## Multi-headed Lattice Green Function (N = 4, M = 3)

## ODE for $\tilde{R}_{3,4}(z)$ in Theorem 4.2

```
166\,498\,086\,762\,886\,201\,344\,z^4\,+\,781\,156\,297\,810\,381\,520\,240\,640\,z^5\,-\,
          1\,637\,147\,560\,168\,901\,326\,135\,099\,392\,z^6+732\,231\,540\,023\,730\,620\,969\,986\,818\,048\,z^7+
          149697886463404317932617973366784z^8 - 96719208505536419841142621892247552z^9 +
          17 986 272 310 903 846 816 671 667 502 362 656 768 z<sup>10</sup> -
          1 365 121 772 758 889 406 361 975 817 513 893 625 856 z<sup>11</sup> -
          266 982 380 934 205 139 034 767 194 888 213 610 102 784 z<sup>12</sup> +
          3\,451\,537\,920\,763\,342\,815\,087\,344\,036\,232\,525\,206\,519\,808\,z^{13} +
          57 677 198 298 608 369 079 887 772 175 160 628 008 189 952 z<sup>14</sup> -
          655787571926373008384765243492546204588310528z^{15}
          241 642 117 251 606 275 763 798 810 128 911 651 647 258 624 z^{16}) \theta_{7}^{8} +
        (1512 + 1985050368 z + 6350035230720 z^2 + 557203438524432384 z^3 +
          587\,179\,135\,837\,113\,679\,872\,z^4 - 6 373 155 470 045 165 823 983 616 z^5 +
          1292611484404060407000465408z^{6} + 5768579650367308639843349692416z^{7} -
          1\,382\,427\,013\,757\,614\,047\,365\,601\,869\,955\,072\,z^8 -\,358\,992\,894\,157\,107\,285\,438\,601\,960\,070\,578\,176\,z^9 +\,
          53 119 839 081 791 910 699 130 795 605 268 365 312 z<sup>10</sup> -
          20 033 688 987 472 672 410 340 587 286 921 796 911 104 z<sup>11</sup> -
          1\,070\,184\,586\,795\,418\,191\,307\,064\,402\,944\,853\,050\,130\,432\,z^{12} +
          47 857 531 084 168 298 968 105 213 313 896 536 229 281 792 z<sup>13</sup> +
          270 884 195 420 762 288 774 538 561 958 062 360 455 282 688 z<sup>14</sup> -
          5 786 930 575 834 638 459 669 201 820 980 856 382 306 648 064 z<sup>15</sup> +
          1 933 136 938 012 850 206 110 390 481 031 293 213 178 068 992 z^{16} \theta_{7}^{7} +
        (-1113 - 1950259584z - 6216682061824z^2 - 988841093180162048z^3 -
          1\,233\,270\,686\,793\,691\,299\,840\,z^4+9\,715\,936\,946\,399\,911\,434\,256\,384\,z^5+
          7752074094169934619780055040z^6 + 3292303250603115641274504314880z^7 -
          81 315 393 847 369 615 391 288 576 991 201 067 008 z<sup>10</sup> -
          46 854 607 085 100 541 227 643 541 228 521 577 775 104 z<sup>11</sup> -
          484722323648843742960229713378845655040000z^{12}
          230 657 489 060 094 958 856 963 994 975 994 903 435 149 312 z<sup>13</sup> +
          720\,571\,084\,999\,990\,630\,257\,768\,886\,154\,063\,035\,548\,827\,648\,z^{14} -
          22\,027\,520\,447\,398\,165\,760\,511\,055\,419\,885\,169\,351\,394\,852\,864\,z^{15} +
          6 663 616 997 264 781 396 236 424 132 096 584 504 800 444 416 z^{16}) \theta_z^6 +
        (357 + 789151104z + 10558964416512z^2 + 679933490467176448z^3 + 287227915264289931264z^4 -
          14427253172957536013254656z^{5} - 3092284696452480308007141376z^{6} -
          905 311 608 923 360 926 047 701 827 584 z^7 - 14 621 774 397 415 013 636 807 083 954 274 304 z^8 -
          1520148020883568461909863756948570112z^9 +
          386 791 883 303 174 384 286 527 316 852 952 006 656 z<sup>10</sup> -
          55 223 977 247 937 670 737 556 473 181 100 776 095 744 z<sup>11</sup> +
          3\,961\,198\,864\,716\,838\,655\,960\,160\,400\,530\,693\,479\,727\,104\,z^{12} +
          595\,909\,152\,288\,030\,158\,390\,074\,172\,187\,987\,674\,363\,068\,416\,z^{13} +
          1 344 613 983 895 642 776 006 711 946 247 382 448 056 827 904 z<sup>14</sup> -
          46737381677309460398827589903959671231607734272z^{15} +
          12 917 784 851 408 785 491 873 078 058 141 402 044 309 700 608 z^{16}) \theta_z^5 +
        (-42 - 103505472 z - 4717152813056 z^2 - 229951271138492416 z^3 -
```

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33452654058350313472z^{4} + 3519629264891117955973120z^{5} +
  8712355168877862347467653120z^6 - 3183462774294546535677280911360z^7 -
  12\,662\,024\,101\,532\,041\,005\,416\,571\,287\,371\,776\,z^8-91\,457\,574\,708\,638\,075\,983\,201\,720\,533\,516\,288\,z^9+
  586 373 716 393 067 719 463 798 499 745 499 971 584 z<sup>10</sup> -
  45 402 712 266 053 628 419 392 613 379 787 913 691 136 z<sup>11</sup> +
  9 548 218 855 973 838 530 825 534 106 648 111 229 173 760 z<sup>12</sup> +
  910 171 319 762 953 713 098 938 394 074 694 947 157 573 632 z^{13} +
  1 695 248 459 973 650 411 462 298 355 247 964 543 229 362 176 z<sup>14</sup> -
  60 316 818 440 945 087 853 828 024 483 516 860 568 289 935 360 z<sup>15</sup> +
  15 391 679 930 285 039 325 517 386 362 534 096 505 705 332 736 z^{16}) \theta_z^4 +
(212352z + 41049243648z^2 - 8757517736738816z^3 + 20173834021513461760z^4 -
  285697925187496921006080z^{5} + 5192831041959280753355259904z^{6} -
  1\,854\,167\,396\,972\,337\,514\,541\,117\,079\,552\,z^7\, – 7\,611\,731\,718\,211\,226\,366\,541\,506\,826\,731\,520\,z^8\, +
  1\,132\,092\,616\,093\,392\,427\,901\,870\,092\,654\,215\,168\,z^9 +
  573 360 955 845 607 449 871 633 552 338 403 196 928 z<sup>10</sup> -
  31 196 974 018 934 147 496 719 981 299 967 906 021 376 z<sup>11</sup> +
  10\,343\,484\,480\,536\,631\,324\,792\,615\,211\,856\,556\,940\,328\,960\,z^{12} +
  850\,871\,471\,160\,179\,197\,799\,997\,784\,539\,641\,065\,269\,886\,976\,z^{13} +
  1 366 110 491 586 634 438 250 598 685 972 842 495 483 052 032 z<sup>14</sup> -
  48\,462\,627\,453\,914\,171\,613\,580\,503\,834\,409\,896\,539\,840\,315\,392\,z^{15} +
  11 533 376 887 988 124 536 976 314 041 777 845 706 747 281 408 z^{16}) \theta_z^3 +
(102\,144\,z - 45\,890\,052\,096\,z^2 - 4\,372\,668\,181\,905\,408\,z^3 + 25\,373\,328\,015\,678\,767\,104\,z^4 +
  35\,860\,603\,273\,980\,739\,059\,712\,z^5+2\,609\,042\,215\,039\,715\,330\,989\,490\,176\,z^6-
  1\,135\,805\,724\,897\,588\,664\,940\,548\,325\,376\,z^7 - 2\,255\,387\,710\,140\,891\,706\,830\,918\,298\,632\,192\,z^8 +
  1 208 690 199 949 684 174 443 411 490 448 867 328 z<sup>9</sup> +
  334 887 474 030 943 944 488 261 929 148 495 167 488 z<sup>10</sup> -
  17 877 519 858 996 120 053 115 971 187 944 045 150 208 z<sup>11</sup> +
  6 128 654 166 961 763 785 820 170 570 933 495 910 105 088 z<sup>12</sup> +
  476\ 288\ 752\ 718\ 822\ 257\ 140\ 265\ 748\ 951\ 023\ 617\ 535\ 115\ 264\ z^{13}\ +
  668 324 955 523 996 949 091 967 282 097 867 454 465 179 648 z<sup>14</sup> -
  23 673 396 984 425 987 991 182 604 909 788 067 572 173 766 656 z<sup>15</sup> +
  5 303 121 535 030 477 312 378 786 039 652 035 049 432 285 184 z^{16}) \theta_7^2 +
(18\,816\,z - 15\,679\,168\,512\,z^2 - 1\,105\,852\,812\,492\,800\,z^3 + 9\,253\,977\,260\,438\,847\,488\,z^4 +
  36631485914913630584832z^5 + 726314268655758437624315904z^6 -
  358748918263218897800795258880z^7 - 286918040829362957086349485670400z^8 +
  503 093 135 988 065 408 878 446 537 502 359 552 z<sup>9</sup> +
  108 054 624 128 516 395 031 347 156 140 800 606 208 z<sup>10</sup> -
  6 462 199 176 714 597 967 385 137 880 595 550 961 664 z<sup>11</sup> +
  1\,918\,694\,308\,581\,208\,774\,434\,293\,647\,882\,766\,231\,011\,328\,z^{12}\,+
  145 968 214 956 821 518 855 061 140 764 657 290 013 835 264 z<sup>13</sup> +
  180 192 916 196 793 417 299 520 478 544 551 103 988 498 432 z<sup>14</sup> -
  6\,427\,990\,896\,954\,765\,589\,117\,223\,123\,312\,617\,887\,797\,084\,160\,z^{15}\,+
  1 366 788 225 704 397 997 288 987 019 791 656 529 629 806 592 z^{16} \theta_z +
(1344 z - 1639464960 z^2 - 114267116273664 z^3 + 1211269289902866432 z^4 +
  5\,825\,052\,469\,481\,755\,901\,952\,z^5\,+\,84\,152\,329\,059\,287\,491\,751\,706\,624\,z^6\,-
  48\,938\,139\,253\,071\,191\,076\,992\,188\,416\,z^7 – 3\,045\,898\,181\,345\,513\,899\,617\,530\,413\,056\,z^8 +
  78\,022\,182\,208\,697\,643\,235\,066\,215\,175\,028\,736\,z^9+
  14 678 268 634 598 917 861 557 009 824 329 236 480 z<sup>10</sup> -
  991 390 991 530 383 611 754 057 315 362 342 436 864 z<sup>11</sup> +
  247 958 505 832 498 167 951 336 010 415 935 397 560 320 z<sup>12</sup> +
  18 747 996 529 475 474 000 600 656 049 610 020 358 193 152 z<sup>13</sup> +
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20 499 222 726 707 352 515 629 191 626 716 497 397 678 080 z<sup>14</sup> - $742\ 685\ 376\ 897\ 284\ 273\ 453\ 811\ 376\ 847\ 779\ 469\ 564\ 313\ 600\ z^{15}\ +$  $151\,026\,323\,282\,253\,922\,352\,374\,256\,330\,569\,782\,279\,536\,640\,\,z^{16})$