Myopia in dynamic spatial games*

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Abstract

We design an experiment to evaluate behavior in a dynamic spatial game representing the incentives faced by drivers for a ride-hailing service while waiting to be matched with a rider. The design is unique in that it allows us to observe not only participants' choices, but also the considerations that went into those choices. The results of the experiment show that a large majority of player choices are consistent with myopic best responding. A myopic best response maximizes a player's flow payoff at the time of the decision but is not necessarily optimal as it ignores strategic considerations regarding the future choices of opponents. Given the observed prevalence of this behavior and the challenges of equilibrium analysis, which we detail, we argue in favor of computational models of spatial competition built upon myopic agents. Myopic behavior in our model results in quite efficient outcomes, suggesting that ride-hailing companies may benefit from sharing with drivers the locations of other nearby drivers to allow them to compete spatially.

1 Introduction

When Uber drivers are waiting for ride requests, they often sign out of the driver app and into the passenger app. They do this because they want to see where nearby Uber drivers are, and Uber shows nearby drivers only to passengers, not drivers. If in the passenger app an Uber driver finds that she is surrounded by other drivers in near proximity, she knows she may have to wait a while for a request or move to a different location—when passengers

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request rides on Uber, they are matched, roughly speaking, with the nearest available driver,[1] and therefore a driver's catchment region is small when she is surrounded by others.

An efficient spatial allocation of idle Uber drivers would minimize the expected wait time for passengers, which we assume proportional to the expected distance to nearest driver. Supposing passengers are uniformly distributed over a unit disk and drivers travel as the crow flies, Figure 1 shows three possible spatial allocations, each with six Uber drivers. The points represent drivers, the black borders represent the catchment regions, and the shading represents the distance from a point to its nearest driver. The allocation on the left yields an expected distance of about 0.364, while the center and right allocations yield expected distances of about 0.285 and 0.282, respectively. In this sense, the allocation on the right is the most efficient—in fact, it is the optimal allocation of six drivers in this space.

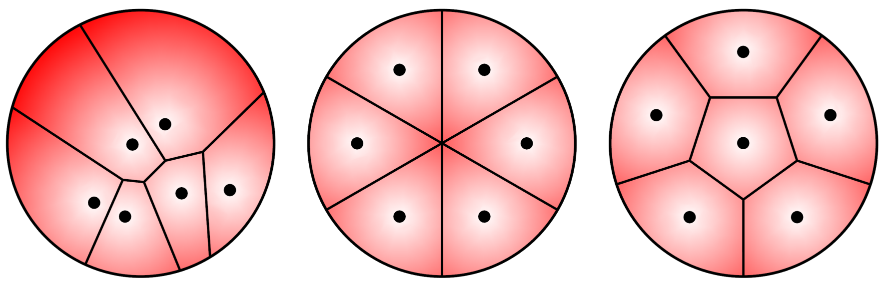


Figure 1: Three six-driver spatial allocations of varying efficiency

Why do ride-hailing apps make it difficult for drivers to see the locations of their competitors? Intuition might suggest that competitive markets would yield reasonably efficient spatial allocations—if there were high expected demand in an area with no driver in close proximity, an idle driver would reduce their expected idle time by moving to this area. This intuition has been both supported and questioned in theoretical literature dating back to Hotelling (1929).

We develop an experiment to examine behavior in a dynamic spatial game. In the experiment, we provide players with calculator software to compute flow payoffs for any possible spatial allocation. As players use the calculator to consider choices before making them, we observe not only their choices in the actual game but also, in the calculator data, indications of the allocations that they consider in making those choices. Looking at the choices, we find that a large majority are consistent with myopic best responding. Of the choices that are not, the calculator data suggests that most are attributable to error. Furthermore, the number of times a player chooses a myopically optimal move has a statistically significant positive effect on a player's payment, suggesting that myopic optimization is a good rule of thumb and that learning may reinforce it.[2]

The spatial game in our experiment was designed to roughly match the ride-hailing motivation. It is a dynamic game with reversible decisions, no price competition, and fairly low

[1]The actual matching algorithm is proprietary and more sophisticated. For instance, rides may be offered to drivers who are completing another trip and about to become available in some proximity to the request.

[2]Myopic optimization can also be embedded in richer strategies: a driver may learn broad patterns of driver supply and passenger demand to decide where to drive, then myopically optimize within that region.

stakes. Assuming myopia in location choices with irreversible decisions, price competition, and higher stakes would be far stronger—businesses do extensive market research in location choices. However, sophisticated market research will still yield decisions that are consistent with myopic optimization unless reasoning on potential future entry, exit, or competitor relocation induces choices that do not maximize myopic profits.

To evaluate mechanisms that yield spatial allocations and test related policies such as exclusive territories and zoning, we seek a model of spatial competition that is both tractable and rich enough to be predictive in applied settings. Static analyses in location theory reveal many of the challenges in developing such a model: static spatial games may have multiple Nash equilibria, none at all, or be intractable even in relatively simple environments—we discuss this in Section 4.1.

Our experiment results support the modeling of spatial competition as a dynamic game with agents that optimize myopically. With this assumption, a dynamic game can be modeled as a sequence of static, individual optimizations, allowing for simulation and agent-based models even in complex environments. In cases where agents following a myopic best response dynamic converge to a fixed point, that fixed point is a Nash equilibrium of the corresponding static game. Where there is no convergence, the dynamic path itself serves as a prediction in that we can evaluate measures of inefficiency at different moments in time and compare averages or dynamics. In the presence of complexity, agents make choices using heuristics and rules of thumb. Agent-based models involve identifying these and building them into a model to generate predictions through simulation and computation. Where theory is traditionally deductive in that results are derived directly from assumptions, agent-based modeling can be described as inductive in that one makes assumptions on agents and then watches phenomena emerge through agent interaction, as we discuss in Section 4.2.

The rest of this paper is organized as follows: Section 2 presents our model of a dynamic spatial game. In Section 3, we present our experiment results on the prevalence of myopic best-responding. In Section 4, we discuss the challenges of modeling spatial competition through static equilibrium analyses and propose agent-based models with behavioral assumptions as an alternative. As an application, we run simple agent-based models both in our experimental environment and on an actual transportation network to show that ride-hailing companies may benefit from allowing each driver to see the locations of other drivers. We conclude in Section 5.

2 Model

Environment

Let $\mathcal{I} = \{1, \ldots, I\}$ denote the set of players. A graph G is given by the pair $G = (\mathcal{N}, A)$, where $\mathcal{N} = \{1, \ldots, N\}$ is a set of nodes and $A \in [0, \infty]^{N \times N}$ is a weighted adjacency matrix. If there is an edge between nodes m and n, $a_{m,n} \in \mathbb{R}_{\geq 0}$ denotes the distance or weight. Otherwise, $a_{m,n} = \infty$. We embed G in $\mathbb{R}^2$ and assign each $n \in \mathcal{N}$ coordinates $(x_n, y_n) \in \mathbb{R}^2$. G is an undirected graph so that $a_{n,m} = a_{m,n}$ for all nodes $m, n \in \mathcal{N}$. We also assume that G is connected so that every two nodes in G have a path between them and that each node has a self-loop, $a_{m,m} = 0$, so that players may remain in their current positions across periods.

Node n's set of neighbors $\mathcal{B}_n = \{m \in \mathcal{N} : a_{n,m} < \infty\}$ includes all nodes with which n shares an edge.

In a dynamic game of spatial competition on a graph G, players sequentially choose their locations on the graph. We consider games with T periods. A *spatial allocation* s_t is an I-tuple $(s_{t,1}, s_{t,2}, \ldots, s_{t,I}) \in \prod_{i\in\mathcal{I}} \mathcal{N}$ that records the locations of players on the nodes of G in period t. Thus, s_t is a list of I nodes, where the i^{th} element, $s_{t,i}$, is the location of player i in period t. Let $\mathcal{S} = \prod_{i\in\mathcal{I}} \mathcal{N}$ be the set of all possible spatial allocations and define $s_{t,-i} \in \prod_{j\in\mathcal{I}\setminus\{i\}} \mathcal{N}$ as the spatial allocation of i's opponents in period t. This notation allows us to consider the movement of player i holding the positions of her opponents fixed. We will refer to the 2-tuple $(n, s_{t,-i})$ as the spatial allocation in which player i is positioned at node n and her opponents are positioned according to $s_{t,-i}$.

We define a multiplicity function, $\psi : \mathcal{N} \times \mathcal{S} \to \mathbb{N}_{\geq 1}$, that returns the cardinality of the set of players who are closest to a particular node: $\psi(n, s_t) = |\arg\min_{i\in\mathcal{I}}\{d(n, s_{t,i})\}|$, where the distance function $d(\cdot)$ finds the length of the shortest path between two given nodes. For example, $\psi(n, s_t) = 2$ implies that the two closest players are equidistant from node n in the spatial allocation s_t. We also allow colocation—where multiple players are located at n, each is a distance of zero from n, and the multiplicity function therefore gives the number of players at n.

In each period t, we use a Voronoi diagram, $\mathrm{Vor}(s_t)$, to calculate each player's market share. A Voronoi diagram on a space and a set of points divides a space into cells, with each cell representing the region that is closer to a particular point than to any other point. $\mathrm{Vor}(s_t)$ partitions $\mathcal{N}$ into Voronoi cells $V_i(s_t) = \{n \in \mathcal{N} \mid d(n, s_{t,i}) \leq d(n, s_{t,j}), \forall j \neq i\}$ for each player $i \in \mathcal{I}$. At period t in the game, the market share of player i is the number of cells to which she is the closest player, including evenly divided shares of cells to which she and other players are equidistant:

$$\pi_i(s_t) = \sum_{n\in V_i(s_t)} \frac{1}{\psi(n, s_t)} \,.$$

Equilibrium

A spatial allocation s_t is a *global* Nash equilibrium of a static location game if for all $i \in \mathcal{I}$ and for all $n \in \mathcal{N}$, $\pi_i(s_t) \geq \pi_i(n, s_{t,-i})$. It is a *local* Nash equilibrium of a static location game if for all $i \in \mathcal{I}$ and for all $b \in \mathcal{B}_{s_{t,i}}$, $\pi_i(s_t) \geq \pi_i(b, s_{t,-i})$. A global Nash equilibrium requires that there is no profitable deviation for any player to any other node, whereas a local equilibrium precludes only profitable deviations for any player to any adjacent node. Obviously the set of local equilibria contains the set of global equilibria.

Efficiency

To measure the *spatial inefficiency* of an allocation for the purposes of the ride-hailing motivation, we first calculate the average distance from each node to its nearest driver in the spatial allocation[3], s_t:

$$\bar{d}(s_t) = \frac{1}{N} \left(\sum_{n \in \mathcal{N}} \min_{i \in \mathcal{I}} \{ d(n, s_{t,i}) \} \right) . \tag{1}$$

Define an optimal spatial allocation[4] s_t^* as one which minimizes (1):

$$s^* \in \arg\min_{s_t \in \mathcal{S}} \bar{d}(s_t) . \tag{2}$$

Then, define the spatial inefficiency of a spatial allocation s_t as the percentage difference between the average distance of an allocation and that of an optimized allocation:

$$\xi(s_t) = \frac{\bar{d}(s_t) - \bar{d}(s^*)}{\bar{d}(s^*)} . \tag{3}$$

3 Experiment

In this section, we present results from an experiment designed to test the validity of the behavioral assumption that agents myopically best respond (MBR) in the context of a dynamic spatial game. If an agent maximizes her instantaneous flow playoffs at the moment of her decision, ignoring potential future opponent movements, we say that her choices satisfy the MBR assumption.

Insofar as there are costs associated with complex strategic reasoning, myopia could be rational. Abstracting from these costs, we know that behaving in accordance with the MBR assumption is likely to be suboptimal. Our initial hypothesis was that agents would choose to myopically best respond—it is a reasonably good strategy and does not require complex strategic reasoning. Indeed, we find evidence of this.

Our experiment involves participants playing a two-dimensional, discrete-time, dynamic spatial game. Previewing our results, we observe 2178 decisions and find that only 307 of them are suggestive of higher-order reasoning (SHO) that would violate the MBR behavioral assumption. Further, regression analysis shows that players that make more SHO choices earn no more money than those who make fewer, while players who behave most in accordance with the behavioral assumption do earn more money, suggesting that repetition and learning might work in favor of the assumption's validity.

3.1 Related literature

There are two experimental literatures that are relevant to our work. The first seeks to offer an experimental answer to the question of whether competition yields spatially efficient outcomes. Brown-Kruse et al. (1993) tests Hotelling's linear-city model in a repeated game with two firms and examines the role of communication between players. Without communication, the two firms locate near the center of the market. With communication,

[3]This definition of efficiency assumes one consumer per node but is easily extended to allow for non-uniform distributions of consumers across nodes.

[4]Auerbach and Dix (2018) explores how to compute s_t^* in continuous spaces.

they locate one-fourth and three-fourths of the way along the linear market and maximize joint profits. Kruse and Schenk (2000) extends Brown-Kruse et al. (1993) to consider non-uniform customer distributions. Players generally chose symmetric strategies with uniform, unimodal, and bimodal distributions, but, even with communication, they struggled to reach the profit-maximizing allocation with non-uniform customer distributions. In order to study the role of complexity in location games, Kruse and Schenk (2000) also considers Hotelling's linear city model with a simplified decision environment in which they only allowed players to choose between two locations: the center or one edge of the market. Players in the simplified decision environment reached the profit-maximizing allocation more often than those with the full range of potential locations, and allowing communication between players, even in the more complex environment, led more often to the profit-maximizing allocation. Collins and Sherstyuk (2000) considers a similar model to that in Brown-Kruse et al. (1993) but with three firms and finds that players randomize locations and avoid both the center and edges of the market, highlighting the role of risk aversion as agents choose low-risk locations instead of the risk-neutral equilibrium predictions.

The second relevant literature is that on depth of reasoning. The MBR assumption is similar to assuming that all agents are *level-1* in that they fail to reason to any extent about future opponent play. Depth of reasoning is typically studied within a Keynesian beauty contest, first described in Keynes (1936). Nagel (1995) proposed the *level-k* model of depth of reasoning and experimentally identified heterogeneity in this depth among the experiment's participants. Halpern and Pass (2015) develop a framework for reasoning about strategic agents performing possibly costly computation. Alaoui and Penta (2016, 2017a,b) offer a model of, as well as experimental support for, endogenous agent depth of reasoning motivated by an axiomatized cost-benefit analysis. *Level-k* models are typically applied to static games, though Rampal (2017) is a recent extension to dynamic games. While assuming *level-1* behavior would be a very strong assumption in a Keynesian beauty contest, our spatial game on the Euclidean plane is far more complex due to the underlying geometry.

3.2 Design

We use a grid in our experimental environment. An $N \times N$ grid is a graph $G = (\mathcal{N}, A)$ whose set of nodes are numbered 1 through N^2. We embed G in $\mathbb{R}^2$ and assign each node coordinates (i, j) with $1 \leq i \leq N$ and $1 \leq j \leq N$. Then, there is an edge between two nodes m and n with coordinates (i, j) and (k, l), respectively, if and only if $|i - k| + |j - l| \leq 1$. For A, this means that $a_{m,n} = 1$ if and only if the previous condition holds, otherwise $a_{m,n} = \infty$. For all $n \in \mathcal{N}$, $a_{n,n} = 0$.

In the experiment, five participants (players, henceforth) played a location game on a 21×21 grid. Players were given the opportunity, one by one, to move one square in any cardinal direction. Turn order was random. Colocation was not allowed. We used the same initial allocation of players, shown in Figure 2, for each session. Players are labeled by number, 1 through 5. There are also eight computer players, each labeled with C, who do not move and are positioned along the perimeter of the grid. We include these static computer players because we want to abstract from issues resulting from the presence of boundaries. Having the static computer players makes the game played by the actual players theoretically similar to that played in a particular region of the unbounded Euclidean plane.

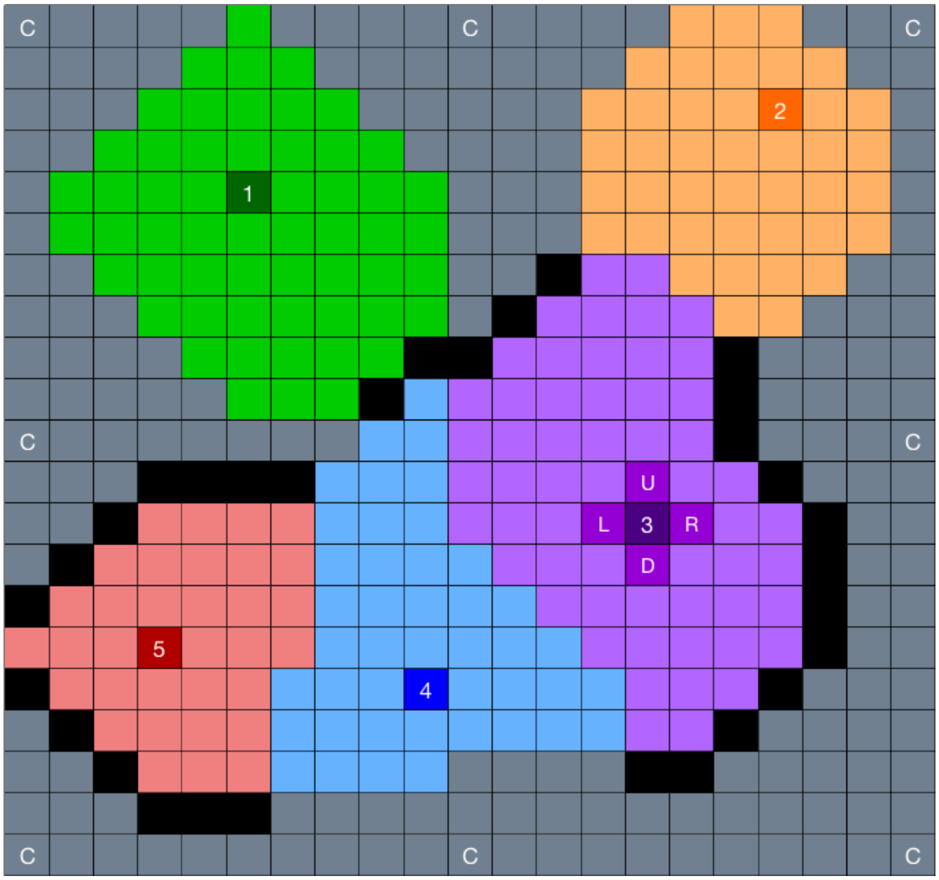


Figure 2: Initial allocation of players on grid

Each player has a color assigned to them. The player's current location is represented by a single cell in the grid with a dark shade of that color and the player's number. The player's Voronoi region, calculated with the ℓ_1 norm (Manhattan distance), is represented by an area of the grid in a lighter shade of the player's color. Black cells are equidistant from two or more players, at least one of which is not a computer. Grey cells are closer to a computer player than any non-computer player.

We conducted the experiment with two pieces of software that we developed. Our main console software, shown in Figure 3, shows the players' locations in the current iteration, the player whose turn it is to move, and a grid with the players' Voronoi regions. The grid also highlights up to five move options, including the four cardinal directions and an option to remain at the current location. In the experiment, the main console was projected for all players to view throughout the experimental session.

Our second piece of software, shown in Figure 4, is calculator software that each player used on her own lab computer throughout the experiment. The calculator allows players to enter in an allocation of players, calculate the area of the players' Voronoi regions for that allocation, and see the grid of the players' Voronoi regions. We provided the calculator for two reasons: First, while it is simple arithmetic to work out which Voronoi region a given square belongs to for a given allocation, it is very time-consuming to calculate the area of a Voronoi region by hand. We wanted to alleviate that burden. Second, because players were

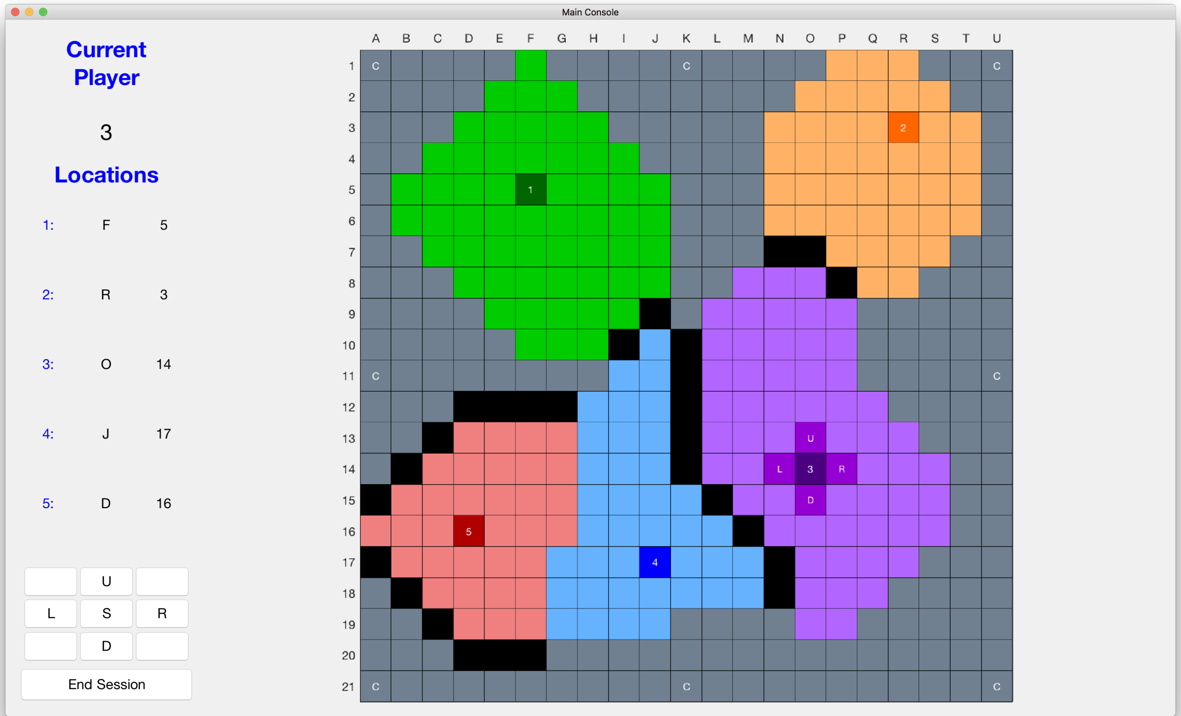


Figure 3: Main console software

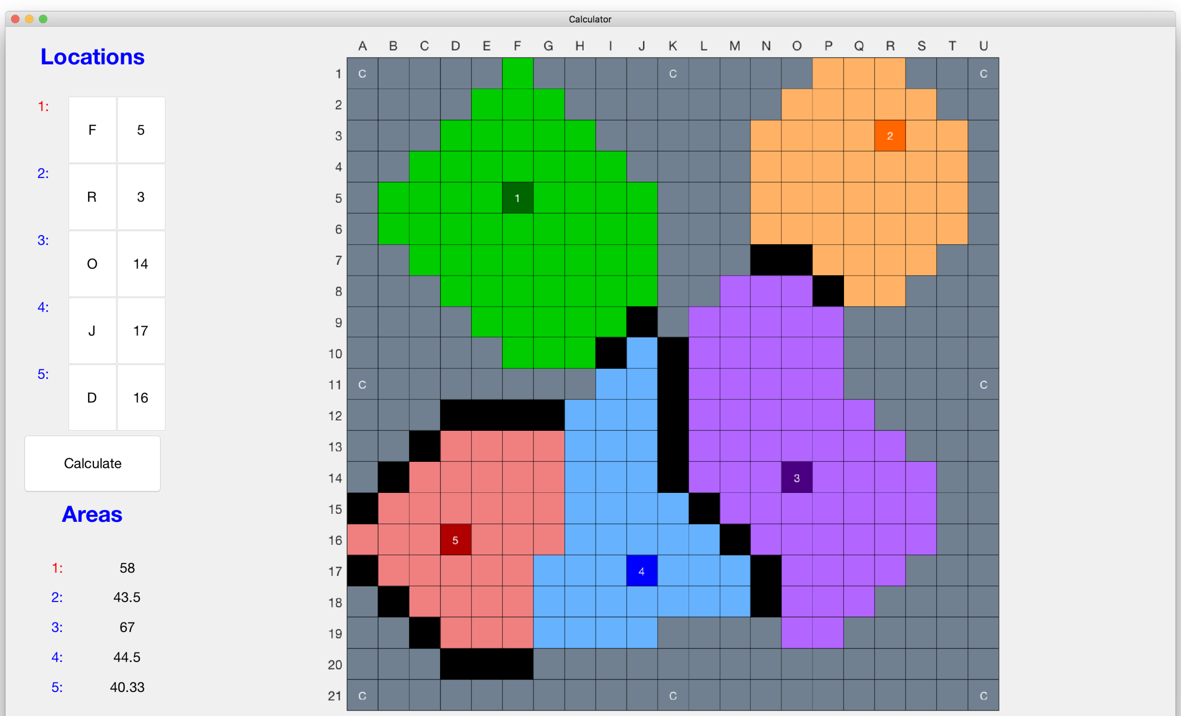


Figure 4: Calculator software

using the calculator to consider their choices during each turn, and between their turns in many cases, we have data on not only the choices they made in the actual game but also all of the allocations they considered in making their choices.

The game is played as follows. In each iteration, the player number whose turn it is in that iteration is announced. This player has up to two minutes to decide where to move.[5] The player then communicates her decision to the experiment leader and the experiment leader updates the main console accordingly. This process repeats for the duration of the experiment session.

In each turn of the game, a player chooses to move to a square within one unit of her current location. If there are no opponents within one unit of the current player's position and she is not on a boundary, she has five squares to choose from. For example, in Figure 3, we see the squares Player 3 can choose to move to are highlighted in a dark shade of purple. Positioning the current player in one of these squares within one unit of her current location and keeping the opponents in place creates up to five potential allocations of players for the next iteration. We define each of these potential allocations as a *move option*. We define a move option's flow payment as the area of the current player's Voronoi region after the move is made. In order to classify all move options, we rank the move options by their flow payments: the FP1 move is the move option with the highest flow payment, the FP2 move is that with the second-highest flow payment, etc. An MBR agent would always choose the FP1 move.

We ran the experiment at the Behavioral Research Insights Through Experiments (BRITE) Lab at the University of Wisconsin-Madison in June, 2017. Players were recruited from a pool of students maintained by the BRITE Lab. We conducted 18 experimental sessions, each with 5 players, for a total of 90 players. Players were shown an instructional video at the beginning of each experimental session to explain how the game is played and how their payments would be calculated. A transcript of the instructions and a link to the video are included in Appendix A.

Players' payments were proportional to the average area of their Voronoi regions over all iterations in the experimental session, where an iteration is defined by the allocation at a given time and a new iteration is entered upon every selected move. The number of iterations per session varied based on speed of play. Turn order was determined randomly. Experimental sessions were scheduled to last 90 minutes, including time for the instructional video. The average time spent playing the game was 68 minutes, and the average number of iterations per experimental session was 121. The mean player payment was \$20.[6]

[5]The time limit was only to prevent a player from deliberately stalling the experiment indefinitely, not to put any time pressure on play. The limit was reached only a handful of times. In each of those occasions, the experiment leader then asked the player where she wanted to move and the player responded immediately.

[6]Each iteration is scored by calculating the size of each player's Voronoi region in the Voronoi diagram of the players over the grid. Let $m_{i,t}$ denote player i's score for iteration t. It is calculated as the number of squares in the grid that are closest to player i in iteration t, i.e., the size of player i's Voronoi region. A square that is equidistant from multiple players is divided evenly amongst those players for scoring purposes, as described in Section 2. Player i's current session score in iteration τ is $M_{i,\tau} = \frac{1}{\tau} \cdot \frac{1}{21^2} \cdot \sum_{t=1}^{\tau} m_{i,t}$. The session score calculates the average percentage of the 21×21 grid controlled by player i over the τ iterations. After the last iteration ($t = T$), player i was paid $\$180 \cdot M_{i,T}$. While \$180 was the technical session prize pool, the actual amount paid out to players was close to \$100 given that the computer agents win a significant portion of the pool.

3.3 Results

There were a total of 2178 main console moves across the 18 sessions. Each of the 90 experiment participants used the calculator between 11 and 786 times, averaging 275 per participant. Table 1 shows the distribution of flow payment move rankings for all moves as well as for those made after the first ten minutes of each session. The percentage of FP1 moves increases with the exclusion of the first ten minutes, suggesting that there was some noise in the beginning of each session as players learned how to play the game and how to use the calculator.

Table 1: Distribution of move FP rankings

	All moves		**After 10**	
FP ranking	**Freq.**	**%**	**Freq.**	**%**
1	1315	60	1103	63
2	439	20	339	20
3	229	11	174	10
4	126	6	74	4
5	69	3	50	3

Result 1. *Players chose the FP1 move 60% of the time.*

This does not imply that a majority of players behaved in accordance with the MBR behavioral assumption. Nor does it suggest that a majority of moves were made by players behaving in accordance with the MBR assumption. Players selecting a FP1 move may be engaging in sophisticated consideration of anticipated future movements that happens to motivate them to pick the same choice as an MBR player. Alternatively, an agent might just select a move randomly and happen to select the FP1 move. Similarly for the moves selected that were not FP1, the analysis above does not suggest the reasoning that motivated these choices.[7] However, we use data from players' calculators to learn about reasoning processes.

If players were engaging in higher-order reasoning on their opponents' subsequent behavior, we would expect them to run calculations that considered potential subsequent movements from opponents. We saw this only rarely. Players tended to keep their opponents in place relative to the current allocation of players in the main console and tested allocations with only their own locations adjusted. In fact, our second result of note is that a majority of calculations made were on move options. Table 2 summarizes the positions of opponents in calculations.

Result 2. *In 82% of calculations, all opponents were positioned as they were in the current iteration. 76% of calculations were of move options.*

[7] In some sense the modeler may not care how agents arrive at FP1 moves—if players are systematically making these choices, they are predictable regardless of how they reached them. On the other hand, such a coincidence of reasoning and predictable choice is less transportable to other models than a behavioral assumption that explains the coincidence.

Table 2: Positioning of opponents in calculations

	Freq.	**%**
Total	24 765	
Opponents in place	20 341	82
Move option	18 869	76

We can also consider whether players made forward-looking calculations with respect to their own locations. As players could only move one square per turn, a calculation with the player moved more than one square tested a non-feasible move. A player positioning herself more than one unit away from her position in the current iteration could suggest some level of higher-order reasoning. Table 3 shows the distances that the player who made the calculation was from her current position in the iteration. Very few forward-looking calculations were made. Instead, a majority of the calculations were either updating to the current iteration or testing a move option of the current iteration. Again, we see that opponents were often in place relative to the current iteration.

Table 3: Distribution of player distances from their iteration position in calculations

Distance	**Freq.**	**%**	**Of which opponents in place**	**%**
0	12 046	49	9546	79
1	10 733	43	9323	87
2	1205	5	957	79
3	360	2	253	70
≥4	421	2	262	62

Result 3. *In 92% of calculations, the calculating player was within one square of her position at the the time of calculation.*[8]

We now turn to an analysis of non-FP1 moves to address the question of how many of them are suggestive of higher-order reasoning on opponent responses. Here, we define the relevant calculation interval for a move to be the timespan from the last change in the allocation up until the move.[9] The set of relevant calculations, then, is the set of all calculations made during a move's relevant interval. To partition the set of non-FP1 moves, we first determine whether the FP1 move was calculated during the relevant interval. In a majority of non-FP1 moves, the FP1 move was not calculated, suggesting to us that these choices are likely attributable to a failure to consider or calculate the FP1 move rather than higher-order reasoning.

To further refine this partitioning, we then determine whether the move that was chosen was actually calculated. For moves where the FP1 move was not calculated but the move

[8]The frequency of zero-distance calculations is high partially because players' calculators do not automatically update to the iteration positions after a move is made—many of these calculations should be attributed to players updating their calculators to a new iteration.

[9]This is to account for instances where players before the player in question choose to remain in place.

chosen was calculated, we can ask whether the player chose the highest-scoring calculated move (HSCM) of all moves calculated in the relevant interval. Although the FP1 move was not chosen, the moves in this subset suggest MBR-like behavior. See Table 4 for the full partitioning of non-FP1 moves.[10]

Table 4: Partitioning the 863 non-FP1 moves

	FP1 not calculated 535	**FP1 calculated** 328
Choice not calculated	325	21
No calculations	(205)	
Some calculations	(120)	
Choice calculated	210	307
Chose HSCM	(126)	
Did not choose HSCM	(84)	

Within the set of non-FP1 moves, we define a move to be suggestive of higher-order reasoning (SHO) if the player calculated both the FP1 move and the move she ultimately chose, but did not choose the FP1 move. For these 307 moves (out of 2178 total moves), the player was accurately using the calculator and deviating from the FP1 move.[11] Table 5 shows how many players, and in what quantities, were responsible for the SHO moves. Approximately 24% of SHO moves were made by less than 7% percent of players.

Another thought is that players may be following the MBR assumption roughly but then deciding by intuition when flow payments of two or more moves are very close. To get at this, we look at the flow payment differences between the FP1 move and that of the chosen moves. The average difference for non-FP1, non-SHO moves, between the move selected and the FP1 move, was 1.74 grid squares. The average score difference for non-FP1, SHO moves was only .82 grid squares, suggesting that players are relying on intuition and higher-order reasoning mostly when the flow payments are close.[12] Table 6 compares the score differences.

Result 4. *Differences in flow payments between SHO moves and their FP1 alternatives were significantly smaller than those between non-SHO, non-FP1 moves and their FP1 alternatives.*

Finally, we can examine the determinants of success in the experiment through regression. Table 7 shows the impact of several behaviors on a player's session score. Since there is

[10] We also saw a slight bias for non-FP1 moves towards the center—of the 863 non-FP1 moves, 362 were towards the center, 249 were towards the perimeter, and 252 remained in place.

[10] Although we made it clear in each session that the calculators did not automatically update for each new iteration (by design), we saw quite a few instances where players were doing MBR-like calculations but with the opponents in the position of a previous (outdated) iteration. This accounts for many of the 120 cases where some calculations were made but neither the choice nor the FP1 move was calculated.

[11] It is also possible that they simply forgot the results of some of their calculations. They were not given pen and paper, or any other means to record their calculations.

[12] SHO moves with very small flow payment differences could also potentially be attributed to errors in noticing the small differences.

Table 5: Number of SHO moves by player

# SHO Moves	# Players	% Players	% Total SHO
0	19	21	0
1	15	17	5
2	11	12	7
3	9	10	9
4	11	12	14
5	6	7	10
6	4	4	8
7	4	4	9
8	3	3	8
9	2	2	6
≥10	6	7	24

Table 6: Score differences for non-FP1 moves

	SHO %	non-SHO %
Score Diff > 1	22	49
Score Diff ≤ 1	78	51
Score Diff < .5	33	14
Score Diff < .25	17	6

expected variation in the session scores between player numbers because of the unequal areas in the initial allocation, we calculate the mean session score for each player number and determine the difference from this mean for each player. We use this as our measure of player performance and the dependent variable in the regression analysis. Moreover, because turn order was randomly determined and sessions were limited by total amount of time playing the game, rather than total number of iterations, players had different numbers of turns. We control for this in the regressions.

In the first column, we find that players who chose more FP1 moves achieved higher session scores than those choosing fewer. In the second and third columns, we find no statistically significant relationship between the number of SHO moves chosen and a player's session score. The fourth column shows that players who made more calculations earned higher session scores. But when we include both the number of calculations made, and the number of FP1 moves, it is the latter that maintains its significance. In the third and seventh columns we run similar regressions but restrict our sample to the moves of players who make at least three SHO moves.

Result 5. *The number of SHO moves a player chose had no statistically significant impact on her performance. The number of FP1 moves had a significant positive impact.*

The more players behaved in this simplistic MBR manner, the better they did. If players are to learn from their performance, it would lead them to select FP1 moves more often, not

Table 7: Difference from mean score (in thousandths, by player number)

	(1) All	(2) All	(3) #SHO≥ 3	(4) All	(5) All	(6) All	(7) #SHO≥ 3
# Turns	−0.171	0.0218	−0.0375	−0.0288	−0.179	−0.00974	−0.0730
	(0.120)	(0.0836)	(0.132)	(0.0873)	(0.117)	(0.0779)	(0.130)
# FP1	0.441*				0.391*		
	(0.174)				(0.169)		
# SHO		−0.221	−0.392			−0.375	−0.381
		(0.279)	(0.495)			(0.272)	(0.446)
# Calcs.				0.0137*	0.0112	0.0162*	0.0147
				(0.00628)	(0.00624)	(0.00708)	(0.00947)
N	90	90	45	90	90	90	45
adj. R^2	0.062	-0.013	-0.023	0.034	0.088	0.051	0.021

Robust standard errors in parentheses. * $p < 0.05$

less. Of course, MBR seems to do well against approximately MBR opponents—we have not shown that it does well against opponents doing more sophisticated reasoning.

We think that the MBR assumption may be realistic in an applied setting. For Uber drivers, given the reversibility of their decisions and the low stakes, sophisticated reasoning on the movements of other drivers is unlikely to be worthwhile. It is a stronger assumption in games with irreversible decisions and large stakes. Businesses do extensive market research before deciding where to place new facilities, but this due diligence may still be analogous to MBR agents considering move options in our experiment unless businesses explicitly reason on potential future entry, exit, or firm relocation and this reasoning alters their choices.

We cannot argue affirmatively that players myopically best respond, but we do fail to find significant evidence that they violate the assumption. And our regression results suggest that learning may push players toward, not away from, MBR behavior. Of course, this negative result is good news for those seeking to do agent-based modeling with an MBR assumption. The irony is that the underlying complexity of spatial competition that makes equilibrium analysis difficult may also make players behave quite predictably, thereby facilitating agent-based modeling. Given the results of our experiment, we think it best to model agents in a spatial agent-based model as noisy MBR agents.[13] These agents would usually choose FP1 moves, but would randomly choose non-FP1 moves, doing so more often when non-FP1 moves are closer in flow payments and in these cases usually selecting moves with relatively high flow payments. This extends easily to agent-based models with spatial and price competition as we describe in Section 4.2. Our finding that agent behavior is quite predictable in a complex dynamic spatial game may also be relevant to other complex dynamic games.

[13]Models in which all agents myopically best respond are also useful both as a benchmark and as a way of computing Nash equilibria in analogous static games.

4 Discussion

4.1 Challenges of static models

Spatial games are typically modeled as static games despite the fact that mechanisms generating empirical spatial allocations are likely continuous-time dynamic games.[14] Analyses vary in the number of firms, the space, and the customer distribution.[15] Each firm's profit is proportional to the mass of customers that is closer to the firm than any other. If two or more firms are colocated, they split equally their joint mass of customers. Each firm simultaneously chooses its location, s_i, to maximize its profits. A Nash equilibrium spatial allocation, s, is such that no firm could increase its profits by unilaterally deviating to another location. In discussing equilibrium analysis on static spatial games, we illustrate three key impediments to using these models for applied prediction: i) multiplicity, ii) non-existence, and iii) intractability.[16]

Analysis of spatial competition dates back to Hotelling (1929).[17] Hotelling's canonical main-street, or linear-city, model has two firms competing on location. His key result, which came to be known both as Hotelling's Law and the principle of minimum differentiation, is that firms may be incentivized to make their products as similar as possible. In a spatial model, this manifests as colocation. Spatial competition also relates to monopolistic competition, à la Chamberlin (1933), except that the product differentiation comes from the location of the sellers. Many real markets involve sellers in different locations selling differentiated products at different prices—high dimensionality makes it very difficult to make theoretical predictions in such a rich model.

Model 1 of Eaton and Lipsey (1975) has a bounded line as its space and uniform customer density—it is a fairly straight-forward extension of Hotelling (1929), without prices, to cover

[14]Sequential games with irreversible decisions are commonly modeled as static games. In our setting, there may be uncertainty about timing in the game, and beliefs on that timing. See Penta and Zuazo-Garin (2017) for analysis on rationalizability in this context.

[15]For reviews of this literature, see Graitson (1982) and Gabszewicz and Thisse (1992).

[16]The challenges of theoretical analysis also motivate the Structure-Conduct-Performance (SCP) paradigm developed in Mason (1939, 1948), which looks for empirical evidence of relationships between industry structure and outcomes. See Bain (1951, 1956) for across-industry analyses and Stigler et al. (1983) for a critique that favored price theory models. In some sense, Von Neumann and Morgenstern (1944) developed game theory as an alternative to the SCP paradigm.

[17]Hotelling (1929) was a response to Bertrand (1883) and extensions in Edgeworth (1897). In turn, Bertrand (1883) was a paradox proposed in critique of Cournot (1838) and Walras (1883). Cournot's duopoly with firms choosing quantities yields lower quantities and higher prices than the social optimum. Bertrand's model is similar but with firms choosing prices, not quantities. Intuitively this should not matter given that prices determine quantities and vice versa, yet Bertrand's model predicts no deadweight loss in the duopoly, thus the paradox. Hotelling's critique of Bertrand (1883) focused on what he viewed as the unrealistic discontinuity in the Bertrand model where one seller goes from serving no customers to serving all of them as she moves her price from minimally above her rival's to below: "…a discontinuity, like a vacuum, is abhorred by nature" (Hotelling, 1929, p.44). Ironically, Hotelling's analysis, which had firms competing on both price and location, was incorrect because he failed to take account of discontinuities in his firms' best response functions. This error was mentioned in Shubik (1959) and Vickrey (1964) but not corrected until d'Aspremont et al. (1979), which showed that no equilibrium existed in Hotelling's original model. While Hotelling's predictions were incorrect for his price-location model, they are correct for the same model without prices, and most now remember Hotelling's contribution as one of pure spatial competition.

$n \geq 2$ firms. For $n = 3$, there is no equilibrium. For $n = 4$ and $n = 5$, there exists a unique equilibrium with two firms colocated near each boundary, and one additional firm in the middle in the $n = 5$ case. For $n \geq 6$, there is multiplicity—while the two firms nearest each boundary must be colocated, each interior firm may be colocated or uniquely located. Flexibility in the equilibrium positioning of the interior, individually-located firms means that there are infinitely many equilibria for any $n \geq 6$. For exposition, assume here that n is even. Again we take spatial efficiency as the inverse of the expected distance from a customer to her nearest firm. Then, in the optimal allocation, which is not an equilibrium, the n firms evenly divide the line. The most efficient equilibrium has all but the boundary firms uniquely located. The least efficient has all firms colocated in pairs. We show the three for $n = 10$ in Figure 7.[18]

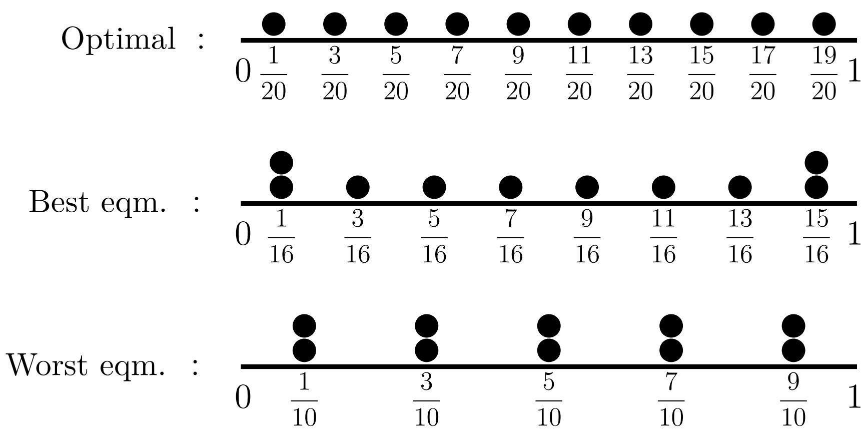


Figure 7: Equilibrium spatial allocations with ten firms

As we abstract from the boundary behavior by increasing n, the expected distance in the best equilibrium allocation converges to that of the optimal allocation, while that of the worst equilibrium converges to twice that of the optimal allocation. The multiplicity of equilibria means that equilibrium analysis gives no conclusive answer to the question of spatial efficiency even in this simple setting of uniform customer density of a line.[19]

Model 3 in Eaton and Lipsey (1975) departs from the assumption of a uniform customer density and finds a non-trivial necessary condition for the existence of equilibrium: the number of firms cannot exceed twice the number of modes of the (assumed continuous) customer density function. In particular, this means that for any $n \geq 3$, there exists no equilibrium if customer density is single-peaked—imagine a linear city with population density highest at the center and tapering toward the boundary in each direction. Where multiplicity impeded prediction with uniform customer density functions, here non-existence is the challenge.

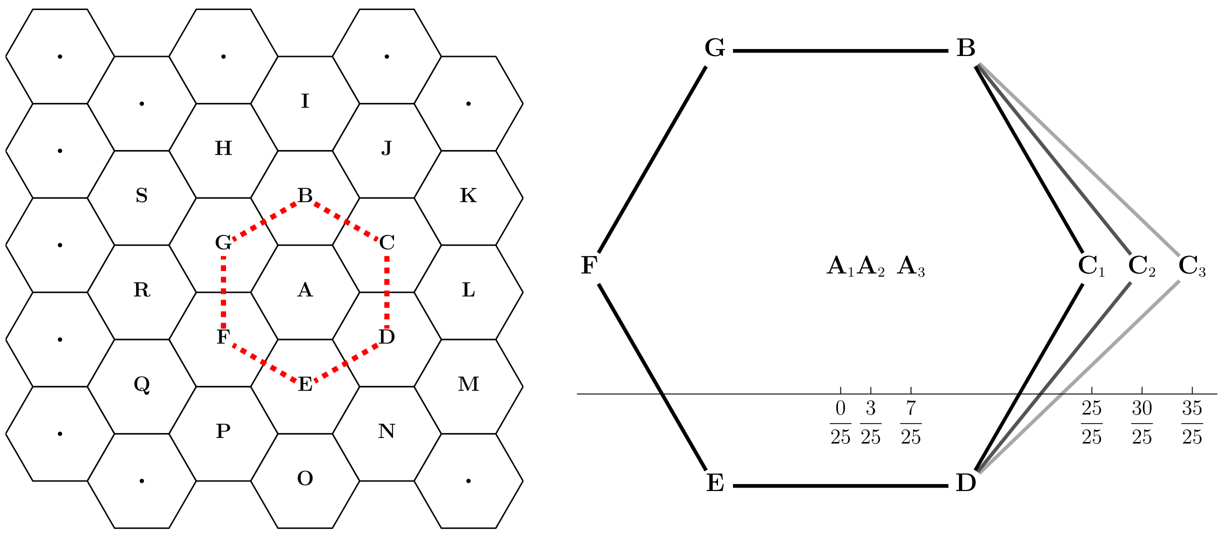
Spatial competition in applied settings usually takes place in two-dimensional spaces. Lösch (1954, p.94-97) considers the unbounded Euclidean plane with a uniform customer distribution and suggests the optimality and stability of an offset grid configuration of firms,

[18]We thank Liyang Liu for his assistance in extending the Eaton and Lipsey (1975) characterization to arbitrary n.

[19]One might favor the worst equilibrium as a prediction given that it is the only strict equilibrium.

the Voronoi diagram[20] of which is a hexagonal covering as shown in Figure 8a—both the letters and points represent firms and the hexagons are Voronoi cells.[21]

Lösch's equilibrium is supported numerically in Eaton and Lipsey (1975) and proven analytically in Okabe and Aoyagi (1991).[22] Its optimality is proven in Bollobas and Stern (1972). In this setting, equilibrium and optimality coincide.



(a) Hexagonal covering

(b) A's optimal position conditional on C

Figure 8: Löschian equilibrium hexagonal covering and individual incentives

But Eaton and Lipsey (1972, 1976, 1978) find many flaws in the Löschian model and argue that competition need not yield that hexagonal covering. Further, even if one were to accept the prediction of the hexagonal covering in this setting, there is no evidence that this prediction generalizes beyond an unbounded Euclidean space with uniform customer density. Boundaries in the linear model implied inefficient colocation near them in equilibrium, while departing from uniform customer density raised questions of existence. The former issue extends fairly intuitively to two dimensions—a uniquely-located firm near a boundary is incentivized to move away from the boundary as it gains customers towards the interior without losing its essentially captive customers between it and the boundary.[23] As for the latter, little is known.

[20]A Voronoi diagram on a space and a set of points divides a space into cells, with each cell representing the region that is closer to a particular point than to any other point. It is a formalization of what we called *catchment regions* in describing Figure 1.

[21]Lösch (1954) attempts game-theoretic equilibrium analysis before the tools were well understood—his equilibrium conditions are a mix of behavioral postulates and conditions that purportedly follow from them. One of the equilibrium conditions, incorporating prices, was a zero-profit condition justified by free entry. But Eaton and Lipsey (1978) shows that the neoclassical result of free entry yielding zero profits does not survive an extension to space with scale effects, calling into question many spatial analyses that used that assumption—Eaton and Lipsey (1978, p.455) offers a list of such analyses in a footnote.

[22]Okabe and Aoyagi (1991) also proves the existence of a square-covering equilibrium. See Knoblauch (2002) for a particularly elegant proof.

[23]This is more nuanced in 2D where Voronoi cell walls pivot with any movement such that only those customers on the line of the direction of movement and between the firm and boundary are truly captive.

In fact, beyond the equilibrium characterization, very little is known even about the unbounded, uniform case. In equilibrium, each firm is located optimally given the locations of all other firms. Yet for a particular firm, A, we have no analytical solution for the optimal location choice given the locations of all other firms except in a few special cases. This problem, maximizing a Voronoi region, is an open problem in computational geometry.[24] If we constrain firm A to select within a particular region[25] meeting minor technical assumptions, Dehne et al. (2005) proves that there exists a unique location that maximizes the area of A's region. The same paper shows that if the convex hull of A's neighbors happens to form a regular polygon with $n > 4$ sides, then A's optimal location is at the center of the polygon. Where the convex hull of opponents is an irregular polygon, we can use area formulas in that paper to find the optimal location for A numerically. As we show in Figure 8b, which looks at how A's optimal position changes as one of its neighbors, C, is moved, a firm's optimal location appears to be *at the center* of the convex hull of its neighbors.[26] But it does not coincide with any of the myriad notions of polygon centrality that we have considered. This leaves us without an analytical solution to the problem beyond the very specific case in which the convex hull of A's neighbors forms a regular polygon.

There is also an obvious heuristic argument against the presumption of equilibrium existence in two-dimensional spaces. Consider Figure 8a. Firm C has six neighbors, with the convex hull of their positions forming a regular hexagon, as is the case for Firm A in Figure 8b. Consideration of the latter figure suggests that if we assume that i) C is located optimally and ii) the locations of C and her other neighbors (B, J, K, L, and D) are given, then we know A's position. But the same argument is true for each of A's neighbors: B along with her other neighbors (G, H, I, J, and C) pins down A, as does G, F, E, and D, each with their neighbors. And this is true for all firms: if a firm has k Voronoi neighbors, then its position must, in equilibrium, solve k equations.[27] As such, the existence of equilibrium requires a solution to a significantly overdetermined system. Such systems can have solutions, of course. Lösch's hexagonal covering on the unbounded Euclidean plane with uniform customer density is a perfect example—each of the six equations pinning down A yields the same location. But we suspect that equilibrium existence is non-generic, a pleasant quirk of unboundedness and uniform customer density.[28] Unfortunately, we cannot speak to the rank of the system without an analytical solution to the problem of maximizing a Voronoi region.

Related models combine competition on price with that on location or product char-

[24] For examples in that literature, see Cheong et al. (2004, 2007) and Fekete and Meijer (2005).

[25] A neighborship cell, which is a set of points such that A would have the same set of Voronoi neighbors locating at any one of those points.

[26] Uber drivers appear to have worked this out intuitively. There are several how-to videos on YouTube in which an experienced Uber driver shows the process of switching into the passenger app and then advises the audience that they will get a ride sooner if they move to the center of an unoccupied region, essentially mimicking our numerical analysis in Figure 8b.

[27] In the linear case, each firm has at most two Voronoi neighbors. And even in this *less overdetermined* system, we know that equilibrium existence is not guaranteed, as shown in Eaton and Lipsey (1975).

[28] We are not the first to make such a conjecture. In Eaton and Lipsey (1975), the authors conjecture that there exists no equilibrium on a disk with uniform density for $n > 2$ firms. Shaked (1975) proves that conjecture for $n = 3$. Dasgupta and Maskin (1986) connects existence issues in location games to those in other games with discontinuities in payoff functions.

acteristic: Capozza and Van Order (1978) offers a generalization of some earlier Löschian models. Salop (1979) adds an outside good/industry to the Hotelling model. Novshek (1980) considers alternatives to Nash equilibrium. Economides (1984, 1986a,b) experiment with the addition of reservation prices, the adjustment of the convexity of utility functions, and an expansion to two dimensions. In models of non-spatial product differentiation: Rosen (1974) offers a perfect competition model, with a continuum of firms, of hedonic pricing where products are differentiated and priced based on attributes—note that Rosen's proof of equilibrium existence is exclusive to the one-dimensional case. Gabszewicz and Thisse (1979) models quality where customers have the same tastes but varying incomes.

Intractability in computing *Markov perfect equilibria* for stochastic dynamic games motivates the *stationary equilibrium* of Hopenhayn (1992), justified rigorously in Adlakha et al. (2015), in which agents optimize assuming the distribution of other agents' states remains constant at its long-run average. In MBR, agents optimize given the current distribution of others' states.

In summary, there exists a rich theoretical literature on spatial competition, but using it to generate predictions in an applied setting is impeded by multiplicity of equilibria, potential equilibrium non-existence, and tractability issues. This motivates our agenda to pursue alternative approaches, including agent-based models.

4.2 Agent-based models

An agent-based model (ABM) is a computational model for simulating the interactions of autonomous agents to assess their effects on the system. While this inductive approach to modeling comes largely from computer science, it is becoming increasingly prevalent in the social sciences. Axtell (2000) describes three distinct uses of ABM in the social sciences. Tesfatsion (2006) argues that it is a new, constructive approach to theory. Farmer and Foley (2009) argue for its use in macroeconomics given the complexity of macroeconomic systems. Hommes (2008) surveys their use in finance, particularly asset pricing. More related to our question, Crooks et al. (2008) and Crooks and Heppenstall (2012) look at the particular challenges of spatial ABM, and economists are starting to apply these models to Hotelling-like environments.[29]

We can build agent-based models based upon the myopic best responding that we found in the experiment. Agent myopia is a behavioral assumption in dynamic environments under which an agent views the positions of her opponents as fixed when deciding whether or not she would profit from changing her own position—that is, she simply maximizes her

[29]See van Leeuwen and Lijesen (2016). NetLogo, software for ABM, even includes a Hotelling model in its model library (Ottino et al., 2009).

instantaneous payoff flow.[30] To define agent myopia, consider a dynamic continuous-time[31] spatial game with payoff flows where each agent periodically makes decisions. Assume that at any given moment at most one agent makes a decision and that each agent can revise her decision in a future time period. In this setting, if an agent takes the locations of opponents as given, she is effectively not strategic—she simply maximizes her instantaneous payoff flow given the current state at the moment of her decision. We say that this agent follows a *myopic best response* (MBR) dynamic.

In motivating spatial competition with the ride-hailing application and designing a related experiment, we focus on a very specific environment without price competition.[32] Most applications of spatial competition also involve competition on prices. We can include prices in spatial ABM's with the MBR dynamic in two ways. The simplest would be to extend the behavioral assumption to also cover prices—when an agent chooses a location and price, she takes her opponents' locations and prices as given and maximizes her instantaneous payoff flow at the moment of the decision. Alternatively, we could construct hybrid models where agents take their opponents' locations as fixed but anticipate prices resulting from a Nash equilibrium in a static price game, given the locations, immediately following their location choice.[33]

The MBR dynamic has important connections to Nash equilibrium. One interpretation of Nash equilibrium as a prediction is that it could result from an evolutive process in an environment in which agents know little about the structure of the game but best respond given their limited information and learn from outcomes.[34] In this sense, Nash equilibrium in a static game represents a fixed point under the MBR dynamic in a dynamic game. Brown (1951) computes equilibria in static games algorithmically by applying the MBR dynamic iteratively in fictitious dynamic play. Similarly, fixed points in the MBR dynamic on our dynamic games represent Nash equilibira in their static analogs.[35]

The consequences of MBR dynamics on a lattice with local interactions are studied in Blume (1993, 1995). The MBR dynamic is also common in evolutionary game theory,[36] although that literature focuses on population games with many players. Agent myopia and the MBR dynamic are tremendously powerful in that they allow us to model complex continuous-time games as sequences of static individual optimizations. Agent-based models in the social sciences commonly exploit behavioral assumptions to allow the representation of

[30] Myopia is a dynamic-game analog of zero conjectural variance (ZCV), which historically has meant simply that Nash equilibrium is being applied as a solution concept to a static game. In that setting, when one checks a potential equilibrium by looking for a profitable unilateral deviation, the possibility of an opponent response is ruled out by the fact that the game is static. The first use of ZCV that we find is in Eaton and Lipsey (1972). The authors employ the term in critique of Löschian models in which, they argue, equilibrium is not well defined due to the absence of an explicit assumption on conjectural variance. While ZCV comes automatically in Nash equilibrium analysis on a static games, Lösch does not appeal to Nash equilibrium—his work was contemporaneous with that of Nash. Therefore, equilibrium in the Löschian model is not well defined without an explicit assumption on conjectural variance.

[31] This argument also applies to discrete-time games so long as at most one agent has a choice at any time.

[32] Since Uber sets prices centrally, drivers cannot compete on price.

[33] We can only do this if pure-strategy price equilibria exist and are computable. Relevant conditions for existence are given in Caplin and Nalebuff (1991).

[34] See Binmore (1987, 1988) and Gilboa and Matsui (1991).

[35] In this sense, our dynamic agent-based models may yield further insight on static games.

[36] See Sandholm (2010, Chapter 6) for best response dynamics in that setting.

individual agents as automata. Because of the assumed relative simplicity of agent behavior, we can work in much richer environments than those used in game-theoretic analyses without losing tractability, and this may allow us to better compare mechanisms that generate spatial allocations and to evaluate relevant policies such as zoning and exclusive territories.

4.3 Application 1: an ABM in the experiment environment

Recall that we designed the experiment to represent the incentives of idle drivers in a ride-hailing setting. Because we use a grid and the ℓ_1 norm for distance, the experimental environment represents competition on an urban road grid with uniformly distributed expected demand. As we described in Section 1, a ride-hailing service may wish for idle drivers to position themselves so as to minimize expected wait times for passengers. An obvious question then is whether the myopic best responding that we find in the experiment yields spatial allocations that do well by this metric, measured by $\xi(s_t)$ as defined in Section 2.

We evaluate the efficiency of the dynamic paths of our experimental sessions and compare those with the paths of MBR simulations starting from various initializations. In our MBR simulations, each player chooses the move that maximizes her flow playoff (FP1) at the time of her decision.

Figure 9 shows the results. Avg. Exp. takes the spatial inefficiency from each experimental session at iteration t and averages them. The initialized allocation in the experiment was quite efficient, but players' early choices increased efficiency further. Avg. Sim1 mimics the experiment with 18 sessions but replaces the observed behavior with that of simulated MBR agents. In this setting, MBR usually led players to optimal or near-optimal spatial allocations. Avg. Sim2 and Avg. Sim3 are equivalent but were run from different, less efficient, initial allocations.

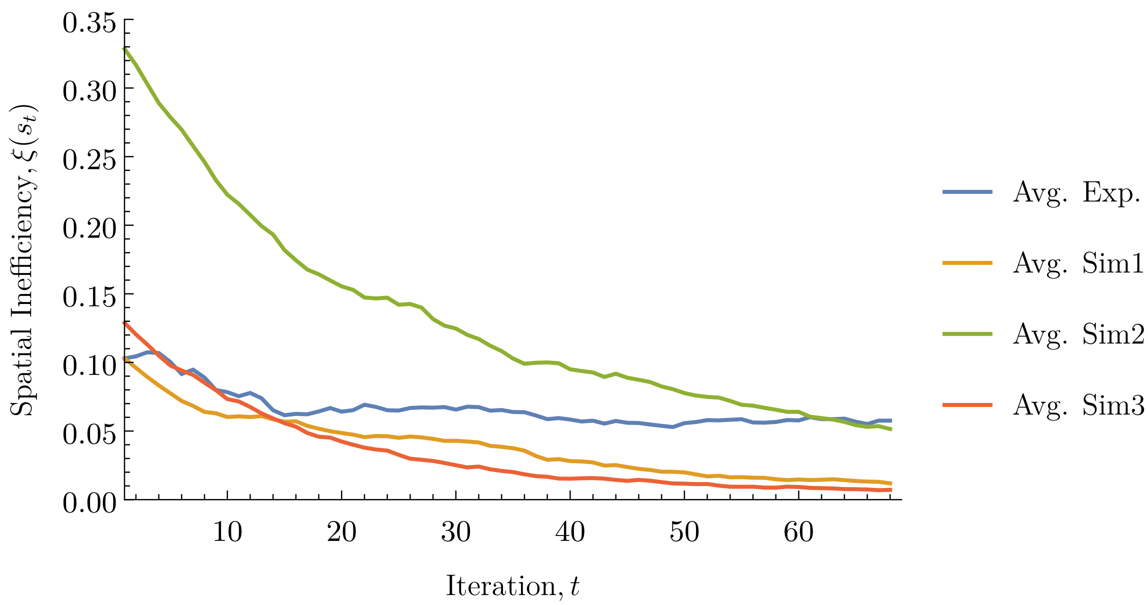


Figure 9: Spatial inefficiency in experiment and simulations

The players in our experiment did choose the FP1 move a majority of time. However, the average spatial inefficiency in the experiments remained substantially higher than that of allocations achieved by agents who perfectly implement MBR in simulations.

4.4 Application 2: an ABM in a ride-hailing environment

We now explore whether the myopic best responding that we find in the experiment yields efficient spatial allocations on a more realistic transportation network. Suppose $\mathcal{I}$ is the set of drivers for a ride-hailing service. A dynamic game of spatial competition represents drivers competing for passengers on a transportation network $G = (\mathcal{N}, A)$. The nodes of this network represent intersections, and the edges represent roads. The length of the edges are proportional to the length of the roads in the city.

We recreate the road network of the City of Oldenburg, Germany using data from Brinkhoff (2002). There are 6,105 nodes and 7,029 edges, and the average degree of the nodes is 2.3. We compute an approximately optimal allocation of 60 drivers in Oldenburg using a myopic (greedy) heuristic, as in Kuehn and Hamburger (1963). Call this optimal allocation s^*—we find that $\bar{d}(s^*) = 480.18$ meters. Figure 10 shows the transportation network and the approximately optimal allocation of drivers, with each driver represented by a black diamond.

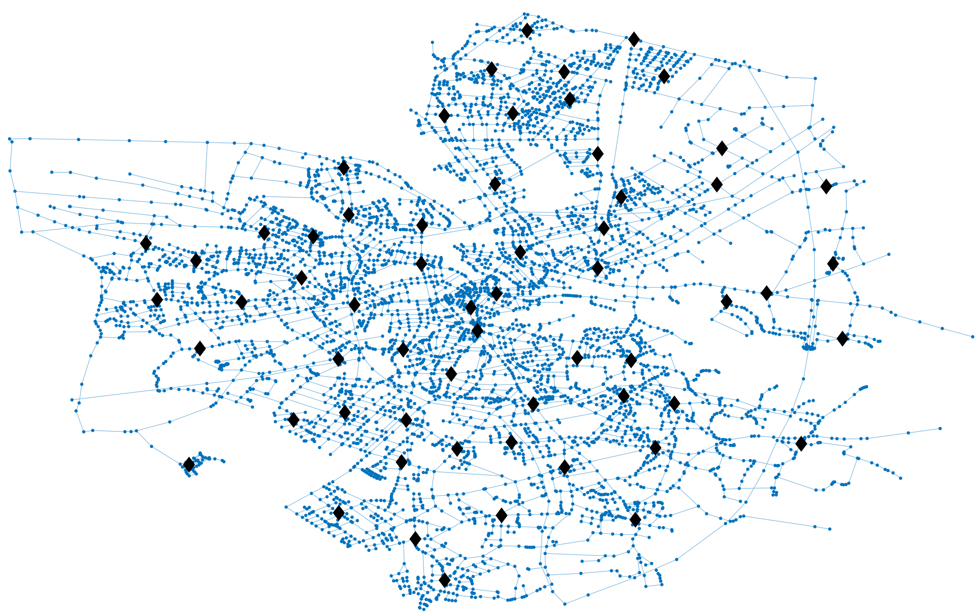


Figure 10: Transportation network in Oldenberg with approximately optimal allocation

To simulate the game played between MBR drivers, we initialize the set $\mathcal{I}$ of drivers on the nodes of the transportation network and run the MBR algorithm for $T = 5000$ iterations. Figure 11 shows spatial inefficiency along the dynamic path of one simulation. We observe this same tendency in all simulation runs.

Figures 12 and 13 show the initial and final allocations, respectively. The spatial inefficiency of the initial allocation is $\xi(s_1) = 2.02$ and the spatial inefficiency of the final

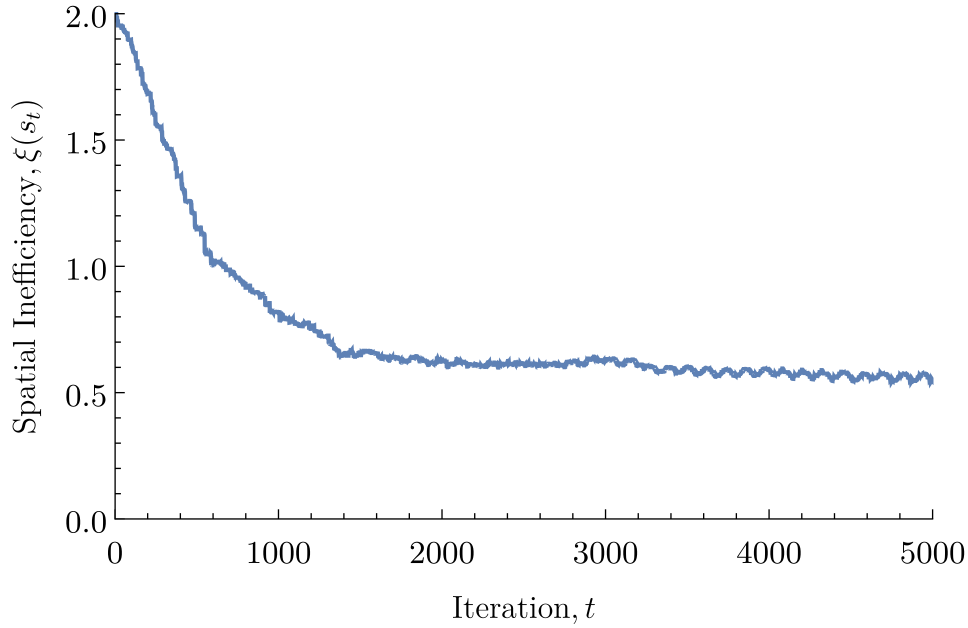


Figure 11: Spatial inefficiency along dynamic path of simulation with $T = 5000$

allocation is $\xi(s_{5000}) = 0.55$. Thus, the decisions made by MBR drivers resulted in a large decrease in spatial inefficiency, which implies a large decrease in expected consumer wait times.[37] We note that spatial inefficiency did not decrease monotonically along the dynamic path.

Given that the interests of myopic best responding drivers coincide well with the interests of the ride-hailing service even in this setting with a more realistic transportation network, we believe that ride-hailing services may wish to allow drivers to see the locations of nearby drivers and assist them in best responding to their neighbors.[38]

In the simulation discussed above, MBR agents did not converge to an optimal spatial allocation. Within 5000 iterations, drivers converged to a cycle of spatial allocations in which spatial inefficiency was near .55 for all allocations in the cycle. This result suggests that allowing agents to myopically best respond may reduce consumer wait times but it need not converge to an optimal outcome.

We believe one contributing factor to inefficiency in fixed points (or cycles) under the MBR dynamic is boundary behavior, where individuals near the periphery have incentives to move inwards as they can do so without sacrificing market share on the periphery. To isolate this, consider the approximately optimal allocation of drivers s^* on Oldenburg's trans-

[37] When we simulate MBR drivers starting from the approximately optimal allocation s^*, spatial inefficiency increases from $\xi(s^*) = 0$ to $\xi(s_{1000}) = .08$ after 1000 iterations. Therefore, individual incentives appear to push drivers away from the periphery and towards the center, which increases inefficiency.

[38] Afeche et al. (2018) shows that there are gains to centralized platform control over driver repositioning even when drivers have perfect information. Potential gains increase with cross-location imbalances in demand and travel time. Platform control over repositioning would be possible with employee drivers but may not be compatible with the independent contractor classification.

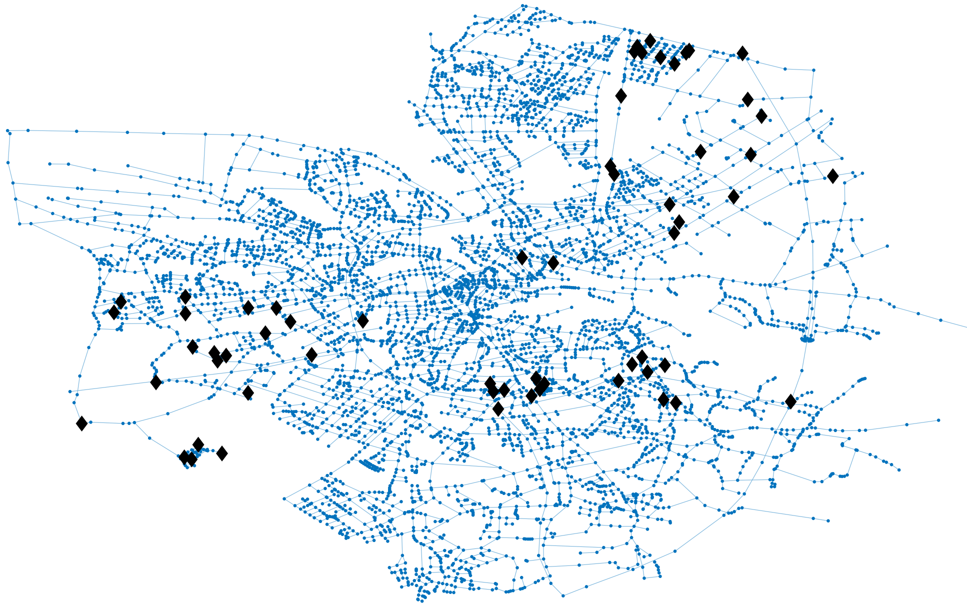


Figure 12: Initial allocation of 60 drivers in Oldenburg; $\xi(s_1) = 2.02$

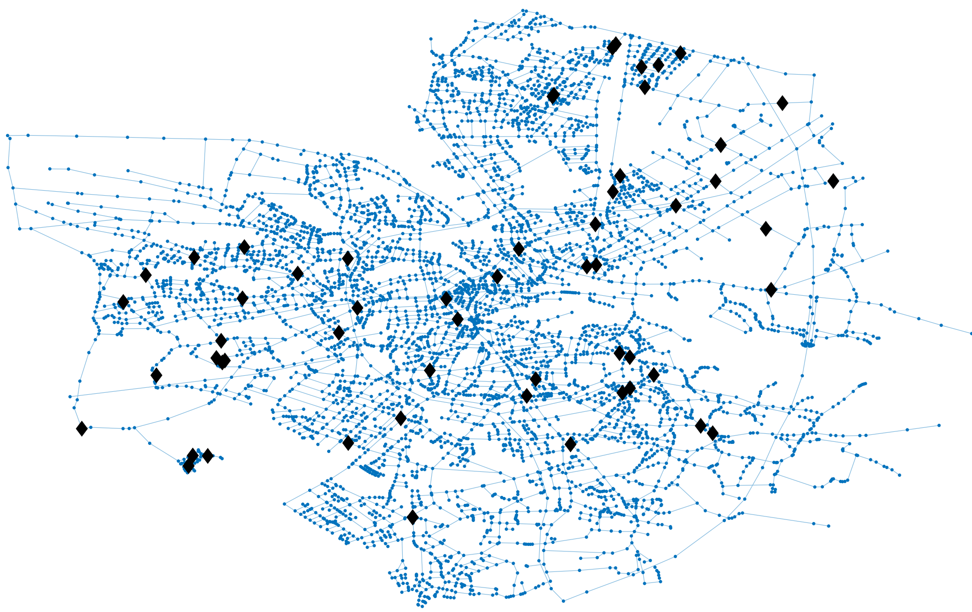


Figure 13: Final allocation of 60 drivers in Oldenburg; $\xi(s_{5000}) = 0.55$

portation network, and suppose we fix the positions of the drivers that are located on the outer periphery of the network. Let $\mathcal{I}_{outer}$ be the set of 14 drivers on the outer periphery. We create a new spatial allocation in which the drivers in $\mathcal{I}_{outer}$ are at their locations in s^*, then we generate a random allocation of the remaining drivers. Finally, we simulate another game played between MBR drivers on Oldenburg's transportation network, but we only allow drivers in the set $\mathcal{I} \setminus \mathcal{I}_{outer}$ to move in the simulation.

In this setting, the spatial inefficiency of the initial allocation is $\xi(s_1) = .34$ and that of the final allocation is $\xi(s_{1500}) = 0.2$. Fixing drivers on the periphery and allowing for MBR movement in the center of the transportation network resulted in significantly lower final spatial inefficiency, suggesting that boundary behavior is indeed a contributing factor to inefficiency in MBR simulations.

This result suggests another interesting policy for ride-hailing platforms. As these platforms start to incorporate autonomous vehicles alongside regular vehicles, they may want to have the autonomous vehicles target the periphery, allowing spatial competition between drivers in the interior.[39]

5 Conclusion

In this paper, we show the prevalence of myopic best responding in an experiment with a dynamic spatial game that represents the incentives of idle drivers in a ride-hailing context. We argue that, in light of this experimental finding and the challenges of static analyses that we detail, agent-based modeling may be a fruitful approach in developing predictive models in complex spatial environments. In applications, we show that agent-based models in our experimental environment and in a more realistic transportation network suggest that ride-hailing services may benefit from allowing each idle driver to observe the locations of other idle drivers.

[39] On the other hand, with high fixed costs and low variable costs, autonomous vehicles are most efficient when operated at high utilization levels, which may preclude peripheral deployment.

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A Experiment instructions

Players were shown an instructional video at the beginning of each experimental session to explain how the game is played and how their payments would be calculated. Players were then given the chance to ask questions before the game began. See the instructional video at `http://youtu.be/7hcN24RFI3M`. The following is a transcript of the instructional video:

> Welcome to the BRITE Lab and thank you in advance for participating in our experiment on spatial competition. You are about to play an experimental game of spatial competition. The duration of the experiment will be about 90 minutes. Please do not talk to other participants during the experiment or use the computers in ways other than those described here. After participating in this experiment, please do not discuss it with others who may also participate in the future. After watching these video instructions, you may ask questions.
>
> This is the game board that you will see projected. It may look slightly different, depending on the computer. At the top left, we see that it is currently player 3's turn. You have already been assigned player numbers. The turn order is randomly determined—after each turn, it is as if the next player's turn is decided by rolling a five-sided die. This can result in you having multiple turns in a row or going for long stretches of time without having a turn. Below the turn indicator, you can see a list of the indexed locations of each of the five human players. These indices will be important for operating the calculator on your computer. Reading the top line, we see that player 1 is located at grid coordinate $F5$. F represents the column and 5 represents the row. Sure enough, we can see player 1 at coordinate $F5$.
>
> At the bottom-left, you can see the controls that the experiment leader will use to move you and the other human players around the board. When it is your turn, you will select to move either up, down, left, or right. You may also stay in your current location. Once you have made your decision, you will communicate it to the experiment leader who will then input the choice using these buttons. Suppose player 3 chooses to move up and communicates this to the leader. Now the leader hits the U button for up, and we see that the gameboard is redrawn with player 3 one row higher than before. We also see that player 4 has been randomly selected for the next turn.
>
> So far, we have not discussed how you should decide where to move. We will show you precisely how your cash payment is determined in a moment, but let's first focus on the grid displayed on the game board. The game is played on a 21 by 21 grid. There are actually 13 players in this game, including 1-2-3-4-5 human players and 1-2-3-4-5-6-7-8 computer players surrounding the grid. The computer players will remain in their current positions throughout the game.
>
> Each human player is given a specific color. Player 4 is blue. Her current location is marked in dark blue. Then, all squares in the grid that are closer to player 4 than any other player (including the computer players) are marked with light

blue. For instance, look at grid coordinate $J10$, shaded light blue. It is 1-2-3-4-5-6-7 squares from player 4's current location. This is less than the distance to any other player. Player 3 is 1-2-3-4-5-6-7-8 squares away. Player 1 is nine squares away: 1-2-3-4-5-6-7-8-9. All of the light-blue shaded squares are currently in player 4's area.

Some squares are equally far from two or more players. For instance, $I10$ is 1-2-3-4-5-6-7-8 squares from player 4. It is also 1-2-3-4-5-6-7-8 from player 1. Squares equally far from multiple players (of which at least one of whom is a human player) are shaded in black. For scoring purposes, such a square is split evenly among the players who are equidistant from it. So $I10$ contributes half of a unit to player 4's area, and half of a unit to player 1's area. Sometimes squares may also be split between 3 or more players. Finally, squares shaded grey are closer to the computer players than to any human player.

The basic objective of the game is to have as large of an area as possible since your payment will depend on the average size of your area over the duration of the experiment. Therefore, to maximize your cash payment, it is in your interests to move strategically to have as large of an average area over the course of the experiment as possible.

When it is your turn, you make take up to two minutes to decide your next move. Note that you are not able to move into a square currently occupied by another player (including human and computer players). If player 5 was in square $I17$, for instance, player 4 would not be allowed to move left. Nor would player 5 be allowed to move right into $J17$. Now let's look at the calculator program that is on your computer screen.

We provide you with a calculator to facilitate your decision-making. The calculator can be used to calculate (and show) the areas for any possible allocation of the five human players. Each of you has the calculator in front of you on your computer.

At the top left of the calculator, you may input player locations. Recall that the indices of the current locations are provided on the gameboard that's projected for all to see. Once you have inputted locations, hit calculate to redraw the simulated gameboard for that allocation of players. As an example, let's try moving player 4 down one position from her current location, from $J17$ to $J18$. If we wanted to, we could move multiple players at once by editing all of the locations. You must hit calculate after editing the locations to regenerate the gameboard.

At the bottom left of the calculator, you can see the size of the areas resulting from the locations that you have inputted. These areas include the appropriate portions of squares that are equally far from multiple players. On the grid itself, you can see what the gameboard would look like if players were in the locations that you inputted.

Please note that the calculator does not update itself to the current locations when players move. If you want to reset your calculator to the current locations, you have to do it manually by inputting the location indices listed for each player on the game board. You may use the calculator at any point in the experiment, whether or not it is your turn. You may use it as much or as little as you want. We do collect data from the calculators. But nothing you do with the calculator will directly affect your payment. Only the selected moves by all participants affect payoffs.

Now let's look at how your cash payment is determined.

To calculate payments, call each player's turn one iteration. The size of your area is calculated in each iteration. Then, we calculate your average area size, averaging over all iterations. Finally, we multiply your average by a number, X, to calculate your cash payment, rounding up to the nearest dollar. X is the same for all players.

The multiplier, X, has been selected to target total payments to human players in the game at around \$100. This implies an anticipated average participant payment of \$20. Your actual payment depends on the choices of all participants, so we can make no minimum payment guarantees. Also, your final payment is not necessarily a good measure of your performance in the experiment, as some players start with more favorable positions.

The experiment leader will terminate the game shortly before 90 minutes has elapsed from the experiment start time. We will have as many turns as time allows. This is the end of the instructional video. You may now ask questions to the experiment leader.