Multi-headed Lattice Green Function (N = 5, M = 2)
Find minimal recurrence for the coefficients

In[ ]:=
```
NN = 5;
MM = 2;
```

Generate a sequence from recurrence & initial values
Koutschan’s implementation

In[ ]:=
```
(* given a recurrence rec in f[n], compute the values {f[0],f[1],...,f[bound]}
 where inits are the initial values
 {f[0],...,f[d-1]} with d being the order of the recurrence *)
Clear[UnrollRecurrence];
UnrollRecurrence[rec1_, f_[n_], inits_, bound_] :=
  Module[{i, x, vals = inits, rec = rec1},
   If[Head[rec] =!= Equal, rec = (rec == 0)];
   rec = rec /. n → n - Max[Cases[rec, f[n + a_.] :→ a, Infinity]];
   Do[
    AppendTo[vals, Solve[rec /. n → i /. f[i] → x /. f[a_] :→ vals[[a + 1]], x][[1, 1, 2]]];
    , {i, Length[inits], bound}];
   Return[vals];
  ];
```

In[ ]:=
```
<< RISC`HolonomicFunctions`
```

HolonomicFunctions Package version 1.7.3 (21-Mar-2017)
written by Christoph Koutschan
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria

--> Type ?HolonomicFunctions for help.

In[ ]:=
```
ClearAll[z, w, α, β, ξ];
```

Import known ODE for *R*(*w*)

Guttmann (2010), p. 6

In[•]:=

```
ODEDiv2 = {16 (-5 + z) (-1 + z) z^4 (5 + z)^2 (10 + z) (15 + z) (5 + 3 z)
     (-675 000 + 3 465 000 z - 1 053 375 z^2 + 933 650 z^3 + 449 735 z^4 + 144 776 z^5 + 15 678 z^6) D_z^6 +
    8 z^3 (5 + z) (-354 375 000 000 + 1 774 828 125 000 z - 503 550 000 000 z^2 - 1 289 447 109 375 z^3 +
       254 876 515 625 z^4 - 266 627 903 125 z^5 - 304 623 830 625 z^6 - 87 265 479 875 z^7 -
       4 878 146 975 z^8 + 3 939 663 705 z^9 + 1 048 560 285 z^10 + 97 471 734 z^11 + 3 057 210 z^12) D_z^5 +
    10 z^2 (-5 568 750 000 000 + 23 905 125 000 000 z + 3 393 646 875 000 z^2 -
       39 702 348 750 000 z^3 - 7 716 298 734 375 z^4 - 3 779 011 321 875 z^5 -
       7 801 785 421 250 z^6 - 3 351 125 770 500 z^7 - 382 134 335 775 z^8 + 148 313 757 125 z^9 +
       68 439 921 540 z^10 + 11 725 276 842 z^11 + 923 795 772 z^12 + 27 279 720 z^13) D_z^4 +
    5 z (-13 162 500 000 000 + 45 343 125 000 000 z + 40 530 375 000 000 z^2 -
       190 176 960 000 000 z^3 - 77 498 059 625 000 z^4 - 3 649 915 059 375 z^5 -
       26 918 293 320 000 z^6 - 13 545 524 756 500 z^7 - 465 440 555 100 z^8 + 1 350 059 072 325 z^9 +
       524 857 986 060 z^10 + 92 744 995 638 z^11 + 7 892 060 544 z^12 + 255 864 960 z^13) D_z^3 +
    5 (-3 240 000 000 000 + 5 055 750 000 000 z + 44 457 862 500 000 z^2 - 133 825 053 750 000 z^3 -
       110 925 736 437 500 z^4 + 13 367 806 743 750 z^5 - 6 199 228 765 625 z^6 -
       8 282 515 456 375 z^7 + 1 646 226 060 075 z^8 + 2 287 368 823 475 z^9 +
       810 956 145 330 z^10 + 149 186 684 934 z^11 + 13 819 981 248 z^12 + 496 679 040 z^13) D_z^2 +
    10 (-189 000 000 000 + 4 816 462 500 000 z - 7 268 326 875 000 z^2 - 21 210 430 812 500 z^3 +
       2 664 478 321 875 z^4 + 3 711 617 481 250 z^5 - 135 661 728 250 z^6 + 689 643 286 650 z^7 +
       607 021 304 825 z^8 + 209 673 119 160 z^9 + 40 678 130 502 z^10 + 4 143 853 440 z^11 + 167 064 768 z^12)
     D_z + 30 (27 000 000 000 + 84 037 500 000 z - 346 865 625 000 z^2 - 55 567 000 000 z^3 +
       187 923 165 625 z^4 + 36 477 006 875 z^5 + 21 336 230 625 z^6 + 19 123 388 575 z^7 +
       6 925 739 310 z^8 + 1 443 544 710 z^9 + 163 913 184 z^10 + 7 525 440 z^11)} /. {D_z → Der[z]};
ToOrePolynomial[
 ODEDiv2]
```

Out[ ]=

$$
\begin{aligned}
\{ & (-1\,012\,500\,000\,000\, z^{4} + 5\,231\,250\,000\,000\, z^{5} - \\
& 997\,312\,500\,000\, z^{6} - 2\,191\,406\,250\,000\, z^{7} + 591\,170\,000\,000\, z^{8} - 471\,619\,500\,000\, z^{9} - \\
& 776\,465\,800\,000\, z^{10} - 323\,611\,220\,000\, z^{11} - 55\,071\,280\,000\, z^{12} + 2\,439\,803\,200\, z^{13} + \\
& 2\,659\,336\,800\, z^{14} + 436\,630\,192\, z^{15} + 30\,027\,264\, z^{16} + 752\,544\, z^{17})\, D_z^6 + \\
& (-14\,175\,000\,000\,000\, z^{3} + 68\,158\,125\,000\,000\, z^{4} - 5\,943\,375\,000\,000\, z^{5} - 55\,606\,284\,375\,000\, z^{6} - \\
& 120\,516\,250\,000\, z^{7} - 8\,626\,104\,000\,000\, z^{8} - 14\,317\,976\,450\,000\, z^{9} - \\
& 5\,927\,609\,840\,000\, z^{10} - 893\,249\,718\,000\, z^{11} + 118\,561\,372\,400\, z^{12} + \\
& 73\,459\,721\,040\, z^{13} + 12\,287\,351\,640\, z^{14} + 902\,062\,272\, z^{15} + 24\,457\,680\, z^{16})\, D_z^5 + \\
& (-55\,687\,500\,000\,000\, z^{2} + 239\,051\,250\,000\,000\, z^{3} + 33\,936\,468\,750\,000\, z^{4} - 397\,023\,487\,500\,000\, z^{5} - \\
& 77\,162\,987\,343\,750\, z^{6} - 37\,790\,113\,218\,750\, z^{7} - 78\,017\,854\,212\,500\, z^{8} - \\
& 33\,511\,257\,705\,000\, z^{9} - 3\,821\,343\,357\,750\, z^{10} + 1\,483\,137\,571\,250\, z^{11} + \\
& 684\,399\,215\,400\, z^{12} + 117\,252\,768\,420\, z^{13} + 9\,237\,957\,720\, z^{14} + 272\,797\,200\, z^{15})\, D_z^4 + \\
& (-65\,812\,500\,000\,000\, z + 226\,715\,625\,000\,000\, z^{2} + 202\,651\,875\,000\,000\, z^{3} - 950\,884\,800\,000\,000\, z^{4} - \\
& 387\,490\,298\,125\,000\, z^{5} - 18\,249\,575\,296\,875\, z^{6} - 134\,591\,466\,600\,000\, z^{7} - \\
& 67\,727\,623\,782\,500\, z^{8} - 2\,327\,202\,775\,500\, z^{9} + 6\,750\,295\,361\,625\, z^{10} + \\
& 2\,624\,289\,930\,300\, z^{11} + 463\,724\,978\,190\, z^{12} + 39\,460\,302\,720\, z^{13} + 1\,279\,324\,800\, z^{14})\, D_z^3 + \\
& (-16\,200\,000\,000\,000 + 25\,278\,750\,000\,000\, z + 222\,289\,312\,500\,000\, z^{2} - 669\,125\,268\,750\,000\, z^{3} - \\
& 554\,628\,682\,187\,500\, z^{4} + 66\,839\,033\,718\,750\, z^{5} - 30\,996\,143\,828\,125\, z^{6} - \\
& 41\,412\,577\,281\,875\, z^{7} + 8\,231\,130\,300\,375\, z^{8} + 11\,436\,844\,117\,375\, z^{9} + \\
& 4\,054\,780\,726\,650\, z^{10} + 745\,933\,424\,670\, z^{11} + 69\,099\,906\,240\, z^{12} + 2\,483\,395\,200\, z^{13})\, D_z^2 + \\
& (-1\,890\,000\,000\,000 + 48\,164\,625\,000\,000\, z - 72\,683\,268\,750\,000\, z^{2} - 212\,104\,308\,125\,000\, z^{3} + \\
& 26\,644\,783\,218\,750\, z^{4} + 37\,116\,174\,812\,500\, z^{5} - 1\,356\,617\,282\,500\, z^{6} + \\
& 6\,896\,432\,866\,500\, z^{7} + 6\,070\,213\,048\,250\, z^{8} + 2\,096\,731\,191\,600\, z^{9} + \\
& 406\,781\,305\,020\, z^{10} + 41\,438\,534\,400\, z^{11} + 1\,670\,647\,680\, z^{12})\, D_z + \\
& (810\,000\,000\,000 + 2\,521\,125\,000\,000\, z - 10\,405\,968\,750\,000\, z^{2} - 1\,667\,010\,000\,000\, z^{3} + \\
& 5\,637\,694\,968\,750\, z^{4} + 1\,094\,310\,206\,250\, z^{5} + 640\,086\,918\,750\, z^{6} + 573\,701\,657\,250\, z^{7} + \\
& 207\,772\,179\,300\, z^{8} + 43\,306\,341\,300\, z^{9} + 4\,917\,395\,520\, z^{10} + 225\,763\,200\, z^{11}) \}
\end{aligned}
$$

In[ ]:=
```
ODENormalized = -DFiniteSubstitute[ToOrePolynomial[ODEDiv2],
    {z → w * 2^MM * Binomial[NN, MM]}, Algebra → OreAlgebra[Der[w]]];
ToOrePolynomial[ODENormalized]
```

Out[ ]=

$$
\begin{aligned}
\{ & (81\,w^4 - 16\,740\,w^5 + 127\,656\,w^6 + 11\,220\,000\,w^7 - 121\,071\,616\,w^8 + 3\,863\,506\,944\,w^9 + 254\,432\,313\,344\,w^{10} + \\
& \quad 4\,241\,636\,982\,784\,w^{11} + 28\,873\,211\,248\,640\,w^{12} - 51\,166\,381\,604\,864\,w^{13} - 2\,230\,813\,395\,517\,440\,w^{14} - \\
& \quad 14\,650\,878\,086\,610\,944\,w^{15} - 40\,301\,911\,521\,361\,920\,w^{16} - 40\,401\,898\,360\,012\,800\,w^{17})\;D_w^6 + \\
& (1134\,w^3 - 218\,106\,w^4 + 760\,752\,w^5 + 284\,704\,176\,w^6 + 24\,681\,728\,w^7 + 70\,665\,043\,968\,w^8 + \\
& \quad 4\,691\,714\,523\,136\,w^9 + 77\,694\,367\,694\,848\,w^{10} + 468\,320\,108\,150\,784\,w^{11} - \\
& \quad 2\,486\,412\,192\,514\,048\,w^{12} - 61\,622\,480\,359\,391\,232\,w^{13} - 412\,295\,105\,064\,468\,480\,w^{14} - \\
& \quad 1\,210\,727\,486\,623\,580\,160\,w^{15} - 1\,313\,061\,696\,700\,416\,000\,w^{16})\;D_w^5 + \\
& (4455\,w^2 - 764\,964\,w^3 - 4\,343\,868\,w^4 + 2\,032\,760\,256\,w^5 + 15\,802\,979\,808\,w^6 + 309\,576\,607\,488\,w^7 + \\
& \quad 25\,564\,890\,468\,352\,w^8 + 439\,238\,756\,990\,976\,w^9 + 2\,003\,484\,466\,348\,032\,w^{10} - \\
& \quad 31\,103\,649\,238\,220\,800\,w^{11} - 574\,115\,673\,349\,816\,320\,w^{12} - 3\,934\,350\,044\,760\,637\,440\,w^{13} - \\
& \quad 12\,398\,976\,965\,384\,601\,600\,w^{14} - 14\,645\,688\,155\,504\,640\,000\,w^{15})\;D_w^4 + \\
& (5265\,w - 725\,490\,w^2 - 25\,939\,440\,w^3 + 4\,868\,530\,176\,w^4 + 79\,358\,013\,056\,w^5 + 149\,500\,520\,832\,w^6 + \\
& \quad 44\,102\,931\,775\,488\,w^7 + 887\,719\,510\,441\,984\,w^8 + 1\,220\,124\,488\,761\,344\,w^9 - \\
& \quad 141\,563\,954\,182\,225\,920\,w^{10} - 2\,201\,413\,950\,363\,402\,240\,w^{11} - 15\,560\,028\,247\,377\,838\,080\,w^{12} - \\
& \quad 52\,962\,721\,772\,706\,201\,600\,w^{13} - 68\,683\,227\,212\,021\,760\,000\,w^{14})\;D_w^3 + \\
& (1296 - 80\,892\,w - 28\,453\,032\,w^2 + 3\,425\,921\,376\,w^3 + 113\,587\,954\,112\,w^4 - 547\,545\,364\,224\,w^5 + \\
& \quad 10\,156\,816\,409\,600\,w^6 + 542\,802\,932\,948\,992\,w^7 - 4\,315\,482\,842\,923\,008\,w^8 - \\
& \quad 239\,848\,005\,144\,412\,160\,w^9 - 3\,401\,396\,604\,182\,200\,320\,w^{10} - 25\,029\,372\,374\,616\,637\,440\,w^{11} - \\
& \quad 92\,744\,324\,205\,458\,227\,200\,w^{12} - 133\,326\,264\,588\,042\,240\,000\,w^{13})\;D_w^2 + \\
& (6048 - 6\,165\,072\,w + 372\,138\,336\,w^2 + 43\,438\,962\,304\,w^3 - 218\,274\,064\,128\,w^4 - \\
& \quad 12\,162\,228\,162\,560\,w^5 + 17\,781\,454\,045\,184\,w^6 - 3\,615\,716\,994\,711\,552\,w^7 - \\
& \quad 127\,301\,594\,345\,635\,840\,w^8 - 1\,758\,865\,604\,770\,529\,280\,w^9 - 13\,649\,315\,638\,164\,848\,640\,w^{10} - \\
& \quad 55\,617\,859\,388\,178\,432\,000\,w^{11} - 89\,692\,214\,359\,228\,416\,000\,w^{12})\;D_w + \\
& (-103\,680 - 12\,908\,160\,w + 2\,131\,142\,400\,w^2 + 13\,656\,145\,920\,w^3 - 1\,847\,359\,887\,360\,w^4 - \\
& \quad 14\,343\,342\,735\,360\,w^5 - 335\,589\,890\,457\,600\,w^6 - 12\,031\,395\,779\,051\,520\,w^7 - 174\,291\,936\,545\,341\,440\,w^8 - \\
& \quad 1\,453\,119\,684\,319\,641\,600\,w^9 - 6\,600\,016\,543\,717\,785\,600\,w^{10} - 12\,120\,569\,508\,003\,840\,000\,w^{11}) \}
\end{aligned}
$$

In[ ]:=
```
ODENormalizedinD = ODENormalized[[1]];
ToOrePolynomial[ODENormalizedinD]
```

Out[ ]=

$$\left(81\,w^4-16\,740\,w^5+127\,656\,w^6+11\,220\,000\,w^7-121\,071\,616\,w^8+3\,863\,506\,944\,w^9+254\,432\,313\,344\,w^{10}+4\,241\,636\,982\,784\,w^{11}+28\,873\,211\,248\,640\,w^{12}-51\,166\,381\,604\,864\,w^{13}-2\,230\,813\,395\,517\,440\,w^{14}-14\,650\,878\,086\,610\,944\,w^{15}-40\,301\,911\,521\,361\,920\,w^{16}-40\,401\,898\,360\,012\,800\,w^{17}\right)D_w^6+$$
$$\left(1134\,w^3-218\,106\,w^4+760\,752\,w^5+284\,704\,176\,w^6+24\,681\,728\,w^7+70\,665\,043\,968\,w^8+4\,691\,714\,523\,136\,w^9+77\,694\,367\,694\,848\,w^{10}+468\,320\,108\,150\,784\,w^{11}-2\,486\,412\,192\,514\,048\,w^{12}-61\,622\,480\,359\,391\,232\,w^{13}-412\,295\,105\,064\,468\,480\,w^{14}-1\,210\,727\,486\,623\,580\,160\,w^{15}-1\,313\,061\,696\,700\,416\,000\,w^{16}\right)D_w^5+$$
$$\left(4455\,w^2-764\,964\,w^3-4\,343\,868\,w^4+2\,032\,760\,256\,w^5+15\,802\,979\,808\,w^6+309\,576\,607\,488\,w^7+25\,564\,890\,468\,352\,w^8+439\,238\,756\,990\,976\,w^9+2\,003\,484\,466\,348\,032\,w^{10}-31\,103\,649\,238\,220\,800\,w^{11}-574\,115\,673\,349\,816\,320\,w^{12}-3\,934\,350\,044\,760\,637\,440\,w^{13}-12\,398\,976\,965\,384\,601\,600\,w^{14}-14\,645\,688\,155\,504\,640\,000\,w^{15}\right)D_w^4+$$
$$\left(5265\,w-725\,490\,w^2-25\,939\,440\,w^3+4\,868\,530\,176\,w^4+79\,358\,013\,056\,w^5+149\,500\,520\,832\,w^6+44\,102\,931\,775\,488\,w^7+887\,719\,510\,441\,984\,w^8+1\,220\,124\,488\,761\,344\,w^9-141\,563\,954\,182\,225\,920\,w^{10}-2\,201\,413\,950\,363\,402\,240\,w^{11}-15\,560\,028\,247\,377\,838\,080\,w^{12}-52\,962\,721\,772\,706\,201\,600\,w^{13}-68\,683\,227\,212\,021\,760\,000\,w^{14}\right)D_w^3+$$
$$\left(1296-80\,892\,w-28\,453\,032\,w^2+3\,425\,921\,376\,w^3+113\,587\,954\,112\,w^4-547\,545\,364\,224\,w^5+10\,156\,816\,409\,600\,w^6+542\,802\,932\,948\,992\,w^7-4\,315\,482\,842\,923\,008\,w^8-239\,848\,005\,144\,412\,160\,w^9-3\,401\,396\,604\,182\,200\,320\,w^{10}-25\,029\,372\,374\,616\,637\,440\,w^{11}-92\,744\,324\,205\,458\,227\,200\,w^{12}-133\,326\,264\,588\,042\,240\,000\,w^{13}\right)D_w^2+$$
$$\left(6048-6\,165\,072\,w+372\,138\,336\,w^2+43\,438\,962\,304\,w^3-218\,274\,064\,128\,w^4-12\,162\,228\,162\,560\,w^5+17\,781\,454\,045\,184\,w^6-3\,615\,716\,994\,711\,552\,w^7-127\,301\,594\,345\,635\,840\,w^8-1\,758\,865\,604\,770\,529\,280\,w^9-13\,649\,315\,638\,164\,848\,640\,w^{10}-55\,617\,859\,388\,178\,432\,000\,w^{11}-89\,692\,214\,359\,228\,416\,000\,w^{12}\right)D_w+$$
$$\left(-103\,680-12\,908\,160\,w+2\,131\,142\,400\,w^2+13\,656\,145\,920\,w^3-1\,847\,359\,887\,360\,w^4-14\,343\,342\,735\,360\,w^5-335\,589\,890\,457\,600\,w^6-12\,031\,395\,779\,051\,520\,w^7-174\,291\,936\,545\,341\,440\,w^8-1\,453\,119\,684\,319\,641\,600\,w^9-6\,600\,016\,543\,717\,785\,600\,w^{10}-12\,120\,569\,508\,003\,840\,000\,w^{11}\right)$$

In[∘]:=
```
ODENormalizedinTheta = ChangeOreAlgebra[w ** ODENormalizedinD, OreAlgebra[Euler[w]]];
ToOrePolynomial[ODENormalizedinTheta]
```

Out[∘]=

$$\left(-16\,740 + \frac{81}{w} + 127\,656\,w + 11\,220\,000\,w^2 - 121\,071\,616\,w^3 + 3\,863\,506\,944\,w^4 + 254\,432\,313\,344\,w^5 + 4\,241\,636\,982\,784\,w^6 + 28\,873\,211\,248\,640\,w^7 - 51\,166\,381\,604\,864\,w^8 - 2\,230\,813\,395\,517\,440\,w^9 - 14\,650\,878\,086\,610\,944\,w^{10} - 40\,301\,911\,521\,361\,920\,w^{11} - 40\,401\,898\,360\,012\,800\,w^{12}\right)\Theta_w^6 +$$

$$\left(32\,994 - \frac{81}{w} - 1\,154\,088\,w + 116\,404\,176\,w^2 + 1\,840\,755\,968\,w^3 + 12\,712\,439\,808\,w^4 + 875\,229\,822\,976\,w^5 + 14\,069\,812\,953\,088\,w^6 + 35\,221\,939\,421\,184\,w^7 - 1\,718\,916\,468\,441\,088\,w^8 - 28\,160\,279\,426\,629\,632\,w^9 - 192\,531\,933\,765\,304\,320\,w^{10} - 606\,198\,813\,803\,151\,360\,w^{11} - 707\,033\,221\,300\,224\,000\,w^{12}\right)\Theta_w^5 +$$

$$\left(-6804 - 1\,100\,628\,w + 139\,418\,496\,w^2 + 5\,265\,075\,168\,w^3 - 68\,675\,741\,952\,w^4 + 274\,491\,871\,232\,w^5 + 22\,834\,223\,579\,136\,w^6 - 225\,493\,659\,025\,408\,w^7 - 10\,588\,669\,749\,493\,760\,w^8 - 147\,510\,008\,374\,886\,400\,w^9 - 1\,056\,723\,631\,477\,882\,880\,w^{10} - 3\,717\,364\,578\,464\,563\,200\,w^{11} - 4\,949\,232\,549\,101\,568\,000\,w^{12}\right)\Theta_w^4 +$$

$$\left(-2916 - 1\,972\,512\,w + 112\,114\,800\,w^2 + 12\,645\,108\,288\,w^3 - 103\,971\,647\,616\,w^4 - 2\,323\,673\,227\,264\,w^5 + 17\,221\,516\,689\,408\,w^6 - 906\,051\,054\,993\,408\,w^7 - 30\,454\,049\,629\,798\,400\,w^8 - 411\,573\,708\,851\,773\,440\,w^9 - 3\,087\,809\,086\,582\,947\,840\,w^{10} - 11\,876\,391\,919\,917\,465\,600\,w^{11} - 17\,675\,830\,532\,505\,600\,000\,w^{12}\right)\Theta_w^3 +$$

$$\left(-486 - 1\,477\,116\,w + 19\,764\,864\,w^2 + 14\,938\,983\,648\,w^3 - 65\,355\,540\,096\,w^4 - 5\,809\,456\,065\,536\,w^5 - 11\,239\,122\,935\,808\,w^6 - 1\,442\,272\,704\,790\,528\,w^7 - 46\,995\,263\,152\,193\,536\,w^8 - 642\,546\,012\,342\,190\,080\,w^9 - 5\,026\,723\,467\,358\,109\,696\,w^{10} - 20\,751\,254\,932\,244\,398\,080\,w^{11} - 33\,796\,187\,978\,150\,707\,200\,w^{12}\right)\Theta_w^2 +$$

$$\left(-588\,384\,w - 26\,784\,000\,w^2 + 8\,870\,110\,848\,w^3 + 3\,152\,918\,784\,w^4 - 5\,433\,252\,877\,312\,w^5 - 29\,346\,613\,223\,424\,w^6 - 1\,105\,994\,726\,572\,032\,w^7 - 37\,493\,528\,964\,104\,192\,w^8 - 526\,844\,782\,379\,532\,288\,w^9 - 4\,270\,876\,640\,893\,992\,960\,w^{10} - 18\,626\,347\,232\,227\,491\,840\,w^{11} - 32\,523\,528\,179\,810\,304\,000\,w^{12}\right)\Theta_w +$$

$$\left(-103\,680\,w - 12\,908\,160\,w^2 + 2\,131\,142\,400\,w^3 + 13\,656\,145\,920\,w^4 - 1\,847\,359\,887\,360\,w^5 - 14\,343\,342\,735\,360\,w^6 - 335\,589\,890\,457\,600\,w^7 - 12\,031\,395\,779\,051\,520\,w^8 - 174\,291\,936\,545\,341\,440\,w^9 - 1\,453\,119\,684\,319\,641\,600\,w^{10} - 6\,600\,016\,543\,717\,785\,600\,w^{11} - 12\,120\,569\,508\,003\,840\,000\,w^{12}\right)$$

Recurrence for {r(0), r(1), r(2), ...}.

In[ ]:=
```
RECNormalized = DFiniteDE2RE[ToOrePolynomial[ODENormalized], {w}, {α}];
ToOrePolynomial[RECNormalized]
```

Out[ ]=

$$\begin{aligned}
\{&(360\,896\,796 + 168\,881\,193\,\alpha + 32\,922\,045\,\alpha^2 + 3\,422\,250\,\alpha^3 + 200\,070\,\alpha^4 + 6237\,\alpha^5 + 81\,\alpha^6)\,S_\alpha^{13} + \\
&(-41\,921\,605\,728 - 21\,620\,168\,784\,\alpha - 4\,642\,657\,398\,\alpha^2 - 531\,352\,548\,\alpha^3 - \\
&\quad 34\,185\,564\,\alpha^4 - 1\,172\,286\,\alpha^5 - 16\,740\,\alpha^6)\,S_\alpha^{12} + (21\,358\,349\,568 + 32\,260\,896\,432\,\alpha + \\
&\quad 11\,808\,635\,220\,\alpha^2 + 1\,951\,356\,096\,\alpha^3 + 167\,120\,172\,\alpha^4 + 7\,271\,208\,\alpha^5 + 127\,656\,\alpha^6)\,S_\alpha^{11} + \\
&(24\,368\,413\,098\,240 + 13\,143\,885\,737\,280\,\alpha + 2\,934\,076\,066\,464\,\alpha^2 + 346\,493\,030\,640\,\alpha^3 + \\
&\quad 22\,789\,627\,296\,\alpha^4 + 789\,604\,176\,\alpha^5 + 11\,220\,000\,\alpha^6)\,S_\alpha^{10} + \\
&(89\,406\,840\,410\,496 + 36\,194\,544\,731\,520\,\alpha + 4\,419\,031\,357\,152\,\alpha^2 - 72\,024\,012\,864\,\alpha^3 - \\
&\quad 59\,002\,919\,712\,\alpha^4 - 4\,697\,111\,296\,\alpha^5 - 121\,071\,616\,\alpha^6)\,S_\alpha^{9} + \\
&(1\,090\,683\,194\,271\,744 + 858\,294\,128\,931\,072\,\alpha + 273\,529\,398\,463\,872\,\alpha^2 + 45\,396\,677\,193\,600\,\alpha^3 + \\
&\quad 4\,148\,788\,516\,608\,\alpha^4 + 198\,160\,773\,120\,\alpha^5 + 3\,863\,506\,944\,\alpha^6)\,S_\alpha^{8} + \\
&(44\,181\,186\,456\,002\,560 + 36\,112\,854\,612\,190\,208\,\alpha + 12\,191\,512\,074\,195\,968\,\alpha^2 + \\
&\quad 2\,179\,630\,381\,965\,312\,\alpha^3 + 217\,915\,285\,983\,232\,\alpha^4 + 11\,561\,386\,983\,424\,\alpha^5 + 254\,432\,313\,344\,\alpha^6)\,S_\alpha^{7} + \\
&(340\,022\,650\,507\,689\,984 + 310\,494\,679\,891\,156\,992\,\alpha + 118\,079\,159\,394\,557\,952\,\alpha^2 + \\
&\quad 23\,954\,247\,311\,327\,232\,\alpha^3 + 2\,735\,412\,582\,875\,136\,\alpha^4 + 166\,768\,744\,333\,312\,\alpha^5 + 4\,241\,636\,982\,784\,\alpha^6) \\
&\,S_\alpha^{6} + (265\,100\,186\,543\,063\,040 + 455\,211\,891\,191\,513\,088\,\alpha + 265\,856\,692\,348\,977\,152\,\alpha^2 + \\
&\quad 75\,572\,588\,741\,394\,432\,\alpha^3 + 11\,482\,509\,044\,744\,192\,\alpha^4 + 901\,418\,276\,880\,384\,\alpha^5 + 28\,873\,211\,248\,640\,\alpha^6) \\
&\,S_\alpha^{5} + (-7\,543\,436\,316\,985\,262\,080 - 7\,100\,528\,800\,467\,255\,296\,\alpha - \\
&\quad 2\,725\,541\,599\,826\,149\,376\,\alpha^2 - 540\,392\,369\,026\,498\,560\,\alpha^3 - \\
&\quad 57\,246\,930\,703\,482\,880\,\alpha^4 - 2\,946\,909\,626\,957\,824\,\alpha^5 - 51\,166\,381\,604\,864\,\alpha^6)\,S_\alpha^{4} + \\
&(-39\,067\,752\,078\,130\,544\,640 - 46\,083\,130\,998\,367\,715\,328\,\alpha - 22\,625\,963\,564\,995\,706\,880\,\alpha^2 - \\
&\quad 5\,920\,758\,191\,326\,494\,720\,\alpha^3 - 871\,074\,008\,169\,185\,280\,\alpha^4 - \\
&\quad 68\,314\,920\,545\,943\,552\,\alpha^5 - 2\,230\,813\,395\,517\,440\,\alpha^6)\,S_\alpha^{3} + \\
&(-78\,810\,495\,709\,882\,613\,760 - 113\,462\,159\,050\,467\,704\,832\,\alpha - 67\,833\,710\,584\,335\,958\,016\,\alpha^2 - \\
&\quad 21\,587\,015\,982\,875\,934\,720\,\alpha^3 - 3\,861\,095\,654\,327\,582\,720\,\alpha^4 - \\
&\quad 368\,342\,470\,804\,635\,648\,\alpha^5 - 14\,650\,878\,086\,610\,944\,\alpha^6)\,S_\alpha^{2} + \\
&(-62\,217\,875\,931\,896\,217\,600 - 113\,900\,296\,708\,470\,865\,920\,\alpha - 85\,351\,134\,973\,636\,116\,480\,\alpha^2 - \\
&\quad 33\,613\,876\,602\,234\,470\,400\,\alpha^3 - 7\,352\,887\,320\,300\,748\,800\,\alpha^4 - \\
&\quad 848\,010\,282\,931\,322\,880\,\alpha^5 - 40\,301\,911\,521\,361\,920\,\alpha^6)\,S_\alpha + \\
&(-12\,120\,569\,508\,003\,840\,000 - 32\,523\,528\,179\,810\,304\,000\,\alpha - 33\,796\,187\,978\,150\,707\,200\,\alpha^2 - \\
&\quad 17\,675\,830\,532\,505\,600\,000\,\alpha^3 - 4\,949\,232\,549\,101\,568\,000\,\alpha^4 - \\
&\quad 707\,033\,221\,300\,224\,000\,\alpha^5 - 40\,401\,898\,360\,012\,800\,\alpha^6)\,\}
\end{aligned}$$

In[ ]:=
```
RECNormalizedinS = RECNormalized[[1]];
ToOrePolynomial[RECNormalizedinS]
```

Out[ ]=

$$
\begin{aligned}
&\left(360\,896\,796 + 168\,881\,193\,\alpha + 32\,922\,045\,\alpha^2 + 3\,422\,250\,\alpha^3 + 200\,070\,\alpha^4 + 6237\,\alpha^5 + 81\,\alpha^6\right) S_\alpha^{13} + \\
&\left(-41\,921\,605\,728 - 21\,620\,168\,784\,\alpha - 4\,642\,657\,398\,\alpha^2 - 531\,352\,548\,\alpha^3 - \right.\\
&\left. 34\,185\,564\,\alpha^4 - 1\,172\,286\,\alpha^5 - 16\,740\,\alpha^6\right) S_\alpha^{12} + \left(21\,358\,349\,568 + 32\,260\,896\,432\,\alpha + \right.\\
&\left. 11\,808\,635\,220\,\alpha^2 + 1\,951\,356\,096\,\alpha^3 + 167\,120\,172\,\alpha^4 + 7\,271\,208\,\alpha^5 + 127\,656\,\alpha^6\right) S_\alpha^{11} + \\
&\left(24\,368\,413\,098\,240 + 13\,143\,885\,737\,280\,\alpha + 2\,934\,076\,066\,464\,\alpha^2 + 346\,493\,030\,640\,\alpha^3 + \right.\\
&\left. 22\,789\,627\,296\,\alpha^4 + 789\,604\,176\,\alpha^5 + 11\,220\,000\,\alpha^6\right) S_\alpha^{10} + \\
&\left(89\,406\,840\,410\,496 + 36\,194\,544\,731\,520\,\alpha + 4\,419\,031\,357\,152\,\alpha^2 - 72\,024\,012\,864\,\alpha^3 - \right.\\
&\left. 59\,002\,919\,712\,\alpha^4 - 4\,697\,111\,296\,\alpha^5 - 121\,071\,616\,\alpha^6\right) S_\alpha^{9} + \\
&\left(1\,090\,683\,194\,271\,744 + 858\,294\,128\,931\,072\,\alpha + 273\,529\,398\,463\,872\,\alpha^2 + \right.\\
&\left. 45\,396\,677\,193\,600\,\alpha^3 + 4\,148\,788\,516\,608\,\alpha^4 + 198\,160\,773\,120\,\alpha^5 + 3\,863\,506\,944\,\alpha^6\right) S_\alpha^{8} + \\
&\left(44\,181\,186\,456\,002\,560 + 36\,112\,854\,612\,190\,208\,\alpha + 12\,191\,512\,074\,195\,968\,\alpha^2 + \right.\\
&\left. 2\,179\,630\,381\,965\,312\,\alpha^3 + 217\,915\,285\,983\,232\,\alpha^4 + 11\,561\,386\,983\,424\,\alpha^5 + 254\,432\,313\,344\,\alpha^6\right) S_\alpha^{7} + \\
&\left(340\,022\,650\,507\,689\,984 + 310\,494\,679\,891\,156\,992\,\alpha + 118\,079\,159\,394\,557\,952\,\alpha^2 + \right.\\
&\left. 23\,954\,247\,311\,327\,232\,\alpha^3 + 2\,735\,412\,582\,875\,136\,\alpha^4 + 166\,768\,744\,333\,312\,\alpha^5 + 4\,241\,636\,982\,784\,\alpha^6\right) S_\alpha^{6} + \\
&\left(265\,100\,186\,543\,063\,040 + 455\,211\,891\,191\,513\,088\,\alpha + 265\,856\,692\,348\,977\,152\,\alpha^2 + \right.\\
&\left. 75\,572\,588\,741\,394\,432\,\alpha^3 + 11\,482\,509\,044\,744\,192\,\alpha^4 + 901\,418\,276\,880\,384\,\alpha^5 + 28\,873\,211\,248\,640\,\alpha^6\right) \\
&S_\alpha^{5} + \left(-7\,543\,436\,316\,985\,262\,080 - 7\,100\,528\,800\,467\,255\,296\,\alpha - \right.\\
&2\,725\,541\,599\,826\,149\,376\,\alpha^2 - 540\,392\,369\,026\,498\,560\,\alpha^3 - \\
&\left. 57\,246\,930\,703\,482\,880\,\alpha^4 - 2\,946\,909\,626\,957\,824\,\alpha^5 - 51\,166\,381\,604\,864\,\alpha^6\right) S_\alpha^{4} + \\
&\left(-39\,067\,752\,078\,130\,544\,640 - 46\,083\,130\,998\,367\,715\,328\,\alpha - 22\,625\,963\,564\,995\,706\,880\,\alpha^2 - \right.\\
&5\,920\,758\,191\,326\,494\,720\,\alpha^3 - 871\,074\,008\,169\,185\,280\,\alpha^4 - \\
&\left. 68\,314\,920\,545\,943\,552\,\alpha^5 - 2\,230\,813\,395\,517\,440\,\alpha^6\right) S_\alpha^{3} + \\
&\left(-78\,810\,495\,709\,882\,613\,760 - 113\,462\,159\,050\,467\,704\,832\,\alpha - 67\,833\,710\,584\,335\,958\,016\,\alpha^2 - \right.\\
&21\,587\,015\,982\,875\,934\,720\,\alpha^3 - 3\,861\,095\,654\,327\,582\,720\,\alpha^4 - \\
&\left. 368\,342\,470\,804\,635\,648\,\alpha^5 - 14\,650\,878\,086\,610\,944\,\alpha^6\right) S_\alpha^{2} + \\
&\left(-62\,217\,875\,931\,896\,217\,600 - 113\,900\,296\,708\,470\,865\,920\,\alpha - 85\,351\,134\,973\,636\,116\,480\,\alpha^2 - \right.\\
&33\,613\,876\,602\,234\,470\,400\,\alpha^3 - 7\,352\,887\,320\,300\,748\,800\,\alpha^4 - \\
&\left. 848\,010\,282\,931\,322\,880\,\alpha^5 - 40\,301\,911\,521\,361\,920\,\alpha^6\right) S_\alpha + \\
&\left(-12\,120\,569\,508\,003\,840\,000 - 32\,523\,528\,179\,810\,304\,000\,\alpha - 33\,796\,187\,978\,150\,707\,200\,\alpha^2 - \right.\\
&17\,675\,830\,532\,505\,600\,000\,\alpha^3 - 4\,949\,232\,549\,101\,568\,000\,\alpha^4 - \\
&\left. 707\,033\,221\,300\,224\,000\,\alpha^5 - 40\,401\,898\,360\,012\,800\,\alpha^6\right)
\end{aligned}
$$

In[ ]:=
```
RecNormalizedOrder = OrePolynomialDegree[RECNormalizedinS, S[α]]
```

Out[ ]= 13

Write recurrence explicitly.

In[ ]:=
```
ClearAll[Seq];
SeqNormalized = ApplyOreOperator[RECNormalizedinS, Seq[α]]
```

Out[ ]=

$$(-12\,120\,569\,508\,003\,840\,000 - 32\,523\,528\,179\,810\,304\,000\,\alpha - 33\,796\,187\,978\,150\,707\,200\,\alpha^2 - 17\,675\,830\,532\,505\,600\,000\,\alpha^3 - 4\,949\,232\,549\,101\,568\,000\,\alpha^4 - 707\,033\,221\,300\,224\,000\,\alpha^5 - 40\,401\,898\,360\,012\,800\,\alpha^6)\,\text{Seq}[\alpha] +$$
$$(-62\,217\,875\,931\,896\,217\,600 - 113\,900\,296\,708\,470\,865\,920\,\alpha - 85\,351\,134\,973\,636\,116\,480\,\alpha^2 - 33\,613\,876\,602\,234\,470\,400\,\alpha^3 - 7\,352\,887\,320\,300\,748\,800\,\alpha^4 - 848\,010\,282\,931\,322\,880\,\alpha^5 - 40\,301\,911\,521\,361\,920\,\alpha^6)\,\text{Seq}[1+\alpha] +$$
$$(-78\,810\,495\,709\,882\,613\,760 - 113\,462\,159\,050\,467\,704\,832\,\alpha - 67\,833\,710\,584\,335\,958\,016\,\alpha^2 - 21\,587\,015\,982\,875\,934\,720\,\alpha^3 - 3\,861\,095\,654\,327\,582\,720\,\alpha^4 - 368\,342\,470\,804\,635\,648\,\alpha^5 - 14\,650\,878\,086\,610\,944\,\alpha^6)\,\text{Seq}[2+\alpha] +$$
$$(-39\,067\,752\,078\,130\,544\,640 - 46\,083\,130\,998\,367\,715\,328\,\alpha - 22\,625\,963\,564\,995\,706\,880\,\alpha^2 - 5\,920\,758\,191\,326\,494\,720\,\alpha^3 - 871\,074\,008\,169\,185\,280\,\alpha^4 - 68\,314\,920\,545\,943\,552\,\alpha^5 - 2\,230\,813\,395\,517\,440\,\alpha^6)\,\text{Seq}[3+\alpha] +$$
$$(-7\,543\,436\,316\,985\,262\,080 - 7\,100\,528\,800\,467\,255\,296\,\alpha - 2\,725\,541\,599\,826\,149\,376\,\alpha^2 - 540\,392\,369\,026\,498\,560\,\alpha^3 - 57\,246\,930\,703\,482\,880\,\alpha^4 - 2\,946\,909\,626\,957\,824\,\alpha^5 - 51\,166\,381\,604\,864\,\alpha^6)\,\text{Seq}[4+\alpha] +$$
$$(265\,100\,186\,543\,063\,040 + 455\,211\,891\,191\,513\,088\,\alpha + 265\,856\,692\,348\,977\,152\,\alpha^2 + 75\,572\,588\,741\,394\,432\,\alpha^3 + 11\,482\,509\,044\,744\,192\,\alpha^4 + 901\,418\,276\,880\,384\,\alpha^5 + 28\,873\,211\,248\,640\,\alpha^6)\,\text{Seq}[5+\alpha] +$$
$$(340\,022\,650\,507\,689\,984 + 310\,494\,679\,891\,156\,992\,\alpha + 118\,079\,159\,394\,557\,952\,\alpha^2 + 23\,954\,247\,311\,327\,232\,\alpha^3 + 2\,735\,412\,582\,875\,136\,\alpha^4 + 166\,768\,744\,333\,312\,\alpha^5 + 4\,241\,636\,982\,784\,\alpha^6)\,\text{Seq}[6+\alpha] +$$
$$(44\,181\,186\,456\,002\,560 + 36\,112\,854\,612\,190\,208\,\alpha + 12\,191\,512\,074\,195\,968\,\alpha^2 + 2\,179\,630\,381\,965\,312\,\alpha^3 + 217\,915\,285\,983\,232\,\alpha^4 + 11\,561\,386\,983\,424\,\alpha^5 + 254\,432\,313\,344\,\alpha^6)\,\text{Seq}[7+\alpha] +$$
$$(1\,090\,683\,194\,271\,744 + 858\,294\,128\,931\,072\,\alpha + 273\,529\,398\,463\,872\,\alpha^2 + 45\,396\,677\,193\,600\,\alpha^3 + 4\,148\,788\,516\,608\,\alpha^4 + 198\,160\,773\,120\,\alpha^5 + 3\,863\,506\,944\,\alpha^6)\,\text{Seq}[8+\alpha] +$$
$$(89\,406\,840\,410\,496 + 36\,194\,544\,731\,520\,\alpha + 4\,419\,031\,357\,152\,\alpha^2 - 72\,024\,012\,864\,\alpha^3 - 59\,002\,919\,712\,\alpha^4 - 4\,697\,111\,296\,\alpha^5 - 121\,071\,616\,\alpha^6)\,\text{Seq}[9+\alpha] +$$
$$(24\,368\,413\,098\,240 + 13\,143\,885\,737\,280\,\alpha + 2\,934\,076\,066\,464\,\alpha^2 + 346\,493\,030\,640\,\alpha^3 + 22\,789\,627\,296\,\alpha^4 + 789\,604\,176\,\alpha^5 + 11\,220\,000\,\alpha^6)\,\text{Seq}[10+\alpha] +$$
$$(21\,358\,349\,568 + 32\,260\,896\,432\,\alpha + 11\,808\,635\,220\,\alpha^2 + 1\,951\,356\,096\,\alpha^3 + 167\,120\,172\,\alpha^4 + 7\,271\,208\,\alpha^5 + 127\,656\,\alpha^6)\,\text{Seq}[11+\alpha] +$$
$$(-41\,921\,605\,728 - 21\,620\,168\,784\,\alpha - 4\,642\,657\,398\,\alpha^2 - 531\,352\,548\,\alpha^3 - 34\,185\,564\,\alpha^4 - 1\,172\,286\,\alpha^5 - 16\,740\,\alpha^6)\,\text{Seq}[12+\alpha] +$$
$$(360\,896\,796 + 168\,881\,193\,\alpha + 32\,922\,045\,\alpha^2 + 3\,422\,250\,\alpha^3 + 200\,070\,\alpha^4 + 6237\,\alpha^5 + 81\,\alpha^6)\,\text{Seq}[13+\alpha]$$

Initial values of {r(0), r(1), r(2), ...}

In[ ]:=
```
SeqListIni = {1};

sympoly = SymmetricPolynomial[MM, Table[Indexed[ξ, i] + Indexed[ξ, i]^-1, {i, 1, NN}]];

MAX = 15;

sympolypower = 1;

For[n = 1, n ≤ MAX, n++,
  sympolypower = Expand[sympolypower * sympoly];
  p = Coefficient[Expand[sympolypower * Product[Indexed[ξ, i], {i, 1, NN}]],
    Product[Indexed[ξ, i], {i, 1, NN}]];
  SeqListIni = Append[SeqListIni, p];
 ];

SeqListIni

seq[n_] := SeqListIni[[n + 1]];
```

Out[ ]=
```
{1, 0, 40, 480, 11 880, 281 280, 7 506 400, 210 268 800, 6 166 993 000,
 187 069 411 200, 5 833 030 976 640, 186 014 056 166 400, 6 044 435 339 896 800,
 199 561 060 892 793 600, 6 679 216 425 794 140 800, 226 213 441 773 789 550 080}
```

Verify recurrence by initial values

In[ ]:=
```
Table[SeqNormalized /. {Seq → seq, α → n}, {n, 0, MAX - RecNormalizedOrder}]
```

Out[ ]=
```
{0, 0, 0}
```

Generate more terms in the sequence

SeqList[[n]] = r(n)

In[ ]:=
```
Bound = 200;

SeqList = UnrollRecurrence[SeqNormalized, Seq[α], SeqListIni, Bound];

seq[n_] := SeqList[[n + 1]];
```

Let’s guess (and prove!) a shorter recurrence.

In[ ]:=
```
<< RISC`Guess`
```

Package GeneratingFunctions version 0.8 written by Christian Mallinger
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria

Guess Package version 0.52
written by Manuel Kauers

In[∘]:= **SeqGuess = GuessMinRE[Take[SeqList, 100], Seq[α]]**

Out[∘]=
$$\left(55\,425\,003\,520\,000 + \frac{2\,283\,606\,788\,915\,200\,\alpha}{11} + \frac{33\,506\,706\,472\,622\,080\,\alpha^2}{99} + \frac{31\,740\,118\,281\,247\,744\,\alpha^3}{99} + \frac{19\,582\,945\,761\,359\,872\,\alpha^4}{99} + \frac{8\,329\,211\,188\,054\,016\,\alpha^5}{99} + \frac{2\,513\,185\,416\,501\,248\,\alpha^6}{99} + \frac{543\,555\,762\,154\,496\,\alpha^7}{99} + \frac{83\,825\,403\,885\,568\,\alpha^8}{99} + \frac{9\,006\,371\,467\,264\,\alpha^9}{99} + \frac{640\,944\,627\,712\,\alpha^{10}}{99} + \frac{823\,132\,160\,\alpha^{11}}{3} + 5\,242\,880\,\alpha^{12}\right) \mathrm{Seq}[\alpha] +$$
$$\left(\frac{5\,140\,098\,017\,536\,000}{33} + \frac{46\,256\,062\,410\,951\,680\,\alpha}{99} + \frac{188\,106\,511\,562\,207\,104\,\alpha^2}{297} + \frac{762\,306\,794\,937\,484\,096\,\alpha^3}{1485} + \frac{411\,649\,468\,733\,659\,328\,\alpha^4}{1485} + \frac{17\,343\,425\,915\,532\,736\,\alpha^5}{165} + \frac{14\,210\,267\,482\,398\,784\,\alpha^6}{495} + \frac{2\,817\,510\,448\,709\,888\,\alpha^7}{495} + \frac{1\,207\,869\,321\,663\,232\,\alpha^8}{1485} + \frac{121\,355\,508\,279\,296\,\alpha^9}{1485} + \frac{8\,139\,401\,433\,088\,\alpha^{10}}{1485} + \frac{1\,983\,643\,648\,\alpha^{11}}{9} + \frac{12\,058\,624\,\alpha^{12}}{3}\right) \mathrm{Seq}[1+\alpha] +$$
$$\left(82\,434\,527\,124\,000 + \frac{21\,902\,860\,415\,414\,720\,\alpha}{99} + \frac{80\,398\,590\,690\,611\,318\,\alpha^2}{297} + \frac{296\,467\,937\,716\,727\,606\,\alpha^3}{1485} + \frac{29\,350\,523\,706\,501\,110\,\alpha^4}{297} + 34\,584\,428\,808\,394\,\alpha^5 + \frac{4\,342\,545\,420\,574\,924\,\alpha^6}{495} + \frac{804\,378\,844\,067\,528\,\alpha^7}{495} + \frac{64\,781\,892\,974\,608\,\alpha^8}{297} + \frac{6\,144\,492\,239\,168\,\alpha^9}{297} + \frac{1\,954\,371\,306\,368\,\alpha^{10}}{1485} + \frac{453\,705\,728\,\alpha^{11}}{9} + \frac{2\,637\,824\,\alpha^{12}}{3}\right) \mathrm{Seq}[2+\alpha] +$$
$$\left(\frac{233\,617\,870\,750\,100}{99} + \frac{1\,664\,390\,482\,848\,064\,\alpha}{297} + \frac{36\,326\,147\,683\,420\,421\,\alpha^2}{5940} + \frac{6\,021\,394\,052\,113\,444\,\alpha^3}{1485} + \frac{10\,804\,312\,886\,041\,049\,\alpha^4}{5940} + \frac{863\,344\,369\,243\,894\,\alpha^5}{1485} + \frac{403\,088\,696\,550\,349\,\alpha^6}{2970} + \frac{3\,146\,399\,954\,636\,\alpha^7}{135} + \frac{4\,337\,524\,095\,614\,\alpha^8}{1485} + \frac{386\,746\,652\,416\,\alpha^9}{1485} + \frac{23\,280\,058\,144\,\alpha^{10}}{1485} + \frac{5\,146\,624\,\alpha^{11}}{9} + \frac{28\,672\,\alpha^{12}}{3}\right)$$
$$\mathrm{Seq}[3+\alpha] + \left(-\frac{2\,752\,227\,484\,912\,825}{594} - \frac{13\,197\,763\,070\,949\,587\,\alpha}{1188} - \frac{96\,334\,328\,804\,795\,381\,\alpha^2}{7920} - \frac{382\,173\,446\,266\,805\,161\,\alpha^3}{47\,520} - \frac{42\,491\,952\,903\,828\,403\,\alpha^4}{11\,880} - \frac{17\,856\,423\,230\,617\,273\,\alpha^5}{15\,840} - \frac{557\,701\,194\,403\,033\,\alpha^6}{2160} - \frac{102\,887\,973\,582\,425\,\alpha^7}{2376} - \frac{3\,483\,633\,682\,193\,\alpha^8}{660} - \frac{135\,437\,449\,361\,\alpha^9}{297} - \frac{7\,872\,812\,906\,\alpha^{10}}{297} - \frac{8\,377\,600\,\alpha^{11}}{9} - \frac{44\,800\,\alpha^{12}}{3}\right) \mathrm{Seq}[4+\alpha] +$$

$$\left(-\frac{956\,051\,495\,370\,875}{1584}-\frac{6\,600\,841\,412\,724\,055\,\alpha}{4752}-\frac{1\,445\,178\,003\,986\,117\,\alpha^2}{990}-\frac{3\,913\,023\,967\,166\,771\,\alpha^3}{4224}-\frac{75\,161\,757\,558\,445\,403\,\alpha^4}{190\,080}-\frac{842\,079\,566\,274\,951\,\alpha^5}{7040}-\frac{37\,024\,238\,924\,151\,\alpha^6}{1408}-\frac{201\,173\,515\,009\,163\,\alpha^7}{47\,520}-\frac{7\,847\,523\,796\,583\,\alpha^8}{15\,840}-\frac{40\,693\,095\,779\,\alpha^9}{990}-\frac{3\,408\,405\,608\,\alpha^{10}}{1485}-\frac{697\,088\,\alpha^{11}}{9}-\frac{3584\,\alpha^{12}}{3}\right)\mathrm{Seq}[5+\alpha]+$$

$$\left(-\frac{512\,342\,015\,535}{352}-\frac{10\,788\,453\,993\,483\,\alpha}{3520}-\frac{1\,893\,722\,135\,997\,\alpha^2}{640}-\frac{73\,174\,664\,752\,223\,\alpha^3}{42\,240}-\frac{17\,365\,678\,003\,627\,\alpha^4}{25\,344}-\frac{2\,292\,048\,906\,487\,\alpha^5}{11\,880}-\frac{5\,026\,114\,320\,389\,\alpha^6}{126\,720}-\frac{207\,343\,482\,731\,\alpha^7}{34\,560}-\frac{125\,987\,799\,409\,\alpha^8}{190\,080}-\frac{330\,530\,147\,\alpha^9}{6336}-\frac{131\,959\,031\,\alpha^{10}}{47\,520}-\frac{808\,\alpha^{11}}{9}-\frac{4\,\alpha^{12}}{3}\right)\mathrm{Seq}[6+\alpha]+$$

$$\left(\frac{1\,548\,536\,390\,765}{1408}+\frac{1\,151\,180\,739\,667\,\alpha}{480}+\frac{201\,300\,324\,201\,781\,\alpha^2}{84\,480}+\frac{60\,256\,292\,068\,651\,\alpha^3}{42\,240}+\frac{145\,277\,813\,269\,057\,\alpha^4}{253\,440}+\frac{10\,321\,738\,171\,027\,\alpha^5}{63\,360}+\frac{773\,696\,451\,943\,\alpha^6}{23\,040}+\frac{213\,727\,216\,579\,\alpha^7}{42\,240}+\frac{140\,202\,899\,923\,\alpha^8}{253\,440}+\frac{678\,064\,061\,\alpha^9}{15\,840}+\frac{141\,024\,791\,\alpha^{10}}{63\,360}+\frac{419\,\alpha^{11}}{6}+\alpha^{12}\right)\mathrm{Seq}[7+\alpha]$$

```
In[ ]:= SeqGuess = SeqGuess * 253 440 * 3;
```

In[ ]:=
```
RECGuess = ToOrePolynomial[{ReplaceAll[SeqGuess, Seq[n_] :> S[α]^(n-α)]}]
```

Out[ ]=

$$\{(836\,209\,651\,013\,100 + 1\,823\,470\,291\,632\,528\,\alpha + 1\,811\,702\,917\,816\,029\,\alpha^2 + 1\,084\,613\,257\,235\,718\,\alpha^3 + 435\,833\,439\,807\,171\,\alpha^4 + 123\,860\,858\,052\,324\,\alpha^5 + 25\,531\,982\,914\,119\,\alpha^6 + 3\,847\,089\,898\,422\,\alpha^7 + 420\,608\,699\,769\,\alpha^8 + 32\,547\,074\,928\,\alpha^9 + 1\,692\,297\,492\,\alpha^{10} + 53\,095\,680\,\alpha^{11} + 760\,320\,\alpha^{12})\,S_\alpha^7 +$$

$$(-1\,106\,658\,753\,555\,600 - 2\,330\,306\,062\,592\,328\,\alpha - 2\,249\,741\,897\,564\,436\,\alpha^2 - 1\,317\,143\,965\,540\,014\,\alpha^3 - 520\,970\,340\,108\,810\,\alpha^4 - 146\,691\,130\,015\,168\,\alpha^5 - 30\,156\,685\,922\,334\,\alpha^6 - 4\,561\,556\,620\,082\,\alpha^7 - 503\,951\,197\,636\,\alpha^8 - 39\,663\,617\,640\,\alpha^9 - 2\,111\,344\,496\,\alpha^{10} - 68\,259\,840\,\alpha^{11} - 1\,013\,760\,\alpha^{12})\,S_\alpha^6 +$$

$$(-458\,904\,717\,778\,020\,000 - 1\,056\,134\,626\,035\,848\,800\,\alpha - 1\,109\,896\,707\,061\,337\,856\,\alpha^2 - 704\,344\,314\,090\,018\,780\,\alpha^3 - 300\,647\,030\,233\,781\,612\,\alpha^4 - 90\,944\,593\,157\,694\,708\,\alpha^5 - 19\,993\,089\,019\,041\,540\,\alpha^6 - 3\,218\,776\,240\,146\,608\,\alpha^7 - 376\,681\,142\,235\,984\,\alpha^8 - 31\,252\,297\,558\,272\,\alpha^9 - 1\,745\,103\,671\,296\,\alpha^{10} - 58\,889\,994\,240\,\alpha^{11} - 908\,328\,960\,\alpha^{12})\,S_\alpha^5 +$$

$$(-3\,522\,851\,180\,688\,416\,000 - 8\,446\,568\,365\,407\,735\,680\,\alpha - 9\,248\,095\,565\,260\,356\,576\,\alpha^2 - 6\,114\,775\,140\,268\,882\,576\,\alpha^3 - 2\,719\,484\,985\,845\,017\,792\,\alpha^4 - 857\,108\,315\,069\,629\,104\,\alpha^5 - 196\,310\,820\,429\,867\,616\,\alpha^6 - 32\,924\,151\,546\,376\,000\,\alpha^7 - 4\,013\,146\,001\,886\,336\,\alpha^8 - 346\,719\,870\,364\,160\,\alpha^9 - 20\,154\,401\,039\,360\,\alpha^{10} - 707\,739\,648\,000\,\alpha^{11} - 11\,354\,112\,000\,\alpha^{12})\,S_\alpha^4 +$$

$$(1\,794\,185\,247\,360\,768\,000 + 4\,260\,839\,636\,091\,043\,840\,\alpha + 4\,649\,746\,903\,477\,813\,888\,\alpha^2 + 3\,082\,953\,754\,682\,083\,328\,\alpha^3 + 1\,382\,952\,049\,413\,254\,272\,\alpha^4 + 442\,032\,317\,052\,873\,728\,\alpha^5 + 103\,190\,706\,316\,889\,344\,\alpha^6 + 17\,720\,524\,544\,509\,952\,\alpha^7 + 2\,220\,812\,336\,954\,368\,\alpha^8 + 198\,014\,286\,036\,992\,\alpha^9 + 11\,919\,389\,769\,728\,\alpha^{10} + 434\,786\,795\,520\,\alpha^{11} + 7\,266\,631\,680\,\alpha^{12})\,S_\alpha^3 +$$

$$(62\,676\,619\,662\,919\,680\,000 + 168\,213\,967\,990\,385\,049\,600\,\alpha + 205\,820\,392\,167\,964\,974\,080\,\alpha^2 + 151\,791\,584\,110\,964\,534\,272\,\alpha^3 + 75\,137\,340\,688\,642\,841\,600\,\alpha^4 + 26\,295\,232\,911\,598\,126\,080\,\alpha^5 + 6\,670\,149\,766\,003\,083\,264\,\alpha^6 + 1\,235\,525\,904\,487\,723\,008\,\alpha^7 + 165\,841\,646\,014\,996\,480\,\alpha^8 + 15\,729\,900\,132\,270\,080\,\alpha^9 + 1\,000\,638\,108\,860\,416\,\alpha^{10} + 38\,329\,059\,901\,440\,\alpha^{11} + 668\,530\,114\,560\,\alpha^{12})\,S_\alpha^2 +$$

$$(118\,427\,858\,324\,029\,440\,000 + 355\,246\,559\,316\,108\,902\,400\,\alpha + 481\,552\,669\,599\,250\,186\,240\,\alpha^2 + 390\,301\,079\,007\,991\,857\,152\,\alpha^3 + 210\,764\,527\,991\,633\,575\,936\,\alpha^4 + 79\,918\,506\,618\,774\,847\,488\,\alpha^5 + 21\,826\,970\,852\,964\,532\,224\,\alpha^6 + 4\,327\,696\,049\,218\,387\,968\,\alpha^7 + 618\,429\,092\,691\,574\,784\,\alpha^8 + 62\,134\,020\,238\,999\,552\,\alpha^9 + 4\,167\,373\,533\,741\,056\,\alpha^{10} + 167\,578\,215\,383\,040\,\alpha^{11} + 3\,056\,137\,666\,560\,\alpha^{12})\,S_\alpha +$$

$$(42\,140\,738\,676\,326\,400\,000 + 157\,842\,901\,249\,818\,624\,000\,\alpha + 257\,331\,505\,709\,737\,574\,400\,\alpha^2 + 243\,764\,108\,399\,982\,673\,920\,\alpha^3 + 150\,397\,023\,447\,243\,816\,960\,\alpha^4 + 63\,968\,341\,924\,254\,842\,880\,\alpha^5 + 19\,301\,263\,998\,729\,584\,640\,\alpha^6 + 4\,174\,508\,253\,346\,529\,280\,\alpha^7 + 643\,779\,101\,841\,162\,240\,\alpha^8 + 69\,168\,932\,868\,587\,520\,\alpha^9 + 4\,922\,454\,740\,828\,160\,\alpha^{10} + 208\,614\,614\,630\,400\,\alpha^{11} + 3\,986\,266\,521\,600\,\alpha^{12})\}$$

In[◦]:=
```
ClearAll[Seq];
SeqGuess = ApplyOreOperator[RECGuess[[1]], Seq[α]]
```

Out[◦]=

$$
\begin{aligned}
&\big(42\,140\,738\,676\,326\,400\,000 + 157\,842\,901\,249\,818\,624\,000\,\alpha + 257\,331\,505\,709\,737\,574\,400\,\alpha^{2} + \\
&\quad 243\,764\,108\,399\,982\,673\,920\,\alpha^{3} + 150\,397\,023\,447\,243\,816\,960\,\alpha^{4} + 63\,968\,341\,924\,254\,842\,880\,\alpha^{5} + \\
&\quad 19\,301\,263\,998\,729\,584\,640\,\alpha^{6} + 4\,174\,508\,253\,346\,529\,280\,\alpha^{7} + 643\,779\,101\,841\,162\,240\,\alpha^{8} + \\
&\quad 69\,168\,932\,868\,587\,520\,\alpha^{9} + 4\,922\,454\,740\,828\,160\,\alpha^{10} + 208\,614\,614\,630\,400\,\alpha^{11} + 3\,986\,266\,521\,600\,\alpha^{12}\big) \\
&\ \mathrm{Seq}[\alpha] + \big(118\,427\,858\,324\,029\,440\,000 + 355\,246\,559\,316\,108\,902\,400\,\alpha + \\
&\quad 481\,552\,669\,599\,250\,186\,240\,\alpha^{2} + 390\,301\,079\,007\,991\,857\,152\,\alpha^{3} + \\
&\quad 210\,764\,527\,991\,633\,575\,936\,\alpha^{4} + 79\,918\,506\,618\,774\,847\,488\,\alpha^{5} + 21\,826\,970\,852\,964\,532\,224\,\alpha^{6} + \\
&\quad 4\,327\,696\,049\,218\,387\,968\,\alpha^{7} + 618\,429\,092\,691\,574\,784\,\alpha^{8} + 62\,134\,020\,238\,999\,552\,\alpha^{9} + \\
&\quad 4\,167\,373\,533\,741\,056\,\alpha^{10} + 167\,578\,215\,383\,040\,\alpha^{11} + 3\,056\,137\,666\,560\,\alpha^{12}\big)\ \mathrm{Seq}[1+\alpha] + \\
&\ \big(62\,676\,619\,662\,919\,680\,000 + 168\,213\,967\,990\,385\,049\,600\,\alpha + 205\,820\,392\,167\,964\,974\,080\,\alpha^{2} + \\
&\quad 151\,791\,584\,110\,964\,534\,272\,\alpha^{3} + 75\,137\,340\,688\,642\,841\,600\,\alpha^{4} + 26\,295\,232\,911\,598\,126\,080\,\alpha^{5} + \\
&\quad 6\,670\,149\,766\,003\,083\,264\,\alpha^{6} + 1\,235\,525\,904\,487\,723\,008\,\alpha^{7} + 165\,841\,646\,014\,996\,480\,\alpha^{8} + \\
&\quad 15\,729\,900\,132\,270\,080\,\alpha^{9} + 1\,000\,638\,108\,860\,416\,\alpha^{10} + 38\,329\,059\,901\,440\,\alpha^{11} + 668\,530\,114\,560\,\alpha^{12}\big) \\
&\ \mathrm{Seq}[2+\alpha] + \big(1\,794\,185\,247\,360\,768\,000 + 4\,260\,839\,636\,091\,043\,840\,\alpha + 4\,649\,746\,903\,477\,813\,888\,\alpha^{2} + \\
&\quad 3\,082\,953\,754\,682\,083\,328\,\alpha^{3} + 1\,382\,952\,049\,413\,254\,272\,\alpha^{4} + 442\,032\,317\,052\,873\,728\,\alpha^{5} + \\
&\quad 103\,190\,706\,316\,889\,344\,\alpha^{6} + 17\,720\,524\,544\,509\,952\,\alpha^{7} + 2\,220\,812\,336\,954\,368\,\alpha^{8} + \\
&\quad 198\,014\,286\,036\,992\,\alpha^{9} + 11\,919\,389\,769\,728\,\alpha^{10} + 434\,786\,795\,520\,\alpha^{11} + 7\,266\,631\,680\,\alpha^{12}\big)\ \mathrm{Seq}[3+\alpha] + \\
&\ \big(-3\,522\,851\,180\,688\,416\,000 - 8\,446\,568\,365\,407\,735\,680\,\alpha - 9\,248\,095\,565\,260\,356\,576\,\alpha^{2} - \\
&\quad 6\,114\,775\,140\,268\,882\,576\,\alpha^{3} - 2\,719\,484\,985\,845\,017\,792\,\alpha^{4} - 857\,108\,315\,069\,629\,104\,\alpha^{5} - \\
&\quad 196\,310\,820\,429\,867\,616\,\alpha^{6} - 32\,924\,151\,546\,376\,000\,\alpha^{7} - 4\,013\,146\,001\,886\,336\,\alpha^{8} - \\
&\quad 346\,719\,870\,364\,160\,\alpha^{9} - 20\,154\,401\,039\,360\,\alpha^{10} - 707\,739\,648\,000\,\alpha^{11} - 11\,354\,112\,000\,\alpha^{12}\big)\ \mathrm{Seq}[4+\alpha] + \\
&\ \big(-458\,904\,717\,778\,020\,000 - 1\,056\,134\,626\,035\,848\,800\,\alpha - 1\,109\,896\,707\,061\,337\,856\,\alpha^{2} - \\
&\quad 704\,344\,314\,090\,018\,780\,\alpha^{3} - 300\,647\,030\,233\,781\,612\,\alpha^{4} - 90\,944\,593\,157\,694\,708\,\alpha^{5} - \\
&\quad 19\,993\,089\,019\,041\,540\,\alpha^{6} - 3\,218\,776\,240\,146\,608\,\alpha^{7} - 376\,681\,142\,235\,984\,\alpha^{8} - \\
&\quad 31\,252\,297\,558\,272\,\alpha^{9} - 1\,745\,103\,671\,296\,\alpha^{10} - 58\,889\,994\,240\,\alpha^{11} - 908\,328\,960\,\alpha^{12}\big)\ \mathrm{Seq}[5+\alpha] + \\
&\ \big(-1\,106\,658\,753\,555\,600 - 2\,330\,306\,062\,592\,328\,\alpha - 2\,249\,741\,897\,564\,436\,\alpha^{2} - 1\,317\,143\,965\,540\,014\,\alpha^{3} - \\
&\quad 520\,970\,340\,108\,810\,\alpha^{4} - 146\,691\,130\,015\,168\,\alpha^{5} - 30\,156\,685\,922\,334\,\alpha^{6} - 4\,561\,556\,620\,082\,\alpha^{7} - \\
&\quad 503\,951\,197\,636\,\alpha^{8} - 39\,663\,617\,640\,\alpha^{9} - 2\,111\,344\,496\,\alpha^{10} - 68\,259\,840\,\alpha^{11} - 1\,013\,760\,\alpha^{12}\big)\ \mathrm{Seq}[6+\alpha] + \\
&\ \big(836\,209\,651\,013\,100 + 1\,823\,470\,291\,632\,528\,\alpha + 1\,811\,702\,917\,816\,029\,\alpha^{2} + 1\,084\,613\,257\,235\,718\,\alpha^{3} + \\
&\quad 435\,833\,439\,807\,171\,\alpha^{4} + 123\,860\,858\,052\,324\,\alpha^{5} + 25\,531\,982\,914\,119\,\alpha^{6} + 3\,847\,089\,898\,422\,\alpha^{7} + \\
&\quad 420\,608\,699\,769\,\alpha^{8} + 32\,547\,074\,928\,\alpha^{9} + 1\,692\,297\,492\,\alpha^{10} + 53\,095\,680\,\alpha^{11} + 760\,320\,\alpha^{12}\big)\ \mathrm{Seq}[7+\alpha]
\end{aligned}
$$

In[∘]:=
```
RECCompare = DFinitePlus[RECNormalized, RECGuess];
ToOrePolynomial[RECCompare]
```

Out[∘]=
$$\{(360\,896\,796 + 168\,881\,193\,\alpha + 32\,922\,045\,\alpha^2 + 3\,422\,250\,\alpha^3 + 200\,070\,\alpha^4 + 6237\,\alpha^5 + 81\,\alpha^6)\,S_\alpha^{13} + (-41\,921\,605\,728 - 21\,620\,168\,784\,\alpha - 4\,642\,657\,398\,\alpha^2 - 531\,352\,548\,\alpha^3 - 34\,185\,564\,\alpha^4 - 1\,172\,286\,\alpha^5 - 16\,740\,\alpha^6)\,S_\alpha^{12} + (21\,358\,349\,568 + 32\,260\,896\,432\,\alpha + 11\,808\,635\,220\,\alpha^2 + 1\,951\,356\,096\,\alpha^3 + 167\,120\,172\,\alpha^4 + 7\,271\,208\,\alpha^5 + 127\,656\,\alpha^6)\,S_\alpha^{11} + (24\,368\,413\,098\,240 + 13\,143\,885\,737\,280\,\alpha + 2\,934\,076\,066\,464\,\alpha^2 + 346\,493\,030\,640\,\alpha^3 + 22\,789\,627\,296\,\alpha^4 + 789\,604\,176\,\alpha^5 + 11\,220\,000\,\alpha^6)\,S_\alpha^{10} + (89\,406\,840\,410\,496 + 36\,194\,544\,731\,520\,\alpha + 4\,419\,031\,357\,152\,\alpha^2 - 72\,024\,012\,864\,\alpha^3 - 59\,002\,919\,712\,\alpha^4 - 4\,697\,111\,296\,\alpha^5 - 121\,071\,616\,\alpha^6)\,S_\alpha^9 + (1\,090\,683\,194\,271\,744 + 858\,294\,128\,931\,072\,\alpha + 273\,529\,398\,463\,872\,\alpha^2 + 45\,396\,677\,193\,600\,\alpha^3 + 4\,148\,788\,516\,608\,\alpha^4 + 198\,160\,773\,120\,\alpha^5 + 3\,863\,506\,944\,\alpha^6)\,S_\alpha^8 + (44\,181\,186\,456\,002\,560 + 36\,112\,854\,612\,190\,208\,\alpha + 12\,191\,512\,074\,195\,968\,\alpha^2 + 2\,179\,630\,381\,965\,312\,\alpha^3 + 217\,915\,285\,983\,232\,\alpha^4 + 11\,561\,386\,983\,424\,\alpha^5 + 254\,432\,313\,344\,\alpha^6)\,S_\alpha^7 + (340\,022\,650\,507\,689\,984 + 310\,494\,679\,891\,156\,992\,\alpha + 118\,079\,159\,394\,557\,952\,\alpha^2 + 23\,954\,247\,311\,327\,232\,\alpha^3 + 2\,735\,412\,582\,875\,136\,\alpha^4 + 166\,768\,744\,333\,312\,\alpha^5 + 4\,241\,636\,982\,784\,\alpha^6)\,S_\alpha^6 + (265\,100\,186\,543\,063\,040 + 455\,211\,891\,191\,513\,088\,\alpha + 265\,856\,692\,348\,977\,152\,\alpha^2 + 75\,572\,588\,741\,394\,432\,\alpha^3 + 11\,482\,509\,044\,744\,192\,\alpha^4 + 901\,418\,276\,880\,384\,\alpha^5 + 28\,873\,211\,248\,640\,\alpha^6)\,S_\alpha^5 + (-7\,543\,436\,316\,985\,262\,080 - 7\,100\,528\,800\,467\,255\,296\,\alpha - 2\,725\,541\,599\,826\,149\,376\,\alpha^2 - 540\,392\,369\,026\,498\,560\,\alpha^3 - 57\,246\,930\,703\,482\,880\,\alpha^4 - 2\,946\,909\,626\,957\,824\,\alpha^5 - 51\,166\,381\,604\,864\,\alpha^6)\,S_\alpha^4 + (-39\,067\,752\,078\,130\,544\,640 - 46\,083\,130\,998\,367\,715\,328\,\alpha - 22\,625\,963\,564\,995\,706\,880\,\alpha^2 - 5\,920\,758\,191\,326\,494\,720\,\alpha^3 - 871\,074\,008\,169\,185\,280\,\alpha^4 - 68\,314\,920\,545\,943\,552\,\alpha^5 - 2\,230\,813\,395\,517\,440\,\alpha^6)\,S_\alpha^3 + (-78\,810\,495\,709\,882\,613\,760 - 113\,462\,159\,050\,467\,704\,832\,\alpha - 67\,833\,710\,584\,335\,958\,016\,\alpha^2 - 21\,587\,015\,982\,875\,934\,720\,\alpha^3 - 3\,861\,095\,654\,327\,582\,720\,\alpha^4 - 368\,342\,470\,804\,635\,648\,\alpha^5 - 14\,650\,878\,086\,610\,944\,\alpha^6)\,S_\alpha^2 + (-62\,217\,875\,931\,896\,217\,600 - 113\,900\,296\,708\,470\,865\,920\,\alpha - 85\,351\,134\,973\,636\,116\,480\,\alpha^2 - 33\,613\,876\,602\,234\,470\,400\,\alpha^3 - 7\,352\,887\,320\,300\,748\,800\,\alpha^4 - 848\,010\,282\,931\,322\,880\,\alpha^5 - 40\,301\,911\,521\,361\,920\,\alpha^6)\,S_\alpha + (-12\,120\,569\,508\,003\,840\,000 - 32\,523\,528\,179\,810\,304\,000\,\alpha - 33\,796\,187\,978\,150\,707\,200\,\alpha^2 - 17\,675\,830\,532\,505\,600\,000\,\alpha^3 - 4\,949\,232\,549\,101\,568\,000\,\alpha^4 - 707\,033\,221\,300\,224\,000\,\alpha^5 - 40\,401\,898\,360\,012\,800\,\alpha^6)\}$$

In[∘]:=
```
RECCompareOrder = OrePolynomialDegree[RECNormalizedinS, S[α]]
```

Out[∘]= 13

The above argument means that if the sequence generated by “RECGuess” matches with that by “RECNormalized” for the first “RECCompareOrder” terms, then the two sequences are identical.

Hence, we get a rigorous proof of the shorter recurrence “RECGuess” by the following verification!

In[ ]:=
```
SeqListIni
```

Out[ ]=
```
{1, 0, 40, 480, 11 880, 281 280, 7 506 400, 210 268 800, 6 166 993 000,
 187 069 411 200, 5 833 030 976 640, 186 014 056 166 400, 6 044 435 339 896 800,
 199 561 060 892 793 600, 6 679 216 425 794 140 800, 226 213 441 773 789 550 080}
```

In[ ]:=
```
CheckNum = RECCompareOrder + 20 + Length@SeqListIni

SeqGuessList = UnrollRecurrence[SeqGuess, Seq[α], SeqListIni, CheckNum];
SeqGuessList - Take[SeqList, Length@SeqGuessList]
```

Out[ ]=
```
49
```

Out[ ]=
```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

Transform guessed recurrence for $r(n)$ back to ODE for $R(w)$

In[∘]:=
```
RECGuessOrder = OrePolynomialDegree[RECGuess[[1]], S[α]]
RECGuessDetails = RECGuess[[1, 1]]
```

Out[∘]= 7

Out[∘]=
$$\{\{836209651013100 + 1823470291632528\,\alpha + 1811702917816029\,\alpha^2 + 1084613257235718\,\alpha^3 + 435833439807171\,\alpha^4 + 123860858052324\,\alpha^5 + 25531982914119\,\alpha^6 + 3847089898422\,\alpha^7 + 420608699769\,\alpha^8 + 32547074928\,\alpha^9 + 1692297492\,\alpha^{10} + 53095680\,\alpha^{11} + 760320\,\alpha^{12}, \{7\}\},$$
$$\{-1106658753555600 - 2330306062592328\,\alpha - 2249741897564436\,\alpha^2 - 1317143965540014\,\alpha^3 - 520970340108810\,\alpha^4 - 146691130015168\,\alpha^5 - 30156685922334\,\alpha^6 - 4561556620082\,\alpha^7 - 503951197636\,\alpha^8 - 39663617640\,\alpha^9 - 2111344496\,\alpha^{10} - 68259840\,\alpha^{11} - 1013760\,\alpha^{12}, \{6\}\},$$
$$\{-458904717778020000 - 1056134626035848800\,\alpha - 1109896707061337856\,\alpha^2 - 704344314090018780\,\alpha^3 - 300647030233781612\,\alpha^4 - 90944593157694708\,\alpha^5 - 19993089019041540\,\alpha^6 - 3218776240146608\,\alpha^7 - 376681142235984\,\alpha^8 - 31252297558272\,\alpha^9 - 1745103671296\,\alpha^{10} - 58889994240\,\alpha^{11} - 908328960\,\alpha^{12}, \{5\}\},$$
$$\{-3522851180688416000 - 8446568365407735680\,\alpha - 9248095565260356576\,\alpha^2 - 6114775140268882576\,\alpha^3 - 2719484985845017792\,\alpha^4 - 857108315069629104\,\alpha^5 - 196310820429867616\,\alpha^6 - 32924151546376000\,\alpha^7 - 4013146001886336\,\alpha^8 - 346719870364160\,\alpha^9 - 20154401039360\,\alpha^{10} - 707739648000\,\alpha^{11} - 11354112000\,\alpha^{12}, \{4\}\},$$
$$\{1794185247360768000 + 4260839636091043840\,\alpha + 4649746903477813888\,\alpha^2 + 3082953754682083328\,\alpha^3 + 1382952049413254272\,\alpha^4 + 442032317052873728\,\alpha^5 + 103190706316889344\,\alpha^6 + 17720524544509952\,\alpha^7 + 2220812336954368\,\alpha^8 + 198014286036992\,\alpha^9 + 11919389769728\,\alpha^{10} + 434786795520\,\alpha^{11} + 7266631680\,\alpha^{12}, \{3\}\},$$
$$\{62676619662919680000 + 168213967990385049600\,\alpha + 205820392167964974080\,\alpha^2 + 151791584110964534272\,\alpha^3 + 75137340688642841600\,\alpha^4 + 26295232911598126080\,\alpha^5 + 6670149766003083264\,\alpha^6 + 1235525904487723008\,\alpha^7 + 165841646014996480\,\alpha^8 + 15729900132270080\,\alpha^9 + 1000638108860416\,\alpha^{10} + 38329059901440\,\alpha^{11} + 668530114560\,\alpha^{12}, \{2\}\},$$
$$\{118427858324029440000 + 355246559316108902400\,\alpha + 481552669599250186240\,\alpha^2 + 390301079007991857152\,\alpha^3 + 210764527991633575936\,\alpha^4 + 79918506618774847488\,\alpha^5 + 21826970852964532224\,\alpha^6 + 4327696049218387968\,\alpha^7 + 618429092691574784\,\alpha^8 + 62134020238999552\,\alpha^9 + 4167373533741056\,\alpha^{10} + 167578215383040\,\alpha^{11} + 3056137666560\,\alpha^{12}, \{1\}\},$$
$$\{42140738676326400000 + 157842901249818624000\,\alpha + 257331505709737574400\,\alpha^2 + 243764108399982673920\,\alpha^3 + 150397023447243816960\,\alpha^4 + 63968341924254842880\,\alpha^5 + 19301263998729584640\,\alpha^6 + 4174508253346529280\,\alpha^7 + 643779101841162240\,\alpha^8 + 69168932868587520\,\alpha^9 + 4922454740828160\,\alpha^{10} + 208614614630400\,\alpha^{11} + 3986266521600\,\alpha^{12}, \{0\}\}\}$$

In[•]:=
```
ODEGuessinTheta =
  35 * Sum[w^(RECGuessOrder-RECGuessDetails[[i,2]][[1]]) ** Expand[RECGuessDetails[[i, 1]] /.
        {α → Euler[w] - RECGuessDetails[[i, 2]][[1]]}], {i, 1, Length@RECGuessDetails}];
ToOrePolynomial[ODEGuessinTheta]
```

Out[•]=

$$
\begin{aligned}
&(26\,611\,200 - 35\,481\,600\, w - 31\,791\,513\,600\, w^2 - 397\,393\,920\,000\, w^3 + 254\,332\,108\,800\, w^4 + \\
&\quad 23\,398\,554\,009\,600\, w^5 + 106\,964\,818\,329\,600\, w^6 + 139\,519\,328\,256\,000\, w^7)\, \theta_w^{12} + \\
&(-376\,992\,000 + 165\,580\,800\, w - 153\,658\,982\,400\, w^2 - 5\,695\,979\,520\,000\, w^3 + 6\,061\,581\,926\,400\, w^4 + \\
&\quad 779\,951\,800\,320\,000\, w^5 + 4\,581\,659\,718\,451\,200\, w^6 + 7\,301\,511\,512\,064\,000\, w^7)\, \theta_w^{11} + \\
&(2\,198\,175\,420 - 521\,108\,560\, w - 171\,387\,023\,360\, w^2 - 35\,132\,957\,977\,600\, w^3 + 66\,073\,165\,742\,080\, w^4 + \\
&\quad 11\,686\,175\,944\,540\,160\, w^5 + 88\,400\,138\,768\,220\,160\, w^6 + 172\,285\,915\,928\,985\,600\, w^7)\, \theta_w^{10} + \\
&(-6\,812\,368\,920 + 1\,275\,544\,200\, w + 286\,661\,428\,480\, w^2 - 122\,108\,772\,441\,600\, w^3 + 437\,089\,259\,192\,320\, w^4 + \\
&\quad 104\,052\,134\,611\,353\,600\, w^5 + 1\,015\,165\,776\,135\,454\,720\, w^6 + 2\,420\,912\,650\,400\,563\,200\, w^7)\, \theta_w^{9} + \\
&(12\,091\,028\,775 - 2\,119\,116\,860\, w + 786\,690\,738\,960\, w^2 - 261\,163\,240\,652\,160\, w^3 + 1\,965\,596\,285\,219\,840\, w^4 + \\
&\quad 613\,154\,593\,324\,851\,200\, w^5 + 7\,721\,598\,575\,798\,517\,760\, w^6 + 22\,532\,268\,564\,440\,678\,400\, w^7)\, \theta_w^{8} + \\
&(-12\,276\,531\,750 + 2\,365\,861\,610\, w + 335\,310\,078\,320\, w^2 - 346\,312\,107\,001\,280\, w^3 + 6\,376\,246\,006\,036\,480 \\
&\quad w^4 + 2\,519\,538\,414\,570\,311\,680\, w^5 + 40\,945\,924\,679\,986\,708\,480\, w^6 + 146\,107\,788\,867\,128\,524\,800\, w^7)\, \theta_w^{7} + \\
&(6\,597\,132\,255 + 2\,431\,170\, w - 799\,779\,507\,100\, w^2 - 258\,344\,957\,905\,440\, w^3 + 15\,393\,654\,259\,531\,520\, w^4 + \\
&\quad 7\,405\,299\,615\,413\,073\,920\, w^5 + 155\,064\,230\,352\,727\,736\,320\, w^6 + 675\,544\,239\,955\,535\,462\,400\, w^7)\, \theta_w^{6} + \\
&(-1\,447\,054\,980 - 2\,277\,768\,080\, w - 1\,263\,494\,739\,780\, w^2 - 58\,112\,944\,742\,160\, w^3 + \\
&\quad 27\,905\,552\,458\,309\,120\, w^4 + 15\,694\,979\,051\,138\,329\,600\, w^5 + \\
&\quad 422\,099\,245\,327\,615\,098\,880\, w^6 + 2\,238\,891\,967\,348\,919\,500\,800\, w^7)\, \theta_w^{5} + \\
&(-141\,135\,750\, w - 681\,984\,849\,420\, w^2 + 54\,647\,132\,314\,880\, w^3 + 37\,474\,413\,054\,700\,160\, w^4 + \\
&\quad 23\,824\,615\,805\,600\,332\,800\, w^5 + 818\,639\,721\,446\,197\,002\,240\, w^6 + 5\,263\,895\,820\,653\,533\,593\,600\, w^7)\, \theta_w^{4} + \\
&(567\,912\,870\, w + 837\,776\,100\, w^2 + 23\,175\,319\,582\,800\, w^3 + 35\,903\,670\,578\,782\,720\, w^4 + \\
&\quad 25\,282\,158\,181\,891\,179\,520\, w^5 + 1\,101\,523\,487\,158\,827\,909\,120\, w^6 + 8\,531\,743\,793\,999\,393\,587\,200\, w^7)\, \theta_w^{3} + \\
&(102\,702\,600\, w + 169\,532\,984\,040\, w^2 - 26\,142\,472\,268\,640\, w^3 + 22\,934\,895\,664\,419\,200\, w^4 + \\
&\quad 17\,815\,126\,092\,016\,081\,920\, w^5 + 974\,126\,533\,702\,973\,521\,920\, w^6 + 9\,006\,602\,699\,840\,815\,104\,000\, w^7)\, \theta_w^{2} + \\
&(73\,660\,641\,600\, w^2 - 22\,240\,390\,677\,120\, w^3 + 8\,640\,759\,758\,677\,760\, w^4 + 7\,485\,648\,868\,235\,059\,200\, w^5 + \\
&\quad 506\,899\,116\,068\,231\,577\,600\, w^6 + 5\,524\,501\,543\,743\,651\,840\,000\, w^7)\, \theta_w + \\
&(10\,535\,616\,000\, w^2 - 4\,760\,984\,390\,400\, w^3 + 1\,434\,492\,881\,203\,200\, w^4 + 1\,417\,250\,921\,393\,664\,000\, w^5 + \\
&\quad 116\,846\,929\,489\,231\,872\,000\, w^6 + 1\,474\,925\,853\,671\,424\,000\,000\, w^7)
\end{aligned}
$$

In[∘]:=
```
ODEGuessinD =
  ChangeOreAlgebra[ToOrePolynomial[w^-1 ** ODEGuessinTheta], OreAlgebra[Der[w]]];
ToOrePolynomial[ODEGuessinD]
```

Out[∘]=

$$
\begin{aligned}
&\left(26\,611\,200\, w^{11} - 35\,481\,600\, w^{12} - 31\,791\,513\,600\, w^{13} - 397\,393\,920\,000\, w^{14} + 254\,332\,108\,800\, w^{15} + 23\,398\,554\,009\,600\, w^{16} + 106\,964\,818\,329\,600\, w^{17} + 139\,519\,328\,256\,000\, w^{18}\right) D_w^{12} + \\
&\left(1\,379\,347\,200\, w^{10} - 2\,176\,204\,800\, w^{11} - 2\,251\,898\,880\,000\, w^{12} - 31\,923\,978\,240\,000\, w^{13} + 22\,847\,501\,107\,200\, w^{14} + 2\,324\,256\,364\,953\,600\, w^{15} + 11\,641\,337\,728\,204\,800\, w^{16} + 16\,509\,787\,176\,960\,000\, w^{17}\right) D_w^{11} + \\
&\left(26\,835\,711\,420\, w^{9} - 51\,910\,292\,560\, w^{10} - 62\,827\,161\,743\,360\, w^{11} - 1\,025\,968\,465\,177\,600\, w^{12} + 833\,096\,417\,198\,080\, w^{13} + 94\,478\,059\,548\,508\,160\, w^{14} + 522\,766\,438\,535\,004\,160\, w^{15} + 811\,749\,503\,768\,985\,600\, w^{16}\right) D_w^{10} + \\
&\left(249\,444\,244\,980\, w^{8} - 621\,281\,157\,000\, w^{9} - 893\,057\,844\,734\,720\, w^{10} - 17\,133\,897\,795\,033\,600\, w^{11} + 16\,076\,756\,566\,097\,920\, w^{12} + 2\,051\,977\,172\,049\,100\,800\, w^{13} + 12\,667\,630\,323\,808\,337\,920\, w^{14} + 21\,714\,817\,700\,541\,235\,200\, w^{15}\right) D_w^{9} + \\
&\left(1\,168\,711\,655\,055\, w^{7} - 4\,022\,463\,444\,860\, w^{8} - 6\,998\,611\,509\,534\,960\, w^{9} - 161\,871\,397\,145\,189\,760\, w^{10} + 179\,712\,949\,474\,472\,960\, w^{11} + 26\,110\,492\,634\,024\,960\,000\, w^{12} + 182\,008\,082\,212\,541\,562\,880\, w^{13} + 347\,828\,857\,903\,487\,385\,600\, w^{14}\right) D_w^{8} + \\
&\left(2\,677\,402\,633\,710\, w^{6} - 14\,197\,781\,586\,870\, w^{7} - 30\,631\,001\,240\,044\,240\, w^{8} - 884\,446\,926\,000\,128\,960\, w^{9} + 1\,199\,287\,784\,759\,715\,840\, w^{10} + 201\,061\,602\,810\,490\,490\,880\, w^{11} + 1\,606\,226\,048\,899\,314\,483\,200\, w^{12} + 3\,463\,249\,824\,411\,903\,590\,400\, w^{13}\right) D_w^{7} + \\
&\left(2\,676\,030\,768\,075\, w^{5} - 26\,279\,944\,459\,300\, w^{6} - 72\,598\,934\,307\,475\,660\, w^{7} - 2\,750\,445\,697\,955\,635\,680\, w^{8} + 4\,761\,556\,126\,173\,208\,320\, w^{9} + 936\,459\,766\,209\,410\,795\,520\, w^{10} + 8\,736\,845\,034\,708\,691\,353\,600\, w^{11} + 21\,571\,098\,154\,827\,841\,536\,000\, w^{12}\right) D_w^{6} + \\
&\left(891\,262\,190\,175\, w^{4} - 23\,279\,495\,413\,930\, w^{5} - 86\,201\,435\,666\,885\,000\, w^{6} - 4\,622\,979\,700\,434\,872\,560\, w^{7} + 10\,909\,729\,540\,084\,381\,440\, w^{8} + 2\,568\,256\,667\,132\,156\,851\,200\, w^{9} + 28\,681\,784\,044\,178\,536\,857\,600\, w^{10} + 82\,634\,205\,305\,324\,961\,792\,000\, w^{11}\right) D_w^{5} + \\
&\left(47\,254\,482\,450\, w^{3} - 8\,224\,388\,292\,260\, w^{4} - 44\,063\,917\,300\,942\,560\, w^{5} - 3\,802\,957\,302\,589\,932\,480\, w^{6} + 13\,580\,891\,613\,973\,129\,600\, w^{7} + 3\,921\,961\,620\,254\,838\,451\,200\, w^{8} + 54\,220\,230\,773\,678\,540\,390\,400\, w^{9} + 186\,864\,119\,990\,862\,741\,504\,000\, w^{10}\right) D_w^{4} + \\
&\left(1\,735\,653\,150\, w^{2} - 746\,231\,041\,080\, w^{3} - 7\,108\,904\,277\,977\,040\, w^{4} - 1\,274\,771\,999\,582\,231\,360\, w^{5} + 8\,295\,259\,035\,022\,624\,000\, w^{6} + 3\,025\,813\,412\,925\,657\,446\,400\, w^{7} + 54\,340\,898\,122\,692\,860\,313\,600\, w^{8} + 231\,781\,876\,171\,978\,309\,632\,000\, w^{9}\right) D_w^{3} + \\
&\left(-131\,695\,200\, w + 2\,384\,396\,280\, w^{2} - 164\,063\,646\,677\,280\, w^{3} - 119\,170\,999\,659\,783\,680\, w^{4} + 2\,092\,030\,359\,232\,665\,600\, w^{5} + 995\,375\,156\,357\,298\,124\,800\, w^{6} + 25\,016\,836\,878\,817\,394\,688\,000\, w^{7} + 138\,753\,870\,452\,815\,822\,848\,000\, w^{8}\right) D_w^{2} + \\
&\left(-614\,577\,600\, w - 1\,449\,402\,968\,160\, w^{2} - 1\,057\,828\,765\,208\,320\, w^{3} + 157\,104\,266\,404\,646\,400\, w^{4} + 100\,757\,062\,283\,099\,443\,200\, w^{5} + 4\,028\,128\,111\,851\,798\,528\,000\, w^{6} + 31\,412\,420\,762\,570\,588\,160\,000\, w^{7}\right) D_w + \\
&\left(10\,535\,616\,000\, w - 4\,760\,984\,390\,400\, w^{2} + 1\,434\,492\,881\,203\,200\, w^{3} + 1\,417\,250\,921\,393\,664\,000\, w^{4} + 116\,846\,929\,489\,231\,872\,000\, w^{5} + 1\,474\,925\,853\,671\,424\,000\,000\, w^{6}\right)
\end{aligned}
$$

In[ ]:=
```
ODEGuess = {ODEGuessinD};
ToOrePolynomial[ODEGuess]
```

Out[ ]=

$$\{(26\,611\,200\,w^{11} - 35\,481\,600\,w^{12} - 31\,791\,513\,600\,w^{13} - 397\,393\,920\,000\,w^{14} + 254\,332\,108\,800\,w^{15} + 23\,398\,554\,009\,600\,w^{16} + 106\,964\,818\,329\,600\,w^{17} + 139\,519\,328\,256\,000\,w^{18})\,D_w^{12} +$$
$$(1\,379\,347\,200\,w^{10} - 2\,176\,204\,800\,w^{11} - 2\,251\,898\,880\,000\,w^{12} - 31\,923\,978\,240\,000\,w^{13} + 22\,847\,501\,107\,200\,w^{14} + 2\,324\,256\,364\,953\,600\,w^{15} + 11\,641\,337\,728\,204\,800\,w^{16} + 16\,509\,787\,176\,960\,000\,w^{17})\,D_w^{11} +$$
$$(26\,835\,711\,420\,w^{9} - 51\,910\,292\,560\,w^{10} - 62\,827\,161\,743\,360\,w^{11} - 1\,025\,968\,465\,177\,600\,w^{12} + 833\,096\,417\,198\,080\,w^{13} + 94\,478\,059\,548\,508\,160\,w^{14} + 522\,766\,438\,535\,004\,160\,w^{15} + 811\,749\,503\,768\,985\,600\,w^{16})\,D_w^{10} +$$
$$(249\,444\,244\,980\,w^{8} - 621\,281\,157\,000\,w^{9} - 893\,057\,844\,734\,720\,w^{10} - 17\,133\,897\,795\,033\,600\,w^{11} + 16\,076\,756\,566\,097\,920\,w^{12} + 2\,051\,977\,172\,049\,100\,800\,w^{13} + 12\,667\,630\,323\,808\,337\,920\,w^{14} + 21\,714\,817\,700\,541\,235\,200\,w^{15})\,D_w^{9} +$$
$$(1\,168\,711\,655\,055\,w^{7} - 4\,022\,463\,444\,860\,w^{8} - 6\,998\,611\,509\,534\,960\,w^{9} - 161\,871\,397\,145\,189\,760\,w^{10} + 179\,712\,949\,474\,472\,960\,w^{11} + 26\,110\,492\,634\,024\,960\,000\,w^{12} + 182\,008\,082\,212\,541\,562\,880\,w^{13} + 347\,828\,857\,903\,487\,385\,600\,w^{14})\,D_w^{8} +$$
$$(2\,677\,402\,633\,710\,w^{6} - 14\,197\,781\,586\,870\,w^{7} - 30\,631\,001\,240\,044\,240\,w^{8} - 884\,446\,926\,000\,128\,960\,w^{9} + 1\,199\,287\,784\,759\,715\,840\,w^{10} + 201\,061\,602\,810\,490\,490\,880\,w^{11} + 1\,606\,226\,048\,899\,314\,483\,200\,w^{12} + 3\,463\,249\,824\,411\,903\,590\,400\,w^{13})\,D_w^{7} +$$
$$(2\,676\,030\,768\,075\,w^{5} - 26\,279\,944\,459\,300\,w^{6} - 72\,598\,934\,307\,475\,660\,w^{7} - 2\,750\,445\,697\,955\,635\,680\,w^{8} + 4\,761\,556\,126\,173\,208\,320\,w^{9} + 936\,459\,766\,209\,410\,795\,520\,w^{10} + 8\,736\,845\,034\,708\,691\,353\,600\,w^{11} + 21\,571\,098\,154\,827\,841\,536\,000\,w^{12})\,D_w^{6} +$$
$$(891\,262\,190\,175\,w^{4} - 23\,279\,495\,413\,930\,w^{5} - 86\,201\,435\,666\,885\,000\,w^{6} - 4\,622\,979\,700\,434\,872\,560\,w^{7} + 10\,909\,729\,540\,084\,381\,440\,w^{8} + 2\,568\,256\,667\,132\,156\,851\,200\,w^{9} + 28\,681\,784\,044\,178\,536\,857\,600\,w^{10} + 82\,634\,205\,305\,324\,961\,792\,000\,w^{11})\,D_w^{5} +$$
$$(47\,254\,482\,450\,w^{3} - 8\,224\,388\,292\,260\,w^{4} - 44\,063\,917\,300\,942\,560\,w^{5} - 3\,802\,957\,302\,589\,932\,480\,w^{6} + 13\,580\,891\,613\,973\,129\,600\,w^{7} + 3\,921\,961\,620\,254\,838\,451\,200\,w^{8} + 54\,220\,230\,773\,678\,540\,390\,400\,w^{9} + 186\,864\,119\,990\,862\,741\,504\,000\,w^{10})\,D_w^{4} +$$
$$(1\,735\,653\,150\,w^{2} - 746\,231\,041\,080\,w^{3} - 7\,108\,904\,277\,977\,040\,w^{4} - 1\,274\,771\,999\,582\,231\,360\,w^{5} + 8\,295\,259\,035\,022\,624\,000\,w^{6} + 3\,025\,813\,412\,925\,657\,446\,400\,w^{7} + 54\,340\,898\,122\,692\,860\,313\,600\,w^{8} + 231\,781\,876\,171\,978\,309\,632\,000\,w^{9})\,D_w^{3} +$$
$$(-131\,695\,200\,w + 2\,384\,396\,280\,w^{2} - 164\,063\,646\,677\,280\,w^{3} - 119\,170\,999\,659\,783\,680\,w^{4} + 2\,092\,030\,359\,232\,665\,600\,w^{5} + 995\,375\,156\,357\,298\,124\,800\,w^{6} + 25\,016\,836\,878\,817\,394\,688\,000\,w^{7} + 138\,753\,870\,452\,815\,822\,848\,000\,w^{8})\,D_w^{2} +$$
$$(-614\,577\,600\,w - 1\,449\,402\,968\,160\,w^{2} - 1\,057\,828\,765\,208\,320\,w^{3} + 157\,104\,266\,404\,646\,400\,w^{4} + 100\,757\,062\,283\,099\,443\,200\,w^{5} + 4\,028\,128\,111\,851\,798\,528\,000\,w^{6} + 31\,412\,420\,762\,570\,588\,160\,000\,w^{7})\,D_w +$$
$$(10\,535\,616\,000\,w - 4\,760\,984\,390\,400\,w^{2} + 1\,434\,492\,881\,203\,200\,w^{3} + 1\,417\,250\,921\,393\,664\,000\,w^{4} + 116\,846\,929\,489\,231\,872\,000\,w^{5} + 1\,474\,925\,853\,671\,424\,000\,000\,w^{6})\}$$

Compare with the known ODE

In[∘]:= **ToOrePolynomial[ODEGuessinTheta]**

Out[∘]=
$$\left(26611200-35481600\,w-31791513600\,w^2-397393920000\,w^3+254332108800\,w^4+23398554009600\,w^5+106964818329600\,w^6+139519328256000\,w^7\right)\theta_w^{12}+$$
$$\left(-376992000+165580800\,w-153658982400\,w^2-5695979520000\,w^3+6061581926400\,w^4+779951800320000\,w^5+4581659718451200\,w^6+7301511512064000\,w^7\right)\theta_w^{11}+$$
$$\left(2198175420-521108560\,w-171387023360\,w^2-35132957977600\,w^3+66073165742080\,w^4+11686175944540160\,w^5+88400138768220160\,w^6+172285915928985600\,w^7\right)\theta_w^{10}+$$
$$\left(-6812368920+1275544200\,w+286661428480\,w^2-122108772441600\,w^3+437089259192320\,w^4+104052134611353600\,w^5+1015165776135454720\,w^6+2420912650400563200\,w^7\right)\theta_w^{9}+$$
$$\left(12091028775-2119116860\,w+786690738960\,w^2-261163240652160\,w^3+1965596285219840\,w^4+613154593324851200\,w^5+7721598575798517760\,w^6+22532268564440678400\,w^7\right)\theta_w^{8}+$$
$$\left(-12276531750+2365861610\,w+335310078320\,w^2-346312107001280\,w^3+6376246006036480\,w^4+2519538414570311680\,w^5+40945924679986708480\,w^6+146107788867128524800\,w^7\right)\theta_w^{7}+$$
$$\left(6597132255+2431170\,w-799779507100\,w^2-258344957905440\,w^3+15393654259531520\,w^4+7405299615413073920\,w^5+155064230352727736320\,w^6+675544239955535462400\,w^7\right)\theta_w^{6}+$$
$$\left(-1447054980-2277768080\,w-1263494739780\,w^2-58112944742160\,w^3+27905552458309120\,w^4+15694979051138329600\,w^5+422099245327615098880\,w^6+2238891967348919500800\,w^7\right)\theta_w^{5}+$$
$$\left(-141135750\,w-681984849420\,w^2+54647132314880\,w^3+37474413054700160\,w^4+23824615805600332800\,w^5+818639721446197002240\,w^6+5263895820653533593600\,w^7\right)\theta_w^{4}+$$
$$\left(567912870\,w+837776100\,w^2+23175319582800\,w^3+35903670578782720\,w^4+25282158181891179520\,w^5+1101523487158827909120\,w^6+8531743793999393587200\,w^7\right)\theta_w^{3}+$$
$$\left(102702600\,w+169532984040\,w^2-26142472268640\,w^3+22934895664419200\,w^4+17815126092016081920\,w^5+974126533702973521920\,w^6+9006602699840815104000\,w^7\right)\theta_w^{2}+$$
$$\left(73660641600\,w^2-22240390677120\,w^3+8640759758677760\,w^4+7485648868235059200\,w^5+506899116068231577600\,w^6+5524501543743651840000\,w^7\right)\theta_w+$$
$$\left(10535616000\,w^2-4760984390400\,w^3+1434492881203200\,w^4+1417250921393664000\,w^5+116846929489231872000\,w^6+1474925853671424000000\,w^7\right)$$

In[ ]:= **ToOrePolynomial[ODENormalizedinTheta]**

Out[ ]=
$$\left(-16740+\frac{81}{w}+127656\,w+11220000\,w^2-121071616\,w^3+3863506944\,w^4+254432313344\,w^5+4241636982784\,w^6+28873211248640\,w^7-51166381604864\,w^8-2230813395517440\,w^9-14650878086610944\,w^{10}-40301911521361920\,w^{11}-40401898360012800\,w^{12}\right)\theta_w^6+$$
$$\left(32994-\frac{81}{w}-1154088\,w+116404176\,w^2+1840755968\,w^3+12712439808\,w^4+875229822976\,w^5+14069812953088\,w^6+35221939421184\,w^7-1718916468441088\,w^8-28160279426629632\,w^9-192531933765304320\,w^{10}-606198813803151360\,w^{11}-707033221300224000\,w^{12}\right)\theta_w^5+$$
$$\left(-6804-1100628\,w+139418496\,w^2+5265075168\,w^3-68675741952\,w^4+274491871232\,w^5+22834223579136\,w^6-225493659025408\,w^7-10588669749493760\,w^8-147510008374886400\,w^9-1056723631477882880\,w^{10}-3717364578464563200\,w^{11}-4949232549101568000\,w^{12}\right)\theta_w^4+$$
$$\left(-2916-1972512\,w+112114800\,w^2+12645108288\,w^3-103971647616\,w^4-2323673227264\,w^5+17221516689408\,w^6-906051054993408\,w^7-30454049629798400\,w^8-411573708851773440\,w^9-3087809086582947840\,w^{10}-11876391919917465600\,w^{11}-17675830532505600000\,w^{12}\right)\theta_w^3+$$
$$\left(-486-1477116\,w+19764864\,w^2+14938983648\,w^3-65355540096\,w^4-5809456065536\,w^5-11239122935808\,w^6-1442272704790528\,w^7-46995263152193536\,w^8-642546012342190080\,w^9-5026723467358109696\,w^{10}-20751254932244398080\,w^{11}-33796187978150707200\,w^{12}\right)\theta_w^2+$$
$$\left(-588384\,w-26784000\,w^2+8870110848\,w^3+3152918784\,w^4-5433252877312\,w^5-29346613223424\,w^6-1105994726572032\,w^7-37493528964104192\,w^8-526844782379532288\,w^9-4270876640893992960\,w^{10}-18626347232227491840\,w^{11}-32523528179810304000\,w^{12}\right)\theta_w+$$
$$\left(-103680\,w-12908160\,w^2+2131142400\,w^3+13656145920\,w^4-1847359887360\,w^5-14343342735360\,w^6-335589890457600\,w^7-12031395779051520\,w^8-174291936545341440\,w^9-1453119684319641600\,w^{10}-6600016543717785600\,w^{11}-12120569508003840000\,w^{12}\right)$$