

# Multi-headed Lattice Green Function (N = 5, M = 2)

## REC for $r_{2,5}(n)$ in Theorem 6.2

$$\begin{aligned} \text{Out}[n] = & \left( 42\,140\,738\,676\,326\,400\,000 + 157\,842\,901\,249\,818\,624\,000\,\alpha + 257\,331\,505\,709\,737\,574\,400\,\alpha^2 + \right. \\ & 243\,764\,108\,399\,982\,673\,920\,\alpha^3 + 150\,397\,023\,447\,243\,816\,960\,\alpha^4 + 63\,968\,341\,924\,254\,842\,880\,\alpha^5 + \\ & 19\,301\,263\,998\,729\,584\,640\,\alpha^6 + 4\,174\,508\,253\,346\,529\,280\,\alpha^7 + 643\,779\,101\,841\,162\,240\,\alpha^8 + \\ & 69\,168\,932\,868\,587\,520\,\alpha^9 + 4\,922\,454\,740\,828\,160\,\alpha^{10} + 208\,614\,614\,630\,400\,\alpha^{11} + 3\,986\,266\,521\,600\,\alpha^{12} \Big) \\ & \text{Seq}[\alpha] + \left( 118\,427\,858\,324\,029\,440\,000 + 355\,246\,559\,316\,108\,902\,400\,\alpha + \right. \\ & 481\,552\,669\,599\,250\,186\,240\,\alpha^2 + 390\,301\,079\,007\,991\,857\,152\,\alpha^3 + \\ & 210\,764\,527\,991\,633\,575\,936\,\alpha^4 + 79\,918\,506\,618\,774\,847\,488\,\alpha^5 + 21\,826\,970\,852\,964\,532\,224\,\alpha^6 + \\ & 4\,327\,696\,049\,218\,387\,968\,\alpha^7 + 618\,429\,092\,691\,574\,784\,\alpha^8 + 62\,134\,020\,238\,999\,552\,\alpha^9 + \\ & 4\,167\,373\,533\,741\,056\,\alpha^{10} + 167\,578\,215\,383\,040\,\alpha^{11} + 3\,056\,137\,666\,560\,\alpha^{12} \Big) \text{Seq}[1 + \alpha] + \\ & \left( 62\,676\,619\,662\,919\,680\,000 + 168\,213\,967\,990\,385\,049\,600\,\alpha + 205\,820\,392\,167\,964\,974\,080\,\alpha^2 + \right. \\ & 151\,791\,584\,110\,964\,534\,272\,\alpha^3 + 75\,137\,340\,688\,642\,841\,600\,\alpha^4 + 26\,295\,232\,911\,598\,126\,080\,\alpha^5 + \\ & 6\,670\,149\,766\,003\,083\,264\,\alpha^6 + 1\,235\,525\,904\,487\,723\,008\,\alpha^7 + 165\,841\,646\,014\,996\,480\,\alpha^8 + \\ & 15\,729\,900\,132\,270\,080\,\alpha^9 + 1\,000\,638\,108\,860\,416\,\alpha^{10} + 38\,329\,059\,901\,440\,\alpha^{11} + 668\,530\,114\,560\,\alpha^{12} \Big) \\ & \text{Seq}[2 + \alpha] + \left( 1\,794\,185\,247\,360\,768\,000 + 4\,260\,839\,636\,091\,043\,840\,\alpha + 4\,649\,746\,903\,477\,813\,888\,\alpha^2 + \right. \\ & 3\,082\,953\,754\,682\,083\,328\,\alpha^3 + 1\,382\,952\,049\,413\,254\,272\,\alpha^4 + 442\,032\,317\,052\,873\,728\,\alpha^5 + \\ & 103\,190\,706\,316\,889\,344\,\alpha^6 + 17\,720\,524\,544\,509\,952\,\alpha^7 + 2\,220\,812\,336\,954\,368\,\alpha^8 + \\ & 198\,014\,286\,036\,992\,\alpha^9 + 11\,919\,389\,769\,728\,\alpha^{10} + 434\,786\,795\,520\,\alpha^{11} + 7\,266\,631\,680\,\alpha^{12} \Big) \text{Seq}[3 + \alpha] + \\ & \left( -3\,522\,851\,180\,688\,416\,000 - 8\,446\,568\,365\,407\,735\,680\,\alpha - 9\,248\,095\,565\,260\,356\,576\,\alpha^2 - \right. \\ & 6\,114\,775\,140\,268\,882\,576\,\alpha^3 - 2\,719\,484\,985\,845\,017\,792\,\alpha^4 - 857\,108\,315\,069\,629\,104\,\alpha^5 - \\ & 196\,310\,820\,429\,867\,616\,\alpha^6 - 32\,924\,151\,546\,376\,000\,\alpha^7 - 4\,013\,146\,001\,886\,336\,\alpha^8 - \\ & 346\,719\,870\,364\,160\,\alpha^9 - 20\,154\,401\,039\,360\,\alpha^{10} - 707\,739\,648\,000\,\alpha^{11} - 11\,354\,112\,000\,\alpha^{12} \Big) \text{Seq}[4 + \alpha] + \\ & \left( -458\,904\,717\,778\,020\,000 - 1\,056\,134\,626\,035\,848\,800\,\alpha - 1\,109\,896\,707\,061\,337\,856\,\alpha^2 - \right. \\ & 704\,344\,314\,090\,018\,780\,\alpha^3 - 300\,647\,030\,233\,781\,612\,\alpha^4 - 90\,944\,593\,157\,694\,708\,\alpha^5 - \\ & 19\,993\,089\,019\,041\,540\,\alpha^6 - 3\,218\,776\,240\,146\,608\,\alpha^7 - 376\,681\,142\,235\,984\,\alpha^8 - \\ & 31\,252\,297\,558\,272\,\alpha^9 - 1\,745\,103\,671\,296\,\alpha^{10} - 58\,889\,994\,240\,\alpha^{11} - 908\,328\,960\,\alpha^{12} \Big) \text{Seq}[5 + \alpha] + \\ & \left( -1\,106\,658\,753\,555\,600 - 2\,330\,306\,062\,592\,328\,\alpha - 2\,249\,741\,897\,564\,436\,\alpha^2 - 1\,317\,143\,965\,540\,014\,\alpha^3 - \right. \\ & 520\,970\,340\,108\,810\,\alpha^4 - 146\,691\,130\,015\,168\,\alpha^5 - 30\,156\,685\,922\,334\,\alpha^6 - 4\,561\,556\,620\,082\,\alpha^7 - \\ & 503\,951\,197\,636\,\alpha^8 - 39\,663\,617\,640\,\alpha^9 - 2\,111\,344\,496\,\alpha^{10} - 68\,259\,840\,\alpha^{11} - 1\,013\,760\,\alpha^{12} \Big) \text{Seq}[6 + \alpha] + \\ & \left( 836\,209\,651\,013\,100 + 1\,823\,470\,291\,632\,528\,\alpha + 1\,811\,702\,917\,816\,029\,\alpha^2 + 1\,084\,613\,257\,235\,718\,\alpha^3 + \right. \\ & 435\,833\,439\,807\,171\,\alpha^4 + 123\,860\,858\,052\,324\,\alpha^5 + 25\,531\,982\,914\,119\,\alpha^6 + 3\,847\,089\,898\,422\,\alpha^7 + \\ & 420\,608\,699\,769\,\alpha^8 + 32\,547\,074\,928\,\alpha^9 + 1\,692\,297\,492\,\alpha^{10} + 53\,095\,680\,\alpha^{11} + 760\,320\,\alpha^{12} \Big) \text{Seq}[7 + \alpha] \end{aligned}$$