Multi-headed Lattice Green Function (N = 4, M = 2)
Find minimal recurrence for the coefficients

In[∘]:=
```
NN = 4;
MM = 2;
```

Generate a sequence from recurrence & initial values
Koutschan’s implementation

In[∘]:=
```
(* given a recurrence rec in f[n], compute the values {f[0],f[1],...,f[bound]}
 where inits are the initial values
 {f[0],...,f[d-1]} with d being the order of the recurrence *)
Clear[UnrollRecurrence];
UnrollRecurrence[rec1_, f_[n_], inits_, bound_] :=
  Module[{i, x, vals = inits, rec = rec1},
   If[Head[rec] =!= Equal, rec = (rec == 0)];
   rec = rec /. n → n - Max[Cases[rec, f[n + a_.] ⧴ a, Infinity]];
   Do[
    AppendTo[vals, Solve[rec /. n → i /. f[i] → x /. f[a_] ⧴ vals[[a + 1]], x][[1, 1, 2]]];
    , {i, Length[inits], bound}];
   Return[vals];
  ];
```

In[∘]:=
```
<< RISC`HolonomicFunctions`
```

HolonomicFunctions Package version 1.7.3 (21-Mar-2017)
written by Christoph Koutschan

--> Type ?HolonomicFunctions for help.

In[∘]:=
```
ClearAll[z, w, α, β, ξ];
```

Import known ODE for *R*(*w*)

Guttmann (2010), p. 6

In[∘]:=
```
ODEDiv2 = {(-1 + z) z^3 (2 + z) (3 + z) (6 + z) (8 + z) (4 + 3 z)^2 D_z^4 +
     2 z^2 (4 + 3 z) (-3456 - 2304 z + 3676 z^2 + 4920 z^3 + 2079 z^4 + 356 z^5 + 21 z^6) D_z^3 +
     6 z (-5376 - 5248 z + 11 080 z^2 + 25 286 z^3 + 19 898 z^4 + 7432 z^5 + 1286 z^6 + 81 z^7) D_z^2 +
     12 (-384 + 224 z + 3716 z^2 + 7633 z^3 + 6734 z^4 + 2939 z^5 + 604 z^6 + 45 z^7) D_z +
     12 z (256 + 632 z + 702 z^2 + 382 z^3 + 98 z^4 + 9 z^5)} /. {D_z → Der[z]};
ToOrePolynomial[
 ODEDiv2]
```

Out[∘]=
$$\begin{aligned}\{&(-4608 z^3 - 7488 z^4 - 256 z^5 + 6156 z^6 + 4608 z^7 + 1393 z^8 + 186 z^9 + 9 z^{10})\, D_z^4 +\\ &(-27\,648 z^2 - 39\,168 z^3 + 15\,584 z^4 + 61\,416 z^5 + 46\,152 z^6 + 15\,322 z^7 + 2304 z^8 + 126 z^9)\, D_z^3 +\\ &(-32\,256 z - 31\,488 z^2 + 66\,480 z^3 + 151\,716 z^4 + 119\,388 z^5 + 44\,592 z^6 + 7716 z^7 + 486 z^8)\, D_z^2 +\\ &(-4608 + 2688 z + 44\,592 z^2 + 91\,596 z^3 + 80\,808 z^4 + 35\,268 z^5 + 7248 z^6 + 540 z^7)\, D_z +\\ &(3072 z + 7584 z^2 + 8424 z^3 + 4584 z^4 + 1176 z^5 + 108 z^6)\}\end{aligned}$$

In[∘]:=
```
ODENormalized = -DFiniteSubstitute[ToOrePolynomial[ODEDiv2],
    {z → w * 2^MM * Binomial[NN, MM]}, Algebra → OreAlgebra[Der[w]]];
ToOrePolynomial[ODENormalized]
```

Out[∘]=
$$\begin{aligned}\{&(w^3 + 39 w^4 + 32 w^5 - 18\,468 w^6 - 331\,776 w^7 - 2\,407\,104 w^8 - 7\,713\,792 w^9 - 8\,957\,952 w^{10})\, D_w^4 +\\ &(6 w^2 + 204 w^3 - 1948 w^4 - 184\,248 w^5 - 3\,322\,944 w^6 - 26\,476\,416 w^7 - 95\,551\,488 w^8 - 125\,411\,328 w^9)\, D_w^3 +\\ &(7 w + 164 w^2 - 8310 w^3 - 455\,148 w^4 - 8\,595\,936 w^5 - 77\,054\,976 w^6 - 319\,997\,952 w^7 - 483\,729\,408 w^8)\, D_w^2 +\\ &(1 - 14 w - 5574 w^2 - 274\,788 w^3 - 5\,818\,176 w^4 - 60\,943\,104 w^5 - 300\,589\,056 w^6 - 537\,477\,120 w^7)\, D_w +\\ &(-384 w - 22\,752 w^2 - 606\,528 w^3 - 7\,921\,152 w^4 - 48\,771\,072 w^5 - 107\,495\,424 w^6)\}\end{aligned}$$

In[∘]:=
```
ODENormalizedinD = ODENormalized[[1]];
ToOrePolynomial[ODENormalizedinD]
```

Out[∘]=
$$\begin{aligned}&(w^3 + 39 w^4 + 32 w^5 - 18\,468 w^6 - 331\,776 w^7 - 2\,407\,104 w^8 - 7\,713\,792 w^9 - 8\,957\,952 w^{10})\, D_w^4 +\\ &(6 w^2 + 204 w^3 - 1948 w^4 - 184\,248 w^5 - 3\,322\,944 w^6 - 26\,476\,416 w^7 - 95\,551\,488 w^8 - 125\,411\,328 w^9)\, D_w^3 +\\ &(7 w + 164 w^2 - 8310 w^3 - 455\,148 w^4 - 8\,595\,936 w^5 - 77\,054\,976 w^6 - 319\,997\,952 w^7 - 483\,729\,408 w^8)\, D_w^2 +\\ &(1 - 14 w - 5574 w^2 - 274\,788 w^3 - 5\,818\,176 w^4 - 60\,943\,104 w^5 - 300\,589\,056 w^6 - 537\,477\,120 w^7)\, D_w +\\ &(-384 w - 22\,752 w^2 - 606\,528 w^3 - 7\,921\,152 w^4 - 48\,771\,072 w^5 - 107\,495\,424 w^6)\end{aligned}$$

In[∘]:=
```
ODENormalizedinTheta = ChangeOreAlgebra[w ** ODENormalizedinD, OreAlgebra[Euler[w]]];
ToOrePolynomial[ODENormalizedinTheta]
```

Out[∘]=
$$\begin{aligned}&(1 + 39 w + 32 w^2 - 18\,468 w^3 - 331\,776 w^4 - 2\,407\,104 w^5 - 7\,713\,792 w^6 - 8\,957\,952 w^7)\, \theta_w^4 +\\ &(-30 w - 2140 w^2 - 73\,440 w^3 - 1\,332\,288 w^4 - 12\,033\,792 w^5 - 49\,268\,736 w^6 - 71\,663\,616 w^7)\, \theta_w^3 +\\ &(-19 w - 2114 w^2 - 105\,552 w^3 - 2\,276\,640 w^4 - 24\,103\,872 w^5 - 118\,195\,200 w^6 - 206\,032\,896 w^7)\, \theta_w^2 +\\ &(-4 w - 1352 w^2 - 77\,328 w^3 - 1\,877\,472 w^4 - 22\,398\,336 w^5 - 125\,411\,328 w^6 - 250\,822\,656 w^7)\, \theta_w +\\ &(-384 w^2 - 22\,752 w^3 - 606\,528 w^4 - 7\,921\,152 w^5 - 48\,771\,072 w^6 - 107\,495\,424 w^7)\end{aligned}$$

Recurrence for {r(0), r(1), r(2), ...}.

In[ ]:=
```
RECNormalized = DFiniteDE2RE[ToOrePolynomial[ODENormalized], {w}, {α}];
ToOrePolynomial[RECNormalized]
```

Out[ ]=
$$\{(2401 + 1372\alpha + 294\alpha^2 + 28\alpha^3 + \alpha^4)\, S_\alpha^7 + (43356 + 30224\alpha + 7865\alpha^2 + 906\alpha^3 + 39\alpha^4)\, S_\alpha^6 + (-307494 - 166992\alpha - 29414\alpha^2 - 1500\alpha^3 + 32\alpha^4)\, S_\alpha^5 + (-11448864 - 9174672\alpha - 2759760\alpha^2 - 368928\alpha^3 - 18468\alpha^4)\, S_\alpha^4 + (-89574336 - 87340896\alpha - 32183136\alpha^2 - 5313600\alpha^3 - 331776\alpha^4)\, S_\alpha^3 + (-283917312 - 340246656\alpha - 154077120\alpha^2 - 31290624\alpha^3 - 2407104\alpha^4)\, S_\alpha^2 + (-349360128 - 540463104\alpha - 312284160\alpha^2 - 80123904\alpha^3 - 7713792\alpha^4)\, S_\alpha + (-107495424 - 250822656\alpha - 206032896\alpha^2 - 71663616\alpha^3 - 8957952\alpha^4)\}$$

In[ ]:=
```
RECNormalizedinS = RECNormalized[[1]];
ToOrePolynomial[RECNormalizedinS]
```

Out[ ]=
$$(2401 + 1372\alpha + 294\alpha^2 + 28\alpha^3 + \alpha^4)\, S_\alpha^7 + (43356 + 30224\alpha + 7865\alpha^2 + 906\alpha^3 + 39\alpha^4)\, S_\alpha^6 + (-307494 - 166992\alpha - 29414\alpha^2 - 1500\alpha^3 + 32\alpha^4)\, S_\alpha^5 + (-11448864 - 9174672\alpha - 2759760\alpha^2 - 368928\alpha^3 - 18468\alpha^4)\, S_\alpha^4 + (-89574336 - 87340896\alpha - 32183136\alpha^2 - 5313600\alpha^3 - 331776\alpha^4)\, S_\alpha^3 + (-283917312 - 340246656\alpha - 154077120\alpha^2 - 31290624\alpha^3 - 2407104\alpha^4)\, S_\alpha^2 + (-349360128 - 540463104\alpha - 312284160\alpha^2 - 80123904\alpha^3 - 7713792\alpha^4)\, S_\alpha + (-107495424 - 250822656\alpha - 206032896\alpha^2 - 71663616\alpha^3 - 8957952\alpha^4)$$

In[ ]:=
```
RecNormalizedOrder = OrePolynomialDegree[RECNormalizedinS, S[α]]
```

Out[ ]= 7

Write recurrence explicitly.

In[ ]:=
```
ClearAll[Seq];
SeqNormalized = ApplyOreOperator[RECNormalizedinS, Seq[α]]
```

Out[ ]=
$$(-107495424 - 250822656\alpha - 206032896\alpha^2 - 71663616\alpha^3 - 8957952\alpha^4)\, \mathrm{Seq}[\alpha] + (-349360128 - 540463104\alpha - 312284160\alpha^2 - 80123904\alpha^3 - 7713792\alpha^4)\, \mathrm{Seq}[1+\alpha] + (-283917312 - 340246656\alpha - 154077120\alpha^2 - 31290624\alpha^3 - 2407104\alpha^4)\, \mathrm{Seq}[2+\alpha] + (-89574336 - 87340896\alpha - 32183136\alpha^2 - 5313600\alpha^3 - 331776\alpha^4)\, \mathrm{Seq}[3+\alpha] + (-11448864 - 9174672\alpha - 2759760\alpha^2 - 368928\alpha^3 - 18468\alpha^4)\, \mathrm{Seq}[4+\alpha] + (-307494 - 166992\alpha - 29414\alpha^2 - 1500\alpha^3 + 32\alpha^4)\, \mathrm{Seq}[5+\alpha] + (43356 + 30224\alpha + 7865\alpha^2 + 906\alpha^3 + 39\alpha^4)\, \mathrm{Seq}[6+\alpha] + (2401 + 1372\alpha + 294\alpha^2 + 28\alpha^3 + \alpha^4)\, \mathrm{Seq}[7+\alpha]$$

Initial values of {r(0), r(1), r(2), ...}

In[∘]:=
```
SeqListIni = {1};

sympoly = SymmetricPolynomial[MM, Table[Indexed[ξ, i] + Indexed[ξ, i]^-1, {i, 1, NN}]];

MAX = 10;

sympolypower = 1;

For[n = 1, n ≤ MAX, n++,
  sympolypower = Expand[sympolypower * sympoly];
  p = Coefficient[Expand[sympolypower * Product[Indexed[ξ, i], {i, 1, NN}]],
    Product[Indexed[ξ, i], {i, 1, NN}]];
  SeqListIni = Append[SeqListIni, p];
 ];

SeqListIni

seq[n_] := SeqListIni[[n + 1]];
```

Out[∘]=
```
{1, 0, 24, 192, 3384, 51840, 911040, 16369920, 307009080, 5902176000, 116083727424}
```

Verify recurrence by initial values

In[∘]:=
```
Table[SeqNormalized /. {Seq → seq, α → n}, {n, 0, MAX – RecNormalizedOrder}]
```

Out[∘]=
```
{0, 0, 0, 0}
```

Generate more terms in the sequence
SeqList[[n]] = r(n)

In[∘]:=
```
Bound = 200;

SeqList = UnrollRecurrence[SeqNormalized, Seq[α], SeqListIni, Bound];

seq[n_] := SeqList[[n + 1]];
```

Let’s guess (and prove!) a shorter recurrence.

In[∘]:=
```
<< RISC`Guess`
```

Package GeneratingFunctions version 0.8 written by Christian Mallinger
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria

Guess Package version 0.52
written by Manuel Kauers

In[◦]:=
```
SeqGuess = GuessMinRE[Take[SeqList, 100], Seq[α]]
```

Out[◦]=
$$\left(-\frac{287649792}{35}-\frac{112472064\,\alpha}{5}-\frac{833891328\,\alpha^2}{35}-\frac{441427968\,\alpha^3}{35}-\frac{123641856\,\alpha^4}{35}-497664\,\alpha^5-27648\,\alpha^6\right)\mathrm{Seq}[\alpha]+\left(-\frac{708258816}{35}-\frac{202493952\,\alpha}{5}-\frac{166005504\,\alpha^2}{5}-\frac{498714624\,\alpha^3}{35}-\frac{117891072\,\alpha^4}{35}-414720\,\alpha^5-20736\,\alpha^6\right)\mathrm{Seq}[1+\alpha]+\left(-\frac{75831552}{7}-\frac{643100256\,\alpha}{35}-\frac{452539152\,\alpha^2}{35}-\frac{33779520\,\alpha^3}{7}-1005984\,\alpha^4-110880\,\alpha^5-5040\,\alpha^6\right)\mathrm{Seq}[2+\alpha]+\left(-\frac{55519056}{35}-\frac{84088296\,\alpha}{35}-\frac{10599424\,\alpha^2}{7}-\frac{3557208\,\alpha^3}{7}-\frac{670240\,\alpha^4}{7}-9600\,\alpha^5-400\,\alpha^6\right)\mathrm{Seq}[3+\alpha]+\left(\frac{638976}{35}+\frac{904864\,\alpha}{35}+\frac{76184\,\alpha^2}{5}+\frac{167156\,\alpha^3}{35}+\frac{29341\,\alpha^4}{35}+78\,\alpha^5+3\,\alpha^6\right)\mathrm{Seq}[4+\alpha]+\left(\frac{69000}{7}+\frac{90200\,\alpha}{7}+\frac{48935\,\alpha^2}{7}+\frac{14108\,\alpha^3}{7}+\frac{11402\,\alpha^4}{35}+28\,\alpha^5+\alpha^6\right)\mathrm{Seq}[5+\alpha]$$

In[◦]:=
```
SeqGuess = SeqGuess * 35;
```

In[◦]:=
```
RECGuess = ToOrePolynomial[{ReplaceAll[SeqGuess, Seq[n_] :> S[α]^(n-α)]}]
```

Out[◦]=
$$\{(345000+451000\,\alpha+244675\,\alpha^2+70540\,\alpha^3+11402\,\alpha^4+980\,\alpha^5+35\,\alpha^6)\,S_\alpha^5+(638976+904864\,\alpha+533288\,\alpha^2+167156\,\alpha^3+29341\,\alpha^4+2730\,\alpha^5+105\,\alpha^6)\,S_\alpha^4+(-55519056-84088296\,\alpha-52997120\,\alpha^2-17786040\,\alpha^3-3351200\,\alpha^4-336000\,\alpha^5-14000\,\alpha^6)\,S_\alpha^3+(-379157760-643100256\,\alpha-452539152\,\alpha^2-168897600\,\alpha^3-35209440\,\alpha^4-3880800\,\alpha^5-176400\,\alpha^6)\,S_\alpha^2+(-708258816-1417457664\,\alpha-1162038528\,\alpha^2-498714624\,\alpha^3-117891072\,\alpha^4-14515200\,\alpha^5-725760\,\alpha^6)\,S_\alpha+(-287649792-787304448\,\alpha-833891328\,\alpha^2-441427968\,\alpha^3-123641856\,\alpha^4-17418240\,\alpha^5-967680\,\alpha^6)\}$$

In[◦]:=
```
ClearAll[Seq];
SeqGuess = ApplyOreOperator[RECGuess[[1]], Seq[α]]
```

Out[◦]=
$$(-287649792-787304448\,\alpha-833891328\,\alpha^2-441427968\,\alpha^3-123641856\,\alpha^4-17418240\,\alpha^5-967680\,\alpha^6)\,\mathrm{Seq}[\alpha]+(-708258816-1417457664\,\alpha-1162038528\,\alpha^2-498714624\,\alpha^3-117891072\,\alpha^4-14515200\,\alpha^5-725760\,\alpha^6)\,\mathrm{Seq}[1+\alpha]+(-379157760-643100256\,\alpha-452539152\,\alpha^2-168897600\,\alpha^3-35209440\,\alpha^4-3880800\,\alpha^5-176400\,\alpha^6)\,\mathrm{Seq}[2+\alpha]+(-55519056-84088296\,\alpha-52997120\,\alpha^2-17786040\,\alpha^3-3351200\,\alpha^4-336000\,\alpha^5-14000\,\alpha^6)\,\mathrm{Seq}[3+\alpha]+(638976+904864\,\alpha+533288\,\alpha^2+167156\,\alpha^3+29341\,\alpha^4+2730\,\alpha^5+105\,\alpha^6)\,\mathrm{Seq}[4+\alpha]+(345000+451000\,\alpha+244675\,\alpha^2+70540\,\alpha^3+11402\,\alpha^4+980\,\alpha^5+35\,\alpha^6)\,\mathrm{Seq}[5+\alpha]$$

In[∘]:=
```
RECCompare = DFinitePlus[RECNormalized, RECGuess];
ToOrePolynomial[RECCompare]
```

Out[∘]=
$$\{(2401 + 1372\,\alpha + 294\,\alpha^2 + 28\,\alpha^3 + \alpha^4)\,S_\alpha^7 + (43\,356 + 30\,224\,\alpha + 7865\,\alpha^2 + 906\,\alpha^3 + 39\,\alpha^4)\,S_\alpha^6 + (-307\,494 - 166\,992\,\alpha - 29\,414\,\alpha^2 - 1500\,\alpha^3 + 32\,\alpha^4)\,S_\alpha^5 + (-11\,448\,864 - 9\,174\,672\,\alpha - 2\,759\,760\,\alpha^2 - 368\,928\,\alpha^3 - 18\,468\,\alpha^4)\,S_\alpha^4 + (-89\,574\,336 - 87\,340\,896\,\alpha - 32\,183\,136\,\alpha^2 - 5\,313\,600\,\alpha^3 - 331\,776\,\alpha^4)\,S_\alpha^3 + (-283\,917\,312 - 340\,246\,656\,\alpha - 154\,077\,120\,\alpha^2 - 31\,290\,624\,\alpha^3 - 2\,407\,104\,\alpha^4)\,S_\alpha^2 + (-349\,360\,128 - 540\,463\,104\,\alpha - 312\,284\,160\,\alpha^2 - 80\,123\,904\,\alpha^3 - 7\,713\,792\,\alpha^4)\,S_\alpha + (-107\,495\,424 - 250\,822\,656\,\alpha - 206\,032\,896\,\alpha^2 - 71\,663\,616\,\alpha^3 - 8\,957\,952\,\alpha^4)\}$$

In[∘]:=
```
RECCompareOrder = OrePolynomialDegree[RECNormalizedinS, S[α]]
```

Out[∘]= 7

The above argument means that if the sequence generated by “RECGuess” matches with that by “RECNormalized” for the first “RECCompareOrder” terms, then the two sequences are identical.

Hence, we get a rigorous proof of the shorter recurrence “RECGuess” by the following verification!

In[∘]:=
```
SeqListIni
```

Out[∘]= {1, 0, 24, 192, 3384, 51 840, 911 040, 16 369 920, 307 009 080, 5 902 176 000, 116 083 727 424}

In[∘]:=
```
CheckNum = RECCompareOrder + 20 + Length@SeqListIni

SeqGuessList = UnrollRecurrence[SeqGuess, Seq[α], SeqListIni, CheckNum];
SeqGuessList - Take[SeqList, Length@SeqGuessList]
```

Out[∘]= 38

Out[∘]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

Transform guessed recurrence for $r(n)$ back to ODE for $R(w)$

In[∘]:=
```
RECGuessOrder = OrePolynomialDegree[RECGuess[[1]], S[α]]
RECGuessDetails = RECGuess[[1, 1]]
```

Out[∘]= 5

Out[∘]=
$$\{\{345\,000 + 451\,000\,\alpha + 244\,675\,\alpha^2 + 70\,540\,\alpha^3 + 11\,402\,\alpha^4 + 980\,\alpha^5 + 35\,\alpha^6, \{5\}\}, \{638\,976 + 904\,864\,\alpha + 533\,288\,\alpha^2 + 167\,156\,\alpha^3 + 29\,341\,\alpha^4 + 2730\,\alpha^5 + 105\,\alpha^6, \{4\}\}, \{-55\,519\,056 - 84\,088\,296\,\alpha - 52\,997\,120\,\alpha^2 - 17\,786\,040\,\alpha^3 - 3\,351\,200\,\alpha^4 - 336\,000\,\alpha^5 - 14\,000\,\alpha^6, \{3\}\}, \{-379\,157\,760 - 643\,100\,256\,\alpha - 452\,539\,152\,\alpha^2 - 168\,897\,600\,\alpha^3 - 35\,209\,440\,\alpha^4 - 3\,880\,800\,\alpha^5 - 176\,400\,\alpha^6, \{2\}\}, \{-708\,258\,816 - 1\,417\,457\,664\,\alpha - 1\,162\,038\,528\,\alpha^2 - 498\,714\,624\,\alpha^3 - 117\,891\,072\,\alpha^4 - 14\,515\,200\,\alpha^5 - 725\,760\,\alpha^6, \{1\}\}, \{-287\,649\,792 - 787\,304\,448\,\alpha - 833\,891\,328\,\alpha^2 - 441\,427\,968\,\alpha^3 - 123\,641\,856\,\alpha^4 - 17\,418\,240\,\alpha^5 - 967\,680\,\alpha^6, \{0\}\}\}$$

In[∘]:=
```
ODEGuessinTheta = Sum[w^(RECGuessOrder-RECGuessDetails[[i,2]][[1]]) **
    Expand[RECGuessDetails[[i, 1]] /. {α → Euler[w] - RECGuessDetails[[i, 2]][[1]]}],
   {i, 1, Length@RECGuessDetails}];
ToOrePolynomial[ODEGuessinTheta]
```

Out[∘]=
$$\left(35+105 w-14000 w^2-176400 w^3-725760 w^4-967680 w^5\right) \theta_w^6+\left(-70+210 w-84000 w^2-1764000 w^3-10160640 w^4-17418240 w^5\right) \theta_w^5+\left(27-59 w-201200 w^2-6985440 w^3-56201472 w^4-123641856 w^5\right) \theta_w^4+\left(100 w-251640 w^2-14230080 w^3-157787136 w^4-441427968 w^5\right) \theta_w^3+\left(152 w-177560 w^2-16052112 w^3-238975488 w^4-833891328 w^5\right) \theta_w^2+\left(32 w-67056 w^2-9607968 w^3-186181632 w^4-787304448 w^5\right) \theta_w+\left(-10368 w^2-2388096 w^3-58226688 w^4-287649792 w^5\right)$$

In[∘]:=
```
ODEGuessinD =
  ChangeOreAlgebra[ToOrePolynomial[w^-1 ** ODEGuessinTheta], OreAlgebra[Der[w]]];
ToOrePolynomial[ODEGuessinD]
```

Out[∘]=
$$\left(35 w^5+105 w^6-14000 w^7-176400 w^8-725760 w^9-967680 w^{10}\right) D_w^6+\left(455 w^4+1785 w^5-294000 w^6-4410000 w^7-21047040 w^8-31933440 w^9\right) D_w^5+\left(1602 w^3+8866 w^4-1951200 w^5-36091440 w^6-204982272 w^7-360723456 w^8\right) D_w^4+\left(1562 w^2+14446 w^3-4818840 w^4-116118720 w^5-814330368 w^6-1705826304 w^7\right) D_w^3+\left(224 w+6444 w^2-4034880 w^3-139568832 w^4-1280655360 w^5-3314939904 w^6\right) D_w^2+\left(-8+540 w-795456 w^2-48816000 w^3-650032128 w^4-2204651520 w^5\right) D_w+\left(-10368 w-2388096 w^2-58226688 w^3-287649792 w^4\right)$$

In[∘]:=
```
ODEGuess = {ODEGuessinD};
ToOrePolynomial[ODEGuess]
```

Out[∘]=
$$\left\{\left(35 w^5+105 w^6-14000 w^7-176400 w^8-725760 w^9-967680 w^{10}\right) D_w^6+\left(455 w^4+1785 w^5-294000 w^6-4410000 w^7-21047040 w^8-31933440 w^9\right) D_w^5+\left(1602 w^3+8866 w^4-1951200 w^5-36091440 w^6-204982272 w^7-360723456 w^8\right) D_w^4+\left(1562 w^2+14446 w^3-4818840 w^4-116118720 w^5-814330368 w^6-1705826304 w^7\right) D_w^3+\left(224 w+6444 w^2-4034880 w^3-139568832 w^4-1280655360 w^5-3314939904 w^6\right) D_w^2+\left(-8+540 w-795456 w^2-48816000 w^3-650032128 w^4-2204651520 w^5\right) D_w+\left(-10368 w-2388096 w^2-58226688 w^3-287649792 w^4\right)\right\}$$

Compare with the known ODE

In[∘]:=
```
ToOrePolynomial[ODEGuessinTheta]
```

Out[∘]=
$$\left(35+105 w-14000 w^2-176400 w^3-725760 w^4-967680 w^5\right) \theta_w^6+\left(-70+210 w-84000 w^2-1764000 w^3-10160640 w^4-17418240 w^5\right) \theta_w^5+\left(27-59 w-201200 w^2-6985440 w^3-56201472 w^4-123641856 w^5\right) \theta_w^4+\left(100 w-251640 w^2-14230080 w^3-157787136 w^4-441427968 w^5\right) \theta_w^3+\left(152 w-177560 w^2-16052112 w^3-238975488 w^4-833891328 w^5\right) \theta_w^2+\left(32 w-67056 w^2-9607968 w^3-186181632 w^4-787304448 w^5\right) \theta_w+\left(-10368 w^2-2388096 w^3-58226688 w^4-287649792 w^5\right)$$

In[ ]:= **ToOrePolynomial[ODENormalizedinTheta]**

Out[ ]=

$$\left(1+39\,w+32\,w^2-18\,468\,w^3-331\,776\,w^4-2\,407\,104\,w^5-7\,713\,792\,w^6-8\,957\,952\,w^7\right)\,\theta_w^4+\left(-30\,w-2140\,w^2-73\,440\,w^3-1\,332\,288\,w^4-12\,033\,792\,w^5-49\,268\,736\,w^6-71\,663\,616\,w^7\right)\,\theta_w^3+\left(-19\,w-2114\,w^2-105\,552\,w^3-2\,276\,640\,w^4-24\,103\,872\,w^5-118\,195\,200\,w^6-206\,032\,896\,w^7\right)\,\theta_w^2+\left(-4\,w-1352\,w^2-77\,328\,w^3-1\,877\,472\,w^4-22\,398\,336\,w^5-125\,411\,328\,w^6-250\,822\,656\,w^7\right)\,\theta_w+\left(-384\,w^2-22\,752\,w^3-606\,528\,w^4-7\,921\,152\,w^5-48\,771\,072\,w^6-107\,495\,424\,w^7\right)$$