Multi-headed Lattice Green Function (N = 5, M = 4)

In[・]:=
```
NN = 5;
MM = 4;
```

Recall some basic definitions in the paper:

$$P_{M,N}(z) := \frac{1}{(2\pi)^N} \int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} \frac{1}{1 - \frac{z}{\binom{N}{M}} \sigma_M(\cos\theta_1, \ldots, \cos\theta_N)} d\theta_1 \ldots d\theta_N$$

$$R_{M,N}(z) := P_{M,N}\left(2^M \binom{N}{M} z\right) \text{ and } R_{M,N}(z) = \sum_{n\geq 0} r_{M,N}(n)\, z^n$$

Also, for M odd or $M = N$, we always have $r(2n+1) = 0$. Hence, define
$\tilde{r}_{M,N}(n) := r_{M,N}(2n)$ and $\tilde{R}_{M,N}(z) := \sum_{n\geq 0} \tilde{r}_{M,N}(n)\, z^n = \sum_{n\geq 0} r_{M,N}(2n)\, z^n$

Our goal is to find:

Case 1. M even and $M \neq N$:

- recurrences (REC) for $r(n)$ or differential equations (ODE) for $R(z)$.

Case 2. M odd or $M = N$:

- recurrences (REC) for $\tilde{r}(n)$ or differential equations (ODE) for $\tilde{R}(z)$.

Command: UnrollRecurrence

Generate a sequence from recurrence & initial values (Koutschan's implementation).

In[・]:=
```
(* given a recurrence rec in f[n], compute the values {f[0],f[1],...,f[bound]}
 where inits are the initial values
 {f[0],...,f[d-1]} with d being the order of the recurrence *)
Clear[UnrollRecurrence];
UnrollRecurrence[rec1_, f_[n_], inits_, bound_] :=
  Module[{i, x, vals = inits, rec = rec1},
   If[Head[rec] =!= Equal, rec = (rec == 0)];
   rec = rec /. n → n - Max[Cases[rec, f[n + a_.] :> a, Infinity]];
   Do[
    AppendTo[vals, Solve[rec /. n → i /. f[i] → x /. f[a_] :> vals[[a + 1]], x][[1, 1, 2]]];
    , {i, Length[inits], bound}];
   Return[vals];
  ];
```

Load RISC packages.

In[ ]:=
```
<< RISC`HolonomicFunctions`
<< RISC`Asymptotics`
<< RISC`Guess`
```



```
--> Type  ?HolonomicFunctions  for help.
```


Apply creative telescoping to the even-indexed subsequence $\tilde{r}_e(n) := r(2n)$.

In[ ]:=
```
ClearAll[k1, k2, k3, k4, k5, z, w, α, β];
```

In[ ]:=
```
k5 = α - k1 - k2 - k3 - k4;
summandEVEN = Binomial[2 α, 2 k1] Binomial[2 α - 2 k1, 2 k2] Binomial[2 α - 2 k1 - 2 k2, 2 k3]
   Binomial[2 α - 2 k1 - 2 k2 - 2 k3, 2 k4] Binomial[2 (α - k1), α - k1] Binomial[2 (α - k2), α - k2]
   Binomial[2 (α - k3), α - k3] Binomial[2 (α - k4), α - k4] Binomial[2 (α - k5), α - k5];
```

In[ ]:=
```
Timing[ann0EVEN = Annihilator[summandEVEN, {S[k1], S[k2], S[k3], S[k4], S[α]}];]
```

Out[ ]=
```
{0.078125, Null}
```

In[ ]:=
```
Timing[ann1EVEN = FindCreativeTelescoping[ann0EVEN, S[k1] - 1][[1]];]
```

Out[ ]=
```
{433.984, Null}
```

In[ ]:=
```
Timing[ann2EVEN = FindCreativeTelescoping[ann1EVEN, S[k2] - 1][[1]];]
```

Out[ ]=
```
{12354.5, Null}
```

In[ ]:=
```
Timing[ann3EVEN = FindCreativeTelescoping[ann2EVEN, S[k3] - 1][[1]];]
```

Out[ ]=
```
{39765., Null}
```

In[•]:=
```
Timing[ann4EVEN = FindCreativeTelescoping[ann3EVEN, S[k4] - 1][[1]];]
```
Out[•]=
```
{44146.1, Null}
```

Alternatively, you may import the value of {ann1EVEN, ..., ann4EVEN} from an external file.

In[•]:=
```
{ann1EVEN, ann2EVEN, ann3EVEN, ann4EVEN} =
  ToExpression[Import[NotebookDirectory[] <> "Data-N5M4-Sum-EVEN.txt"]];
```

ann4EVEN gives a REC for $\tilde{r}_e(n)$.

Apply creative telescoping to the odd-indexed subsequence $\tilde{r}_o(n) := r(2n+1)$.

In[•]:=
```
ClearAll[k1, k2, k3, k4, k5, z, w, α, β];
```

In[•]:=
```
k5 = α + (1 - NN)/2 - k1 - k2 - k3 - k4;
summandODD = Binomial[2 α + 1, 2 k1 + 1]
   Binomial[(2 α + 1) - (2 k1 + 1), 2 k2 + 1] Binomial[(2 α + 1) - (2 k1 + 1) - (2 k2 + 1), 2 k3 + 1]
   Binomial[(2 α + 1) - (2 k1 + 1) - (2 k2 + 1) - (2 k3 + 1), 2 k4 + 1]
   Binomial[2 (α - k1), α - k1] Binomial[2 (α - k2), α - k2] Binomial[2 (α - k3), α - k3]
   Binomial[2 (α - k4), α - k4] Binomial[2 (α - k5), α - k5];
```

In[•]:=
```
Timing[ann0ODD = Annihilator[summandODD, {S[k1], S[k2], S[k3], S[k4], S[α]}];]
```
Out[•]=
```
{0.09375, Null}
```

In[•]:=
```
Timing[ann1ODD = FindCreativeTelescoping[ann0ODD, S[k1] - 1][[1]];]
```
Out[•]=
```
{419.172, Null}
```

In[•]:=
```
Timing[ann2ODD = FindCreativeTelescoping[ann1ODD, S[k2] - 1][[1]];]
```
Out[•]=
```
{15208.2, Null}
```

In[•]:=
```
Timing[ann3ODD = FindCreativeTelescoping[ann2ODD, S[k3] - 1][[1]];]
```
Out[•]=
```
{35861.1, Null}
```

In[•]:=
```
Timing[ann4ODD = FindCreativeTelescoping[ann3ODD, S[k4] - 1][[1]];]
```
Out[•]=
```
{42672., Null}
```

Alternatively, you may import the value of {ann1ODD, ..., ann4ODD} from an external file.

In[•]:=
```
{ann1ODD, ann2ODD, ann3ODD, ann4ODD} =
  ToExpression[Import[NotebookDirectory[] <> "Data-N5M4-Sum-ODD.txt"]];
```

ann4ODD gives a REC for $\tilde{r}_o(n)$.

Compute the REC for $r(n)$.

REC: Order 12

ODE: Order 71

We first store the RECs for $\tilde{r}_e(n)$ and $\tilde{r}_o(n)$.

In[•]:=
```
RECNormalizedinSEVEN = ann4EVEN[[1]];
RecNormalizedOrderEVEN = OrePolynomialDegree[RECNormalizedinSEVEN]
```

Out[∘]= 6

```
In[∘]:= RECNormalizedinSODD = ann4ODD[[1]];
RecNormalizedOrderODD = OrePolynomialDegree[RECNormalizedinSODD]
```

Out[∘]= 6

Then we derive the RECs for sequences
$\{r(0),\ 0,\ r(2),\ 0,\ \ldots\}$ and
$\{0,\ r(1),\ 0,\ r(3),\ \ldots\}$,
and compute the REC for their linear combination, including
$\{r(0),\ 0,\ r(2),\ 0,\ \ldots\} + \{0,\ r(1),\ 0,\ r(3),\ \ldots\} = \{r(0),\ r(1),\ r(2),\ r(3),\ \ldots\}$.

```
In[∘]:= RECNormalizedEVENnew =
  OrePolynomialSubstitute[{RECNormalizedinSEVEN}, {α → (α - 0) / 2, S[α] → S[α]^2}];

In[∘]:= RECNormalizedODDnew =
  OrePolynomialSubstitute[{RECNormalizedinSODD}, {α → (α - 1) / 2, S[α] → S[α]^2}];

In[∘]:= RECNormalizedinS = DFinitePlus[RECNormalizedEVENnew, RECNormalizedODDnew][[1]];

In[∘]:= RecNormalizedOrder = OrePolynomialDegree[RECNormalizedinS]
```

Out[∘]= 12

```
In[∘]:= ODENormalizedOrder = Max[Exponent[OrePolynomialListCoefficients[
     α^Max[Exponent[OrePolynomialListCoefficients[RECNormalizedinS] /. {α→α^-1}, α]] * RECNormalizedinS], α]]
```

Out[∘]= 71

We also write this REC explicitly.

```
In[∘]:= ClearAll[Seq];
SeqNormalized = ApplyOreOperator[RECNormalizedinS, Seq[α]];
```

The initial values of $r(n)$ are as follows.

In[∘]:=
```
SeqListIni = {};

MAX = 20;

For[n = 0, n ≤ MAX, n++,
  coord = Select[Tuples[Table[i, {i, 0, n}], NN], Total[#] == n &];
  size = Length@coord;
  p = Sum[Multinomial[Sequence @@ (2 coord[[i]])] *
     Product[Binomial[2 n - 2 coord[[i, j]], n - coord[[i, j]]], {j, 1, NN}], {i, 1, size}];
  SeqListIni = Append[SeqListIni, p];

  coord = Select[Tuples[Table[i, {i, 0, n}], NN], Total[#] == n + (1 - NN) / 2 &];
  size = Length@coord;
  p = Sum[Multinomial[Sequence @@ (2 coord[[i]] + 1)] *
     Product[Binomial[2 n - 2 coord[[i, j]], n - coord[[i, j]]], {j, 1, NN}], {i, 1, size}];
  SeqListIni = Append[SeqListIni, p];
 ];

SeqListIni

seq[n_] := SeqListIni[[n + 1]];
```

Out[∘]=
```
{1, 0, 80, 0, 58 320, 933 120, 107 360 000, 4 032 000 000, 305 742 850 000,
 16 007 947 200 000, 1 092 754 448 110 080, 66 052 872 139 161 600, 4 433 464 272 394 080 000,
 287 105 556 124 600 012 800, 19 441 756 158 387 587 481 600, 1 307 659 624 636 945 150 771 200,
 89 869 341 860 254 106 893 314 000, 6 191 536 013 119 541 254 794 624 000,
 431 788 153 780 445 031 117 712 736 000, 30 259 578 124 053 738 011 950 295 040 000,
 2 137 643 722 042 861 014 846 923 875 678 720, 151 778 757 062 056 398 402 787 590 848 716 800,
 10 840 750 037 089 338 687 405 094 405 540 454 400, 777 883 218 982 271 229 558 388 389 382 825 574 400,
 56 080 935 388 938 320 492 345 601 400 578 969 030 400,
 4 059 518 371 465 289 501 011 809 299 957 269 579 653 120,
 295 006 495 123 163 326 450 011 592 999 699 774 386 176 000,
 21 513 746 057 744 924 699 009 848 676 027 694 742 870 425 600,
 1 574 148 924 348 897 968 127 657 314 112 417 503 459 217 408 000,
 115 532 761 111 124 106 137 388 311 120 877 422 599 980 279 398 400,
 8 503 842 442 314 663 173 760 541 941 753 193 179 094 810 125 926 400,
 627 609 496 898 499 522 225 265 285 115 906 238 911 179 967 692 800 000,
 46 436 433 389 594 145 887 536 322 203 955 919 558 553 470 641 486 850 000,
 3 443 934 036 721 437 625 596 385 616 851 665 233 141 061 945 297 580 800 000,
 255 987 247 247 218 119 955 440 370 898 615 088 710 853 711 642 084 487 200 000,
 19 067 482 593 646 334 342 036 067 557 315 656 461 776 897 366 982 437 990 400 000,
 1 423 081 446 108 803 178 035 349 924 075 427 821 311 627 222 594 248 532 220 000 000,
 106 409 576 497 910 521 328 093 928 056 177 350 881 687 619 362 437 540 913 600 000 000,
 7 970 830 048 553 981 080 058 702 593 590 669 197 116 023 210 365 395 365 879 360 000 000,
 598 079 060 794 011 278 983 455 745 029 821 926 281 050 762 038 228 190 896 727 040 000 000,
 44 947 891 716 233 478 275 997 236 905 855 509 440 585 640 503 537 143 428 499 957 569 600 000,
 3 383 154 085 138 020 637 793 497 624 953 038 417 160 337 631 975 043 003 579 851 781 888 000 000}
```

Now we may numerically verify our REC.

In[∘]:=
```
Table[SeqNormalized /. {Seq → seq, α → n}, {n, 0, 2 MAX - RecNormalizedOrder}]
```

Out[ ]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

Let us the generate a list of $r(n)$.

```
In[ ]:= Bound = 5000;

SeqList = UnrollRecurrence[SeqNormalized, Seq[α], SeqListIni, Bound];

seq[n_] := SeqList[[n + 1]];
```

Guess a Minimal ODE for $R(z)$.

ODEGuessinTheta gives the ODE in Theorem 4.6! (To be displayed at the end of this notebook)

Order 9, Degree 24

```
In[ ]:= ClearAll[Diff];
ODEGuessTmp = GuessMinDE[Take[SeqList, 400], Diff[z]];
DenominatorsLCM = LCM[Sequence @@ Denominator[Flatten[CoefficientList[
        ODEGuessTmp /. {Derivative[k_][Diff][z] → w^k} /. {Diff[z] → 1}, {z, w}]]]];
ODEGuessinD = ToOrePolynomial[ODEGuessTmp * DenominatorsLCM /.
      {Derivative[k_][Diff][z] → Der[z]^k} /. {Diff[z] → 1}];

In[ ]:= ODEGuessinThetaTmp = ChangeOreAlgebra[z ** ODEGuessinD, OreAlgebra[Euler[z]]];
ODEGuessinTheta =
  ODEGuessinThetaTmp * z^Max[Exponent[OrePolynomialListCoefficients[ODEGuessinThetaTmp] /. {z→z^-1}, z]];

In[ ]:= ODEGuessinThetaOrder = OrePolynomialDegree[ODEGuessinTheta, Euler[z]]

Out[ ]= 9

In[ ]:= ODEGuessinThetaDegree = Max[Exponent[OrePolynomialListCoefficients[ODEGuessinTheta], z]]

Out[ ]= 24
```

Get the REC from ODE and write it explicitly.

```
In[ ]:= RECfromODEGuessinS = DFiniteDE2RE[{ODEGuessinD}, {z}, {α}][[1]];

In[ ]:= RECfromODEGuessinSOrder = OrePolynomialDegree[RECfromODEGuessinS, S[α]]

Out[ ]= 24

In[ ]:= ClearAll[Seq];
SeqfromODEGuess = ApplyOreOperator[RECfromODEGuessinS, Seq[α]];

In[ ]:= SeqfromODEGuessList =
  UnrollRecurrence[SeqfromODEGuess, Seq[α], Take[SeqList, RECfromODEGuessinSOrder], 200];
```

Prove the minimal ODE for $R(z)$.

```
In[ ]:= RECCompare = DFinitePlus[{RECNormalizedinS}, {RECfromODEGuessinS}][[1]];

In[ ]:= RECCompareOrder = OrePolynomialDegree[RECCompare, S[α]]

Out[ ]= 30
```

```
In[•]:= CheckNum = RECCompareOrder + 20;
        Take[SeqList, CheckNum] - Take[SeqfromODEGuessList, CheckNum]

Out[•]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
         0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

Guess a Minimal REC for *r*(*n*).

SeqfromRECGuess gives the REC in Theorem 4.7! (To be displayed at the end of this notebook)

REC: Order 6

ODE: Order 27

```
In[•]:= RECGuessTmp = GuessMinRE[Take[SeqList, 200], Seq[α]];
        DenominatorsLCM = LCM[Sequence @@
            Denominator[Flatten[CoefficientList[RECGuessTmp /. {Seq[k_] → w^(k-α)}, {α, w}]]]];

In[•]:= RECGuessinS = ToOrePolynomial[RECGuessTmp * DenominatorsLCM /. {Seq[k_] → S[α]^(k-α)}];

In[•]:= RECGuessinSOrder = OrePolynomialDegree[RECGuessinS, S[α]]

Out[•]= 6

In[•]:= ODEfromRECGuessOrder = Max[Exponent[OrePolynomialListCoefficients[
            α^Max[Exponent[OrePolynomialListCoefficients[RECGuessinS]/.{α→α^-1},α]] * RECGuessinS], α]]

Out[•]= 27
```

We may also write this REC explicitly.

```
In[•]:= ClearAll[Seq];
        SeqfromRECGuess = ApplyOreOperator[RECGuessinS, Seq[α]];

In[•]:= SeqfromRECGuessList =
          UnrollRecurrence[SeqfromRECGuess, Seq[α], Take[SeqList, RECGuessinSOrder], 200];
```

Prove the minimal REC for *r*(*n*).

```
In[•]:= RECCompare = DFinitePlus[{RECNormalizedinS}, {RECGuessinS}][[1]];

In[•]:= RECCompareOrder = LeadingExponent[RECCompare][[1]]

Out[•]= 12

In[•]:= CheckNum = RECCompareOrder + 20;
        Take[SeqList, CheckNum] - Take[SeqfromRECGuessList, CheckNum]

Out[•]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

Compute the asymptotics for *r*(*n*).

```
In[•]:= AsyList = Asymptotics[SeqfromRECGuess, Seq[α]];
        N[AsyList]
```

Out[•]= $\left\{\frac{(-432.)^{\alpha}}{\alpha^{5/2}}, \frac{(-48.)^{\alpha}}{\alpha^{5/2}}, \frac{(-5.33333)^{\alpha}}{\alpha^{5/2}}, \frac{16.^{\alpha}}{\alpha^{9/4}}, \frac{16.^{\alpha}}{\alpha^{7/4}}, \frac{80.^{\alpha}}{\alpha^{5/2}}\right\}$

In[∘]:=
```
Ind = Reverse[Table[Floor[Bound / i], {i, 1, 3}]]
Table[N[seq[Ind[[i]]] / (AsyList[[4]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
Table[N[seq[Ind[[i]]] / (AsyList[[5]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
Table[N[seq[Ind[[i]]] / (AsyList[[6]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
```

Out[∘]= {3333, 5000, 10 000}

Out[∘]= $\{2.157784655879568 \times 10^{2327},\ 2.971843676012373 \times 10^{3492},\ 1.769474996617337 \times 10^{6987}\}$

Out[∘]= $\{3.737579539425117 \times 10^{2325},\ 4.202821631869412 \times 10^{3490},\ 1.769474996617337 \times 10^{6985}\}$

Out[∘]= {0.0352933, 0.0352977, 0.0353021}

Approximate the Polya number.

In[∘]:=
```
AtOne = N[Sum[seq[n] * (1 / (2^MM Binomial[NN, MM]))^n, {n, 0, Bound}], 11]
N[1 - 1/AtOne, 10]
```

Out[∘]= 1.0158559936

Out[∘]= 0.01560850527

Display the ODE in Theorem 4.6

In[∘]:=
```
ODEGuessinTheta
```

Out[∘]=

$$\begin{aligned}
&(1\,968\,300 - 14\,377\,372\,992\, z - 31\,378\,944\,803\,328\, z^{2} - 587\,599\,727\,984\,640\, z^{3} -\\
&\quad 11\,393\,107\,020\,720\,046\,080\, z^{4} - 7\,512\,914\,091\,413\,564\,817\,408\, z^{5} + 299\,638\,067\,426\,947\,151\,953\,920\, z^{6} +\\
&\quad 195\,572\,469\,268\,564\,090\,225\,164\,288\, z^{7} - 25\,066\,230\,988\,181\,914\,756\,830\,986\,240\, z^{8} +\\
&\quad 1\,466\,023\,585\,546\,150\,566\,663\,720\,796\,160\, z^{9} + 71\,839\,838\,988\,731\,444\,762\,798\,769\,307\,648\, z^{10} -\\
&\quad 8\,620\,981\,873\,487\,530\,449\,442\,157\,746\,978\,816\, z^{11} - 107\,877\,900\,379\,022\,416\,281\,433\,704\,771\,878\,912\, z^{12} +\\
&\quad 6\,045\,203\,063\,427\,555\,738\,693\,218\,864\,495\,329\,280\, z^{13} -\\
&\quad 27\,383\,749\,995\,592\,913\,844\,335\,383\,773\,613\,916\,160\, z^{14} +\\
&\quad 44\,159\,405\,750\,235\,818\,360\,995\,501\,107\,081\,904\,128\, z^{15} +\\
&\quad 13\,699\,073\,426\,625\,876\,523\,234\,327\,944\,587\,328\,356\,352\, z^{16} -\\
&\quad 387\,340\,817\,532\,181\,412\,702\,477\,239\,142\,346\,601\,267\,200\, z^{17} -\\
&\quad 93\,561\,082\,878\,589\,380\,479\,405\,717\,324\,153\,487\,360\,000\, z^{18} +\\
&\quad 26\,199\,174\,990\,188\,349\,028\,511\,624\,137\,716\,063\,535\,104\,000\, z^{19} +\\
&\quad 43\,846\,547\,777\,265\,123\,304\,897\,934\,342\,541\,583\,319\,040\,000\, z^{20} -\\
&\quad 73\,654\,449\,615\,358\,974\,329\,157\,395\,854\,519\,173\,120\,000\,000\, z^{21} +\\
&\quad 463\,163\,910\,329\,304\,284\,499\,157\,507\,080\,361\,869\,312\,000\,000\, z^{22} -\\
&\quad 26\,729\,974\,870\,757\,227\,259\,400\,450\,682\,776\,453\,120\,000\,000\, z^{23} +\\
&\quad 25\,483\,039\,017\,248\,114\,833\,274\,026\,825\,089\,024\,000\,000\,000\, z^{24})\ \theta_z^{9} +\\
&(25\,587\,900 - 167\,017\,235\,904\, z - 388\,316\,496\,158\,208\, z^{2} - 71\,432\,378\,741\,514\,240\, z^{3} -\\
&\quad 67\,251\,283\,102\,180\,638\,720\, z^{4} - 106\,035\,468\,401\,043\,255\,066\,624\, z^{5} +\\
&\quad 3\,933\,938\,204\,585\,015\,786\,864\,640\, z^{6} + 3\,550\,830\,229\,428\,629\,160\,044\,003\,328\, z^{7} -\\
&\quad 414\,827\,655\,192\,955\,512\,486\,740\,623\,360\, z^{8} + 14\,705\,350\,966\,486\,458\,929\,924\,160\,880\,640\, z^{9} +\\
&\quad 1\,481\,459\,728\,061\,408\,765\,324\,573\,754\,261\,504\, z^{10} - 193\,671\,651\,198\,531\,963\,311\,300\,948\,925\,612\,032\, z^{11} -\\
&\quad 1\,744\,302\,070\,180\,818\,241\,862\,079\,143\,751\,974\,912\, z^{12} +
\end{aligned}$$

$188\,152\,751\,580\,271\,237\,119\,102\,149\,995\,832\,279\,040\, z^{13} -$
$851\,746\,667\,821\,751\,911\,755\,882\,821\,997\,916\,323\,840\, z^{14} -$
$7\,276\,349\,700\,019\,150\,919\,019\,652\,322\,303\,668\,125\,696\, z^{15} +$
$253\,386\,321\,751\,200\,231\,391\,561\,171\,894\,048\,175\,161\,344\, z^{16} -$
$10\,787\,404\,513\,612\,693\,833\,929\,182\,911\,837\,457\,770\,086\,400\, z^{17} +$
$4\,040\,614\,420\,659\,532\,716\,685\,372\,741\,544\,180\,711\,424\,000\, z^{18} +$
$952\,123\,292\,291\,213\,693\,591\,421\,570\,421\,616\,642\,359\,296\,000\, z^{19} +$
$1\,859\,374\,469\,358\,285\,389\,183\,575\,405\,391\,438\,067\,793\,920\,000\, z^{20} -$
$2\,989\,212\,943\,556\,764\,016\,767\,457\,029\,537\,816\,117\,248\,000\,000\, z^{21} +$
$16\,696\,891\,280\,208\,066\,019\,958\,691\,706\,888\,469\,348\,352\,000\,000\, z^{22} -$
$949\,529\,398\,826\,243\,239\,300\,048\,052\,502\,606\,643\,200\,000\,000\, z^{23} +$
$866\,423\,326\,586\,435\,904\,331\,316\,912\,053\,026\,816\,000\,000\,000\, z^{24}\big)\, \theta_z^8 +$
$\big(143\,193\,825 - 790\,043\,593\,572\, z - 2\,145\,035\,751\,247\,872\, z^2 - 795\,395\,093\,950\,794\,240\, z^3 +$
$138\,793\,827\,460\,464\,156\,672\, z^4 - 662\,587\,314\,792\,999\,489\,110\,016\, z^5 +$
$39\,361\,134\,961\,606\,763\,469\,864\,960\, z^6 + 27\,836\,403\,113\,349\,843\,191\,460\,790\,272\, z^7 -$
$2\,952\,763\,516\,224\,970\,957\,938\,648\,678\,400\, z^8 + 17\,715\,419\,750\,558\,753\,940\,033\,123\,647\,488\, z^9 +$
$17\,561\,306\,674\,573\,793\,971\,071\,316\,861\,648\,896\, z^{10} -$
$2\,013\,528\,827\,982\,674\,648\,891\,849\,053\,239\,771\,136\, z^{11} -$
$6\,702\,623\,954\,407\,224\,045\,318\,611\,701\,352\,890\,368\, z^{12} +$
$2\,530\,409\,575\,669\,922\,681\,509\,808\,952\,688\,204\,840\,960\, z^{13} -$
$14\,158\,098\,288\,888\,638\,170\,539\,242\,906\,998\,183\,821\,312\, z^{14} -$
$212\,428\,785\,403\,743\,639\,945\,027\,731\,918\,002\,317\,164\,544\, z^{15} +$
$1\,740\,698\,869\,942\,495\,071\,397\,034\,886\,461\,543\,638\,106\,112\, z^{16} -$
$133\,590\,920\,120\,774\,486\,375\,251\,925\,639\,058\,930\,479\,923\,200\, z^{17} +$
$117\,853\,225\,542\,428\,730\,412\,209\,928\,347\,232\,071\,843\,840\,000\, z^{18} +$
$15\,514\,196\,975\,326\,917\,364\,062\,104\,829\,911\,826\,016\,763\,904\,000\, z^{19} +$
$33\,354\,396\,540\,999\,090\,913\,407\,702\,429\,888\,059\,277\,312\,000\,000\, z^{20} -$
$52\,066\,193\,814\,874\,539\,659\,294\,278\,620\,155\,473\,821\,696\,000\,000\, z^{21} +$
$266\,344\,647\,348\,956\,449\,504\,115\,441\,740\,129\,838\,825\,472\,000\,000\, z^{22} -$
$14\,770\,234\,221\,615\,586\,187\,256\,793\,881\,539\,625\,615\,360\,000\,000\, z^{23} +$
$13\,051\,386\,184\,451\,845\,257\,422\,171\,891\,507\,920\,896\,000\,000\,000\, z^{24}\big)\, \theta_z^7 +$
$\big(449\,264\,475 - 1\,868\,417\,168\,706\, z - 7\,066\,918\,189\,948\,896\, z^2 - 3\,878\,860\,874\,135\,507\,712\, z^3 +$
$2\,443\,017\,166\,286\,127\,538\,176\, z^4 - 2\,362\,380\,901\,647\,109\,273\,976\,832\, z^5 +$
$264\,291\,578\,493\,831\,924\,621\,115\,392\, z^6 + 131\,515\,000\,106\,996\,346\,624\,232\,390\,656\, z^7 -$
$12\,203\,514\,471\,539\,125\,045\,628\,510\,404\,608\, z^8 - 497\,477\,693\,141\,507\,581\,640\,821\,121\,744\,896\, z^9 +$
$135\,583\,802\,348\,946\,404\,997\,942\,857\,089\,155\,072\, z^{10} -$
$12\,808\,764\,617\,716\,898\,741\,372\,092\,142\,783\,037\,440\, z^{11} +$
$49\,330\,783\,558\,989\,988\,805\,231\,075\,790\,417\,821\,696\, z^{12} +$
$19\,447\,613\,803\,553\,405\,698\,986\,356\,520\,917\,382\,725\,632\, z^{13} -$
$144\,309\,617\,133\,065\,277\,844\,132\,480\,453\,941\,129\,117\,696\, z^{14} -$
$2\,710\,631\,803\,634\,179\,381\,793\,377\,498\,344\,095\,616\,073\,728\, z^{15} +$
$2\,964\,870\,370\,772\,586\,860\,585\,478\,733\,994\,873\,327\,714\,304\, z^{16} -$
$974\,601\,771\,553\,101\,425\,549\,162\,295\,455\,430\,182\,594\,150\,400\, z^{17} +$
$1\,199\,025\,070\,777\,376\,290\,858\,573\,274\,294\,378\,978\,869\,248\,000\, z^{18} +$
$148\,002\,858\,072\,532\,733\,903\,382\,318\,856\,274\,090\,724\,950\,016\,000\, z^{19} +$
$338\,543\,845\,114\,612\,124\,673\,021\,108\,297\,229\,880\,933\,744\,640\,000\, z^{20} -$
$515\,893\,327\,627\,243\,954\,647\,854\,653\,604\,064\,361\,709\,568\,000\,000\, z^{21} +$
$2\,467\,681\,175\,207\,979\,642\,662\,110\,323\,549\,952\,322\,568\,192\,000\,000\, z^{22} -$
$132\,231\,011\,763\,661\,328\,740\,679\,532\,066\,610\,694\,062\,080\,000\,000\, z^{23} +$
$114\,331\,247\,240\,822\,245\,206\,661\,000\,977\,438\,474\,240\,000\,000\,000\, z^{24}\big)\, \theta_z^6 +$
$\big(861\,131\,250 - 1\,933\,234\,949\,826\, z - 15\,613\,778\,270\,821\,824\, z^2 - 10\,309\,316\,152\,243\,684\,608\, z^3 +$

$10\,208\,121\,855\,056\,887\,836\,672\, z^4 - 5\,177\,478\,897\,928\,951\,338\,663\,936\, z^5 +$
$1\,051\,071\,333\,282\,686\,988\,259\,688\,448\, z^6 + 434\,605\,161\,566\,794\,661\,885\,094\,395\,904\, z^7 -$
$33\,621\,302\,900\,734\,471\,517\,776\,319\,086\,592\, z^8 - 3\,862\,797\,242\,368\,863\,995\,804\,954\,019\,233\,792\, z^9 +$
$682\,523\,911\,500\,282\,487\,919\,453\,997\,436\,502\,016\, z^{10} -$
$54\,885\,706\,010\,533\,092\,142\,603\,108\,401\,761\,222\,656\, z^{11} +$
$628\,295\,030\,317\,792\,870\,701\,259\,949\,069\,570\,146\,304\, z^{12} +$
$94\,615\,620\,246\,695\,935\,456\,646\,477\,297\,107\,177\,832\,448\, z^{13} -$
$943\,374\,946\,910\,174\,228\,088\,452\,295\,651\,235\,014\,377\,472\, z^{14} -$
$20\,239\,505\,490\,425\,686\,638\,866\,526\,052\,588\,532\,136\,935\,424\, z^{15} -$
$32\,106\,654\,409\,284\,054\,688\,300\,061\,132\,285\,669\,604\,851\,712\, z^{16} -$
$4\,661\,186\,834\,850\,235\,941\,611\,447\,026\,185\,947\,054\,918\,860\,800\, z^{17} +$
$6\,305\,858\,935\,526\,636\,963\,613\,625\,345\,233\,919\,710\,068\,736\,000\, z^{18} +$
$907\,835\,208\,762\,550\,426\,257\,263\,133\,272\,785\,468\,161\,785\,856\,000\, z^{19} +$
$2\,163\,393\,059\,021\,746\,024\,723\,195\,689\,099\,497\,341\,043\,343\,360\,000\, z^{20} -$
$3\,223\,849\,259\,321\,090\,413\,099\,936\,188\,064\,249\,820\,479\,488\,000\,000\, z^{21} +$
$14\,634\,823\,166\,233\,773\,519\,713\,879\,943\,050\,302\,888\,869\,888\,000\,000\, z^{22} -$
$751\,559\,666\,050\,831\,529\,712\,855\,803\,432\,617\,566\,535\,680\,000\,000\, z^{23} +$
$641\,926\,999\,294\,401\,201\,636\,284\,114\,667\,753\,701\,376\,000\,000\,000\, z^{24}\big)\ \theta_z^5 +$
$\big(1\,027\,452\,600 + 650\,073\,935\,826\, z - 24\,331\,013\,564\,272\,416\, z^2 - 15\,445\,917\,700\,094\,672\,640\, z^3 +$
$23\,767\,563\,566\,040\,632\,524\,800\, z^4 - 7\,076\,237\,807\,199\,353\,917\,833\,216\, z^5 +$
$2\,464\,344\,308\,447\,820\,339\,127\,123\,968\, z^6 + 1\,063\,844\,885\,293\,196\,366\,240\,201\,834\,496\, z^7 -$
$68\,128\,813\,392\,655\,213\,983\,420\,372\,746\,240\, z^8 - 14\,470\,974\,465\,501\,609\,824\,429\,971\,257\,425\,920\, z^9 +$
$2\,255\,366\,779\,025\,336\,638\,703\,621\,614\,721\,826\,816\, z^{10} -$
$163\,072\,913\,432\,831\,468\,367\,882\,056\,800\,812\,400\,640\, z^{11} +$
$2\,997\,803\,475\,346\,983\,445\,074\,315\,347\,099\,287\,289\,856\, z^{12} +$
$303\,567\,717\,202\,213\,863\,472\,216\,828\,600\,831\,893\,831\,680\, z^{13} -$
$4\,047\,013\,525\,578\,461\,281\,906\,717\,598\,427\,157\,616\,394\,240\, z^{14} -$
$96\,118\,533\,388\,650\,019\,713\,306\,746\,783\,626\,734\,499\,528\,704\, z^{15} -$
$255\,335\,975\,497\,145\,631\,364\,487\,812\,420\,478\,765\,151\,289\,344\, z^{16} -$
$15\,296\,132\,053\,875\,053\,696\,100\,212\,626\,928\,213\,683\,706\,265\,600\, z^{17} +$
$18\,246\,597\,120\,323\,314\,085\,822\,020\,233\,756\,671\,647\,678\,464\,000\, z^{18} +$
$3\,704\,565\,159\,264\,503\,664\,807\,121\,494\,574\,141\,576\,203\,730\,944\,000\, z^{19} +$
$9\,074\,841\,977\,197\,794\,226\,345\,092\,230\,554\,593\,305\,642\,926\,080\,000\, z^{20} -$
$13\,228\,108\,961\,936\,918\,861\,318\,774\,597\,098\,203\,034\,681\,344\,000\,000\, z^{21} +$
$57\,616\,494\,888\,579\,146\,564\,434\,704\,083\,771\,647\,001\,100\,288\,000\,000\, z^{22} -$
$2\,814\,241\,188\,839\,209\,555\,160\,491\,293\,254\,859\,148\,492\,800\,000\,000\, z^{23} +$
$2\,395\,763\,624\,685\,018\,201\,440\,807\,167\,653\,926\,928\,384\,000\,000\,000\, z^{24}\big)\ \theta_z^4 +$
$\big(740\,080\,800 + 4\,005\,475\,160\,382\, z - 26\,752\,082\,313\,555\,648\, z^2 - 11\,242\,751\,383\,759\,253\,760\, z^3 +$
$35\,720\,292\,244\,900\,235\,563\,008\, z^4 - 6\,065\,066\,346\,145\,944\,588\,877\,824\, z^5 +$
$3\,345\,061\,729\,427\,473\,554\,847\,825\,920\, z^6 + 1\,894\,327\,573\,331\,010\,120\,361\,121\,415\,168\, z^7 -$
$108\,746\,455\,608\,013\,062\,145\,350\,062\,571\,520\, z^8 - 32\,390\,016\,084\,427\,669\,590\,502\,250\,075\,127\,808\, z^9 +$
$4\,885\,336\,097\,506\,818\,994\,189\,729\,806\,914\,945\,024\, z^{10} -$
$332\,529\,853\,189\,481\,862\,490\,202\,166\,008\,777\,539\,584\, z^{11} +$
$7\,957\,400\,712\,436\,373\,498\,355\,760\,634\,495\,162\,646\,528\, z^{12} +$
$645\,438\,310\,936\,945\,974\,171\,829\,835\,738\,420\,084\,736\,000\, z^{13} -$
$11\,380\,150\,415\,633\,780\,615\,071\,928\,566\,811\,032\,654\,184\,448\, z^{14} -$
$295\,206\,510\,184\,985\,427\,490\,073\,480\,082\,097\,150\,000\,889\,856\, z^{15} -$
$898\,378\,340\,748\,269\,192\,996\,272\,984\,886\,327\,758\,125\,268\,992\, z^{16} -$
$34\,684\,745\,325\,811\,914\,042\,962\,369\,649\,474\,365\,098\,347\,724\,800\, z^{17} +$
$25\,494\,877\,999\,429\,408\,875\,059\,256\,957\,683\,509\,820\,915\,712\,000\, z^{18} +$
$10\,041\,801\,219\,994\,880\,276\,401\,244\,788\,909\,988\,006\,389\,088\,256\,000\, z^{19} +$

$$25\,067\,347\,396\,414\,652\,503\,438\,294\,678\,613\,111\,532\,316\,262\,400\,000\, z^{20} -$$
$$35\,735\,302\,066\,158\,223\,488\,956\,018\,306\,355\,698\,054\,725\,632\,000\,000\, z^{21} +$$
$$150\,583\,689\,478\,560\,082\,309\,614\,319\,654\,150\,666\,891\,296\,768\,000\,000\, z^{22} -$$
$$6\,945\,686\,778\,879\,323\,272\,921\,783\,244\,323\,834\,130\,595\,840\,000\,000\, z^{23} +$$
$$5\,943\,864\,920\,522\,147\,046\,229\,253\,977\,370\,938\,834\,944\,000\,000\,000\, z^{24}\Big)\, \theta_z^3 +$$
$$\Big(291\,308\,400 + 4\,327\,052\,213\,376\, z - 19\,804\,951\,861\,835\,904\, z^2 + 219\,328\,532\,451\,021\,312\, z^3 +$$
$$35\,514\,623\,326\,732\,836\,470\,784\, z^4 - 3\,650\,278\,542\,693\,304\,807\,784\,448\, z^5 +$$
$$2\,357\,609\,804\,697\,617\,638\,375\,292\,928\, z^6 + 2\,284\,090\,498\,817\,390\,244\,103\,930\,773\,504\, z^7 -$$
$$135\,279\,193\,095\,431\,387\,395\,370\,130\,604\,032\, z^8 - 44\,421\,617\,669\,647\,576\,628\,773\,186\,399\,371\,264\, z^9 +$$
$$6\,731\,574\,539\,350\,949\,339\,005\,449\,772\,222\,906\,368\, z^{10} -$$
$$444\,264\,448\,160\,624\,489\,412\,697\,369\,540\,244\,275\,200\, z^{11} +$$
$$12\,330\,489\,911\,679\,126\,449\,955\,761\,826\,793\,808\,461\,824\, z^{12} +$$
$$881\,925\,579\,961\,332\,026\,441\,980\,244\,408\,237\,385\,842\,688\, z^{13} -$$
$$20\,267\,206\,896\,219\,283\,314\,649\,057\,825\,416\,356\,837\,720\,064\, z^{14} -$$
$$569\,740\,910\,227\,815\,321\,993\,037\,912\,994\,997\,302\,326\,198\,272\, z^{15} -$$
$$1\,779\,740\,679\,903\,215\,090\,218\,062\,224\,628\,300\,162\,325\,807\,104\, z^{16} -$$
$$52\,569\,118\,468\,391\,792\,326\,478\,962\,488\,127\,157\,953\,993\,113\,600\, z^{17} +$$
$$1\,451\,146\,953\,987\,214\,945\,082\,180\,259\,246\,377\,722\,183\,680\,000\, z^{18} +$$
$$17\,418\,909\,098\,958\,603\,815\,097\,479\,056\,179\,850\,099\,738\,804\,224\,000\, z^{19} +$$
$$44\,056\,786\,669\,226\,693\,020\,411\,427\,408\,040\,785\,954\,482\,421\,760\,000\, z^{20} -$$
$$61\,406\,240\,551\,984\,929\,599\,279\,515\,827\,575\,992\,514\,248\,704\,000\,000\, z^{21} +$$
$$251\,939\,775\,441\,184\,947\,187\,729\,213\,430\,058\,184\,656\,027\,648\,000\,000\, z^{22} -$$
$$10\,897\,710\,329\,468\,549\,993\,283\,540\,156\,647\,403\,442\,667\,520\,000\,000\, z^{23} +$$
$$9\,453\,516\,646\,138\,192\,392\,531\,605\,664\,037\,066\,506\,240\,000\,000\,000\, z^{24}\Big)\, \theta_z^2 +$$
$$\Big(47\,239\,200 + 2\,006\,920\,198\,008\, z - 8\,896\,275\,005\,061\,888\, z^2 + 6\,068\,775\,605\,179\,834\,368\, z^3 +$$
$$21\,592\,494\,904\,802\,476\,474\,368\, z^4 - 2\,070\,943\,659\,269\,789\,689\,184\,256\, z^5 +$$
$$508\,990\,334\,442\,221\,895\,703\,068\,672\, z^6 + 1\,645\,356\,895\,363\,886\,731\,757\,415\,825\,408\, z^7 -$$
$$114\,092\,231\,346\,388\,839\,876\,595\,084\,689\,408\, z^8 - 34\,854\,174\,079\,338\,717\,741\,834\,632\,070\,955\,008\, z^9 +$$
$$5\,391\,219\,180\,182\,910\,013\,667\,323\,640\,750\,800\,896\, z^{10} -$$
$$349\,879\,345\,484\,004\,176\,369\,490\,015\,495\,754\,088\,448\, z^{11} +$$
$$10\,421\,889\,625\,445\,277\,183\,432\,841\,679\,950\,596\,538\,368\, z^{12} +$$
$$707\,518\,047\,056\,858\,825\,894\,051\,502\,973\,057\,826\,291\,712\, z^{13} -$$
$$20\,793\,407\,939\,094\,108\,348\,704\,800\,718\,204\,390\,225\,215\,488\, z^{14} -$$
$$629\,685\,151\,762\,573\,551\,815\,521\,708\,666\,435\,793\,829\,494\,784\, z^{15} -$$
$$1\,930\,793\,766\,565\,323\,892\,002\,668\,180\,323\,066\,516\,623\,851\,520\, z^{16} -$$
$$48\,248\,039\,919\,208\,006\,716\,948\,992\,896\,627\,977\,926\,737\,920\,000\, z^{17} -$$
$$39\,899\,465\,736\,212\,014\,953\,965\,531\,811\,189\,157\,080\,858\,624\,000\, z^{18} +$$
$$17\,535\,154\,948\,554\,606\,749\,217\,297\,744\,053\,519\,048\,402\,534\,400\,000\, z^{19} +$$
$$44\,763\,107\,271\,248\,675\,832\,384\,158\,553\,351\,794\,937\,613\,516\,800\,000\, z^{20} -$$
$$60\,989\,883\,869\,673\,646\,630\,860\,485\,832\,163\,159\,682\,580\,480\,000\,000\, z^{21} +$$
$$244\,859\,130\,660\,973\,130\,028\,081\,246\,340\,193\,403\,210\,301\,440\,000\,000\, z^{22} -$$
$$9\,863\,991\,571\,517\,380\,030\,786\,517\,850\,680\,700\,017\,049\,600\,000\,000\, z^{23} +$$
$$8\,746\,714\,696\,058\,467\,589\,996\,180\,287\,043\,036\,774\,400\,000\,000\,000\, z^{24}\Big)\, \theta_z +$$
$$\Big(333\,047\,697\,408\, z - 1\,872\,897\,434\,966\,016\, z^2 + 2\,977\,127\,512\,452\,956\,160\, z^3 +$$
$$6\,063\,429\,379\,839\,486\,197\,760\, z^4 - 841\,378\,018\,777\,452\,462\,735\,360\, z^5 -$$
$$177\,662\,479\,350\,188\,199\,817\,248\,768\, z^6 + 532\,188\,511\,244\,875\,329\,523\,528\,237\,056\, z^7 -$$
$$46\,468\,194\,557\,583\,422\,535\,635\,168\,133\,120\, z^8 - 12\,129\,481\,477\,266\,120\,825\,633\,907\,345\,981\,440\, z^9 +$$
$$1\,922\,168\,850\,385\,325\,476\,972\,118\,541\,336\,576\,000\, z^{10} -$$
$$122\,905\,090\,504\,449\,842\,544\,597\,830\,488\,521\,965\,568\, z^{11} +$$
$$3\,693\,913\,281\,036\,487\,994\,072\,047\,630\,802\,302\,795\,776\, z^{12} +$$
$$255\,898\,359\,172\,294\,308\,439\,162\,255\,416\,680\,691\,793\,920\, z^{13} -$$

9 383 560 634 074 603 744 909 701 439 786 415 837 675 520 z^{14} −
304 479 107 629 950 202 888 249 997 079 869 068 054 364 160 z^{15} −
897 976 262 418 207 078 877 329 057 815 825 497 246 924 800 z^{16} −
20 318 331 412 941 821 720 614 342 296 829 572 113 498 112 000 z^{17} −
34 744 511 680 906 604 713 962 246 466 155 268 810 997 760 000 z^{18} +
7 802 356 464 780 521 871 748 733 579 721 789 105 242 112 000 000 z^{19} +
20 050 212 474 748 975 261 015 644 998 031 116 001 607 680 000 000 z^{20} −
26 705 230 963 097 524 390 614 329 783 301 547 701 043 200 000 000 z^{21} +
105 329 290 936 390 351 965 199 230 328 560 978 336 153 600 000 000 z^{22} −
3 923 945 710 099 317 224 594 398 360 128 920 223 744 000 000 000 z^{23} +
3 587 135 914 162 316 664 577 589 182 300 422 144 000 000 000 000 z^{24})

Display the REC in Theorem 4.7

In[∘]:= -SeqfromRECGuess

Out[∘]= − (− 2 364 822 061 925 891 270 067 722 649 600 000 − 24 311 763 241 480 737 290 507 853 496 320 000 α −
118 884 714 388 336 585 062 289 753 767 936 000 α^2 −
368 251 136 151 853 255 846 369 719 798 988 800 α^3 − 811 793 640 582 985 414 140 746 797 028 474 880
α^4 − 1 356 499 120 040 750 577 583 138 444 526 223 360 α^5 −
1 786 835 040 377 781 128 110 811 754 937 712 640 α^6 −
1 904 958 007 246 824 509 445 186 467 125 002 240 α^7 −
1 674 545 402 297 600 373 785 511 713 251 000 320 α^8 −
1 230 194 808 706 317 371 163 067 050 208 788 480 α^9 −
762 791 807 513 049 677 466 384 009 532 538 880 α^{10} −
402 079 430 499 218 110 643 393 128 200 929 280 α^{11} −
181 085 303 893 806 582 831 390 648 576 245 760 α^{12} −
69 909 566 044 762 687 837 271 137 604 075 520 α^{13} −
23 174 037 389 797 607 720 091 614 796 840 960 α^{14} −
6 597 237 647 955 223 324 018 009 760 071 680 α^{15} −
1 610 851 715 462 724 269 782 004 410 613 760 α^{16} − 336 382 193 033 012 242 367 855 858 810 880 α^{17} −
59 795 770 083 083 316 221 336 805 703 680 α^{18} − 8 987 061 025 545 721 077 834 511 810 560 α^{19} −
1 131 237 375 988 193 565 613 353 861 120 α^{20} − 117 704 523 870 056 936 584 154 972 160 α^{21} −
9 941 030 662 497 120 749 554 237 440 α^{22} − 664 040 244 922 741 425 721 835 520 α^{23} −
33 746 986 442 943 554 031 452 160 α^{24} − 1 225 566 587 608 656 091 545 600 α^{25} −
28 320 365 528 012 449 382 400 α^{26} − 312 808 771 118 086 225 920 α^{27}) Seq[α] −
(− 880 540 948 213 763 261 498 004 602 880 000 − 8 086 612 414 279 581 582 690 097 299 456 000 α −
35 535 843 625 080 580 938 628 852 403 404 800 α^2 − 99 482 199 073 846 865 130 149 987 053 731 840 α^3 −
199 278 215 238 194 877 084 174 219 759 058 944 α^4 −
304 147 288 569 704 121 767 283 668 058 636 288 α^5 −
367 726 422 460 034 552 713 877 456 306 307 072 α^6 −
361 508 986 147 801 089 153 130 211 095 805 952 α^7 −
294 331 319 744 750 632 422 172 167 712 997 376 α^8 −
201 108 607 972 501 732 293 906 606 562 934 784 α^9 −
116 437 788 942 848 727 536 075 769 222 856 704 α^{10} −
57 524 299 296 878 619 402 424 939 339 382 784 α^{11} −
24 367 165 878 769 872 656 509 536 747 061 248 α^{12} − 8 877 402 295 660 764 714 512 245 808 234 496 α^{13} −
2 785 748 984 068 408 698 625 918 477 467 648 α^{14} − 752 972 653 647 501 430 958 086 738 673 664 α^{15} −
175 049 743 314 674 169 771 167 299 534 848 α^{16} − 34 895 534 864 837 208 484 258 292 957 184 α^{17} −
5 936 277 532 573 962 980 718 997 929 984 α^{18} − 855 818 515 821 739 179 539 429 326 848 α^{19} −
103 560 073 600 267 246 364 541 321 216 α^{20} − 10 380 185 487 431 012 018 005 475 328 α^{21} −
846 180 664 706 397 472 693 420 032 α^{22} − 54 656 640 176 185 180 963 209 216 α^{23} −

$2\,690\,612\,916\,385\,314\,156\,576\,768\,\alpha^{24} - 94\,804\,345\,329\,795\,433\,758\,720\,\alpha^{25} -$
$2\,128\,785\,749\,082\,227\,343\,360\,\alpha^{26} - 22\,881\,382\,331\,785\,936\,896\,\alpha^{27}\Big)\ \mathrm{Seq}[1+\alpha] -$
$\Big(664\,078\,540\,666\,702\,251\,488\,371\,015\,680\,000 + 5\,805\,956\,958\,011\,506\,960\,041\,778\,348\,032\,000\,\alpha +$
$24\,298\,272\,789\,380\,152\,495\,188\,221\,126\,246\,400\,\alpha^{2} + 64\,810\,405\,629\,301\,547\,428\,216\,819\,254\,558\,720\,\alpha^{3} +$
$123\,755\,374\,367\,469\,269\,296\,809\,845\,353\,611\,264\,\alpha^{4} +$
$180\,149\,375\,502\,996\,189\,202\,275\,648\,542\,982\,144\,\alpha^{5} +$
$207\,865\,771\,244\,125\,682\,287\,781\,841\,861\,722\,112\,\alpha^{6} +$
$195\,153\,222\,041\,523\,657\,876\,484\,723\,267\,989\,504\,\alpha^{7} +$
$151\,846\,270\,858\,495\,120\,363\,896\,477\,860\,167\,680\,\alpha^{8} +$
$99\,230\,231\,828\,276\,421\,932\,960\,434\,682\,314\,752\,\alpha^{9} + 54\,993\,115\,047\,787\,497\,911\,079\,580\,675\,899\,392\,\alpha^{10} +$
$26\,028\,017\,908\,489\,825\,928\,212\,462\,245\,453\,824\,\alpha^{11} +$
$10\,572\,113\,416\,646\,586\,933\,511\,582\,698\,766\,336\,\alpha^{12} + 3\,696\,722\,231\,163\,815\,760\,173\,082\,026\,344\,448\,\alpha^{13} +$
$1\,114\,468\,173\,061\,041\,282\,670\,805\,399\,093\,248\,\alpha^{14} + 289\,688\,969\,845\,746\,113\,335\,461\,572\,931\,584\,\alpha^{15} +$
$64\,831\,091\,647\,102\,802\,811\,533\,842\,055\,168\,\alpha^{16} + 12\,454\,053\,009\,083\,005\,771\,527\,566\,163\,968\,\alpha^{17} +$
$2\,043\,760\,292\,966\,696\,499\,523\,264\,184\,320\,\alpha^{18} + 284\,532\,912\,366\,921\,324\,027\,166\,588\,928\,\alpha^{19} +$
$33\,284\,416\,956\,384\,385\,896\,458\,223\,616\,\alpha^{20} + 3\,228\,606\,478\,351\,534\,833\,828\,626\,432\,\alpha^{21} +$
$254\,974\,947\,491\,313\,890\,128\,560\,128\,\alpha^{22} + 15\,972\,126\,457\,377\,261\,067\,698\,176\,\alpha^{23} +$
$763\,333\,007\,662\,980\,725\,211\,136\,\alpha^{24} + 26\,138\,887\,552\,462\,651\,129\,856\,\alpha^{25} +$
$570\,997\,443\,951\,748\,710\,400\,\alpha^{26} + 5\,976\,795\,675\,008\,958\,464\,\alpha^{27}\Big)\ \mathrm{Seq}[2+\alpha] -$
$\Big(-36\,337\,840\,931\,616\,555\,318\,702\,833\,664\,000 - 310\,343\,693\,247\,202\,072\,877\,171\,431\,833\,600\,\alpha -$
$1\,268\,062\,726\,217\,635\,641\,408\,454\,051\,430\,400\,\alpha^{2} - 3\,300\,521\,955\,790\,071\,740\,463\,976\,232\,263\,680\,\alpha^{3} -$
$6\,146\,984\,578\,367\,464\,065\,862\,054\,879\,242\,240\,\alpha^{4} - 8\,723\,512\,529\,514\,925\,026\,222\,139\,080\,468\,480\,\alpha^{5} -$
$9\,808\,817\,646\,565\,897\,068\,529\,809\,213\,239\,808\,\alpha^{6} - 8\,970\,447\,157\,798\,999\,809\,214\,350\,039\,412\,224\,\alpha^{7} -$
$6\,796\,618\,106\,855\,403\,262\,931\,535\,421\,469\,184\,\alpha^{8} - 4\,323\,600\,610\,674\,086\,572\,350\,145\,316\,732\,416\,\alpha^{9} -$
$2\,331\,860\,127\,398\,843\,166\,087\,931\,718\,971\,904\,\alpha^{10} - 1\,073\,804\,990\,271\,736\,796\,663\,841\,511\,156\,224\,\alpha^{11} -$
$424\,279\,297\,446\,148\,516\,898\,147\,199\,947\,264\,\alpha^{12} - 144\,293\,344\,557\,135\,741\,340\,883\,292\,465\,664\,\alpha^{13} -$
$42\,304\,696\,119\,152\,808\,149\,756\,544\,291\,840\,\alpha^{14} - 10\,693\,366\,157\,119\,575\,923\,154\,101\,714\,944\,\alpha^{15} -$
$2\,327\,102\,570\,668\,214\,059\,453\,238\,664\,192\,\alpha^{16} - 434\,708\,874\,971\,795\,823\,099\,840\,116\,736\,\alpha^{17} -$
$69\,373\,988\,097\,051\,870\,247\,906\,934\,784\,\alpha^{18} - 9\,393\,304\,762\,567\,159\,143\,035\,764\,736\,\alpha^{19} -$
$1\,068\,815\,757\,774\,279\,757\,481\,902\,080\,\alpha^{20} - 100\,861\,570\,825\,855\,881\,262\,923\,776\,\alpha^{21} -$
$7\,750\,770\,733\,439\,394\,600\,976\,384\,\alpha^{22} - 472\,551\,963\,878\,997\,639\,561\,216\,\alpha^{23} -$
$21\,986\,541\,883\,647\,884\,001\,280\,\alpha^{24} - 733\,188\,729\,988\,561\,502\,208\,\alpha^{25} -$
$15\,602\,375\,112\,618\,147\,840\,\alpha^{26} - 159\,149\,910\,074\,064\,896\,\alpha^{27}\Big)\ \mathrm{Seq}[3+\alpha] -$
$\Big(-1\,737\,772\,868\,400\,007\,324\,872\,130\,560\,000 - 14\,528\,609\,204\,414\,291\,845\,066\,255\,564\,800\,\alpha -$
$58\,083\,087\,258\,852\,534\,411\,685\,975\,019\,520\,\alpha^{2} - 147\,846\,850\,915\,658\,722\,383\,612\,355\,430\,400\,\alpha^{3} -$
$269\,164\,023\,324\,400\,460\,962\,054\,275\,740\,928\,\alpha^{4} - 373\,240\,816\,513\,597\,979\,905\,593\,440\,661\,888\,\alpha^{5} -$
$409\,908\,879\,949\,766\,514\,326\,399\,060\,864\,064\,\alpha^{6} - 366\,016\,393\,873\,249\,701\,940\,597\,734\,061\,344\,\alpha^{7} -$
$270\,676\,671\,846\,416\,971\,917\,873\,052\,917\,920\,\alpha^{8} - 168\,013\,318\,310\,785\,666\,403\,759\,927\,887\,584\,\alpha^{9} -$
$88\,393\,926\,598\,940\,439\,065\,183\,725\,045\,600\,\alpha^{10} - 39\,697\,363\,634\,496\,672\,642\,069\,844\,386\,912\,\alpha^{11} -$
$15\,293\,672\,611\,896\,263\,618\,803\,193\,519\,136\,\alpha^{12} - 5\,070\,491\,874\,452\,377\,148\,797\,920\,831\,072\,\alpha^{13} -$
$1\,449\,002\,022\,519\,967\,409\,403\,051\,116\,512\,\alpha^{14} - 356\,957\,682\,436\,813\,381\,749\,659\,746\,304\,\alpha^{15} -$
$75\,700\,244\,148\,872\,939\,301\,421\,992\,640\,\alpha^{16} - 13\,779\,371\,789\,456\,905\,170\,877\,563\,840\,\alpha^{17} -$
$2\,142\,685\,081\,818\,193\,152\,012\,367\,872\,\alpha^{18} - 282\,685\,926\,147\,777\,894\,282\,083\,328\,\alpha^{19} -$
$31\,341\,335\,886\,140\,485\,043\,322\,880\,\alpha^{20} - 2\,881\,942\,426\,887\,984\,021\,438\,464\,\alpha^{21} -$
$215\,812\,414\,752\,103\,173\,455\,872\,\alpha^{22} - 12\,823\,036\,513\,484\,289\,343\,488\,\alpha^{23} -$
$581\,508\,878\,853\,457\,575\,936\,\alpha^{24} - 18\,903\,053\,117\,719\,314\,432\,\alpha^{25} -$
$392\,186\,219\,850\,629\,120\,\alpha^{26} - 3\,900\,964\,176\,134\,144\,\alpha^{27}\Big)\ \mathrm{Seq}[4+\alpha] -$
$\Big(36\,446\,102\,109\,669\,030\,849\,285\,120\,000 + 301\,794\,930\,778\,773\,719\,063\,321\,856\,000\,\alpha +$
$1\,194\,401\,836\,156\,084\,887\,609\,064\,224\,000\,\alpha^{2} + 3\,008\,156\,975\,709\,477\,795\,289\,491\,275\,520\,\alpha^{3} +$
$5\,415\,770\,546\,395\,539\,670\,222\,530\,489\,360\,\alpha^{4} + 7\,422\,453\,554\,874\,065\,600\,190\,474\,289\,032\,\alpha^{5} +$
$8\,052\,206\,383\,842\,449\,223\,124\,682\,104\,644\,\alpha^{6} + 7\,098\,162\,826\,794\,167\,361\,280\,152\,144\,294\,\alpha^{7} +$

$5\,179\,144\,111\,408\,801\,590\,076\,035\,892\,950\,\alpha^8 + 3\,169\,950\,795\,733\,038\,711\,522\,140\,215\,280\,\alpha^9 +$
$1\,643\,499\,248\,947\,095\,475\,104\,215\,404\,004\,\alpha^{10} + 726\,910\,788\,718\,026\,537\,302\,273\,862\,144\,\alpha^{11} +$
$275\,635\,972\,025\,251\,416\,199\,969\,761\,656\,\alpha^{12} + 89\,889\,728\,147\,001\,421\,773\,544\,625\,132\,\alpha^{13} +$
$25\,251\,994\,806\,501\,150\,584\,061\,125\,784\,\alpha^{14} + 6\,111\,409\,098\,652\,595\,993\,659\,452\,026\,\alpha^{15} +$
$1\,272\,483\,225\,563\,071\,816\,917\,699\,490\,\alpha^{16} + 227\,273\,250\,419\,552\,627\,170\,585\,084\,\alpha^{17} +$
$34\,655\,941\,701\,831\,856\,557\,922\,624\,\alpha^{18} + 4\,480\,880\,404\,407\,427\,210\,024\,320\,\alpha^{19} +$
$486\,585\,842\,769\,876\,461\,484\,032\,\alpha^{20} + 43\,798\,304\,089\,562\,788\,663\,296\,\alpha^{21} +$
$3\,208\,710\,131\,027\,557\,023\,744\,\alpha^{22} + 186\,416\,522\,833\,559\,945\,216\,\alpha^{23} + 8\,261\,380\,192\,874\,790\,912\,\alpha^{24} +$
$262\,301\,388\,296\,421\,376\,\alpha^{25} + 5\,312\,632\,953\,241\,600\,\alpha^{26} + 51\,561\,082\,388\,480\,\alpha^{27}\big)\ \mathrm{Seq}[5 + \alpha] -$
$\big(154\,404\,486\,709\,237\,819\,219\,968\,000 + 1\,265\,327\,918\,255\,018\,927\,110\,348\,800\,\alpha +$
$4\,953\,641\,658\,930\,095\,511\,385\,751\,040\,\alpha^2 + 12\,335\,446\,851\,783\,544\,166\,937\,390\,720\,\alpha^3 +$
$21\,947\,702\,123\,383\,074\,616\,990\,244\,544\,\alpha^4 + 29\,712\,684\,443\,300\,038\,100\,072\,561\,760\,\alpha^5 +$
$31\,824\,626\,177\,807\,101\,870\,129\,360\,368\,\alpha^6 + 27\,684\,339\,638\,906\,598\,652\,692\,786\,888\,\alpha^7 +$
$19\,923\,668\,408\,873\,674\,929\,361\,243\,572\,\alpha^8 + 12\,021\,754\,897\,932\,453\,908\,473\,126\,194\,\alpha^9 +$
$6\,141\,402\,912\,303\,808\,338\,721\,284\,327\,\alpha^{10} + 2\,675\,090\,519\,652\,464\,763\,702\,625\,995\,\alpha^{11} +$
$998\,451\,712\,547\,824\,111\,144\,656\,513\,\alpha^{12} + 320\,337\,381\,856\,256\,276\,567\,115\,789\,\alpha^{13} +$
$88\,485\,146\,094\,830\,787\,771\,471\,525\,\alpha^{14} + 21\,045\,641\,782\,461\,353\,200\,898\,049\,\alpha^{15} +$
$4\,304\,140\,182\,149\,530\,399\,276\,227\,\alpha^{16} + 754\,678\,659\,252\,915\,954\,749\,073\,\alpha^{17} +$
$112\,910\,766\,050\,133\,819\,763\,020\,\alpha^{18} + 14\,316\,213\,223\,182\,938\,203\,068\,\alpha^{19} +$
$1\,523\,679\,350\,645\,560\,062\,336\,\alpha^{20} + 134\,345\,128\,624\,663\,841\,280\,\alpha^{21} +$
$9\,635\,762\,018\,738\,626\,560\,\alpha^{22} + 547\,760\,583\,383\,666\,688\,\alpha^{23} + 23\,739\,371\,943\,886\,848\,\alpha^{24} +$
$736\,693\,272\,182\,784\,\alpha^{25} + 14\,575\,541\,944\,320\,\alpha^{26} + 138\,110\,042\,112\,\alpha^{27}\big)\ \mathrm{Seq}[6 + \alpha]$