

# Multi-headed Lattice Green Function (N = 5, M = 4)

## ODE for $R_{4,5}(z)$ in Theorem 4.6

$$\begin{aligned} \text{Out}[z] = & (1968300 - 14377372992z - 31378944803328z^2 - 587599727984640z^3 - \\ & 11393107020720046080z^4 - 7512914091413564817408z^5 + 299638067426947151953920z^6 + \\ & 195572469268564090225164288z^7 - 25066230988181914756830986240z^8 + \\ & 1466023585546150566663720796160z^9 + 71839838988731444762798769307648z^{10} - \\ & 8620981873487530449442157746978816z^{11} - 107877900379022416281433704771878912z^{12} + \\ & 6045203063427555738693218864495329280z^{13} - \\ & 27383749995592913844335383773613916160z^{14} + \\ & 44159405750235818360995501107081904128z^{15} + \\ & 13699073426625876523234327944587328356352z^{16} - \\ & 387340817532181412702477239142346601267200z^{17} - \\ & 93561082878589380479405717324153487360000z^{18} + \\ & 26199174990188349028511624137716063535104000z^{19} + \\ & 4384654777265123304897934342541583319040000z^{20} - \\ & 7365444961535897432915739585451917312000000z^{21} + \\ & 463163910329304284499157507080361869312000000z^{22} - \\ & 2672997487075722725940045068277645312000000z^{23} + \\ & 2548303901724811483327402682508902400000000z^{24}) \odot_z^9 + \\ & (25587900 - 167017235904z - 388316496158208z^2 - 71432378741514240z^3 - \\ & 67251283102180638720z^4 - 106035468401043255066624z^5 + \\ & 3933938204585015786864640z^6 + 3550830229428629160044003328z^7 - \\ & 414827655192955512486740623360z^8 + 14705350966486458929924160880640z^9 + \\ & 1481459728061408765324573754261504z^{10} - 193671651198531963311300948925612032z^{11} - \\ & 1744302070180818241862079143751974912z^{12} + \\ & 188152751580271237119102149995832279040z^{13} - \\ & 851746667821751911755882821997916323840z^{14} - \\ & 7276349700019150919019652322303668125696z^{15} + \\ & 253386321751200231391561171894048175161344z^{16} - \\ & 10787404513612693833929182911837457770086400z^{17} + \\ & 4040614420659532716685372741544180711424000z^{18} + \\ & 952123292291213693591421570421616642359296000z^{19} + \\ & 1859374469358285389183575405391438067793920000z^{20} - \\ & 2989212943556764016767457029537816117248000000z^{21} + \\ & 16696891280208066019958691706888469348352000000z^{22} - \\ & 949529398826243239300048052502606643200000000z^{23} + \\ & 866423326586435904331316912053026816000000000z^{24}) \odot_z^8 + \\ & (143193825 - 790043593572z - 2145035751247872z^2 - 795395093950794240z^3 + \\ & 138793827460464156672z^4 - 662587314792999489110016z^5 + \\ & 39361134961606763469864960z^6 + 27836403113349843191460790272z^7 - \\ & 2952763516224970957938648678400z^8 + 17715419750558753940033123647488z^9 + \\ & 17561306674573793971071316861648896z^{10} - \\ & 2013528827982674648891849053239771136z^{11} - \\ & 6702623954407224045318611701352890368z^{12} + \\ & 2530409575669922681509808952688204840960z^{13} - \\ & 1415809828888638170539242906998183821312z^{14} - \\ & 212428785403743639945027731918002317164544z^{15} + \end{aligned}$$

$$\begin{aligned}
& 1\,740\,698\,869\,942\,495\,071\,397\,034\,886\,461\,543\,638\,106\,112\,z^{16} - \\
& 133\,590\,920\,120\,774\,486\,375\,251\,925\,639\,058\,930\,479\,923\,200\,z^{17} + \\
& 117\,853\,225\,542\,428\,730\,412\,209\,928\,347\,232\,071\,843\,840\,000\,z^{18} + \\
& 15\,514\,196\,975\,326\,917\,364\,062\,104\,829\,911\,826\,016\,763\,904\,000\,z^{19} + \\
& 33\,354\,396\,540\,999\,090\,913\,407\,702\,429\,888\,059\,277\,312\,000\,000\,z^{20} - \\
& 52\,066\,193\,814\,874\,539\,659\,294\,278\,620\,155\,473\,821\,696\,000\,000\,z^{21} + \\
& 266\,344\,647\,348\,956\,449\,504\,115\,441\,740\,129\,838\,825\,472\,000\,000\,z^{22} - \\
& 14\,770\,234\,221\,615\,586\,187\,256\,793\,881\,539\,625\,615\,360\,000\,000\,z^{23} + \\
& 13\,051\,386\,184\,451\,845\,257\,422\,171\,891\,507\,920\,896\,000\,000\,000\,z^{24}) \ominus_z^7 + \\
& (449\,264\,475 - 1\,868\,417\,168\,706\,z - 7\,066\,918\,189\,948\,896\,z^2 - 3\,878\,860\,874\,135\,507\,712\,z^3 + \\
& 2\,443\,017\,166\,286\,127\,538\,176\,z^4 - 2\,362\,380\,901\,647\,109\,273\,976\,832\,z^5 + \\
& 264\,291\,578\,493\,831\,924\,621\,115\,392\,z^6 + 131\,515\,000\,106\,996\,346\,624\,232\,390\,656\,z^7 - \\
& 12\,203\,514\,471\,539\,125\,045\,628\,510\,404\,608\,z^8 - 497\,477\,693\,141\,507\,581\,640\,821\,121\,744\,896\,z^9 + \\
& 135\,583\,802\,348\,946\,404\,997\,942\,857\,089\,155\,072\,z^{10} - \\
& 12\,808\,764\,617\,716\,898\,741\,372\,092\,142\,783\,037\,440\,z^{11} + \\
& 49\,330\,783\,558\,989\,988\,805\,231\,075\,790\,417\,821\,696\,z^{12} + \\
& 19\,447\,613\,803\,553\,405\,698\,986\,356\,520\,917\,382\,725\,632\,z^{13} - \\
& 144\,309\,617\,133\,065\,277\,844\,132\,480\,453\,941\,129\,117\,696\,z^{14} - \\
& 2\,710\,631\,803\,634\,179\,381\,793\,377\,498\,344\,095\,616\,073\,728\,z^{15} + \\
& 2\,964\,870\,370\,772\,586\,860\,585\,478\,733\,994\,873\,327\,714\,304\,z^{16} - \\
& 974\,601\,771\,553\,101\,425\,549\,162\,295\,455\,430\,182\,594\,150\,400\,z^{17} + \\
& 1\,199\,025\,070\,777\,376\,290\,858\,573\,274\,294\,378\,978\,869\,248\,000\,z^{18} + \\
& 148\,002\,858\,072\,532\,733\,903\,382\,318\,856\,274\,090\,724\,950\,016\,000\,z^{19} + \\
& 338\,543\,845\,114\,612\,124\,673\,021\,108\,297\,229\,880\,933\,744\,640\,000\,z^{20} - \\
& 515\,893\,327\,627\,243\,954\,647\,854\,653\,604\,064\,361\,709\,568\,000\,000\,z^{21} + \\
& 2\,467\,681\,175\,207\,979\,642\,662\,110\,323\,549\,952\,322\,568\,192\,000\,000\,z^{22} - \\
& 132\,231\,011\,763\,661\,328\,740\,679\,532\,066\,610\,694\,062\,080\,000\,000\,z^{23} + \\
& 114\,331\,247\,240\,822\,245\,206\,661\,000\,977\,438\,474\,240\,000\,000\,000\,z^{24}) \ominus_z^6 + \\
& (861\,131\,250 - 1\,933\,234\,949\,826\,z - 15\,613\,778\,270\,821\,824\,z^2 - 10\,309\,316\,152\,243\,684\,608\,z^3 + \\
& 10\,208\,121\,855\,056\,887\,836\,672\,z^4 - 5\,177\,478\,897\,928\,951\,338\,663\,936\,z^5 + \\
& 1\,051\,071\,333\,282\,686\,988\,259\,688\,448\,z^6 + 434\,605\,161\,566\,794\,661\,885\,094\,395\,904\,z^7 - \\
& 33\,621\,302\,900\,734\,471\,517\,776\,319\,086\,592\,z^8 - 3\,862\,797\,242\,368\,863\,995\,804\,954\,019\,233\,792\,z^9 + \\
& 682\,523\,911\,500\,282\,487\,919\,453\,997\,436\,502\,016\,z^{10} - \\
& 54\,885\,706\,010\,533\,092\,142\,603\,108\,401\,761\,222\,656\,z^{11} + \\
& 628\,295\,030\,317\,792\,870\,701\,259\,949\,069\,570\,146\,304\,z^{12} + \\
& 94\,615\,620\,246\,695\,935\,456\,646\,477\,297\,107\,177\,832\,448\,z^{13} - \\
& 943\,374\,946\,910\,174\,228\,088\,452\,295\,651\,235\,014\,377\,472\,z^{14} - \\
& 20\,239\,505\,490\,425\,686\,638\,866\,526\,052\,588\,532\,136\,935\,424\,z^{15} - \\
& 32\,106\,654\,409\,284\,054\,688\,300\,061\,132\,285\,669\,604\,851\,712\,z^{16} - \\
& 4\,661\,186\,834\,850\,235\,941\,611\,447\,026\,185\,947\,054\,918\,860\,800\,z^{17} + \\
& 6\,305\,858\,935\,526\,636\,963\,613\,625\,345\,233\,919\,710\,068\,736\,000\,z^{18} + \\
& 907\,835\,208\,762\,550\,426\,257\,263\,133\,272\,785\,468\,161\,785\,856\,000\,z^{19} + \\
& 2\,163\,393\,059\,021\,746\,024\,723\,195\,689\,099\,497\,341\,043\,343\,360\,000\,z^{20} - \\
& 3\,223\,849\,259\,321\,090\,413\,099\,936\,188\,064\,249\,820\,479\,488\,000\,000\,z^{21} + \\
& 14\,634\,823\,166\,233\,773\,519\,713\,879\,943\,050\,302\,888\,869\,888\,000\,000\,z^{22} - \\
& 751\,559\,666\,050\,831\,529\,712\,855\,803\,432\,617\,566\,535\,680\,000\,000\,z^{23} + \\
& 641\,926\,999\,294\,401\,201\,636\,284\,114\,667\,753\,701\,376\,000\,000\,000\,z^{24}) \ominus_z^5 + \\
& (1\,027\,452\,600 + 650\,073\,935\,826\,z - 24\,331\,013\,564\,272\,416\,z^2 - 15\,445\,917\,700\,094\,672\,640\,z^3 + \\
& 23\,767\,563\,566\,040\,632\,524\,800\,z^4 - 7\,076\,237\,807\,199\,353\,917\,833\,216\,z^5 + \\
& 2\,464\,344\,308\,447\,820\,339\,127\,123\,968\,z^6 + 1\,063\,844\,885\,293\,196\,366\,240\,201\,834\,496\,z^7 - \\
& 68\,128\,813\,392\,655\,213\,983\,420\,372\,746\,240\,z^8 - 14\,470\,974\,465\,501\,609\,824\,429\,971\,257\,425\,920\,z^9 +
\end{aligned}$$

$$\begin{aligned}
& 2\,255\,366\,779\,025\,336\,638\,703\,621\,614\,721\,826\,816\,z^{10} - \\
& 163\,072\,913\,432\,831\,468\,367\,882\,056\,800\,812\,400\,640\,z^{11} + \\
& 2\,997\,803\,475\,346\,983\,445\,074\,315\,347\,099\,287\,289\,856\,z^{12} + \\
& 303\,567\,717\,202\,213\,863\,472\,216\,828\,600\,831\,893\,831\,680\,z^{13} - \\
& 4\,047\,013\,525\,578\,461\,281\,906\,717\,598\,427\,157\,616\,394\,240\,z^{14} - \\
& 96\,118\,533\,388\,650\,019\,713\,306\,746\,783\,626\,734\,499\,528\,704\,z^{15} - \\
& 255\,335\,975\,497\,145\,631\,364\,487\,812\,420\,478\,765\,151\,289\,344\,z^{16} - \\
& 15\,296\,132\,053\,875\,053\,696\,100\,212\,626\,928\,213\,683\,706\,265\,600\,z^{17} + \\
& 18\,246\,597\,120\,323\,314\,085\,822\,020\,233\,756\,671\,647\,678\,464\,000\,z^{18} + \\
& 3\,704\,565\,159\,264\,503\,664\,807\,121\,494\,574\,141\,576\,203\,730\,944\,000\,z^{19} + \\
& 9\,074\,841\,977\,197\,794\,226\,345\,092\,230\,554\,593\,305\,642\,926\,080\,000\,z^{20} - \\
& 13\,228\,108\,961\,936\,918\,861\,318\,774\,597\,098\,203\,034\,681\,344\,000\,000\,z^{21} + \\
& 57\,616\,494\,888\,579\,146\,564\,434\,704\,083\,771\,647\,001\,100\,288\,000\,000\,z^{22} - \\
& 2\,814\,241\,188\,839\,209\,555\,160\,491\,293\,254\,859\,148\,492\,800\,000\,000\,z^{23} + \\
& 2\,395\,763\,624\,685\,018\,201\,440\,807\,167\,653\,926\,928\,384\,000\,000\,000\,z^{24} \Big) \theta_z^4 + \\
& (740\,080\,800 + 4\,005\,475\,160\,382\,z - 26\,752\,082\,313\,555\,648\,z^2 - 11\,242\,751\,383\,759\,253\,760\,z^3 + \\
& 35\,720\,292\,244\,900\,235\,563\,008\,z^4 - 6\,065\,066\,346\,145\,944\,588\,877\,824\,z^5 + \\
& 3\,345\,061\,729\,427\,473\,554\,847\,825\,920\,z^6 + 1\,894\,327\,573\,331\,010\,120\,361\,121\,415\,168\,z^7 - \\
& 108\,746\,455\,608\,013\,062\,145\,350\,062\,571\,520\,z^8 - 32\,390\,016\,084\,427\,669\,590\,502\,250\,075\,127\,808\,z^9 + \\
& 4\,885\,336\,097\,506\,818\,994\,189\,729\,806\,914\,945\,024\,z^{10} - \\
& 332\,529\,853\,189\,481\,862\,490\,202\,166\,008\,777\,539\,584\,z^{11} + \\
& 7\,957\,400\,712\,436\,373\,498\,355\,760\,634\,495\,162\,646\,528\,z^{12} + \\
& 645\,438\,310\,936\,945\,974\,171\,829\,835\,738\,420\,084\,736\,000\,z^{13} - \\
& 11\,380\,150\,415\,633\,780\,615\,071\,928\,566\,811\,032\,654\,184\,448\,z^{14} - \\
& 295\,206\,510\,184\,985\,427\,490\,073\,480\,082\,097\,150\,000\,889\,856\,z^{15} - \\
& 898\,378\,340\,748\,269\,192\,996\,272\,984\,886\,327\,758\,125\,268\,992\,z^{16} - \\
& 34\,684\,745\,325\,811\,914\,042\,962\,369\,649\,474\,365\,098\,347\,724\,800\,z^{17} + \\
& 25\,494\,877\,999\,429\,408\,875\,059\,256\,957\,683\,509\,820\,915\,712\,000\,z^{18} + \\
& 10\,041\,801\,219\,994\,880\,276\,401\,244\,788\,909\,988\,006\,389\,088\,256\,000\,z^{19} + \\
& 25\,067\,347\,396\,414\,652\,503\,438\,294\,678\,613\,111\,532\,316\,262\,400\,000\,z^{20} - \\
& 35\,735\,302\,066\,158\,223\,488\,956\,018\,306\,355\,698\,054\,725\,632\,000\,000\,z^{21} + \\
& 150\,583\,689\,478\,560\,082\,309\,614\,319\,654\,150\,666\,891\,296\,768\,000\,000\,z^{22} - \\
& 6\,945\,686\,778\,879\,323\,272\,921\,783\,244\,323\,834\,130\,595\,840\,000\,000\,z^{23} + \\
& 5\,943\,864\,920\,522\,147\,046\,229\,253\,977\,370\,938\,834\,944\,000\,000\,000\,z^{24} \Big) \theta_z^3 + \\
& (291\,308\,400 + 4\,327\,052\,213\,376\,z - 19\,804\,951\,861\,835\,904\,z^2 + 219\,328\,532\,451\,021\,312\,z^3 + \\
& 35\,514\,623\,326\,732\,836\,470\,784\,z^4 - 3\,650\,278\,542\,693\,304\,807\,784\,448\,z^5 + \\
& 2\,357\,609\,804\,697\,617\,638\,375\,292\,928\,z^6 + 2\,284\,090\,498\,817\,390\,244\,103\,930\,773\,504\,z^7 - \\
& 135\,279\,193\,095\,431\,387\,395\,370\,130\,604\,032\,z^8 - 44\,421\,617\,669\,647\,576\,628\,773\,186\,399\,371\,264\,z^9 + \\
& 6\,731\,574\,539\,350\,949\,339\,005\,449\,772\,222\,906\,368\,z^{10} - \\
& 444\,264\,448\,160\,624\,489\,412\,697\,369\,540\,244\,275\,200\,z^{11} + \\
& 12\,330\,489\,911\,679\,126\,449\,955\,761\,826\,793\,808\,461\,824\,z^{12} + \\
& 881\,925\,579\,961\,332\,026\,441\,980\,244\,408\,237\,385\,842\,688\,z^{13} - \\
& 20\,267\,206\,896\,219\,283\,314\,649\,057\,825\,416\,356\,837\,720\,064\,z^{14} - \\
& 569\,740\,910\,227\,815\,321\,993\,037\,912\,994\,997\,302\,326\,198\,272\,z^{15} - \\
& 1\,779\,740\,679\,903\,215\,090\,218\,062\,224\,628\,300\,162\,325\,807\,104\,z^{16} - \\
& 52\,569\,118\,468\,391\,792\,326\,478\,962\,488\,127\,157\,953\,993\,113\,600\,z^{17} + \\
& 1\,451\,146\,953\,987\,214\,945\,082\,180\,259\,246\,377\,722\,183\,680\,000\,z^{18} + \\
& 17\,418\,909\,098\,958\,603\,815\,097\,479\,056\,179\,850\,099\,738\,804\,224\,000\,z^{19} + \\
& 44\,056\,786\,669\,226\,693\,020\,411\,427\,408\,040\,785\,954\,482\,421\,760\,000\,z^{20} - \\
& 61\,406\,240\,551\,984\,929\,599\,279\,515\,827\,575\,992\,514\,248\,704\,000\,000\,z^{21} + \\
& 251\,939\,775\,441\,184\,947\,187\,729\,213\,430\,058\,184\,656\,027\,648\,000\,000\,z^{22} -
\end{aligned}$$

$$\begin{aligned}
& 10\,897\,710\,329\,468\,549\,993\,283\,540\,156\,647\,403\,442\,667\,520\,000\,000\,z^{23} + \\
& 9\,453\,516\,646\,138\,192\,392\,531\,605\,664\,037\,066\,506\,240\,000\,000\,000\,z^{24}) \Theta_z^2 + \\
& (47\,239\,200 + 2\,006\,920\,198\,008\,z - 8\,896\,275\,005\,061\,888\,z^2 + 6\,068\,775\,605\,179\,834\,368\,z^3 + \\
& 21\,592\,494\,904\,802\,476\,474\,368\,z^4 - 2\,070\,943\,659\,269\,789\,689\,184\,256\,z^5 + \\
& 508\,990\,334\,442\,221\,895\,703\,068\,672\,z^6 + 1\,645\,356\,895\,363\,886\,731\,757\,415\,825\,408\,z^7 - \\
& 114\,092\,231\,346\,388\,839\,876\,595\,084\,689\,408\,z^8 - 34\,854\,174\,079\,338\,717\,741\,834\,632\,070\,955\,008\,z^9 + \\
& 5\,391\,219\,180\,182\,910\,013\,667\,323\,640\,750\,800\,896\,z^{10} - \\
& 349\,879\,345\,484\,004\,176\,369\,490\,015\,495\,754\,088\,448\,z^{11} + \\
& 10\,421\,889\,625\,445\,277\,183\,432\,841\,679\,950\,596\,538\,368\,z^{12} + \\
& 707\,518\,047\,056\,858\,825\,894\,051\,502\,973\,057\,826\,291\,712\,z^{13} - \\
& 20\,793\,407\,939\,094\,108\,348\,704\,800\,718\,204\,390\,225\,215\,488\,z^{14} - \\
& 629\,685\,151\,762\,573\,551\,815\,521\,708\,666\,435\,793\,829\,494\,784\,z^{15} - \\
& 1\,930\,793\,766\,565\,323\,892\,002\,668\,180\,323\,066\,516\,623\,851\,520\,z^{16} - \\
& 48\,248\,039\,919\,208\,006\,716\,948\,992\,896\,627\,977\,926\,737\,920\,000\,z^{17} - \\
& 39\,899\,465\,736\,212\,014\,953\,965\,531\,811\,189\,157\,080\,858\,624\,000\,z^{18} + \\
& 17\,535\,154\,948\,554\,606\,749\,217\,297\,744\,053\,519\,048\,402\,534\,400\,000\,z^{19} + \\
& 44\,763\,107\,271\,248\,675\,832\,384\,158\,553\,351\,794\,937\,613\,516\,800\,000\,z^{20} - \\
& 60\,989\,883\,869\,673\,646\,630\,860\,485\,832\,163\,159\,682\,580\,480\,000\,000\,z^{21} + \\
& 244\,859\,130\,660\,973\,130\,028\,081\,246\,340\,193\,403\,210\,301\,440\,000\,000\,z^{22} - \\
& 9\,863\,991\,571\,517\,380\,030\,786\,517\,850\,680\,700\,017\,049\,600\,000\,000\,z^{23} + \\
& 8\,746\,714\,696\,058\,467\,589\,996\,180\,287\,043\,036\,774\,400\,000\,000\,000\,z^{24}) \Theta_z + \\
& (333\,047\,697\,408\,z - 1\,872\,897\,434\,966\,016\,z^2 + 2\,977\,127\,512\,452\,956\,160\,z^3 + \\
& 6\,063\,429\,379\,839\,486\,197\,760\,z^4 - 841\,378\,018\,777\,452\,462\,735\,360\,z^5 - \\
& 177\,662\,479\,350\,188\,199\,817\,248\,768\,z^6 + 532\,188\,511\,244\,875\,329\,523\,528\,237\,056\,z^7 - \\
& 46\,468\,194\,557\,583\,422\,535\,635\,168\,133\,120\,z^8 - 12\,129\,481\,477\,266\,120\,825\,633\,907\,345\,981\,440\,z^9 + \\
& 1\,922\,168\,850\,385\,325\,476\,972\,118\,541\,336\,576\,000\,z^{10} - \\
& 122\,905\,090\,504\,449\,842\,544\,597\,830\,488\,521\,965\,568\,z^{11} + \\
& 3\,693\,913\,281\,036\,487\,994\,072\,047\,630\,802\,302\,795\,776\,z^{12} + \\
& 255\,898\,359\,172\,294\,308\,439\,162\,255\,416\,680\,691\,793\,920\,z^{13} - \\
& 9\,383\,560\,634\,074\,603\,744\,909\,701\,439\,786\,415\,837\,675\,520\,z^{14} - \\
& 304\,479\,107\,629\,950\,202\,888\,249\,997\,079\,869\,068\,054\,364\,160\,z^{15} - \\
& 897\,976\,262\,418\,207\,078\,877\,329\,057\,815\,825\,497\,246\,924\,800\,z^{16} - \\
& 20\,318\,331\,412\,941\,821\,720\,614\,342\,296\,829\,572\,113\,498\,112\,000\,z^{17} - \\
& 34\,744\,511\,680\,906\,604\,713\,962\,246\,466\,155\,268\,810\,997\,760\,000\,z^{18} + \\
& 7\,802\,356\,464\,780\,521\,871\,748\,733\,579\,721\,789\,105\,242\,112\,000\,000\,z^{19} + \\
& 20\,050\,212\,474\,748\,975\,261\,015\,644\,998\,031\,116\,001\,607\,680\,000\,000\,z^{20} - \\
& 26\,705\,230\,963\,097\,524\,390\,614\,329\,783\,301\,547\,701\,043\,200\,000\,000\,z^{21} + \\
& 105\,329\,290\,936\,390\,351\,965\,199\,230\,328\,560\,978\,336\,153\,600\,000\,000\,z^{22} - \\
& 3\,923\,945\,710\,099\,317\,224\,594\,398\,360\,128\,920\,223\,744\,000\,000\,000\,z^{23} + \\
& 3\,587\,135\,914\,162\,316\,664\,577\,589\,182\,300\,422\,144\,000\,000\,000\,000\,z^{24})
\end{aligned}$$