

Multi-headed Lattice Green Function (N = 5, M = 4)

REC for $r_{4,5}(n)$ in Theorem 4.7

$$\begin{aligned} \text{Out}[n] = & (2\,364\,822\,061\,925\,891\,270\,067\,722\,649\,600\,000 + \\ & 24\,311\,763\,241\,480\,737\,290\,507\,853\,496\,320\,000\,\alpha + 118\,884\,714\,388\,336\,585\,062\,289\,753\,767\,936\,000\,\alpha^2 + \\ & 368\,251\,136\,151\,853\,255\,846\,369\,719\,798\,988\,800\,\alpha^3 + \\ & 811\,793\,640\,582\,985\,414\,140\,746\,797\,028\,474\,880\,\alpha^4 + 1\,356\,499\,120\,040\,750\,577\,583\,138\,444\,526\,223\,360\,\alpha^5 + \\ & 1\,786\,835\,040\,377\,781\,128\,110\,811\,754\,937\,712\,640\,\alpha^6 + \\ & 1\,904\,958\,007\,246\,824\,509\,445\,186\,467\,125\,002\,240\,\alpha^7 + \\ & 1\,674\,545\,402\,297\,600\,373\,785\,511\,713\,251\,000\,320\,\alpha^8 + \\ & 1\,230\,194\,808\,706\,317\,371\,163\,067\,050\,208\,788\,480\,\alpha^9 + \\ & 762\,791\,807\,513\,049\,677\,466\,384\,009\,532\,538\,880\,\alpha^{10} + \\ & 402\,079\,430\,499\,218\,110\,643\,393\,128\,200\,929\,280\,\alpha^{11} + \\ & 181\,085\,303\,893\,806\,582\,831\,390\,648\,576\,245\,760\,\alpha^{12} + \\ & 69\,909\,566\,044\,762\,687\,837\,271\,137\,604\,075\,520\,\alpha^{13} + \\ & 23\,174\,037\,389\,797\,607\,720\,091\,614\,796\,840\,960\,\alpha^{14} + 6\,597\,237\,647\,955\,223\,324\,018\,009\,760\,071\,680\,\alpha^{15} + \\ & 1\,610\,851\,715\,462\,724\,269\,782\,004\,410\,613\,760\,\alpha^{16} + 336\,382\,193\,033\,012\,242\,367\,855\,858\,810\,880\,\alpha^{17} + \\ & 59\,795\,770\,083\,083\,316\,221\,336\,805\,703\,680\,\alpha^{18} + 8\,987\,061\,025\,545\,721\,077\,834\,511\,810\,560\,\alpha^{19} + \\ & 1\,131\,237\,375\,988\,193\,565\,613\,353\,861\,120\,\alpha^{20} + 117\,704\,523\,870\,056\,936\,584\,154\,972\,160\,\alpha^{21} + \\ & 9\,941\,030\,662\,497\,120\,749\,554\,237\,440\,\alpha^{22} + 664\,040\,244\,922\,741\,425\,721\,835\,520\,\alpha^{23} + \\ & 33\,746\,986\,442\,943\,554\,031\,452\,160\,\alpha^{24} + 1\,225\,566\,587\,608\,656\,091\,545\,600\,\alpha^{25} + \\ & 28\,320\,365\,528\,012\,449\,382\,400\,\alpha^{26} + 312\,808\,771\,118\,086\,225\,920\,\alpha^{27}) \text{Seq}[\alpha] + \\ & (880\,540\,948\,213\,763\,261\,498\,004\,602\,880\,000 + 8\,086\,612\,414\,279\,581\,582\,690\,097\,299\,456\,000\,\alpha + \\ & 35\,535\,843\,625\,080\,580\,938\,628\,852\,403\,404\,800\,\alpha^2 + 99\,482\,199\,073\,846\,865\,130\,149\,987\,053\,731\,840\,\alpha^3 + \\ & 199\,278\,215\,238\,194\,877\,084\,174\,219\,759\,058\,944\,\alpha^4 + \\ & 304\,147\,288\,569\,704\,121\,767\,283\,668\,058\,636\,288\,\alpha^5 + \\ & 367\,726\,422\,460\,034\,552\,713\,877\,456\,306\,307\,072\,\alpha^6 + \\ & 361\,508\,986\,147\,801\,089\,153\,130\,211\,095\,805\,952\,\alpha^7 + \\ & 294\,331\,319\,744\,750\,632\,422\,172\,167\,712\,997\,376\,\alpha^8 + \\ & 201\,108\,607\,972\,501\,732\,293\,906\,606\,562\,934\,784\,\alpha^9 + \\ & 116\,437\,788\,942\,848\,727\,536\,075\,769\,222\,856\,704\,\alpha^{10} + \\ & 57\,524\,299\,296\,878\,619\,402\,424\,939\,339\,382\,784\,\alpha^{11} + \\ & 24\,367\,165\,878\,769\,872\,656\,509\,536\,747\,061\,248\,\alpha^{12} + 8\,877\,402\,295\,660\,764\,714\,512\,245\,808\,234\,496\,\alpha^{13} + \\ & 2\,785\,748\,984\,068\,408\,698\,625\,918\,477\,467\,648\,\alpha^{14} + 752\,972\,653\,647\,501\,430\,958\,086\,738\,673\,664\,\alpha^{15} + \\ & 175\,049\,743\,314\,674\,169\,771\,167\,299\,534\,848\,\alpha^{16} + 34\,895\,534\,864\,837\,208\,484\,258\,292\,957\,184\,\alpha^{17} + \\ & 5\,936\,277\,532\,573\,962\,980\,718\,997\,929\,984\,\alpha^{18} + 855\,818\,515\,821\,739\,179\,539\,429\,326\,848\,\alpha^{19} + \\ & 103\,560\,073\,600\,267\,246\,364\,541\,321\,216\,\alpha^{20} + 10\,380\,185\,487\,431\,012\,018\,005\,475\,328\,\alpha^{21} + \\ & 846\,180\,664\,706\,397\,472\,693\,420\,032\,\alpha^{22} + 54\,656\,640\,176\,185\,180\,963\,209\,216\,\alpha^{23} + \\ & 2\,690\,612\,916\,385\,314\,156\,576\,768\,\alpha^{24} + 94\,804\,345\,329\,795\,433\,758\,720\,\alpha^{25} + \\ & 2\,128\,785\,749\,082\,227\,343\,360\,\alpha^{26} + 22\,881\,382\,331\,785\,936\,896\,\alpha^{27}) \text{Seq}[1 + \alpha] + \\ & (-664\,078\,540\,666\,702\,251\,488\,371\,015\,680\,000 - 5\,805\,956\,958\,011\,506\,960\,041\,778\,348\,032\,000\,\alpha - \\ & 24\,298\,272\,789\,380\,152\,495\,188\,221\,126\,246\,400\,\alpha^2 - 64\,810\,405\,629\,301\,547\,428\,216\,819\,254\,558\,720\,\alpha^3 - \\ & 123\,755\,374\,367\,469\,269\,296\,809\,845\,353\,611\,264\,\alpha^4 - \\ & 180\,149\,375\,502\,996\,189\,202\,275\,648\,542\,982\,144\,\alpha^5 - \\ & 207\,865\,771\,244\,125\,682\,287\,781\,841\,861\,722\,112\,\alpha^6 - 195\,153\,222\,041\,523\,657\,876\,484\,723\,267\,989\,504\,\alpha^7 - \\ & 151\,846\,270\,858\,495\,120\,363\,896\,477\,860\,167\,680\,\alpha^8 - \\ & 99\,230\,231\,828\,276\,421\,932\,960\,434\,682\,314\,752\,\alpha^9 - 54\,993\,115\,047\,787\,497\,911\,079\,580\,675\,899\,392\,\alpha^{10} - \\ & 26\,028\,017\,908\,489\,825\,928\,212\,462\,245\,453\,824\,\alpha^{11} - \end{aligned}$$

$$\begin{aligned}
& 10\,572\,113\,416\,646\,586\,933\,511\,582\,698\,766\,336\,\alpha^{12} - 3\,696\,722\,231\,163\,815\,760\,173\,082\,026\,344\,448\,\alpha^{13} - \\
& 1\,114\,468\,173\,061\,041\,282\,670\,805\,399\,093\,248\,\alpha^{14} - 289\,688\,969\,845\,746\,113\,335\,461\,572\,931\,584\,\alpha^{15} - \\
& 64\,831\,091\,647\,102\,802\,811\,533\,842\,055\,168\,\alpha^{16} - 12\,454\,053\,009\,083\,005\,771\,527\,566\,163\,968\,\alpha^{17} - \\
& 2\,043\,760\,292\,966\,696\,499\,523\,264\,184\,320\,\alpha^{18} - 284\,532\,912\,366\,921\,324\,027\,166\,588\,928\,\alpha^{19} - \\
& 33\,284\,416\,956\,384\,385\,896\,458\,223\,616\,\alpha^{20} - 3\,228\,606\,478\,351\,534\,833\,828\,626\,432\,\alpha^{21} - \\
& 254\,974\,947\,491\,313\,890\,128\,560\,128\,\alpha^{22} - 15\,972\,126\,457\,377\,261\,067\,698\,176\,\alpha^{23} - \\
& 763\,333\,007\,662\,980\,725\,211\,136\,\alpha^{24} - 26\,138\,887\,552\,462\,651\,129\,856\,\alpha^{25} - \\
& 570\,997\,443\,951\,748\,710\,400\,\alpha^{26} - 5\,976\,795\,675\,008\,958\,464\,\alpha^{27}) \operatorname{Seq}[2 + \alpha] + \\
& (36\,337\,840\,931\,616\,555\,318\,702\,833\,664\,000 + 310\,343\,693\,247\,202\,072\,877\,171\,431\,833\,600\,\alpha + \\
& 1\,268\,062\,726\,217\,635\,641\,408\,454\,051\,430\,400\,\alpha^2 + 3\,300\,521\,955\,790\,071\,740\,463\,976\,232\,263\,680\,\alpha^3 + \\
& 6\,146\,984\,578\,367\,464\,065\,862\,054\,879\,242\,240\,\alpha^4 + 8\,723\,512\,529\,514\,925\,026\,222\,139\,080\,468\,480\,\alpha^5 + \\
& 9\,808\,817\,646\,565\,897\,068\,529\,809\,213\,239\,808\,\alpha^6 + 8\,970\,447\,157\,798\,999\,809\,214\,350\,039\,412\,224\,\alpha^7 + \\
& 6\,796\,618\,106\,855\,403\,262\,931\,535\,421\,469\,184\,\alpha^8 + 4\,323\,600\,610\,674\,086\,572\,350\,145\,316\,732\,416\,\alpha^9 + \\
& 2\,331\,860\,127\,398\,843\,166\,087\,931\,718\,971\,904\,\alpha^{10} + 1\,073\,804\,990\,271\,736\,796\,663\,841\,511\,156\,224\,\alpha^{11} + \\
& 424\,279\,297\,446\,148\,516\,898\,147\,199\,947\,264\,\alpha^{12} + 144\,293\,344\,557\,135\,741\,340\,883\,292\,465\,664\,\alpha^{13} + \\
& 42\,304\,696\,119\,152\,808\,149\,756\,544\,291\,840\,\alpha^{14} + 10\,693\,366\,157\,119\,575\,923\,154\,101\,714\,944\,\alpha^{15} + \\
& 2\,327\,102\,570\,668\,214\,059\,453\,238\,664\,192\,\alpha^{16} + 434\,708\,874\,971\,795\,823\,099\,840\,116\,736\,\alpha^{17} + \\
& 69\,373\,988\,097\,051\,870\,247\,906\,934\,784\,\alpha^{18} + 9\,393\,304\,762\,567\,159\,143\,035\,764\,736\,\alpha^{19} + \\
& 1\,068\,815\,757\,774\,279\,757\,481\,902\,080\,\alpha^{20} + 100\,861\,570\,825\,855\,881\,262\,923\,776\,\alpha^{21} + \\
& 7\,750\,770\,733\,439\,394\,600\,976\,384\,\alpha^{22} + 472\,551\,963\,878\,997\,639\,561\,216\,\alpha^{23} + \\
& 21\,986\,541\,883\,647\,884\,001\,280\,\alpha^{24} + 733\,188\,729\,988\,561\,502\,208\,\alpha^{25} + \\
& 15\,602\,375\,112\,618\,147\,840\,\alpha^{26} + 159\,149\,910\,074\,064\,896\,\alpha^{27}) \operatorname{Seq}[3 + \alpha] + \\
& (1\,737\,772\,868\,400\,007\,324\,872\,130\,560\,000 + 14\,528\,609\,204\,414\,291\,845\,066\,255\,564\,800\,\alpha + \\
& 58\,083\,087\,258\,852\,534\,411\,685\,975\,019\,520\,\alpha^2 + 147\,846\,850\,915\,658\,722\,383\,612\,355\,430\,400\,\alpha^3 + \\
& 269\,164\,023\,324\,400\,460\,962\,054\,275\,740\,928\,\alpha^4 + 373\,240\,816\,513\,597\,979\,905\,593\,440\,661\,888\,\alpha^5 + \\
& 409\,908\,879\,949\,766\,514\,326\,399\,060\,864\,064\,\alpha^6 + 366\,016\,393\,873\,249\,701\,940\,597\,734\,061\,344\,\alpha^7 + \\
& 270\,676\,671\,846\,416\,971\,917\,873\,052\,917\,920\,\alpha^8 + 168\,013\,318\,310\,785\,666\,403\,759\,927\,887\,584\,\alpha^9 + \\
& 88\,393\,926\,598\,940\,439\,065\,183\,725\,045\,600\,\alpha^{10} + 39\,697\,363\,634\,496\,672\,642\,069\,844\,386\,912\,\alpha^{11} + \\
& 15\,293\,672\,611\,896\,263\,618\,803\,193\,519\,136\,\alpha^{12} + 5\,070\,491\,874\,452\,377\,148\,797\,920\,831\,072\,\alpha^{13} + \\
& 1\,449\,002\,022\,519\,967\,409\,403\,051\,116\,512\,\alpha^{14} + 356\,957\,682\,436\,813\,381\,749\,659\,746\,304\,\alpha^{15} + \\
& 75\,700\,244\,148\,872\,939\,301\,421\,992\,640\,\alpha^{16} + 13\,779\,371\,789\,456\,905\,170\,877\,563\,840\,\alpha^{17} + \\
& 2\,142\,685\,081\,818\,193\,152\,012\,367\,872\,\alpha^{18} + 282\,685\,926\,147\,777\,894\,282\,083\,328\,\alpha^{19} + \\
& 31\,341\,335\,886\,140\,485\,043\,322\,880\,\alpha^{20} + 2\,881\,942\,426\,887\,984\,021\,438\,464\,\alpha^{21} + \\
& 215\,812\,414\,752\,103\,173\,455\,872\,\alpha^{22} + 12\,823\,036\,513\,484\,289\,343\,488\,\alpha^{23} + \\
& 581\,508\,878\,853\,457\,575\,936\,\alpha^{24} + 18\,903\,053\,117\,719\,314\,432\,\alpha^{25} + \\
& 392\,186\,219\,850\,629\,120\,\alpha^{26} + 3\,900\,964\,176\,134\,144\,\alpha^{27}) \operatorname{Seq}[4 + \alpha] + \\
& (-36\,446\,102\,109\,669\,030\,849\,285\,120\,000 - 301\,794\,930\,778\,773\,719\,063\,321\,856\,000\,\alpha - \\
& 1\,194\,401\,836\,156\,084\,887\,609\,064\,224\,000\,\alpha^2 - 3\,008\,156\,975\,709\,477\,795\,289\,491\,275\,520\,\alpha^3 - \\
& 5\,415\,770\,546\,395\,539\,670\,222\,530\,489\,360\,\alpha^4 - 7\,422\,453\,554\,874\,065\,600\,190\,474\,289\,032\,\alpha^5 - \\
& 8\,052\,206\,383\,842\,449\,223\,124\,682\,104\,644\,\alpha^6 - 7\,098\,162\,826\,794\,167\,361\,280\,152\,144\,294\,\alpha^7 - \\
& 5\,179\,144\,111\,408\,801\,590\,076\,035\,892\,950\,\alpha^8 - 3\,169\,950\,795\,733\,038\,711\,522\,140\,215\,280\,\alpha^9 - \\
& 1\,643\,499\,248\,947\,095\,475\,104\,215\,404\,004\,\alpha^{10} - 726\,910\,788\,718\,026\,537\,302\,273\,862\,144\,\alpha^{11} - \\
& 275\,635\,972\,025\,251\,416\,199\,969\,761\,656\,\alpha^{12} - 89\,889\,728\,147\,001\,421\,773\,544\,625\,132\,\alpha^{13} - \\
& 25\,251\,994\,806\,501\,150\,584\,061\,125\,784\,\alpha^{14} - 6\,111\,409\,098\,652\,595\,993\,659\,452\,026\,\alpha^{15} - \\
& 1\,272\,483\,225\,563\,071\,816\,917\,699\,490\,\alpha^{16} - 227\,273\,250\,419\,552\,627\,170\,585\,084\,\alpha^{17} - \\
& 34\,655\,941\,701\,831\,856\,557\,922\,624\,\alpha^{18} - 4\,480\,880\,404\,407\,427\,210\,024\,320\,\alpha^{19} - \\
& 486\,585\,842\,769\,876\,461\,484\,032\,\alpha^{20} - 43\,798\,304\,089\,562\,788\,663\,296\,\alpha^{21} - \\
& 3\,208\,710\,131\,027\,557\,023\,744\,\alpha^{22} - 186\,416\,522\,833\,559\,945\,216\,\alpha^{23} - 8\,261\,380\,192\,874\,790\,912\,\alpha^{24} - \\
& 262\,301\,388\,296\,421\,376\,\alpha^{25} - 5\,312\,632\,953\,241\,600\,\alpha^{26} - 51\,561\,082\,388\,480\,\alpha^{27}) \operatorname{Seq}[5 + \alpha] + \\
& (-154\,404\,486\,709\,237\,819\,219\,968\,000 - 1\,265\,327\,918\,255\,018\,927\,110\,348\,800\,\alpha - \\
& 4\,953\,641\,658\,930\,095\,511\,385\,751\,040\,\alpha^2 - 12\,335\,446\,851\,783\,544\,166\,937\,390\,720\,\alpha^3 -
\end{aligned}$$

$$\begin{aligned}
& 21\,947\,702\,123\,383\,074\,616\,990\,244\,544\,\alpha^4 - 29\,712\,684\,443\,300\,038\,100\,072\,561\,760\,\alpha^5 - \\
& 31\,824\,626\,177\,807\,101\,870\,129\,360\,368\,\alpha^6 - 27\,684\,339\,638\,906\,598\,652\,692\,786\,888\,\alpha^7 - \\
& 19\,923\,668\,408\,873\,674\,929\,361\,243\,572\,\alpha^8 - 12\,021\,754\,897\,932\,453\,908\,473\,126\,194\,\alpha^9 - \\
& 6\,141\,402\,912\,303\,808\,338\,721\,284\,327\,\alpha^{10} - 2\,675\,090\,519\,652\,464\,763\,702\,625\,995\,\alpha^{11} - \\
& 998\,451\,712\,547\,824\,111\,144\,656\,513\,\alpha^{12} - 320\,337\,381\,856\,256\,276\,567\,115\,789\,\alpha^{13} - \\
& 88\,485\,146\,094\,830\,787\,771\,471\,525\,\alpha^{14} - 21\,045\,641\,782\,461\,353\,200\,898\,049\,\alpha^{15} - \\
& 4\,304\,140\,182\,149\,530\,399\,276\,227\,\alpha^{16} - 754\,678\,659\,252\,915\,954\,749\,073\,\alpha^{17} - \\
& 112\,910\,766\,050\,133\,819\,763\,020\,\alpha^{18} - 14\,316\,213\,223\,182\,938\,203\,068\,\alpha^{19} - \\
& 1\,523\,679\,350\,645\,560\,062\,336\,\alpha^{20} - 134\,345\,128\,624\,663\,841\,280\,\alpha^{21} - \\
& 9\,635\,762\,018\,738\,626\,560\,\alpha^{22} - 547\,760\,583\,383\,666\,688\,\alpha^{23} - 23\,739\,371\,943\,886\,848\,\alpha^{24} - \\
& 736\,693\,272\,182\,784\,\alpha^{25} - 14\,575\,541\,944\,320\,\alpha^{26} - 138\,110\,042\,112\,\alpha^{27} \Big) \text{Seq}[6 + \alpha]
\end{aligned}$$