Multi-headed Lattice Green Function (N = 5, M = 4)

REC for $r_{4,5}(n)$ in Theorem 4.7

Out[ ]=

$$
\begin{aligned}
&-\big(-2\,364\,822\,061\,925\,891\,270\,067\,722\,649\,600\,000 - 24\,311\,763\,241\,480\,737\,290\,507\,853\,496\,320\,000\,\alpha - \\
&118\,884\,714\,388\,336\,585\,062\,289\,753\,767\,936\,000\,\alpha^{2} - \\
&368\,251\,136\,151\,853\,255\,846\,369\,719\,798\,988\,800\,\alpha^{3} - 811\,793\,640\,582\,985\,414\,140\,746\,797\,028\,474\,880 \\
&\alpha^{4} - 1\,356\,499\,120\,040\,750\,577\,583\,138\,444\,526\,223\,360\,\alpha^{5} - \\
&1\,786\,835\,040\,377\,781\,128\,110\,811\,754\,937\,712\,640\,\alpha^{6} - \\
&1\,904\,958\,007\,246\,824\,509\,445\,186\,467\,125\,002\,240\,\alpha^{7} - \\
&1\,674\,545\,402\,297\,600\,373\,785\,511\,713\,251\,000\,320\,\alpha^{8} - \\
&1\,230\,194\,808\,706\,317\,371\,163\,067\,050\,208\,788\,480\,\alpha^{9} - \\
&762\,791\,807\,513\,049\,677\,466\,384\,009\,532\,538\,880\,\alpha^{10} - \\
&402\,079\,430\,499\,218\,110\,643\,393\,128\,200\,929\,280\,\alpha^{11} - \\
&181\,085\,303\,893\,806\,582\,831\,390\,648\,576\,245\,760\,\alpha^{12} - \\
&69\,909\,566\,044\,762\,687\,837\,271\,137\,604\,075\,520\,\alpha^{13} - \\
&23\,174\,037\,389\,797\,607\,720\,091\,614\,796\,840\,960\,\alpha^{14} - \\
&6\,597\,237\,647\,955\,223\,324\,018\,009\,760\,071\,680\,\alpha^{15} - \\
&1\,610\,851\,715\,462\,724\,269\,782\,004\,410\,613\,760\,\alpha^{16} - 336\,382\,193\,033\,012\,242\,367\,855\,858\,810\,880\,\alpha^{17} - \\
&59\,795\,770\,083\,083\,316\,221\,336\,805\,703\,680\,\alpha^{18} - 8\,987\,061\,025\,545\,721\,077\,834\,511\,810\,560\,\alpha^{19} - \\
&1\,131\,237\,375\,988\,193\,565\,613\,353\,861\,120\,\alpha^{20} - 117\,704\,523\,870\,056\,936\,584\,154\,972\,160\,\alpha^{21} - \\
&9\,941\,030\,662\,497\,120\,749\,554\,237\,440\,\alpha^{22} - 664\,040\,244\,922\,741\,425\,721\,835\,520\,\alpha^{23} - \\
&33\,746\,986\,442\,943\,554\,031\,452\,160\,\alpha^{24} - 1\,225\,566\,587\,608\,656\,091\,545\,600\,\alpha^{25} - \\
&28\,320\,365\,528\,012\,449\,382\,400\,\alpha^{26} - 312\,808\,771\,118\,086\,225\,920\,\alpha^{27}\big)\ \mathrm{Seq}[\alpha] - \\
&\big(-880\,540\,948\,213\,763\,261\,498\,004\,602\,880\,000 - 8\,086\,612\,414\,279\,581\,582\,690\,097\,299\,456\,000\,\alpha - \\
&35\,535\,843\,625\,080\,580\,938\,628\,852\,403\,404\,800\,\alpha^{2} - 99\,482\,199\,073\,846\,865\,130\,149\,987\,053\,731\,840\,\alpha^{3} - \\
&199\,278\,215\,238\,194\,877\,084\,174\,219\,759\,058\,944\,\alpha^{4} - \\
&304\,147\,288\,569\,704\,121\,767\,283\,668\,058\,636\,288\,\alpha^{5} - \\
&367\,726\,422\,460\,034\,552\,713\,877\,456\,306\,307\,072\,\alpha^{6} - \\
&361\,508\,986\,147\,801\,089\,153\,130\,211\,095\,805\,952\,\alpha^{7} - \\
&294\,331\,319\,744\,750\,632\,422\,172\,167\,712\,997\,376\,\alpha^{8} - \\
&201\,108\,607\,972\,501\,732\,293\,906\,606\,562\,934\,784\,\alpha^{9} - \\
&116\,437\,788\,942\,848\,727\,536\,075\,769\,222\,856\,704\,\alpha^{10} - \\
&57\,524\,299\,296\,878\,619\,402\,424\,939\,339\,382\,784\,\alpha^{11} - \\
&24\,367\,165\,878\,769\,872\,656\,509\,536\,747\,061\,248\,\alpha^{12} - 8\,877\,402\,295\,660\,764\,714\,512\,245\,808\,234\,496\,\alpha^{13} - \\
&2\,785\,748\,984\,068\,408\,698\,625\,918\,477\,467\,648\,\alpha^{14} - 752\,972\,653\,647\,501\,430\,958\,086\,738\,673\,664\,\alpha^{15} - \\
&175\,049\,743\,314\,674\,169\,771\,167\,299\,534\,848\,\alpha^{16} - 34\,895\,534\,864\,837\,208\,484\,258\,292\,957\,184\,\alpha^{17} - \\
&5\,936\,277\,532\,573\,962\,980\,718\,997\,929\,984\,\alpha^{18} - 855\,818\,515\,821\,739\,179\,539\,429\,326\,848\,\alpha^{19} - \\
&103\,560\,073\,600\,267\,246\,364\,541\,321\,216\,\alpha^{20} - 10\,380\,185\,487\,431\,012\,018\,005\,475\,328\,\alpha^{21} - \\
&846\,180\,664\,706\,397\,472\,693\,420\,032\,\alpha^{22} - 54\,656\,640\,176\,185\,180\,963\,209\,216\,\alpha^{23} - \\
&2\,690\,612\,916\,385\,314\,156\,576\,768\,\alpha^{24} - 94\,804\,345\,329\,795\,433\,758\,720\,\alpha^{25} - \\
&2\,128\,785\,749\,082\,227\,343\,360\,\alpha^{26} - 22\,881\,382\,331\,785\,936\,896\,\alpha^{27}\big)\ \mathrm{Seq}[1+\alpha] - \\
&\big(664\,078\,540\,666\,702\,251\,488\,371\,015\,680\,000 + 5\,805\,956\,958\,011\,506\,960\,041\,778\,348\,032\,000\,\alpha + \\
&24\,298\,272\,789\,380\,152\,495\,188\,221\,126\,246\,400\,\alpha^{2} + 64\,810\,405\,629\,301\,547\,428\,216\,819\,254\,558\,720\,\alpha^{3} + \\
&123\,755\,374\,367\,469\,269\,296\,809\,845\,353\,611\,264\,\alpha^{4} + \\
&180\,149\,375\,502\,996\,189\,202\,275\,648\,542\,982\,144\,\alpha^{5} + \\
&207\,865\,771\,244\,125\,682\,287\,781\,841\,861\,722\,112\,\alpha^{6} + \\
&195\,153\,222\,041\,523\,657\,876\,484\,723\,267\,989\,504\,\alpha^{7} + \\
&151\,846\,270\,858\,495\,120\,363\,896\,477\,860\,167\,680\,\alpha^{8} +
\end{aligned}
$$

$99 230 231 828 276 421 932 960 434 682 314 752 \alpha^{9} + 54 993 115 047 787 497 911 079 580 675 899 392 \alpha^{10} +$
$26 028 017 908 489 825 928 212 462 245 453 824 \alpha^{11} +$
$10 572 113 416 646 586 933 511 582 698 766 336 \alpha^{12} + 3 696 722 231 163 815 760 173 082 026 344 448 \alpha^{13} +$
$1 114 468 173 061 041 282 670 805 399 093 248 \alpha^{14} + 289 688 969 845 746 113 335 461 572 931 584 \alpha^{15} +$
$64 831 091 647 102 802 811 533 842 055 168 \alpha^{16} + 12 454 053 009 083 005 771 527 566 163 968 \alpha^{17} +$
$2 043 760 292 966 696 499 523 264 184 320 \alpha^{18} + 284 532 912 366 921 324 027 166 588 928 \alpha^{19} +$
$33 284 416 956 384 385 896 458 223 616 \alpha^{20} + 3 228 606 478 351 534 833 828 626 432 \alpha^{21} +$
$254 974 947 491 313 890 128 560 128 \alpha^{22} + 15 972 126 457 377 261 067 698 176 \alpha^{23} +$
$763 333 007 662 980 725 211 136 \alpha^{24} + 26 138 887 552 462 651 129 856 \alpha^{25} +$
$570 997 443 951 748 710 400 \alpha^{26} + 5 976 795 675 008 958 464 \alpha^{27}\Big) \mathrm{Seq}[2 + \alpha] -$
$\Big(-36 337 840 931 616 555 318 702 833 664 000 - 310 343 693 247 202 072 877 171 431 833 600 \alpha -$
$1 268 062 726 217 635 641 408 454 051 430 400 \alpha^{2} - 3 300 521 955 790 071 740 463 976 232 263 680 \alpha^{3} -$
$6 146 984 578 367 464 065 862 054 879 242 240 \alpha^{4} - 8 723 512 529 514 925 026 222 139 080 468 480 \alpha^{5} -$
$9 808 817 646 565 897 068 529 809 213 239 808 \alpha^{6} - 8 970 447 157 798 999 809 214 350 039 412 224 \alpha^{7} -$
$6 796 618 106 855 403 262 931 535 421 469 184 \alpha^{8} - 4 323 600 610 674 086 572 350 145 316 732 416 \alpha^{9} -$
$2 331 860 127 398 843 166 087 931 718 971 904 \alpha^{10} - 1 073 804 990 271 736 796 663 841 511 156 224 \alpha^{11} -$
$424 279 297 446 148 516 898 147 199 947 264 \alpha^{12} - 144 293 344 557 135 741 340 883 292 465 664 \alpha^{13} -$
$42 304 696 119 152 808 149 756 544 291 840 \alpha^{14} - 10 693 366 157 119 575 923 154 101 714 944 \alpha^{15} -$
$2 327 102 570 668 214 059 453 238 664 192 \alpha^{16} - 434 708 874 971 795 823 099 840 116 736 \alpha^{17} -$
$69 373 988 097 051 870 247 906 934 784 \alpha^{18} - 9 393 304 762 567 159 143 035 764 736 \alpha^{19} -$
$1 068 815 757 774 279 757 481 902 080 \alpha^{20} - 100 861 570 825 855 881 262 923 776 \alpha^{21} -$
$7 750 770 733 439 394 600 976 384 \alpha^{22} - 472 551 963 878 997 639 561 216 \alpha^{23} -$
$21 986 541 883 647 884 001 280 \alpha^{24} - 733 188 729 988 561 502 208 \alpha^{25} -$
$15 602 375 112 618 147 840 \alpha^{26} - 159 149 910 074 064 896 \alpha^{27}\Big) \mathrm{Seq}[3 + \alpha] -$
$\Big(-1 737 772 868 400 007 324 872 130 560 000 - 14 528 609 204 414 291 845 066 255 564 800 \alpha -$
$58 083 087 258 852 534 411 685 975 019 520 \alpha^{2} - 147 846 850 915 658 722 383 612 355 430 400 \alpha^{3} -$
$269 164 023 324 400 460 962 054 275 740 928 \alpha^{4} - 373 240 816 513 597 979 905 593 440 661 888 \alpha^{5} -$
$409 908 879 949 766 514 326 399 060 864 064 \alpha^{6} - 366 016 393 873 249 701 940 597 734 061 344 \alpha^{7} -$
$270 676 671 846 416 971 917 873 052 917 920 \alpha^{8} - 168 013 318 310 785 666 403 759 927 887 584 \alpha^{9} -$
$88 393 926 598 940 439 065 183 725 045 600 \alpha^{10} - 39 697 363 634 496 672 642 069 844 386 912 \alpha^{11} -$
$15 293 672 611 896 263 618 803 193 519 136 \alpha^{12} - 5 070 491 874 452 377 148 797 920 831 072 \alpha^{13} -$
$1 449 002 022 519 967 409 403 051 116 512 \alpha^{14} - 356 957 682 436 813 381 749 659 746 304 \alpha^{15} -$
$75 700 244 148 872 939 301 421 992 640 \alpha^{16} - 13 779 371 789 456 905 170 877 563 840 \alpha^{17} -$
$2 142 685 081 818 193 152 012 367 872 \alpha^{18} - 282 685 926 147 777 894 282 083 328 \alpha^{19} -$
$31 341 335 886 140 485 043 322 880 \alpha^{20} - 2 881 942 426 887 984 021 438 464 \alpha^{21} -$
$215 812 414 752 103 173 455 872 \alpha^{22} - 12 823 036 513 484 289 343 488 \alpha^{23} -$
$581 508 878 853 457 575 936 \alpha^{24} - 18 903 053 117 719 314 432 \alpha^{25} -$
$392 186 219 850 629 120 \alpha^{26} - 3 900 964 176 134 144 \alpha^{27}\Big) \mathrm{Seq}[4 + \alpha] -$
$\Big(36 446 102 109 669 030 849 285 120 000 + 301 794 930 778 773 719 063 321 856 000 \alpha +$
$1 194 401 836 156 084 887 609 064 224 000 \alpha^{2} + 3 008 156 975 709 477 795 289 491 275 520 \alpha^{3} +$
$5 415 770 546 395 539 670 222 530 489 360 \alpha^{4} + 7 422 453 554 874 065 600 190 474 289 032 \alpha^{5} +$
$8 052 206 383 842 449 223 124 682 104 644 \alpha^{6} + 7 098 162 826 794 167 361 280 152 144 294 \alpha^{7} +$
$5 179 144 111 408 801 590 076 035 892 950 \alpha^{8} + 3 169 950 795 733 038 711 522 140 215 280 \alpha^{9} +$
$1 643 499 248 947 095 475 104 215 404 004 \alpha^{10} + 726 910 788 718 026 537 302 273 862 144 \alpha^{11} +$
$275 635 972 025 251 416 199 969 761 656 \alpha^{12} + 89 889 728 147 001 421 773 544 625 132 \alpha^{13} +$
$25 251 994 806 501 150 584 061 125 784 \alpha^{14} + 6 111 409 098 652 595 993 659 452 026 \alpha^{15} +$
$1 272 483 225 563 071 816 917 699 490 \alpha^{16} + 227 273 250 419 552 627 170 585 084 \alpha^{17} +$
$34 655 941 701 831 856 557 922 624 \alpha^{18} + 4 480 880 404 407 427 210 024 320 \alpha^{19} +$
$486 585 842 769 876 461 484 032 \alpha^{20} + 43 798 304 089 562 788 663 296 \alpha^{21} +$
$3 208 710 131 027 557 023 744 \alpha^{22} + 186 416 522 833 559 945 216 \alpha^{23} + 8 261 380 192 874 790 912 \alpha^{24} +$
$262 301 388 296 421 376 \alpha^{25} + 5 312 632 953 241 600 \alpha^{26} + 51 561 082 388 480 \alpha^{27}\Big) \mathrm{Seq}[5 + \alpha] -$

$$\left(154\,404\,486\,709\,237\,819\,219\,968\,000 + 1\,265\,327\,918\,255\,018\,927\,110\,348\,800\,\alpha + 4\,953\,641\,658\,930\,095\,511\,385\,751\,040\,\alpha^2 + 12\,335\,446\,851\,783\,544\,166\,937\,390\,720\,\alpha^3 + 21\,947\,702\,123\,383\,074\,616\,990\,244\,544\,\alpha^4 + 29\,712\,684\,443\,300\,038\,100\,072\,561\,760\,\alpha^5 + 31\,824\,626\,177\,807\,101\,870\,129\,360\,368\,\alpha^6 + 27\,684\,339\,638\,906\,598\,652\,692\,786\,888\,\alpha^7 + 19\,923\,668\,408\,873\,674\,929\,361\,243\,572\,\alpha^8 + 12\,021\,754\,897\,932\,453\,908\,473\,126\,194\,\alpha^9 + 6\,141\,402\,912\,303\,808\,338\,721\,284\,327\,\alpha^{10} + 2\,675\,090\,519\,652\,464\,763\,702\,625\,995\,\alpha^{11} + 998\,451\,712\,547\,824\,111\,144\,656\,513\,\alpha^{12} + 320\,337\,381\,856\,256\,276\,567\,115\,789\,\alpha^{13} + 88\,485\,146\,094\,830\,787\,771\,471\,525\,\alpha^{14} + 21\,045\,641\,782\,461\,353\,200\,898\,049\,\alpha^{15} + 4\,304\,140\,182\,149\,530\,399\,276\,227\,\alpha^{16} + 754\,678\,659\,252\,915\,954\,749\,073\,\alpha^{17} + 112\,910\,766\,050\,133\,819\,763\,020\,\alpha^{18} + 14\,316\,213\,223\,182\,938\,203\,068\,\alpha^{19} + 1\,523\,679\,350\,645\,560\,062\,336\,\alpha^{20} + 134\,345\,128\,624\,663\,841\,280\,\alpha^{21} + 9\,635\,762\,018\,738\,626\,560\,\alpha^{22} + 547\,760\,583\,383\,666\,688\,\alpha^{23} + 23\,739\,371\,943\,886\,848\,\alpha^{24} + 736\,693\,272\,182\,784\,\alpha^{25} + 14\,575\,541\,944\,320\,\alpha^{26} + 138\,110\,042\,112\,\alpha^{27}\right)\ \mathrm{Seq}[6+\alpha]$$