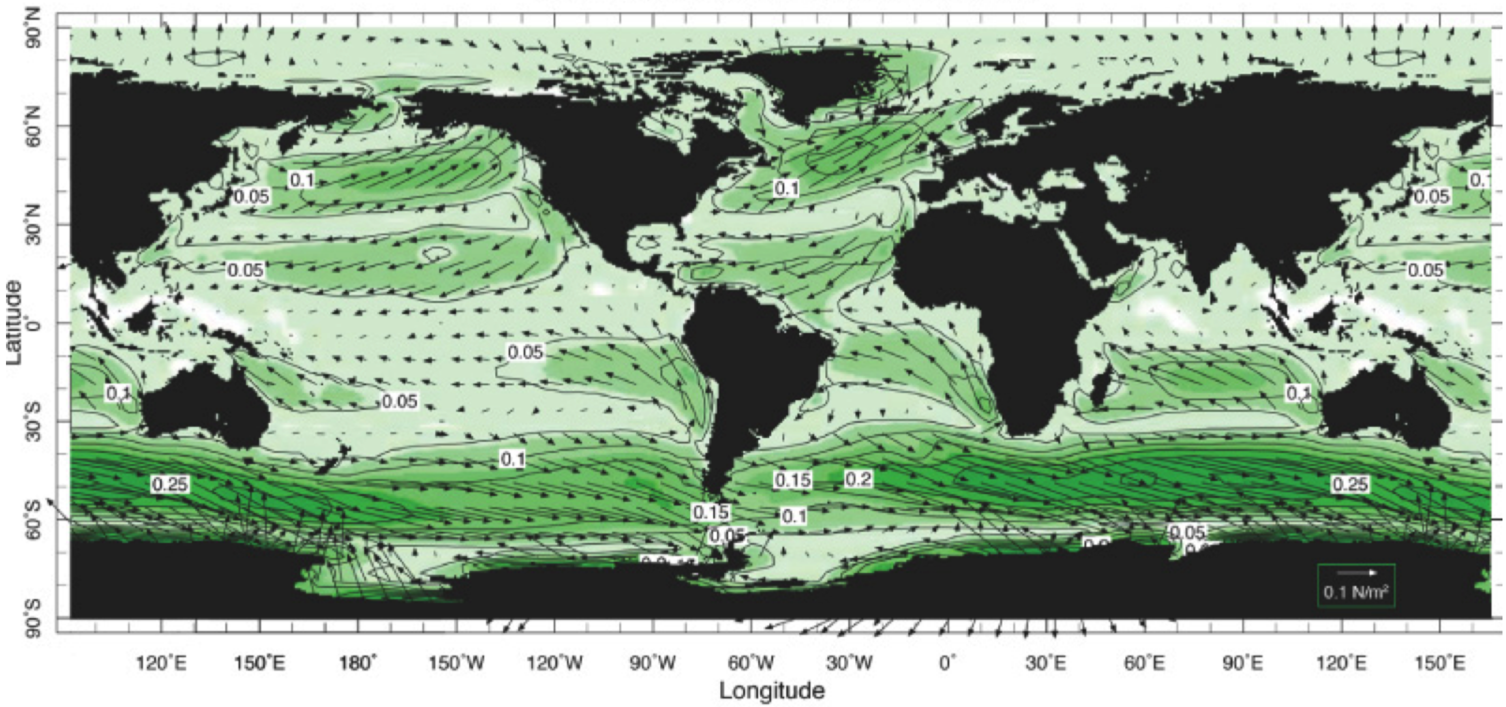
A negative wind-stress curl in the northern hemisphere drives convergent flow and downward Ekman pumping


$f_0 < 0$
In both hemispheres, a cyclonic wind stress curl drives divergence and upward Ekman pumping

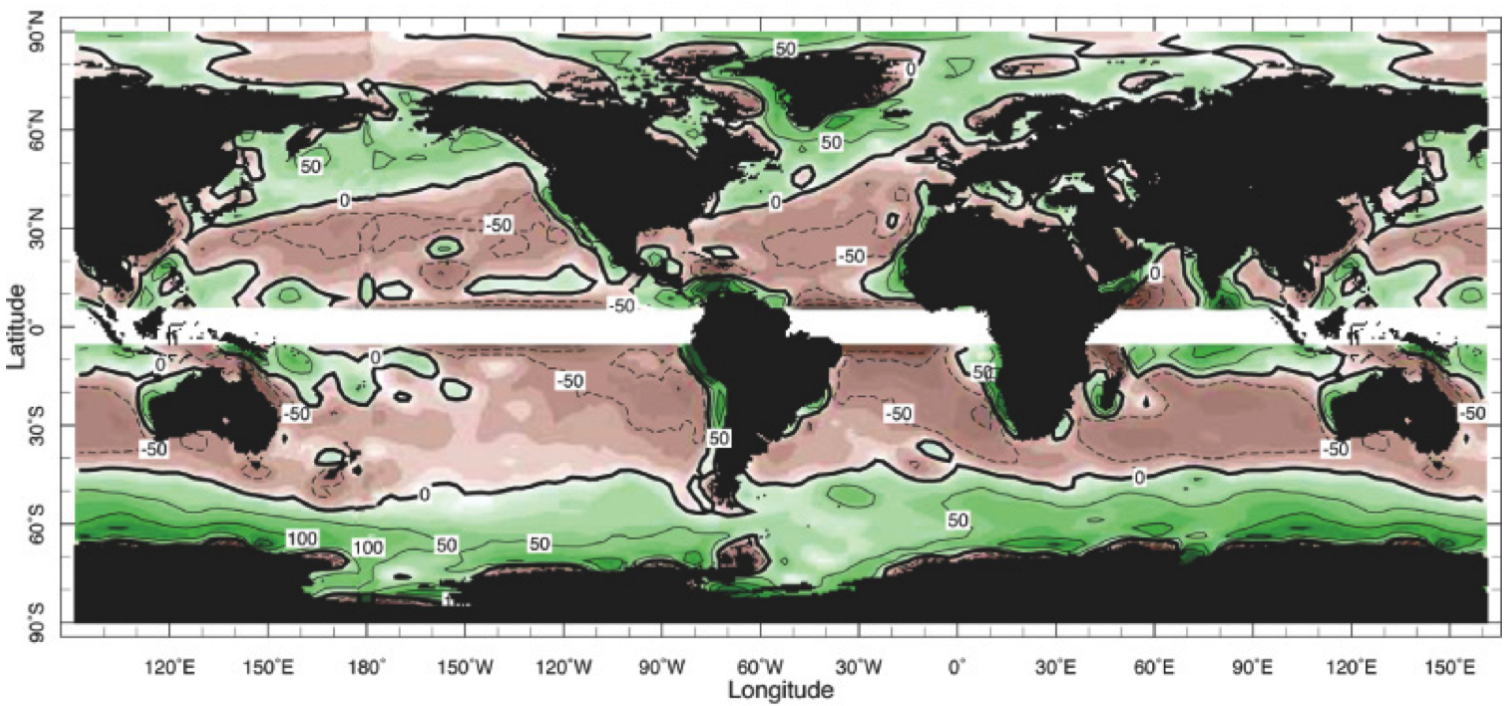


And an anticyclonic wind-stress curl drives convergence and downward Ekman pumping.

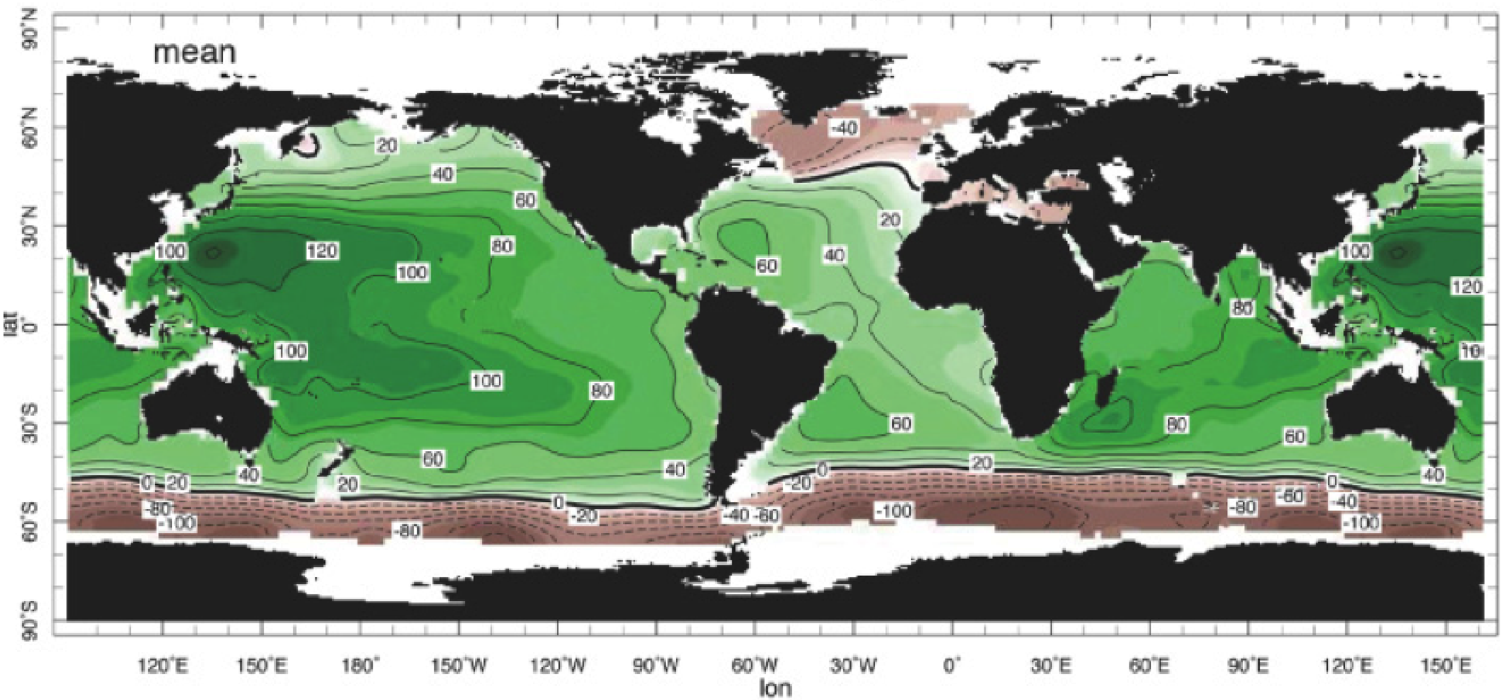
Wind stress (N/m^2)



Ekman pumping (m/year)



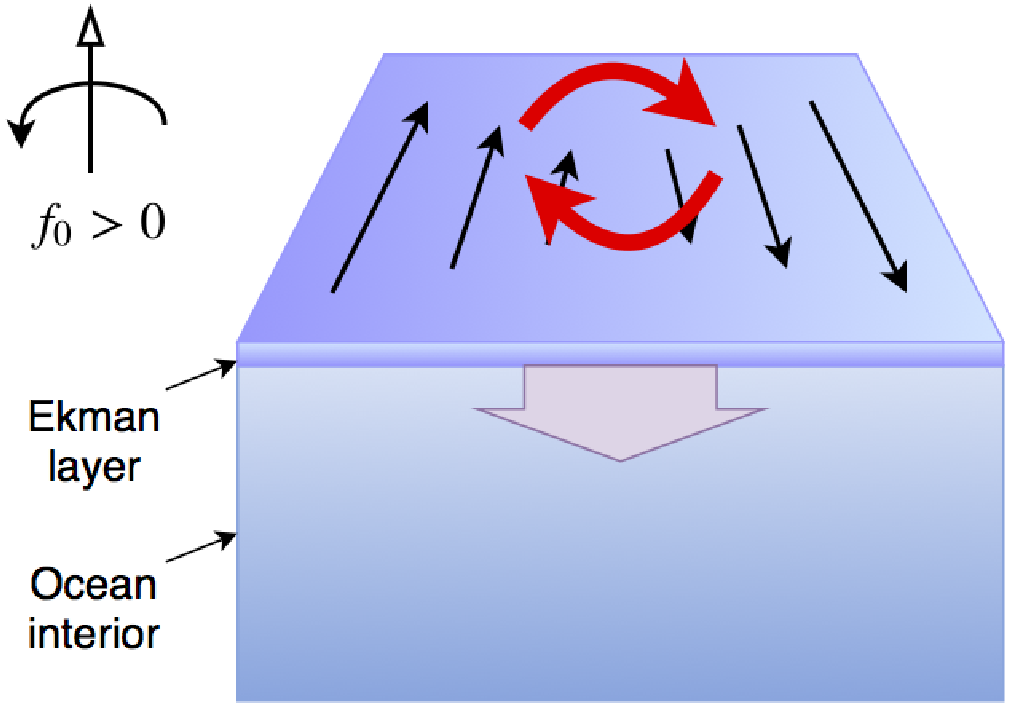
Mean sea-surface height (cm)



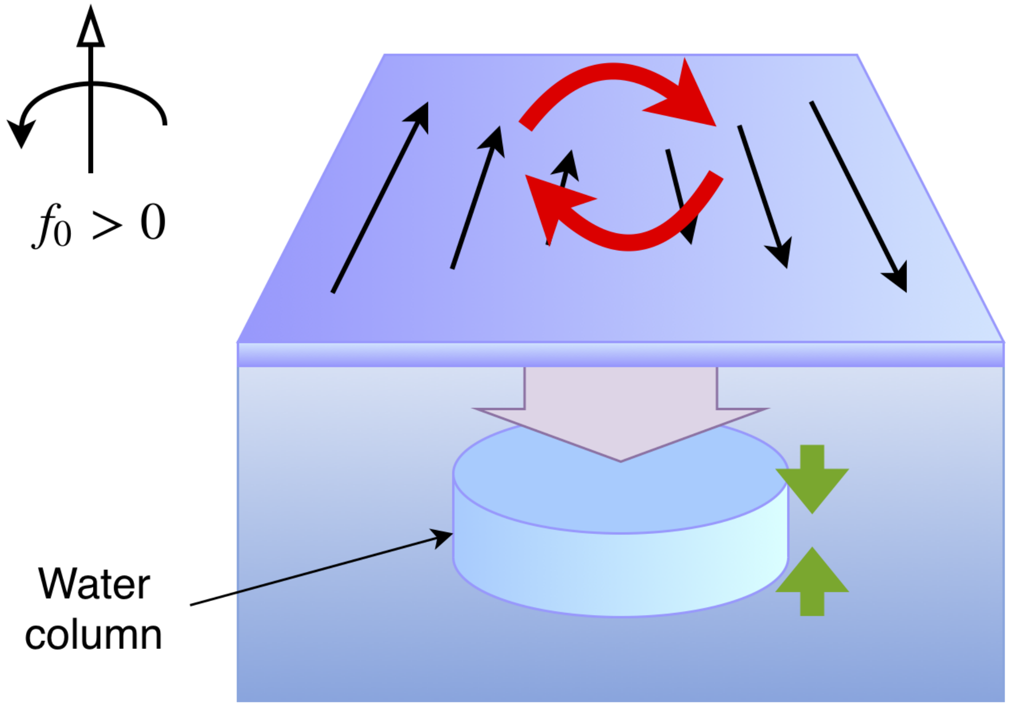
5.3 The wind-driven circulation

"*Definitive* papers are usually written when a subject is no longer interesting."

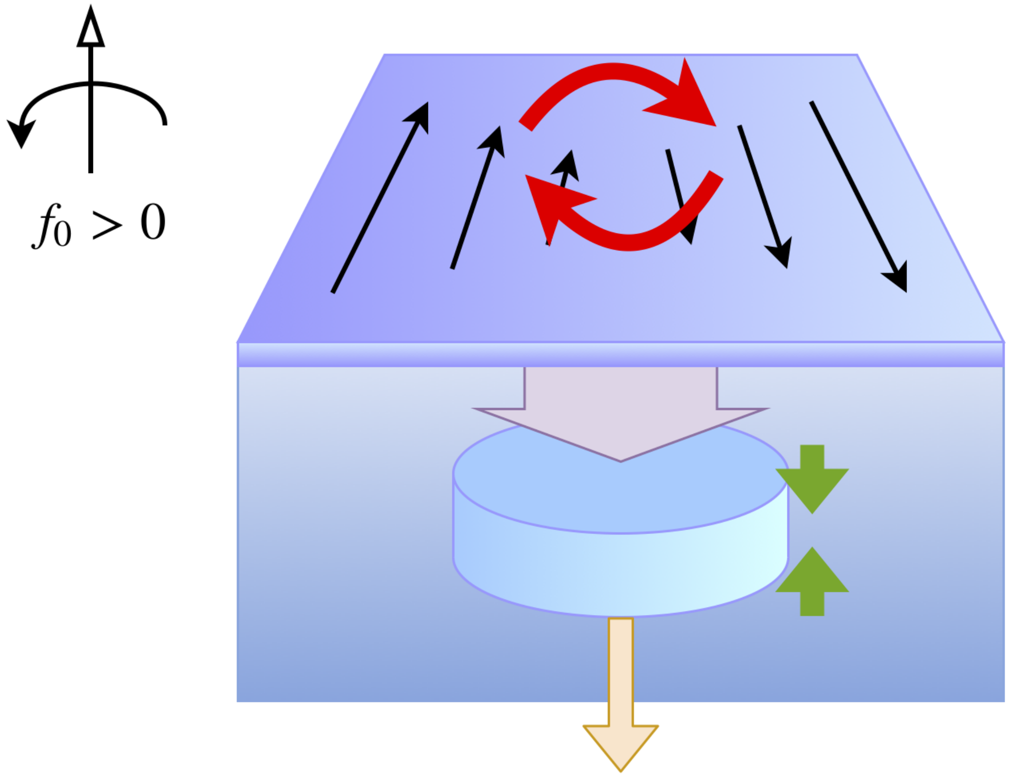
Walter Munk (1917-present)



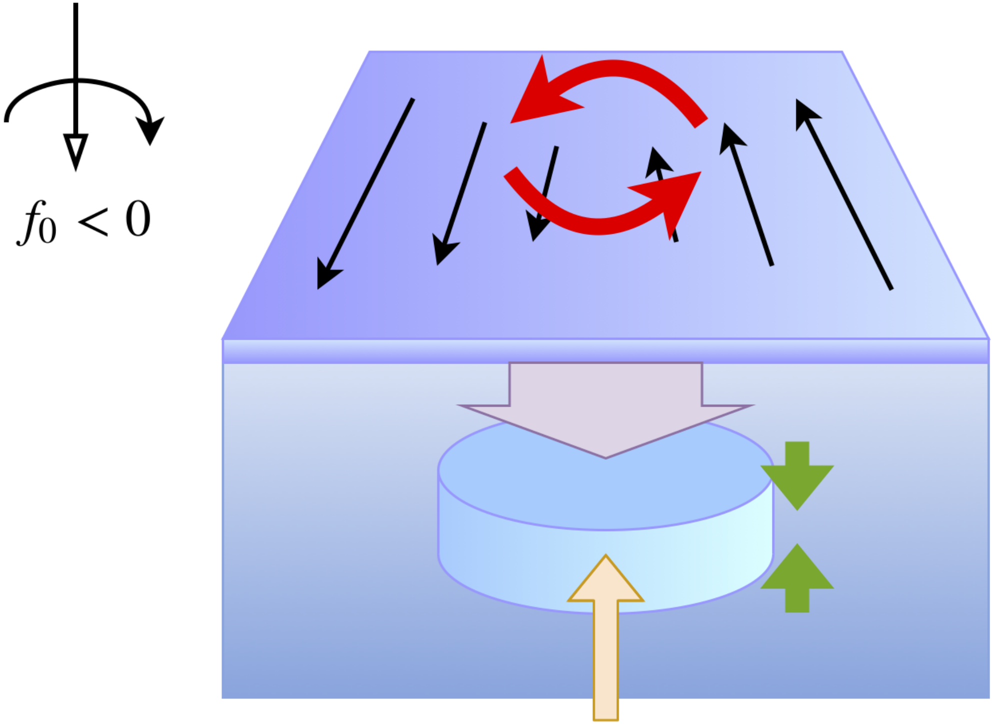
In both hemispheres, an anticyclonic wind-stress curl drives downward Ekman pumping.



Ekman pumping from above squashes water columns in the ocean interior.

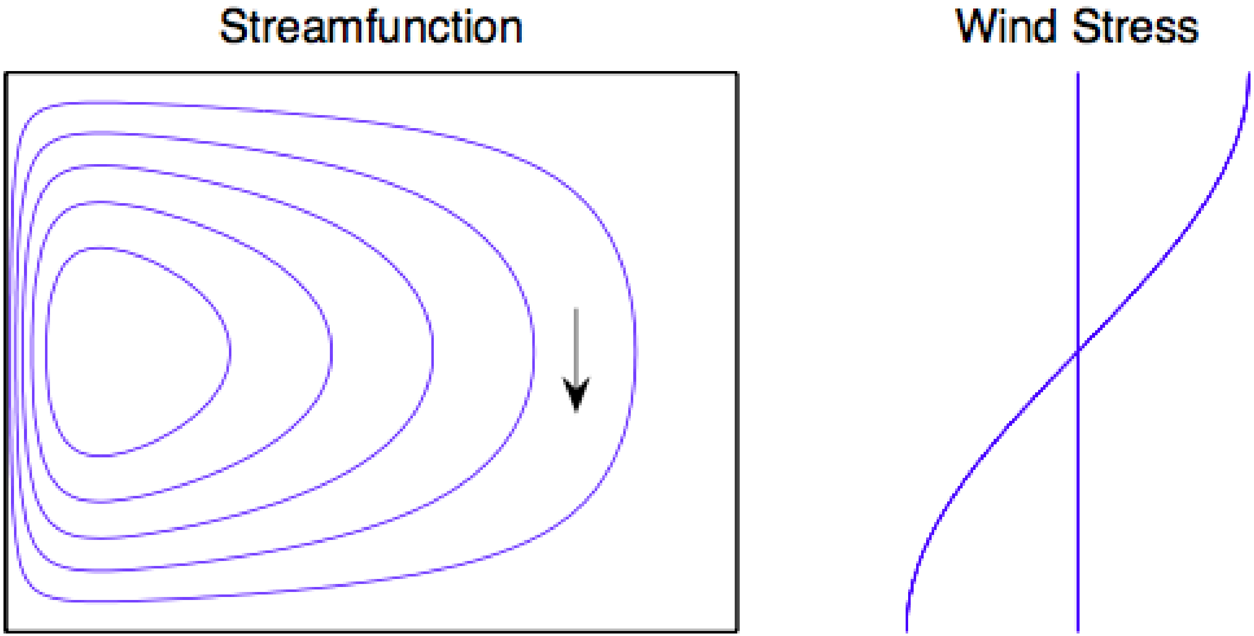


To conserve potential vorticity, the water column must move towards the equator (south in the NH).

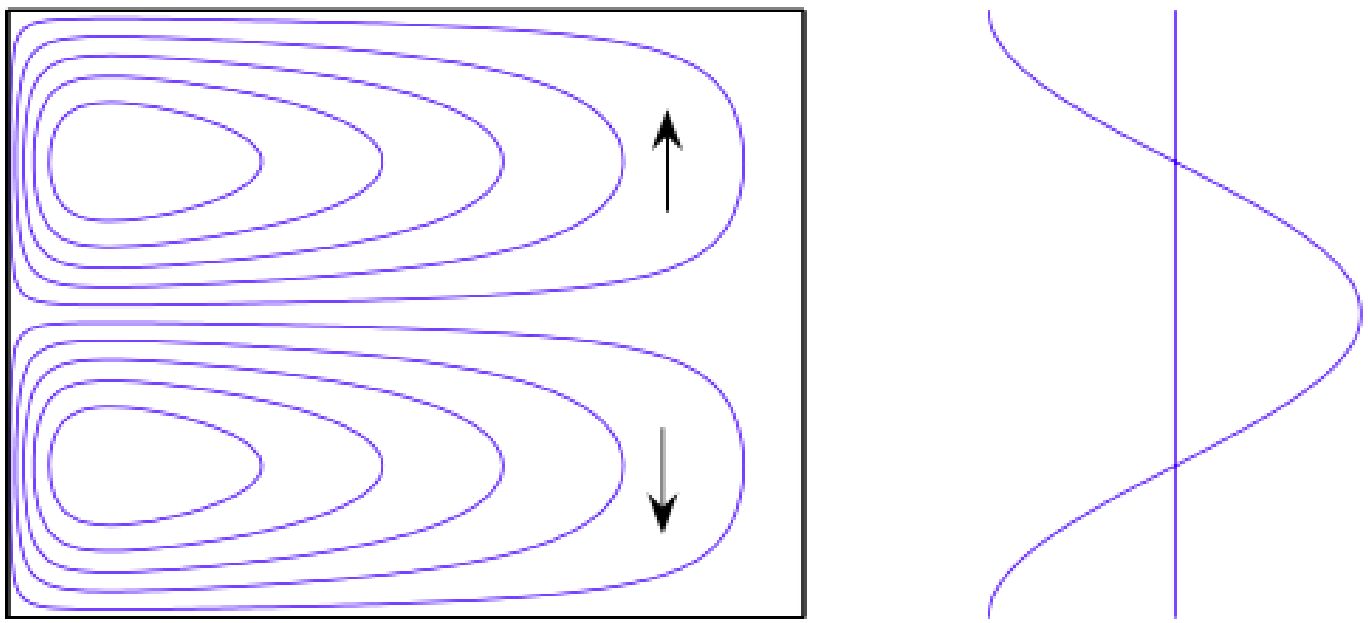


In the SH, anticyclonic wind-stress curl causes water columns to be squashed and move northward.

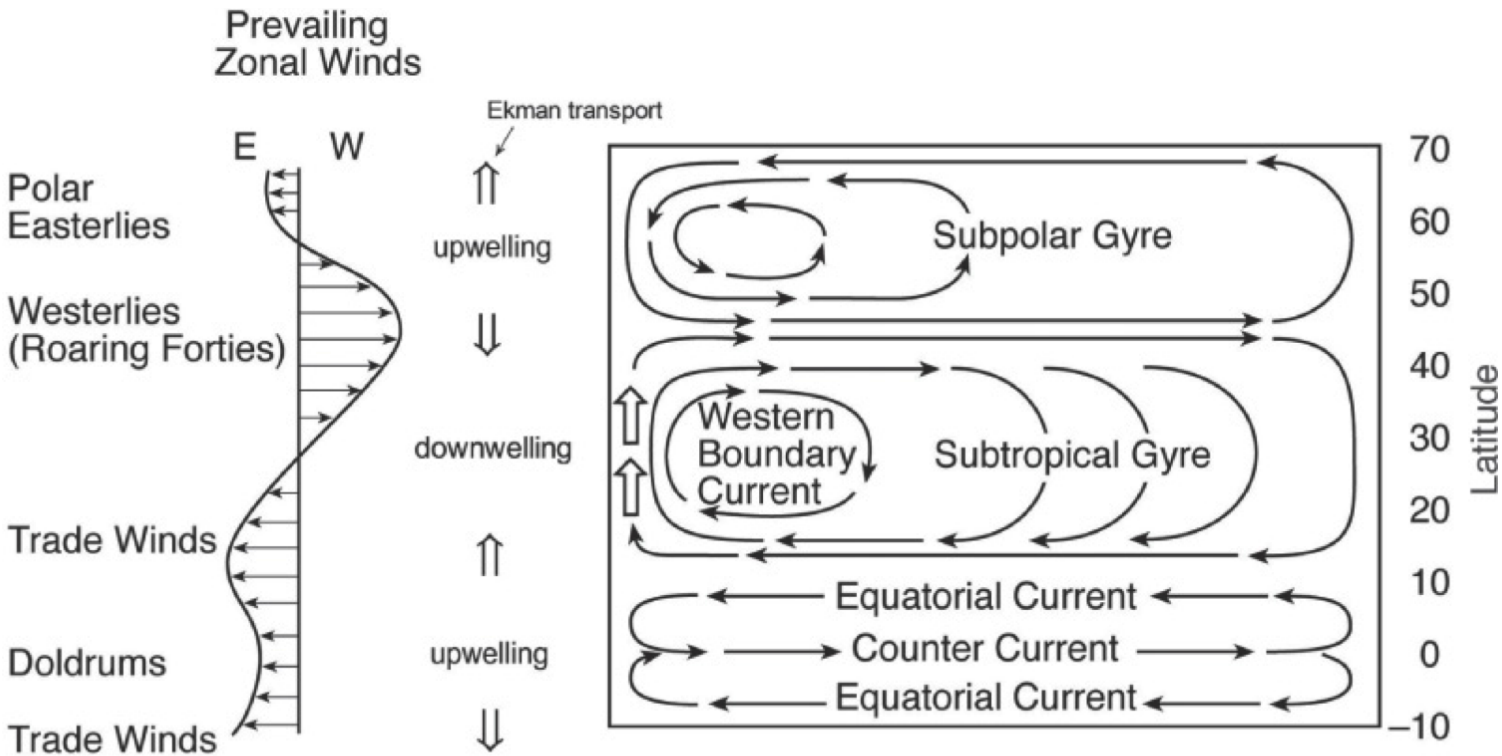
Stommel model (single gyre)



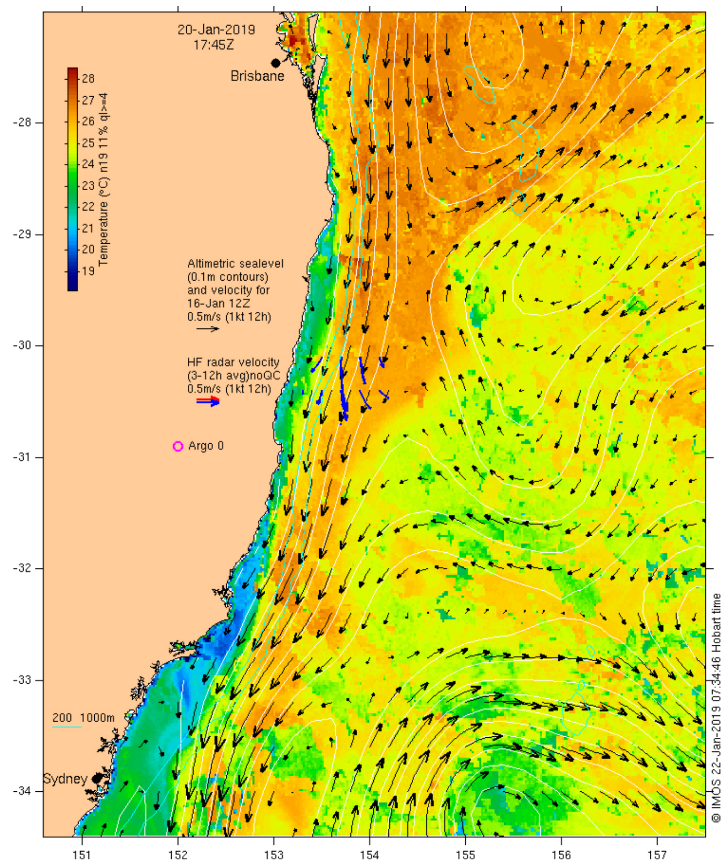
Stommel model (double gyre)

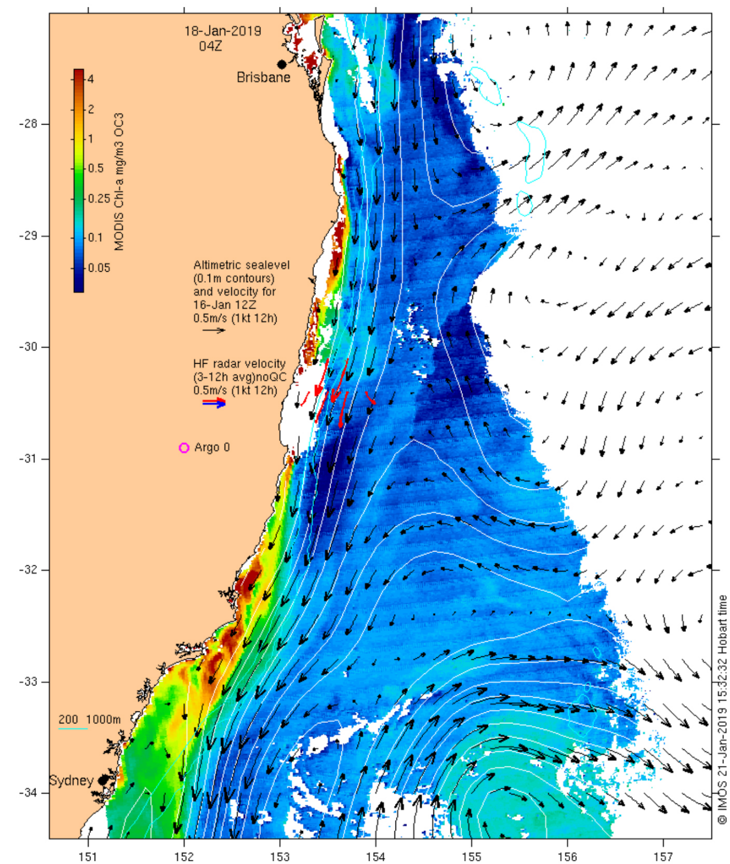


Stommel model (zonal winds)



The East Australian Current





{Integrated Marine Observing System}

Conclusion

"We can count ourselves fortunate to live in a society and at a time when we are actually paid to explore the universe."

Henry Stommel (1920-1992)

Some links:

- {My research website}
- {Mathematics for Planet Earth}
- {Climate Change Research Centre}
- {Honours at UNSW Maths & Stats}
- {PhD/MSc at UNSW Maths & Stats}