Multi-headed Lattice Green Function (N = 4, M = 3)

In[∘]:=
```
NN = 4;
MM = 3;
```

Generate a sequence from recurrence & initial values

Koutschan’s implementation

In[∘]:=
```
(* given a recurrence rec in f[n], compute the values {f[0],f[1],...,f[bound]}
 where inits are the initial values
 {f[0],...,f[d-1]} with d being the order of the recurrence *)
Clear[UnrollRecurrence];
UnrollRecurrence[rec1_, f_[n_], inits_, bound_] :=
  Module[{i, x, vals = inits, rec = rec1},
   If[Head[rec] =!= Equal, rec = (rec == 0)];
   rec = rec /. n → n - Max[Cases[rec, f[n + a_.] :→ a, Infinity]];
   Do[
    AppendTo[vals, Solve[rec /. n → i /. f[i] → x /. f[a_] :→ vals[[a + 1]], x][[1, 1, 2]]];
    , {i, Length[inits], bound}];
   Return[vals];
  ];
```

Marathon begins...

In[∘]:=
```
<< RISC`HolonomicFunctions`
```

HolonomicFunctions Package version 1.7.3 (21-Mar-2017)
written by Christoph Koutschan
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria

```
--> Type ?HolonomicFunctions for help.
```

We work on $R(z/2^M)$.

In[∘]:=
```
ClearAll[x1, x2, x3, x4, z, w, α, β];
```

In[∘]:=
```
SymmetricPolynomial[3, {x1, x2, x3, x4}]
```

Out[∘]= x1 x2 x3 + x1 x2 x4 + x1 x3 x4 + x2 x3 x4

In[∘]:=
```
integrand = 1 / ((1 - z (x1 x2 x3 + x1 x2 x4 + x1 x3 x4 + x2 x3 x4))
      Sqrt[1 - x1^2] Sqrt[1 - x2^2] Sqrt[1 - x3^2] Sqrt[1 - x4^2]);
```

Apply “Creative Telescoping”.

In[ ]:=
```
Timing[ann0 = Annihilator[integrand, {Der[x1], Der[x2], Der[x3], Der[x4], Der[z]}];]
```
Out[ ]= {0.03125, Null}

In[ ]:=
```
Timing[ann1 = FindCreativeTelescoping[ann0, Der[x1]][[1]];]
```
Out[ ]= {0.8125, Null}

In[ ]:=
```
Timing[ann2 = FindCreativeTelescoping[ann1, Der[x2]][[1]];]
```
Out[ ]= {3.89063, Null}

In[ ]:=
```
Timing[ann3 = FindCreativeTelescoping[ann2, Der[x3]][[1]];]
```
Out[ ]= {140.656, Null}

In[ ]:=
```
Timing[ann4 = FindCreativeTelescoping[ann3, Der[x4]][[1]];]
```
Out[ ]= {2214.66, Null}

In[ ]:=
```
ODEDiv2 = ann4;
ToOrePolynomial[ODEDiv2]
```
Out[ ]=

$$
\begin{aligned}
\{ \big(& 756 z^{6} + 10\,282\,923 z^{8} - 183\,086\,010 z^{10} + 426\,675\,786\,552 z^{12} - 9\,924\,059\,317\,284 z^{14} - \\
& 727\,508\,494\,453\,860 z^{16} + 23\,823\,632\,511\,905\,472 z^{18} - 166\,490\,176\,530\,653\,712 z^{20} - \\
& 531\,833\,728\,926\,060\,864 z^{22} + 5\,368\,994\,610\,318\,390\,528 z^{24} - 15\,600\,604\,411\,518\,259\,968 z^{26} + \\
& 18\,500\,849\,896\,655\,637\,504 z^{28} + 56\,535\,718\,246\,917\,525\,504 z^{30} - 11\,420\,181\,006\,188\,101\,632 z^{32} - \\
& 2\,981\,841\,263\,686\,582\,272 z^{34} + 529\,740\,981\,885\,468\,672 z^{36} - 3\,049\,952\,309\,673\,984 z^{38}\big) D_z^8 + \\
\big(& 18\,144 z^{5} + 225\,889\,020 z^{7} - 8\,227\,011\,420 z^{9} + 7\,695\,797\,156\,784 z^{11} - 347\,870\,868\,506\,136 z^{13} - \\
& 8\,499\,310\,723\,133\,712 z^{15} + 629\,441\,764\,844\,830\,560 z^{17} - 7\,284\,971\,257\,240\,402\,944 z^{19} - \\
& 5\,068\,609\,686\,489\,696\,768 z^{21} + 190\,188\,066\,765\,919\,168\,512 z^{23} - 528\,965\,155\,417\,805\,987\,328 z^{25} + \\
& 1\,061\,038\,025\,184\,277\,776\,384 z^{27} + 2\,036\,240\,913\,243\,415\,044\,096 z^{29} - 636\,459\,643\,193\,090\,015\,232 z^{31} - \\
& 111\,500\,324\,720\,609\,329\,152 z^{33} + 24\,182\,038\,075\,841\,445\,888 z^{35} - 134\,197\,901\,625\,655\,296 z^{37}\big) D_z^7 + \\
\big(& 142\,044 z^{4} + 1\,554\,459\,438 z^{6} - 107\,742\,566\,024 z^{8} + 39\,310\,654\,679\,888 z^{10} - \\
& 3\,815\,706\,493\,670\,448 z^{12} + 19\,577\,524\,381\,443\,704 z^{14} + 5\,095\,837\,292\,658\,361\,216 z^{16} - \\
& 102\,368\,891\,922\,231\,289\,440 z^{18} + 156\,219\,538\,176\,862\,310\,400 z^{20} + 2\,585\,436\,757\,900\,095\,247\,872 z^{22} - \\
& 6\,366\,993\,088\,595\,710\,209\,024 z^{24} + 18\,864\,518\,201\,777\,753\,784\,320 z^{26} + \\
& 24\,967\,133\,681\,004\,505\,673\,728 z^{28} - 12\,741\,080\,672\,513\,083\,637\,760 z^{30} - \\
& 1\,530\,364\,540\,095\,289\,425\,920 z^{32} + 408\,420\,960\,678\,296\,158\,208 z^{34} - 2\,172\,497\,974\,360\,276\,992 z^{36}\big) D_z^6 + \\
\big(& 434\,364 z^{3} + 3\,842\,198\,262 z^{5} - 555\,883\,048\,836 z^{7} + 58\,429\,935\,987\,904 z^{9} - \\
& 15\,946\,312\,621\,230\,792 z^{11} + 462\,617\,005\,697\,834\,712 z^{13} + 13\,339\,559\,583\,482\,816\,288 z^{15} - \\
& 585\,337\,402\,592\,941\,457\,952 z^{17} + 2\,603\,481\,482\,373\,802\,463\,232 z^{19} + \\
& 16\,696\,950\,454\,138\,940\,066\,304 z^{21} - 36\,197\,086\,618\,528\,587\,698\,688 z^{23} + \\
& 139\,535\,180\,027\,413\,154\,500\,608 z^{25} + 122\,264\,321\,263\,345\,317\,613\,568 z^{27} - \\
& 117\,892\,911\,224\,732\,364\,767\,232 z^{29} - 9\,843\,439\,461\,571\,492\,380\,672 z^{31} + \\
& 3\,234\,783\,321\,436\,469\,592\,064 z^{33} - 16\,385\,106\,295\,646\,060\,544 z^{35}\big) D_z^5 + \\
\big(& 489\,048 z^{2} + 2\,742\,130\,671 z^{4} - 1\,189\,830\,190\,154 z^{6} + 25\,171\,334\,918\,136 z^{8} - \\
& 23\,605\,302\,146\,643\,332 z^{10} + 1\,587\,145\,538\,991\,806\,220 z^{12} - 401\,548\,614\,981\,169\,568 z^{14} - \\
& 1\,367\,918\,711\,569\,407\,214\,992 z^{16} + 13\,350\,461\,209\,864\,339\,190\,592 z^{18} + \\
& 50\,734\,115\,319\,761\,032\,273\,920 z^{20} - 112\,102\,921\,637\,353\,375\,433\,472 z^{22} + \\
& 456\,343\,592\,538\,122\,549\,500\,928 z^{24} + 182\,033\,020\,334\,177\,103\,949\,824 z^{26} - \\
& 534\,615\,956\,067\,567\,370\,977\,280 z^{28} - 31\,524\,340\,522\,919\,285\,096\,448 z^{30} +
\end{aligned}
$$

$12\,599\,609\,442\,714\,477\,658\,112\, z^{32} - 60\,287\,390\,654\,555\,750\,400\, z^{34}\big)\, D_z^4 +$
$\big(153\,384\, z - 79\,311\,414\, z^3 - 969\,091\,421\,520\, z^5 + 57\,078\,432\,217\,472\, z^7 -$
$7\,463\,767\,247\,981\,216\, z^9 + 1\,993\,974\,547\,224\,448\,008\, z^{11} - 34\,510\,070\,613\,086\,832\,672\, z^{13} -$
$1\,095\,768\,694\,217\,691\,008\,544\, z^{15} + 26\,243\,046\,777\,127\,962\,301\,056\, z^{17} +$
$61\,363\,912\,884\,641\,077\,146\,624\, z^{19} - 200\,034\,631\,208\,217\,374\,948\,352\, z^{21} +$
$632\,203\,788\,415\,405\,110\,233\,088\, z^{23} - 203\,966\,412\,230\,219\,271\,544\,832\, z^{25} -$
$1\,154\,635\,872\,526\,825\,368\,223\,744\, z^{27} - 49\,298\,735\,858\,907\,746\,009\,088\, z^{29} +$
$23\,212\,614\,855\,807\,341\,690\,880\, z^{31} - 103\,830\,543\,129\,001\,328\,640\, z^{33}\big)\, D_z^3 +$
$\big(5376 - 122\,453\,163\, z^2 - 210\,994\,658\,226\, z^4 + 45\,382\,581\,507\,096\, z^6 + 1\,401\,573\,621\,738\,348\, z^8 +$
$802\,090\,748\,535\,233\,988\, z^{10} - 31\,816\,432\,481\,008\,650\,432\, z^{12} -$
$116\,461\,520\,590\,177\,864\,944\, z^{14} + 17\,765\,779\,156\,948\,173\,645\,504\, z^{16} +$
$13\,489\,919\,179\,520\,112\,153\,600\, z^{18} - 180\,085\,805\,188\,644\,580\,639\,488\, z^{20} +$
$330\,121\,108\,081\,811\,674\,764\,288\, z^{22} - 571\,979\,572\,911\,812\,263\,882\,752\, z^{24} -$
$1\,061\,055\,509\,498\,887\,687\,815\,168\, z^{26} - 33\,911\,723\,874\,754\,276\,884\,480\, z^{28} +$
$17\,832\,309\,496\,712\,389\,459\,968\, z^{30} - 73\,473\,351\,140\,046\,274\,560\, z^{32}\big)\, D_z^2 +$
$\big(-2\,872\,149\, z + 1\,476\,999\,090\, z^3 + 7\,225\,802\,989\,032\, z^5 - 96\,816\,971\,270\,508\, z^7 +$
$32\,003\,772\,260\,098\,620\, z^9 - 8\,334\,366\,406\,909\,550\,400\, z^{11} + 47\,904\,013\,183\,865\,871\,600\, z^{13} +$
$2\,744\,674\,065\,604\,424\,390\,976\, z^{15} - 8\,757\,981\,589\,846\,974\,501\,888\, z^{17} -$
$57\,711\,771\,878\,830\,512\,192\,768\, z^{19} + 59\,179\,983\,999\,900\,719\,901\,696\, z^{21} -$
$243\,296\,046\,059\,335\,526\,830\,080\, z^{23} - 320\,106\,024\,990\,960\,568\,320\,000\, z^{25} -$
$7\,802\,155\,892\,694\,106\,767\,360\, z^{27} + 4\,303\,917\,823\,355\,302\,969\,344\, z^{29} - 15\,951\,250\,579\,594\,936\,320\, z^{31}\big)$
$D_z + \big(-5376 + 102\,466\,560\, z^2 + 111\,588\,980\,736\, z^4 - 18\,482\,502\,592\,512\, z^6 -$
$1\,388\,800\,732\,965\,888\, z^8 - 313\,491\,855\,037\,538\,304\, z^{10} + 2\,848\,574\,615\,378\,817\,024\, z^{12} +$
$2\,770\,228\,258\,073\,542\,656\, z^{14} - 1\,108\,761\,895\,930\,820\,788\,224\, z^{16} -$
$3\,259\,230\,359\,952\,995\,450\,880\, z^{18} + 3\,439\,578\,453\,759\,360\,761\,856\, z^{20} -$
$13\,441\,857\,535\,492\,706\,795\,520\, z^{22} - 15\,880\,170\,755\,475\,004\,981\,248\, z^{24} -$
$271\,304\,953\,791\,019\,745\,280\, z^{26} + 153\,583\,736\,249\,017\,958\,400\, z^{28} - 487\,992\,369\,547\,837\,440\, z^{30}\big)\big\}$

Return to $R(z)$

Substitute $z \to z * 2^M$ to move back to $R(z)$

In[ ]:=
```
ODETemp = -DFiniteSubstitute[
    ToOrePolynomial[ODEDiv2], {z → w * 2^MM}, Algebra → OreAlgebra[Der[w]]];
ToOrePolynomial[
 ODETemp]
```

Out[ ]=
$\big\{\big(189\, w^6 + 164\,526\,768\, w^8 - 187\,480\,074\,240\, w^{10} + 27\,962\,624\,347\,471\,872\, w^{12} -$
$41\,624\,521\,690\,721\,550\,336\, w^{14} - 195\,289\,074\,452\,595\,380\,060\,160\, w^{16} +$
$409\,286\,890\,042\,225\,331\,533\,774\,848\, w^{18} - 183\,057\,885\,005\,932\,655\,242\,496\,704\,512\, w^{20} -$
$37\,424\,471\,615\,851\,079\,483\,154\,493\,341\,696\, w^{22} + 24\,179\,802\,126\,384\,104\,960\,285\,655\,473\,061\,888\, w^{24} -$
$4\,496\,568\,077\,725\,961\,704\,167\,916\,875\,590\,664\,192\, w^{26} +$
$341\,280\,443\,189\,722\,351\,590\,493\,954\,378\,473\,406\,464\, w^{28} +$
$66\,745\,595\,233\,551\,284\,758\,691\,798\,722\,053\,402\,525\,696\, w^{30} -$
$862\,884\,480\,190\,835\,703\,771\,836\,009\,058\,131\,301\,629\,952\, w^{32} -$
$14\,419\,299\,574\,652\,092\,269\,971\,943\,043\,790\,157\,002\,047\,488\, w^{34} +$
$163\,946\,892\,981\,593\,252\,096\,191\,310\,873\,136\,551\,147\,077\,632\, w^{36} -$
$60\,410\,529\,312\,901\,568\,940\,949\,702\,532\,227\,912\,911\,814\,656\, w^{38}\big)\, D_w^8 +$
$\big(4536\, w^5 + 3\,614\,224\,320\, w^7 - 8\,424\,459\,694\,080\, w^9 + 504\,351\,762\,466\,996\,224\, w^{11} -$

$1 459 076 175 258 760 249 344 w^{13} - 2 281 516 349 650 087 729 692 672 w^{15} +$
$10 813 727 178 980 279 079 445 463 040 w^{17} - 8 009 910 605 349 768 666 711 582 572 544 w^{19} -$
$356 671 698 365 023 201 845 524 878 589 952 w^{21} + 856 530 906 617 308 581 607 299 333 281 021 952 w^{23} -$
$152 463 825 717 222 883 066 267 070 319 172 780 032 w^{25} +$
$19 572 696 903 048 562 049 704 124 366 058 153 836 544 w^{27} +$
$2 403 968 959 937 145 068 896 902 565 689 921 795 784 704 w^{29} -$
$48 089 530 987 427 549 189 664 014 910 575 944 560 279 552 w^{31} -$
$539 182 485 800 639 727 946 483 686 205 155 576 284 971 008 w^{33} +$
$7 483 978 291 401 930 288 527 957 614 938 251 623 271 497 728 w^{35} -$
$2 658 063 289 767 669 033 401 786 911 418 028 168 119 844 864 w^{37}\Big) D_w^7 +$
$\Big(35 511 w^4 + 24 871 351 008 w^6 - 110 328 387 608 576 w^8 + 2 576 263 065 101 139 968 w^{10} -$
$16 004 233 009 227 934 728 192 w^{12} + 5 255 301 684 683 958 621 569 024 w^{14} +$
$87 545 818 070 819 369 294 699 167 744 w^{16} - 112 555 786 991 037 942 654 133 799 485 440 w^{18} +$
$10 992 972 717 520 438 674 440 203 114 905 600 w^{20} + 11 643 772 019 468 852 238 635 765 432 699 584 512$
$w^{22} - 1 835 160 812 881 290 491 040 827 819 753 635 577 856 w^{24} +$
$347 988 939 342 029 747 059 291 099 605 874 371 461 120 w^{26} +$
$29 475 988 817 125 376 497 496 424 404 366 007 753 244 672 w^{28} -$
$962 688 837 174 624 395 225 377 113 181 371 460 076 175 360 w^{30} -$
$7 400 388 823 775 451 206 202 960 636 361 899 582 873 927 680 w^{32} +$
$126 400 165 026 765 674 644 624 563 232 438 483 970 737 307 648 w^{34} -$
$43 030 755 643 631 525 898 688 145 056 497 788 077 712 867 328 w^{36}\Big) D_w^6 +$
$\Big(108 591 w^3 + 61 475 172 192 w^5 - 569 224 242 008 064 w^7 + 3 829 264 284 903 276 544 w^9 -$
$66 883 682 812 478 795 808 768 w^{11} + 124 182 806 877 852 859 128 348 672 w^{13} +$
$229 171 888 616 408 310 939 744 468 992 w^{15} - 643 585 280 323 140 905 560 678 179 274 752 w^{17} +$
$183 203 722 404 447 551 926 201 261 435 650 048 w^{19} +$
$75 196 379 843 483 764 444 238 710 407 413 366 784 w^{21} -$
$10 433 099 891 654 586 537 757 851 700 312 902 991 872 w^{23} +$
$2 573 969 755 244 678 997 783 625 826 971 948 079 382 528 w^{25} +$
$144 344 233 196 207 101 220 604 021 763 917 084 027 256 832 w^{27} -$
$8 907 736 520 569 643 116 362 401 010 499 694 303 014 551 552 w^{29} -$
$47 599 952 475 629 202 103 568 196 717 849 740 513 365 721 088 w^{31} +$
$1 001 116 948 004 689 014 083 166 014 991 650 208 200 035 139 584 w^{33} -$
$324 540 466 101 235 453 743 017 039 428 761 405 140 496 285 696 w^{35}\Big) D_w^5 +$
$\Big(122 262 w^2 + 43 874 090 736 w^4 - 1 218 386 114 717 696 w^6 + 1 649 628 605 194 960 896 w^8 -$
$99 007 813 214 874 713 980 928 w^{10} + 426 046 136 497 631 282 929 336 320 w^{12} -$
$6 898 552 676 392 875 801 561 792 512 w^{14} - 1 504 042 529 222 927 570 559 252 710 817 792 w^{16} +$
$939 455 189 530 770 299 621 273 614 405 337 088 w^{18} +$
$228 486 142 849 047 557 467 471 257 241 198 264 320 w^{20} -$
$32 311 467 271 240 228 795 157 013 676 770 793 095 168 w^{22} +$
$8 418 053 461 227 938 500 090 563 088 604 167 455 899 648 w^{24} +$
$214 906 658 500 411 634 903 180 607 073 980 774 535 397 376 w^{26} -$
$40 394 439 554 252 654 197 434 202 074 837 977 183 373 230 080 w^{28} -$
$152 442 356 817 923 457 619 504 933 406 101 574 584 885 051 392 w^{30} +$
$3 899 390 252 123 602 186 082 814 236 772 652 015 150 822 326 272 w^{32} -$
$1 194 114 796 085 021 012 732 772 453 387 038 411 890 203 033 600 w^{34}\Big) D_w^4 +$
$\Big(38 346 w - 1 268 982 624 w^3 - 992 349 615 636 480 w^5 + 3 740 692 133 804 244 992 w^7 -$
$31 305 308 823 276 606 193 664 w^9 + 535 253 466 836 588 235 375 771 648 w^{11} -$
$592 878 498 663 434 463 737 857 179 648 w^{13} - 1 204 810 420 645 275 439 700 412 563 718 144 w^{15} +$
$1 846 690 245 102 187 297 016 322 887 918 813 184 w^{17} +$
$276 358 495 201 265 133 334 522 051 227 763 605 504 w^{19} -$
$57 656 056 996 513 431 032 091 962 624 868 391 845 888 w^{21} +$

$11\,662\,101\,487\,328\,601\,498\,797\,600\,487\,747\,969\,727\,070\,208\, w^{23} -$
$240\,801\,037\,186\,790\,192\,401\,131\,615\,868\,141\,719\,171\,104\,768\, w^{25} -$
$87\,241\,819\,909\,434\,053\,197\,948\,400\,079\,840\,315\,821\,139\,165\,184\, w^{27} -$
$238\,394\,058\,616\,780\,626\,497\,995\,697\,634\,958\,361\,516\,102\,909\,952\, w^{29} +$
$7\,183\,956\,336\,628\,635\,589\,493\,026\,781\,628\,401\,423\,108\,831\,969\,280\, w^{31} -$
$2\,056\,575\,786\,242\,212\,411\,979\,731\,039\,872\,145\,581\,894\,543\,605\,760\, w^{33}\big)\, D_w^3 +$
$\big(1344 - 1\,959\,250\,608\, w^2 - 216\,058\,530\,023\,424\, w^4 + 2\,974\,192\,861\,649\,043\,456\, w^6 +$
$5\,878\,625\,847\,951\,639\,969\,792\, w^8 + 215\,309\,595\,836\,436\,867\,635\,478\,528\, w^{10} -$
$546\,602\,147\,925\,297\,178\,794\,145\,087\,488\, w^{12} - 128\,050\,796\,077\,374\,604\,481\,941\,727\,084\,544\, w^{14} +$
$1\,250\,155\,568\,612\,161\,240\,668\,058\,049\,530\,822\,656\, w^{16} +$
$60\,753\,194\,990\,144\,884\,230\,286\,610\,179\,660\,185\,600\, w^{18} -$
$51\,906\,199\,369\,106\,909\,982\,895\,465\,012\,453\,359\,747\,072\, w^{20} +$
$6\,089\,659\,594\,114\,589\,876\,275\,493\,470\,101\,690\,969\,489\,408\, w^{22} -$
$675\,274\,291\,001\,209\,168\,687\,771\,752\,976\,353\,351\,432\,142\,848\, w^{24} -$
$80\,171\,087\,592\,347\,546\,856\,356\,920\,248\,692\,817\,771\,362\,254\,848\, w^{26} -$
$163\,987\,034\,319\,329\,209\,857\,306\,794\,250\,294\,848\,210\,778\,193\,920\, w^{28} +$
$5\,518\,832\,479\,727\,298\,791\,528\,505\,025\,097\,934\,398\,902\,876\,766\,208\, w^{30} -$
$1\,455\,289\,651\,147\,798\,795\,787\,478\,334\,001\,370\,422\,045\,615\,063\,040\, w^{32}\big)\, D_w^2 +$
$\big(-45\,954\,384\, w + 1\,512\,447\,068\,160\, w^3 + 473\,550\,224\,689\,201\,152\, w^5 - 406\,079\,809\,867\,776\,786\,432\, w^7 +$
$8\,590\,947\,200\,359\,723\,664\,670\,720\, w^9 - 143\,183\,324\,602\,230\,189\,592\,254\,873\,600\, w^{11} +$
$52\,671\,019\,512\,795\,328\,863\,369\,009\,561\,600\, w^{13} + 193\,139\,267\,173\,586\,718\,427\,490\,509\,142\,360\,064\, w^{15} -$
$39\,442\,442\,624\,552\,498\,500\,893\,252\,260\,827\,496\,448\, w^{17} -$
$16\,634\,285\,717\,017\,098\,530\,661\,638\,805\,946\,097\,467\,392\, w^{19} +$
$1\,091\,678\,019\,132\,394\,690\,717\,027\,530\,293\,619\,757\,940\,736\, w^{21} -$
$287\,233\,273\,331\,328\,879\,265\,211\,051\,076\,117\,415\,770\,193\,920\, w^{23} -$
$24\,186\,527\,414\,111\,123\,413\,226\,881\,122\,319\,296\,123\,371\,520\,000\, w^{25} -$
$37\,728\,910\,829\,345\,327\,109\,838\,548\,738\,437\,762\,004\,748\,861\,440\, w^{27} +$
$1\,331\,998\,049\,831\,378\,030\,074\,725\,974\,561\,127\,544\,713\,079\,947\,264\, w^{29} -$
$315\,947\,068\,306\,475\,205\,561\,166\,944\,243\,551\,984\,528\,790\,650\,880\, w^{31}\big)\, D_w +$
$\big(-86\,016 + 104\,925\,757\,440\, w^2 + 7\,313\,095\,441\,514\,496\, w^4 - 77\,521\,234\,553\,783\,451\,648\, w^6 -$
$372\,803\,358\,046\,832\,377\,724\,928\, w^8 - 5\,385\,749\,059\,794\,399\,472\,109\,223\,936\, w^{10} +$
$3\,132\,040\,912\,196\,556\,228\,927\,500\,058\,624\, w^{12} + 194\,937\,483\,606\,112\,889\,575\,521\,946\,435\,584\, w^{14} -$
$4\,993\,419\,661\,356\,649\,167\,044\,237\,771\,201\,839\,104\, w^{16} -$
$939\,409\,192\,614\,330\,743\,139\,648\,628\,757\,071\,134\,720\, w^{18} +$
$63\,449\,023\,457\,944\,551\,152\,259\,668\,183\,189\,915\,959\,296\, w^{20} -$
$15\,869\,344\,373\,279\,882\,748\,885\,504\,666\,619\,865\,443\,860\,480\, w^{22} -$
$1\,199\,871\,777\,886\,430\,336\,038\,441\,987\,175\,041\,302\,924\,361\,728\, w^{24} -$
$1\,311\,950\,254\,509\,270\,561\,000\,268\,264\,109\,855\,833\,451\,397\,120\, w^{26} +$
$47\,531\,864\,121\,426\,193\,501\,043\,928\,118\,257\,886\,052\,116\,070\,400\, w^{28} -$
$9\,665\,684\,690\,064\,251\,030\,551\,952\,405\,156\,466\,065\,890\,344\,960\, w^{30}\big)\big\}$

In[•]:=
```
ODE = -DFiniteSubstitute[ToOrePolynomial[ODETemp], {w → z}, Algebra → OreAlgebra[Der[z]]];
ToOrePolynomial[ODE]
```

Out[•]=
$\big\{\big(189\, z^6 + 164\,526\,768\, z^8 - 187\,480\,074\,240\, z^{10} + 27\,962\,624\,347\,471\,872\, z^{12} -$
$41\,624\,521\,690\,721\,550\,336\, z^{14} - 195\,289\,074\,452\,595\,380\,060\,160\, z^{16} +$
$409\,286\,890\,042\,225\,331\,533\,774\,848\, z^{18} - 183\,057\,885\,005\,932\,655\,242\,496\,704\,512\, z^{20} -$
$37\,424\,471\,615\,851\,079\,483\,154\,493\,341\,696\, z^{22} + 24\,179\,802\,126\,384\,104\,960\,285\,655\,473\,061\,888\, z^{24} -$
$4\,496\,568\,077\,725\,961\,704\,167\,916\,875\,590\,664\,192\, z^{26} +$
$341\,280\,443\,189\,722\,351\,590\,493\,954\,378\,473\,406\,464\, z^{28} +$
$66\,745\,595\,233\,551\,284\,758\,691\,798\,722\,053\,402\,525\,696\, z^{30} -$

$862 884 480 190 835 703 771 836 009 058 131 301 629 952 z^{32} -$
$14 419 299 574 652 092 269 971 943 043 790 157 002 047 488 z^{34} +$
$163 946 892 981 593 252 096 191 310 873 136 551 147 077 632 z^{36} -$
$60 410 529 312 901 568 940 949 702 532 227 912 911 814 656 z^{38}\Big)\, D_z^8 +$
$\Big(4536 z^5 + 3 614 224 320 z^7 - 8 424 459 694 080 z^9 + 504 351 762 466 996 224 z^{11} -$
$1 459 076 175 258 760 249 344 z^{13} - 2 281 516 349 650 087 729 692 672 z^{15} +$
$10 813 727 178 980 279 079 445 463 040 z^{17} - 8 009 910 605 349 768 666 711 582 572 544 z^{19} -$
$356 671 698 365 023 201 845 524 878 589 952 z^{21} + 856 530 906 617 308 581 607 299 333 281 021 952 z^{23} -$
$152 463 825 717 222 883 066 267 070 319 172 780 032 z^{25} +$
$19 572 696 903 048 562 049 704 124 366 058 153 836 544 z^{27} +$
$2 403 968 959 937 145 068 896 902 565 689 921 795 784 704 z^{29} -$
$48 089 530 987 427 549 189 664 014 910 575 944 560 279 552 z^{31} -$
$539 182 485 800 639 727 946 483 686 205 155 576 284 971 008 z^{33} +$
$7 483 978 291 401 930 288 527 957 614 938 251 623 271 497 728 z^{35} -$
$2 658 063 289 767 669 033 401 786 911 418 028 168 119 844 864 z^{37}\Big)\, D_z^7 +$
$\Big(35 511 z^4 + 24 871 351 008 z^6 - 110 328 387 608 576 z^8 + 2 576 263 065 101 139 968 z^{10} -$
$16 004 233 009 227 934 728 192 z^{12} + 5 255 301 684 683 958 621 569 024 z^{14} +$
$87 545 818 070 819 369 294 699 167 744 z^{16} - 112 555 786 991 037 942 654 133 799 485 440 z^{18} +$
$10 992 972 717 520 438 674 440 203 114 905 600 z^{20} + 11 643 772 019 468 852 238 635 765 432 699 584 512$
$z^{22} - 1 835 160 812 881 290 491 040 827 819 753 635 577 856 z^{24} +$
$347 988 939 342 029 747 059 291 099 605 874 371 461 120 z^{26} +$
$29 475 988 817 125 376 497 496 424 404 366 007 753 244 672 z^{28} -$
$962 688 837 174 624 395 225 377 113 181 371 460 076 175 360 z^{30} -$
$7 400 388 823 775 451 206 202 960 636 361 899 582 873 927 680 z^{32} +$
$126 400 165 026 765 674 644 624 563 232 438 483 970 737 307 648 z^{34} -$
$43 030 755 643 631 525 898 688 145 056 497 788 077 712 867 328 z^{36}\Big)\, D_z^6 +$
$\Big(108 591 z^3 + 61 475 172 192 z^5 - 569 224 242 008 064 z^7 + 3 829 264 284 903 276 544 z^9 -$
$66 883 682 812 478 795 808 768 z^{11} + 124 182 806 877 852 859 128 348 672 z^{13} +$
$229 171 888 616 408 310 939 744 468 992 z^{15} - 643 585 280 323 140 905 560 678 179 274 752 z^{17} +$
$183 203 722 404 447 551 926 201 261 435 650 048 z^{19} +$
$75 196 379 843 483 764 444 238 710 407 413 366 784 z^{21} -$
$10 433 099 891 654 586 537 757 851 700 312 902 991 872 z^{23} +$
$2 573 969 755 244 678 997 783 625 826 971 948 079 382 528 z^{25} +$
$144 344 233 196 207 101 220 604 021 763 917 084 027 256 832 z^{27} -$
$8 907 736 520 569 643 116 362 401 010 499 694 303 014 551 552 z^{29} -$
$47 599 952 475 629 202 103 568 196 717 849 740 513 365 721 088 z^{31} +$
$1 001 116 948 004 689 014 083 166 014 991 650 208 200 035 139 584 z^{33} -$
$324 540 466 101 235 453 743 017 039 428 761 405 140 496 285 696 z^{35}\Big)\, D_z^5 +$
$\Big(122 262 z^2 + 43 874 090 736 z^4 - 1 218 386 114 717 696 z^6 + 1 649 628 605 194 960 896 z^8 -$
$99 007 813 214 874 713 980 928 z^{10} + 426 046 136 497 631 282 929 336 320 z^{12} -$
$6 898 552 676 392 875 801 561 792 512 z^{14} - 1 504 042 529 222 927 570 559 252 710 817 792 z^{16} +$
$939 455 189 530 770 299 621 273 614 405 337 088 z^{18} +$
$228 486 142 849 047 557 467 471 257 241 198 264 320 z^{20} -$
$32 311 467 271 240 228 795 157 013 676 770 793 095 168 z^{22} +$
$8 418 053 461 227 938 500 090 563 088 604 167 455 899 648 z^{24} +$
$214 906 658 500 411 634 903 180 607 073 980 774 535 397 376 z^{26} -$
$40 394 439 554 252 654 197 434 202 074 837 977 183 373 230 080 z^{28} -$
$152 442 356 817 923 457 619 504 933 406 101 574 584 885 051 392 z^{30} +$
$3 899 390 252 123 602 186 082 814 236 772 652 015 150 822 326 272 z^{32} -$
$1 194 114 796 085 021 012 732 772 453 387 038 411 890 203 033 600 z^{34}\Big)\, D_z^4 +$

$$\left(38\,346\, z - 1\,268\,982\,624\, z^3 - 992\,349\,615\,636\,480\, z^5 + 3\,740\,692\,133\,804\,244\,992\, z^7 -\right.$$
$$31\,305\,308\,823\,276\,606\,193\,664\, z^9 + 535\,253\,466\,836\,588\,235\,375\,771\,648\, z^{11} -$$
$$592\,878\,498\,663\,434\,463\,737\,857\,179\,648\, z^{13} - 1\,204\,810\,420\,645\,275\,439\,700\,412\,563\,718\,144\, z^{15} +$$
$$1\,846\,690\,245\,102\,187\,297\,016\,322\,887\,918\,813\,184\, z^{17} +$$
$$276\,358\,495\,201\,265\,133\,334\,522\,051\,227\,763\,605\,504\, z^{19} -$$
$$57\,656\,056\,996\,513\,431\,032\,091\,962\,624\,868\,391\,845\,888\, z^{21} +$$
$$11\,662\,101\,487\,328\,601\,498\,797\,600\,487\,747\,969\,727\,070\,208\, z^{23} -$$
$$240\,801\,037\,186\,790\,192\,401\,131\,615\,868\,141\,719\,171\,104\,768\, z^{25} -$$
$$87\,241\,819\,909\,434\,053\,197\,948\,400\,079\,840\,315\,821\,139\,165\,184\, z^{27} -$$
$$238\,394\,058\,616\,780\,626\,497\,995\,697\,634\,958\,361\,516\,102\,909\,952\, z^{29} +$$
$$7\,183\,956\,336\,628\,635\,589\,493\,026\,781\,628\,401\,423\,108\,831\,969\,280\, z^{31} -$$
$$\left.2\,056\,575\,786\,242\,212\,411\,979\,731\,039\,872\,145\,581\,894\,543\,605\,760\, z^{33}\right) D_z^3 +$$
$$\left(1344 - 1\,959\,250\,608\, z^2 - 216\,058\,530\,023\,424\, z^4 + 2\,974\,192\,861\,649\,043\,456\, z^6 +\right.$$
$$5\,878\,625\,847\,951\,639\,969\,792\, z^8 + 215\,309\,595\,836\,436\,867\,635\,478\,528\, z^{10} -$$
$$546\,602\,147\,925\,297\,178\,794\,145\,087\,488\, z^{12} - 128\,050\,796\,077\,374\,604\,481\,941\,727\,084\,544\, z^{14} +$$
$$1\,250\,155\,568\,612\,161\,240\,668\,058\,049\,530\,822\,656\, z^{16} +$$
$$60\,753\,194\,990\,144\,884\,230\,286\,610\,179\,660\,185\,600\, z^{18} -$$
$$51\,906\,199\,369\,106\,909\,982\,895\,465\,012\,453\,359\,747\,072\, z^{20} +$$
$$6\,089\,659\,594\,114\,589\,876\,275\,493\,470\,101\,690\,969\,489\,408\, z^{22} -$$
$$675\,274\,291\,001\,209\,168\,687\,771\,752\,976\,353\,351\,432\,142\,848\, z^{24} -$$
$$80\,171\,087\,592\,347\,546\,856\,356\,920\,248\,692\,817\,771\,362\,254\,848\, z^{26} -$$
$$163\,987\,034\,319\,329\,209\,857\,306\,794\,250\,294\,848\,210\,778\,193\,920\, z^{28} +$$
$$5\,518\,832\,479\,727\,298\,791\,528\,505\,025\,097\,934\,398\,902\,876\,766\,208\, z^{30} -$$
$$\left.1\,455\,289\,651\,147\,798\,795\,787\,478\,334\,001\,370\,422\,045\,615\,063\,040\, z^{32}\right) D_z^2 +$$
$$\left(-45\,954\,384\, z + 1\,512\,447\,068\,160\, z^3 + 473\,550\,224\,689\,201\,152\, z^5 - 406\,079\,809\,867\,776\,786\,432\, z^7 +\right.$$
$$8\,590\,947\,200\,359\,723\,664\,670\,720\, z^9 - 143\,183\,324\,602\,230\,189\,592\,254\,873\,600\, z^{11} +$$
$$52\,671\,019\,512\,795\,328\,863\,369\,009\,561\,600\, z^{13} + 193\,139\,267\,173\,586\,718\,427\,490\,509\,142\,360\,064\, z^{15} -$$
$$39\,442\,442\,624\,552\,498\,500\,893\,252\,260\,827\,496\,448\, z^{17} -$$
$$16\,634\,285\,717\,017\,098\,530\,661\,638\,805\,946\,097\,467\,392\, z^{19} +$$
$$1\,091\,678\,019\,132\,394\,690\,717\,027\,530\,293\,619\,757\,940\,736\, z^{21} -$$
$$287\,233\,273\,331\,328\,879\,265\,211\,051\,076\,117\,415\,770\,193\,920\, z^{23} -$$
$$24\,186\,527\,414\,111\,123\,413\,226\,881\,122\,319\,296\,123\,371\,520\,000\, z^{25} -$$
$$37\,728\,910\,829\,345\,327\,109\,838\,548\,738\,437\,762\,004\,748\,861\,440\, z^{27} +$$
$$1\,331\,998\,049\,831\,378\,030\,074\,725\,974\,561\,127\,544\,713\,079\,947\,264\, z^{29} -$$
$$\left.315\,947\,068\,306\,475\,205\,561\,166\,944\,243\,551\,984\,528\,790\,650\,880\, z^{31}\right) D_z +$$
$$\left(-86\,016 + 104\,925\,757\,440\, z^2 + 7\,313\,095\,441\,514\,496\, z^4 - 77\,521\,234\,553\,783\,451\,648\, z^6 -\right.$$
$$372\,803\,358\,046\,832\,377\,724\,928\, z^8 - 5\,385\,749\,059\,794\,399\,472\,109\,223\,936\, z^{10} +$$
$$3\,132\,040\,912\,196\,556\,228\,927\,500\,058\,624\, z^{12} + 194\,937\,483\,606\,112\,889\,575\,521\,946\,435\,584\, z^{14} -$$
$$4\,993\,419\,661\,356\,649\,167\,044\,237\,771\,201\,839\,104\, z^{16} -$$
$$939\,409\,192\,614\,330\,743\,139\,648\,628\,757\,071\,134\,720\, z^{18} +$$
$$63\,449\,023\,457\,944\,551\,152\,259\,668\,183\,189\,915\,959\,296\, z^{20} -$$
$$15\,869\,344\,373\,279\,882\,748\,885\,504\,666\,619\,865\,443\,860\,480\, z^{22} -$$
$$1\,199\,871\,777\,886\,430\,336\,038\,441\,987\,175\,041\,302\,924\,361\,728\, z^{24} -$$
$$1\,311\,950\,254\,509\,270\,561\,000\,268\,264\,109\,855\,833\,451\,397\,120\, z^{26} +$$
$$47\,531\,864\,121\,426\,193\,501\,043\,928\,118\,257\,886\,052\,116\,070\,400\, z^{28} -$$
$$\left.9\,665\,684\,690\,064\,251\,030\,551\,952\,405\,156\,466\,065\,890\,344\,960\, z^{30}\right) \}$$

In[∘]:=
```
ODEinD = ODE[[1]];
ToOrePolynomial[ODEinD]
```

Out[∘]=

$$\left(189\, z^6 + 164\,526\,768\, z^8 - 187\,480\,074\,240\, z^{10} + 27\,962\,624\,347\,471\,872\, z^{12} -\right.$$
$$41\,624\,521\,690\,721\,550\,336\, z^{14} - 195\,289\,074\,452\,595\,380\,060\,160\, z^{16} +$$

$409\,286\,890\,042\,225\,331\,533\,774\,848\, z^{18} - 183\,057\,885\,005\,932\,655\,242\,496\,704\,512\, z^{20} -$
$37\,424\,471\,615\,851\,079\,483\,154\,493\,341\,696\, z^{22} + 24\,179\,802\,126\,384\,104\,960\,285\,655\,473\,061\,888\, z^{24} -$
$4\,496\,568\,077\,725\,961\,704\,167\,916\,875\,590\,664\,192\, z^{26} +$
$341\,280\,443\,189\,722\,351\,590\,493\,954\,378\,473\,406\,464\, z^{28} +$
$66\,745\,595\,233\,551\,284\,758\,691\,798\,722\,053\,402\,525\,696\, z^{30} -$
$862\,884\,480\,190\,835\,703\,771\,836\,009\,058\,131\,301\,629\,952\, z^{32} -$
$14\,419\,299\,574\,652\,092\,269\,971\,943\,043\,790\,157\,002\,047\,488\, z^{34} +$
$163\,946\,892\,981\,593\,252\,096\,191\,310\,873\,136\,551\,147\,077\,632\, z^{36} -$
$60\,410\,529\,312\,901\,568\,940\,949\,702\,532\,227\,912\,911\,814\,656\, z^{38}\big)\, D_z^8 +$
$\big(4536\, z^5 + 3\,614\,224\,320\, z^7 - 8\,424\,459\,694\,080\, z^9 + 504\,351\,762\,466\,996\,224\, z^{11} -$
$1\,459\,076\,175\,258\,760\,249\,344\, z^{13} - 2\,281\,516\,349\,650\,087\,729\,692\,672\, z^{15} +$
$10\,813\,727\,178\,980\,279\,079\,445\,463\,040\, z^{17} - 8\,009\,910\,605\,349\,768\,666\,711\,582\,572\,544\, z^{19} -$
$356\,671\,698\,365\,023\,201\,845\,524\,878\,589\,952\, z^{21} + 856\,530\,906\,617\,308\,581\,607\,299\,333\,281\,021\,952\, z^{23} -$
$152\,463\,825\,717\,222\,883\,066\,267\,070\,319\,172\,780\,032\, z^{25} +$
$19\,572\,696\,903\,048\,562\,049\,704\,124\,366\,058\,153\,836\,544\, z^{27} +$
$2\,403\,968\,959\,937\,145\,068\,896\,902\,565\,689\,921\,795\,784\,704\, z^{29} -$
$48\,089\,530\,987\,427\,549\,189\,664\,014\,910\,575\,944\,560\,279\,552\, z^{31} -$
$539\,182\,485\,800\,639\,727\,946\,483\,686\,205\,155\,576\,284\,971\,008\, z^{33} +$
$7\,483\,978\,291\,401\,930\,288\,527\,957\,614\,938\,251\,623\,271\,497\,728\, z^{35} -$
$2\,658\,063\,289\,767\,669\,033\,401\,786\,911\,418\,028\,168\,119\,844\,864\, z^{37}\big)\, D_z^7 +$
$\big(35\,511\, z^4 + 24\,871\,351\,008\, z^6 - 110\,328\,387\,608\,576\, z^8 + 2\,576\,263\,065\,101\,139\,968\, z^{10} -$
$16\,004\,233\,009\,227\,934\,728\,192\, z^{12} + 5\,255\,301\,684\,683\,958\,621\,569\,024\, z^{14} +$
$87\,545\,818\,070\,819\,369\,294\,699\,167\,744\, z^{16} - 112\,555\,786\,991\,037\,942\,654\,133\,799\,485\,440\, z^{18} +$
$10\,992\,972\,717\,520\,438\,674\,440\,203\,114\,905\,600\, z^{20} +$
$11\,643\,772\,019\,468\,852\,238\,635\,765\,432\,699\,584\,512\, z^{22} -$
$1\,835\,160\,812\,881\,290\,491\,040\,827\,819\,753\,635\,577\,856\, z^{24} +$
$347\,988\,939\,342\,029\,747\,059\,291\,099\,605\,874\,371\,461\,120\, z^{26} +$
$29\,475\,988\,817\,125\,376\,497\,496\,424\,404\,366\,007\,753\,244\,672\, z^{28} -$
$962\,688\,837\,174\,624\,395\,225\,377\,113\,181\,371\,460\,076\,175\,360\, z^{30} -$
$7\,400\,388\,823\,775\,451\,206\,202\,960\,636\,361\,899\,582\,873\,927\,680\, z^{32} +$
$126\,400\,165\,026\,765\,674\,644\,624\,563\,232\,438\,483\,970\,737\,307\,648\, z^{34} -$
$43\,030\,755\,643\,631\,525\,898\,688\,145\,056\,497\,788\,077\,712\,867\,328\, z^{36}\big)\, D_z^6 +$
$\big(108\,591\, z^3 + 61\,475\,172\,192\, z^5 - 569\,224\,242\,008\,064\, z^7 + 3\,829\,264\,284\,903\,276\,544\, z^9 -$
$66\,883\,682\,812\,478\,795\,808\,768\, z^{11} + 124\,182\,806\,877\,852\,859\,128\,348\,672\, z^{13} +$
$229\,171\,888\,616\,408\,310\,939\,744\,468\,992\, z^{15} - 643\,585\,280\,323\,140\,905\,560\,678\,179\,274\,752\, z^{17} +$
$183\,203\,722\,404\,447\,551\,926\,201\,261\,435\,650\,048\, z^{19} +$
$75\,196\,379\,843\,483\,764\,444\,238\,710\,407\,413\,366\,784\, z^{21} -$
$10\,433\,099\,891\,654\,586\,537\,757\,851\,700\,312\,902\,991\,872\, z^{23} +$
$2\,573\,969\,755\,244\,678\,997\,783\,625\,826\,971\,948\,079\,382\,528\, z^{25} +$
$144\,344\,233\,196\,207\,101\,220\,604\,021\,763\,917\,084\,027\,256\,832\, z^{27} -$
$8\,907\,736\,520\,569\,643\,116\,362\,401\,010\,499\,694\,303\,014\,551\,552\, z^{29} -$
$47\,599\,952\,475\,629\,202\,103\,568\,196\,717\,849\,740\,513\,365\,721\,088\, z^{31} +$
$1\,001\,116\,948\,004\,689\,014\,083\,166\,014\,991\,650\,208\,200\,035\,139\,584\, z^{33} -$
$324\,540\,466\,101\,235\,453\,743\,017\,039\,428\,761\,405\,140\,496\,285\,696\, z^{35}\big)\, D_z^5 +$
$\big(122\,262\, z^2 + 43\,874\,090\,736\, z^4 - 1\,218\,386\,114\,717\,696\, z^6 + 1\,649\,628\,605\,194\,960\,896\, z^8 -$
$99\,007\,813\,214\,874\,713\,980\,928\, z^{10} + 426\,046\,136\,497\,631\,282\,929\,336\,320\, z^{12} -$
$6\,898\,552\,676\,392\,875\,801\,561\,792\,512\, z^{14} - 1\,504\,042\,529\,222\,927\,570\,559\,252\,710\,817\,792\, z^{16} +$
$939\,455\,189\,530\,770\,299\,621\,273\,614\,405\,337\,088\, z^{18} +$
$228\,486\,142\,849\,047\,557\,467\,471\,257\,241\,198\,264\,320\, z^{20} -$
$32\,311\,467\,271\,240\,228\,795\,157\,013\,676\,770\,793\,095\,168\, z^{22} +$
$8\,418\,053\,461\,227\,938\,500\,090\,563\,088\,604\,167\,455\,899\,648\, z^{24} +$

$214\,906\,658\,500\,411\,634\,903\,180\,607\,073\,980\,774\,535\,397\,376\, z^{26} -$
$40\,394\,439\,554\,252\,654\,197\,434\,202\,074\,837\,977\,183\,373\,230\,080\, z^{28} -$
$152\,442\,356\,817\,923\,457\,619\,504\,933\,406\,101\,574\,584\,885\,051\,392\, z^{30} +$
$3\,899\,390\,252\,123\,602\,186\,082\,814\,236\,772\,652\,015\,150\,822\,326\,272\, z^{32} -$
$1\,194\,114\,796\,085\,021\,012\,732\,772\,453\,387\,038\,411\,890\,203\,033\,600\, z^{34}\Big)\, D_z^4 +$
$\Big(38\,346\, z - 1\,268\,982\,624\, z^3 - 992\,349\,615\,636\,480\, z^5 + 3\,740\,692\,133\,804\,244\,992\, z^7 -$
$31\,305\,308\,823\,276\,606\,193\,664\, z^9 + 535\,253\,466\,836\,588\,235\,375\,771\,648\, z^{11} -$
$592\,878\,498\,663\,434\,463\,737\,857\,179\,648\, z^{13} - 1\,204\,810\,420\,645\,275\,439\,700\,412\,563\,718\,144\, z^{15} +$
$1\,846\,690\,245\,102\,187\,297\,016\,322\,887\,918\,813\,184\, z^{17} +$
$276\,358\,495\,201\,265\,133\,334\,522\,051\,227\,763\,605\,504\, z^{19} -$
$57\,656\,056\,996\,513\,431\,032\,091\,962\,624\,868\,391\,845\,888\, z^{21} +$
$11\,662\,101\,487\,328\,601\,498\,797\,600\,487\,747\,969\,727\,070\,208\, z^{23} -$
$240\,801\,037\,186\,790\,192\,401\,131\,615\,868\,141\,719\,171\,104\,768\, z^{25} -$
$87\,241\,819\,909\,434\,053\,197\,948\,400\,079\,840\,315\,821\,139\,165\,184\, z^{27} -$
$238\,394\,058\,616\,780\,626\,497\,995\,697\,634\,958\,361\,516\,102\,909\,952\, z^{29} +$
$7\,183\,956\,336\,628\,635\,589\,493\,026\,781\,628\,401\,423\,108\,831\,969\,280\, z^{31} -$
$2\,056\,575\,786\,242\,212\,411\,979\,731\,039\,872\,145\,581\,894\,543\,605\,760\, z^{33}\Big)\, D_z^3 +$
$\Big(1344 - 1\,959\,250\,608\, z^2 - 216\,058\,530\,023\,424\, z^4 + 2\,974\,192\,861\,649\,043\,456\, z^6 +$
$5\,878\,625\,847\,951\,639\,969\,792\, z^8 + 215\,309\,595\,836\,436\,867\,635\,478\,528\, z^{10} -$
$546\,602\,147\,925\,297\,178\,794\,145\,087\,488\, z^{12} - 128\,050\,796\,077\,374\,604\,481\,941\,727\,084\,544\, z^{14} +$
$1\,250\,155\,568\,612\,161\,240\,668\,058\,049\,530\,822\,656\, z^{16} +$
$60\,753\,194\,990\,144\,884\,230\,286\,610\,179\,660\,185\,600\, z^{18} -$
$51\,906\,199\,369\,106\,909\,982\,895\,465\,012\,453\,359\,747\,072\, z^{20} +$
$6\,089\,659\,594\,114\,589\,876\,275\,493\,470\,101\,690\,969\,489\,408\, z^{22} -$
$675\,274\,291\,001\,209\,168\,687\,771\,752\,976\,353\,351\,432\,142\,848\, z^{24} -$
$80\,171\,087\,592\,347\,546\,856\,356\,920\,248\,692\,817\,771\,362\,254\,848\, z^{26} -$
$163\,987\,034\,319\,329\,209\,857\,306\,794\,250\,294\,848\,210\,778\,193\,920\, z^{28} +$
$5\,518\,832\,479\,727\,298\,791\,528\,505\,025\,097\,934\,398\,902\,876\,766\,208\, z^{30} -$
$1\,455\,289\,651\,147\,798\,795\,787\,478\,334\,001\,370\,422\,045\,615\,063\,040\, z^{32}\Big)\, D_z^2 +$
$\Big(-45\,954\,384\, z + 1\,512\,447\,068\,160\, z^3 + 473\,550\,224\,689\,201\,152\, z^5 - 406\,079\,809\,867\,776\,786\,432\, z^7 +$
$8\,590\,947\,200\,359\,723\,664\,670\,720\, z^9 - 143\,183\,324\,602\,230\,189\,592\,254\,873\,600\, z^{11} +$
$52\,671\,019\,512\,795\,328\,863\,369\,009\,561\,600\, z^{13} + 193\,139\,267\,173\,586\,718\,427\,490\,509\,142\,360\,064\, z^{15} -$
$39\,442\,442\,624\,552\,498\,500\,893\,252\,260\,827\,496\,448\, z^{17} -$
$16\,634\,285\,717\,017\,098\,530\,661\,638\,805\,946\,097\,467\,392\, z^{19} +$
$1\,091\,678\,019\,132\,394\,690\,717\,027\,530\,293\,619\,757\,940\,736\, z^{21} -$
$287\,233\,273\,331\,328\,879\,265\,211\,051\,076\,117\,415\,770\,193\,920\, z^{23} -$
$24\,186\,527\,414\,111\,123\,413\,226\,881\,122\,319\,296\,123\,371\,520\,000\, z^{25} -$
$37\,728\,910\,829\,345\,327\,109\,838\,548\,738\,437\,762\,004\,748\,861\,440\, z^{27} +$
$1\,331\,998\,049\,831\,378\,030\,074\,725\,974\,561\,127\,544\,713\,079\,947\,264\, z^{29} -$
$315\,947\,068\,306\,475\,205\,561\,166\,944\,243\,551\,984\,528\,790\,650\,880\, z^{31}\Big)\, D_z +$
$\Big(-86\,016 + 104\,925\,757\,440\, z^2 + 7\,313\,095\,441\,514\,496\, z^4 - 77\,521\,234\,553\,783\,451\,648\, z^6 -$
$372\,803\,358\,046\,832\,377\,724\,928\, z^8 - 5\,385\,749\,059\,794\,399\,472\,109\,223\,936\, z^{10} +$
$3\,132\,040\,912\,196\,556\,228\,927\,500\,058\,624\, z^{12} + 194\,937\,483\,606\,112\,889\,575\,521\,946\,435\,584\, z^{14} -$
$4\,993\,419\,661\,356\,649\,167\,044\,237\,771\,201\,839\,104\, z^{16} -$
$939\,409\,192\,614\,330\,743\,139\,648\,628\,757\,071\,134\,720\, z^{18} +$
$63\,449\,023\,457\,944\,551\,152\,259\,668\,183\,189\,915\,959\,296\, z^{20} -$
$15\,869\,344\,373\,279\,882\,748\,885\,504\,666\,619\,865\,443\,860\,480\, z^{22} -$
$1\,199\,871\,777\,886\,430\,336\,038\,441\,987\,175\,041\,302\,924\,361\,728\, z^{24} -$
$1\,311\,950\,254\,509\,270\,561\,000\,268\,264\,109\,855\,833\,451\,397\,120\, z^{26} +$
$47\,531\,864\,121\,426\,193\,501\,043\,928\,118\,257\,886\,052\,116\,070\,400\, z^{28} -$
$9\,665\,684\,690\,064\,251\,030\,551\,952\,405\,156\,466\,065\,890\,344\,960\, z^{30}\Big)$

In[∘]:=
```
ODEinTheta = ChangeOreAlgebra[z ** ODEinD, OreAlgebra[Euler[z]]];
ToOrePolynomial[ODEinTheta]
```

Out[∘]=

$$
\begin{aligned}
&\Big(\frac{189}{z} + 164\,526\,768\,z - 187\,480\,074\,240\,z^{3} + 27\,962\,624\,347\,471\,872\,z^{5} - \\
&\quad 41\,624\,521\,690\,721\,550\,336\,z^{7} - 195\,289\,074\,452\,595\,380\,060\,160\,z^{9} + \\
&\quad 409\,286\,890\,042\,225\,331\,533\,774\,848\,z^{11} - 183\,057\,885\,005\,932\,655\,242\,496\,704\,512\,z^{13} - \\
&\quad 37\,424\,471\,615\,851\,079\,483\,154\,493\,341\,696\,z^{15} + 24\,179\,802\,126\,384\,104\,960\,285\,655\,473\,061\,888\,z^{17} - \\
&\quad 4\,496\,568\,077\,725\,961\,704\,167\,916\,875\,590\,664\,192\,z^{19} + \\
&\quad 341\,280\,443\,189\,722\,351\,590\,493\,954\,378\,473\,406\,464\,z^{21} + \\
&\quad 66\,745\,595\,233\,551\,284\,758\,691\,798\,722\,053\,402\,525\,696\,z^{23} - \\
&\quad 862\,884\,480\,190\,835\,703\,771\,836\,009\,058\,131\,301\,629\,952\,z^{25} - \\
&\quad 14\,419\,299\,574\,652\,092\,269\,971\,943\,043\,790\,157\,002\,047\,488\,z^{27} + \\
&\quad 163\,946\,892\,981\,593\,252\,096\,191\,310\,873\,136\,551\,147\,077\,632\,z^{29} - \\
&\quad 60\,410\,529\,312\,901\,568\,940\,949\,702\,532\,227\,912\,911\,814\,656\,z^{31}\Big)\,\theta_z^{8} + \\
&\Big(-\frac{756}{z} - 992\,525\,184\,z - 3\,175\,017\,615\,360\,z^{3} - 278\,601\,719\,262\,216\,192\,z^{5} - \\
&\quad 293\,589\,567\,918\,556\,839\,936\,z^{7} + 3\,186\,577\,735\,022\,582\,911\,991\,808\,z^{9} - \\
&\quad 646\,305\,742\,202\,030\,203\,500\,232\,704\,z^{11} - 2\,884\,289\,825\,183\,654\,319\,921\,674\,846\,208\,z^{13} + \\
&\quad 691\,213\,506\,878\,807\,023\,682\,800\,934\,977\,536\,z^{15} + 179\,496\,447\,078\,553\,642\,719\,300\,980\,035\,289\,088\,z^{17} - \\
&\quad 26\,559\,919\,540\,895\,955\,349\,565\,397\,802\,634\,182\,656\,z^{19} + \\
&\quad 10\,016\,844\,493\,736\,336\,205\,170\,293\,643\,460\,898\,455\,552\,z^{21} + \\
&\quad 535\,092\,293\,397\,709\,095\,653\,532\,201\,472\,426\,525\,065\,216\,z^{23} - \\
&\quad 23\,928\,765\,542\,084\,149\,484\,052\,606\,656\,948\,268\,114\,640\,896\,z^{25} - \\
&\quad 135\,442\,097\,710\,381\,144\,387\,269\,280\,979\,031\,180\,227\,641\,344\,z^{27} + \\
&\quad 2\,893\,465\,287\,917\,319\,229\,834\,600\,910\,490\,428\,191\,153\,324\,032\,z^{29} - \\
&\quad 966\,568\,469\,006\,425\,103\,055\,195\,240\,515\,646\,606\,589\,034\,496\,z^{31}\Big)\,\theta_z^{7} + \\
&\Big(\frac{1113}{z} + 1\,950\,259\,584\,z + 6\,216\,682\,061\,824\,z^{3} + 988\,841\,093\,180\,162\,048\,z^{5} + \\
&\quad 1\,233\,270\,686\,793\,691\,299\,840\,z^{7} - 9\,715\,936\,946\,399\,911\,434\,256\,384\,z^{9} - \\
&\quad 7\,752\,074\,094\,169\,934\,619\,780\,055\,040\,z^{11} - 3\,292\,303\,250\,603\,115\,641\,274\,504\,314\,880\,z^{13} + \\
&\quad 6\,432\,398\,522\,881\,878\,319\,620\,478\,709\,268\,480\,z^{15} + \\
&\quad 1\,442\,519\,265\,201\,053\,822\,094\,460\,496\,124\,051\,456\,z^{17} - \\
&\quad 81\,315\,393\,847\,369\,615\,391\,288\,576\,991\,201\,067\,008\,z^{19} + \\
&\quad 46\,854\,607\,085\,100\,541\,227\,643\,541\,228\,521\,577\,775\,104\,z^{21} + \\
&\quad 484\,722\,323\,648\,843\,742\,960\,229\,713\,378\,845\,655\,040\,000\,z^{23} - \\
&\quad 230\,657\,489\,060\,094\,958\,856\,963\,994\,975\,994\,903\,435\,149\,312\,z^{25} - \\
&\quad 720\,571\,084\,999\,990\,630\,257\,768\,886\,154\,063\,035\,548\,827\,648\,z^{27} + \\
&\quad 22\,027\,520\,447\,398\,165\,760\,511\,055\,419\,885\,169\,351\,394\,852\,864\,z^{29} - \\
&\quad 6\,663\,616\,997\,264\,781\,396\,236\,424\,132\,096\,584\,504\,800\,444\,416\,z^{31}\Big)\,\theta_z^{6} + \\
&\Big(-\frac{714}{z} - 1\,578\,302\,208\,z - 21\,117\,928\,833\,024\,z^{3} - 1\,359\,866\,980\,934\,352\,896\,z^{5} - \\
&\quad 574\,455\,830\,528\,579\,862\,528\,z^{7} + 28\,854\,506\,345\,915\,072\,026\,509\,312\,z^{9} + \\
&\quad 6\,184\,569\,392\,904\,960\,616\,014\,282\,752\,z^{11} + 1\,810\,623\,217\,846\,721\,852\,095\,403\,655\,168\,z^{13} + \\
&\quad 29\,243\,548\,794\,830\,027\,273\,614\,167\,908\,548\,608\,z^{15} + 3\,040\,296\,041\,767\,136\,923\,819\,727\,513\,897\,140\,224 \\
&\quad\ z^{17} - 773\,583\,766\,606\,348\,768\,573\,054\,633\,705\,904\,013\,312\,z^{19} + \\
&\quad 110\,447\,954\,495\,875\,341\,475\,112\,946\,362\,201\,552\,191\,488\,z^{21} - \\
&\quad 7\,922\,397\,729\,433\,677\,311\,920\,320\,801\,061\,386\,959\,454\,208\,z^{23} -
\end{aligned}
$$

$$
\begin{aligned}
&1\,191\,818\,304\,576\,060\,316\,780\,148\,344\,375\,975\,348\,726\,136\,832\, z^{25} - \\
&2\,689\,227\,967\,791\,285\,552\,013\,423\,892\,494\,764\,896\,113\,655\,808\, z^{27} + \\
&93\,474\,763\,354\,618\,920\,797\,655\,179\,807\,919\,342\,463\,215\,468\,544\, z^{29} - \\
&25\,835\,569\,702\,817\,570\,983\,746\,156\,116\,282\,804\,088\,619\,401\,216\, z^{31} \Big)\, \Theta_z^5 + \\
&\Big(\frac{168}{z} + 414\,021\,888\, z + 18\,868\,611\,252\,224\, z^3 + 919\,805\,084\,553\,969\,664\, z^5 + \\
&133\,810\,616\,233\,401\,253\,888\, z^7 - 14\,078\,517\,059\,564\,471\,823\,892\,480\, z^9 - \\
&34\,849\,420\,675\,511\,449\,389\,870\,612\,480\, z^{11} + 12\,733\,851\,097\,178\,186\,142\,709\,123\,645\,440\, z^{13} + \\
&50\,648\,096\,406\,128\,164\,021\,666\,285\,149\,487\,104\, z^{15} + 365\,830\,298\,834\,552\,303\,932\,806\,882\,134\,065\,152 \\
&\quad z^{17} - 2\,345\,494\,865\,572\,270\,877\,855\,193\,998\,981\,999\,886\,336\, z^{19} + \\
&181\,610\,849\,064\,214\,513\,677\,570\,453\,519\,151\,654\,764\,544\, z^{21} - \\
&38\,192\,875\,423\,895\,354\,123\,302\,136\,426\,592\,444\,916\,695\,040\, z^{23} - \\
&3\,640\,685\,279\,051\,814\,852\,395\,753\,576\,298\,779\,788\,630\,294\,528\, z^{25} - \\
&6\,780\,993\,839\,894\,601\,645\,849\,193\,420\,991\,858\,172\,917\,448\,704\, z^{27} + \\
&241\,267\,273\,763\,780\,351\,415\,312\,097\,934\,067\,442\,273\,159\,741\,440\, z^{29} - \\
&61\,566\,719\,721\,140\,157\,302\,069\,545\,450\,136\,386\,022\,821\,330\,944\, z^{31} \Big)\, \Theta_z^4 + \\
&\Big(-1\,698\,816\, z - 328\,393\,949\,184\, z^3 + 70\,060\,141\,893\,910\,528\, z^5 - 161\,390\,672\,172\,107\,694\,080\, z^7 + \\
&2\,285\,583\,401\,499\,975\,368\,048\,640\, z^9 - 41\,542\,648\,335\,674\,246\,026\,842\,079\,232\, z^{11} + \\
&14\,833\,339\,175\,778\,700\,116\,328\,936\,636\,416\, z^{13} + 60\,893\,853\,745\,689\,810\,932\,332\,054\,613\,852\,160\, z^{15} - \\
&9\,056\,740\,928\,747\,139\,423\,214\,960\,741\,233\,721\,344\, z^{17} - \\
&4\,586\,887\,646\,764\,859\,598\,973\,068\,418\,707\,225\,575\,424\, z^{19} + \\
&249\,575\,792\,151\,473\,179\,973\,759\,850\,399\,743\,248\,171\,008\, z^{21} - \\
&82\,747\,875\,844\,293\,050\,598\,340\,921\,694\,852\,455\,522\,631\,680\, z^{23} - \\
&6\,806\,971\,769\,281\,433\,582\,399\,982\,276\,317\,128\,522\,159\,095\,808\, z^{25} - \\
&10\,928\,883\,932\,693\,075\,506\,004\,789\,487\,782\,739\,963\,864\,416\,256\, z^{27} + \\
&387\,701\,019\,631\,313\,372\,908\,644\,030\,675\,279\,172\,318\,722\,523\,136\, z^{29} - \\
&92\,267\,015\,103\,904\,996\,295\,810\,512\,334\,222\,765\,653\,978\,251\,264\, z^{31} \Big)\, \Theta_z^3 + \\
&\Big(-1\,634\,304\, z + 734\,240\,833\,536\, z^3 + 69\,962\,690\,910\,486\,528\, z^5 - 405\,973\,248\,250\,860\,273\,664\, z^7 - \\
&573\,769\,652\,383\,691\,824\,955\,392\, z^9 - 41\,744\,675\,440\,635\,445\,295\,831\,842\,816\, z^{11} + \\
&18\,172\,891\,598\,361\,418\,639\,048\,773\,206\,016\, z^{13} + 36\,086\,203\,362\,254\,267\,309\,294\,692\,778\,115\,072\, z^{15} - \\
&19\,339\,043\,199\,194\,946\,791\,094\,583\,847\,181\,877\,248\, z^{17} - \\
&5\,358\,199\,584\,495\,103\,111\,812\,190\,866\,375\,922\,679\,808\, z^{19} + \\
&286\,040\,317\,743\,937\,920\,849\,855\,539\,007\,104\,722\,403\,328\, z^{21} - \\
&98\,058\,466\,671\,388\,220\,573\,122\,729\,134\,935\,934\,561\,681\,408\, z^{23} - \\
&7\,620\,620\,043\,501\,156\,114\,244\,251\,983\,216\,377\,880\,561\,844\,224\, z^{25} - \\
&10\,693\,199\,288\,383\,951\,185\,471\,476\,513\,565\,879\,271\,442\,874\,368\, z^{27} + \\
&378\,774\,351\,750\,815\,807\,858\,921\,678\,556\,609\,081\,154\,780\,266\,496\, z^{29} - \\
&84\,849\,944\,560\,487\,636\,998\,060\,576\,634\,432\,560\,790\,916\,562\,944\, z^{31} \Big)\, \Theta_z^2 + \\
&\Big(-602\,112\, z + 501\,733\,392\,384\, z^3 + 35\,387\,289\,999\,769\,600\, z^5 - 296\,127\,272\,334\,043\,119\,616\, z^7 - \\
&1\,172\,207\,549\,277\,236\,178\,714\,624\, z^9 - 23\,242\,056\,596\,984\,270\,003\,978\,108\,928\, z^{11} + \\
&11\,479\,965\,384\,423\,004\,729\,625\,448\,284\,160\, z^{13} + 9\,181\,377\,306\,539\,614\,626\,763\,183\,541\,452\,800\, z^{15} - \\
&16\,098\,980\,351\,618\,093\,084\,110\,289\,200\,075\,505\,664\, z^{17} - \\
&3\,457\,747\,972\,112\,524\,641\,003\,108\,996\,505\,619\,398\,656\, z^{19} + \\
&206\,790\,373\,654\,867\,134\,956\,324\,412\,179\,057\,630\,773\,248\, z^{21} - \\
&61\,398\,217\,874\,598\,680\,781\,897\,396\,732\,248\,519\,392\,362\,496\, z^{23} - \\
&4\,670\,982\,878\,618\,288\,603\,361\,956\,504\,469\,033\,280\,442\,728\,448\, z^{25} - \\
&5\,766\,173\,318\,297\,389\,353\,584\,655\,313\,425\,635\,327\,631\,949\,824\, z^{27} + \\
&205\,695\,708\,702\,552\,498\,851\,751\,139\,946\,003\,772\,409\,506\,693\,120\, z^{29} -
\end{aligned}
$$

$43\,737\,223\,222\,540\,735\,913\,247\,584\,633\,333\,008\,948\,153\,810\,944\, z^{31}\big)\, \theta_z +$
$\big(-86\,016\, z + 104\,925\,757\,440\, z^3 + 7\,313\,095\,441\,514\,496\, z^5 - 77\,521\,234\,553\,783\,451\,648\, z^7 -$
$372\,803\,358\,046\,832\,377\,724\,928\, z^9 - 5\,385\,749\,059\,794\,399\,472\,109\,223\,936\, z^{11} +$
$3\,132\,040\,912\,196\,556\,228\,927\,500\,058\,624\, z^{13} + 194\,937\,483\,606\,112\,889\,575\,521\,946\,435\,584\, z^{15} -$
$4\,993\,419\,661\,356\,649\,167\,044\,237\,771\,201\,839\,104\, z^{17} -$
$939\,409\,192\,614\,330\,743\,139\,648\,628\,757\,071\,134\,720\, z^{19} +$
$63\,449\,023\,457\,944\,551\,152\,259\,668\,183\,189\,915\,959\,296\, z^{21} -$
$15\,869\,344\,373\,279\,882\,748\,885\,504\,666\,619\,865\,443\,860\,480\, z^{23} -$
$1\,199\,871\,777\,886\,430\,336\,038\,441\,987\,175\,041\,302\,924\,361\,728\, z^{25} -$
$1\,311\,950\,254\,509\,270\,561\,000\,268\,264\,109\,855\,833\,451\,397\,120\, z^{27} +$
$47\,531\,864\,121\,426\,193\,501\,043\,928\,118\,257\,886\,052\,116\,070\,400\, z^{29} -$
$9\,665\,684\,690\,064\,251\,030\,551\,952\,405\,156\,466\,065\,890\,344\,960\, z^{31}\big)$

Since M = 3 is odd, we only need to work on $R(z^{1/2})$.

Normalization: Change $z \to w^{1/2}$

In[∘]:=
```
ODENormalized =
  -DFiniteSubstitute[ToOrePolynomial[ODE], {z → w^(1/2)}, Algebra → OreAlgebra[Der[w]]];
ToOrePolynomial[ODENormalized]
```

Out[∘]=
$\big\{\big(756\, w^7 + 658\,107\,072\, w^8 - 749\,920\,296\,960\, w^9 + 111\,850\,497\,389\,887\,488\, w^{10} -$
$166\,498\,086\,762\,886\,201\,344\, w^{11} - 781\,156\,297\,810\,381\,520\,240\,640\, w^{12} +$
$1\,637\,147\,560\,168\,901\,326\,135\,099\,392\, w^{13} - 732\,231\,540\,023\,730\,620\,969\,986\,818\,048\, w^{14} -$
$149\,697\,886\,463\,404\,317\,932\,617\,973\,366\,784\, w^{15} + 96\,719\,208\,505\,536\,419\,841\,142\,621\,892\,247\,552\, w^{16} -$
$17\,986\,272\,310\,903\,846\,816\,671\,667\,502\,362\,656\,768\, w^{17} +$
$1\,365\,121\,772\,758\,889\,406\,361\,975\,817\,513\,893\,625\,856\, w^{18} +$
$266\,982\,380\,934\,205\,139\,034\,767\,194\,888\,213\,610\,102\,784\, w^{19} -$
$3\,451\,537\,920\,763\,342\,815\,087\,344\,036\,232\,525\,206\,519\,808\, w^{20} -$
$57\,677\,198\,298\,608\,369\,079\,887\,772\,175\,160\,628\,008\,189\,952\, w^{21} +$
$655\,787\,571\,926\,373\,008\,384\,765\,243\,492\,546\,204\,588\,310\,528\, w^{22} -$
$241\,642\,117\,251\,606\,275\,763\,798\,810\,128\,911\,651\,647\,258\,624\, w^{23}\big)\, D_w^8 +$
$\big(19\,656\, w^6 + 16\,441\,947\,648\, w^7 - 27\,347\,803\,545\,600\, w^8 + 2\,574\,610\,488\,392\,417\,280\, w^9 -$
$5\,249\,125\,565\,197\,927\,317\,504\, w^{10} - 15\,499\,220\,868\,645\,516\,742\,754\,304\, w^{11} +$
$44\,547\,520\,200\,325\,176\,724\,782\,317\,568\, w^{12} - 26\,271\,062\,771\,031\,766\,027\,002\,980\,597\,760\, w^{13} -$
$2\,809\,113\,807\,217\,706\,854\,747\,701\,384\,314\,880\, w^{14} + 3\,067\,130\,732\,312\,127\,040\,990\,595\,373\,053\,509\,632$
$w^{15} - 556\,735\,463\,787\,099\,621\,565\,937\,485\,671\,422\,754\,816\, w^{16} +$
$58\,257\,098\,624\,721\,575\,788\,475\,910\,177\,310\,818\,435\,072\, w^{17} +$
$8\,545\,691\,252\,953\,162\,084\,280\,545\,859\,814\,834\,133\,008\,384\, w^{18} -$
$144\,500\,592\,865\,541\,897\,790\,550\,846\,328\,407\,242\,011\,836\,416\, w^{19} -$
$1\,885\,845\,747\,781\,796\,623\,011\,396\,182\,862\,559\,944\,684\,601\,344\, w^{20} +$
$24\,148\,982\,589\,773\,082\,694\,442\,628\,638\,772\,150\,110\,779\,342\,848\, w^{21} -$
$8\,699\,116\,221\,057\,825\,927\,496\,757\,164\,640\,819\,459\,301\,310\,464\, w^{22}\big)\, D_w^7 +$
$\big(170\,457\, w^5 + 135\,320\,683\,008\, w^6 - 326\,612\,856\,774\,656\, w^7 + 19\,039\,801\,189\,877\,153\,792\, w^8 -$
$55\,385\,982\,244\,713\,425\,534\,976\, w^9 - 83\,667\,247\,293\,012\,913\,514\,610\,688\, w^{10} +$
$400\,584\,335\,738\,272\,549\,585\,146\,609\,664\, w^{11} - 319\,206\,065\,554\,628\,942\,256\,001\,341\,456\,384\, w^{12} -$
$4\,356\,271\,987\,473\,775\,255\,778\,262\,937\,239\,552\, w^{13} + 34\,708\,679\,504\,972\,994\,494\,049\,039\,080\,944\,041\,984$
$w^{14} - 5\,981\,180\,449\,265\,422\,993\,307\,698\,840\,330\,303\,438\,848\, w^{15} +$
$830\,684\,467\,375\,891\,243\,937\,081\,441\,712\,575\,017\,385\,984\, w^{16} +$
$93\,975\,911\,974\,851\,192\,743\,656\,656\,015\,485\,579\,995\,119\,616\, w^{17} -$

$2 153 774 728 750 678 426 000 406 988 205 673 869 184 335 872 w^{18} -$
$21 751 273 936 265 824 869 773 226 085 866 099 655 288 291 328 w^{19} +$
$317 992 556 672 340 793 643 911 848 429 500 443 800 325 062 656 w^{20} -$
$111 536 295 884 461 905 077 725 107 728 044 241 319 710 687 232 w^{21}) D_w^6 +$
$(598 458 w^4 + 441 570 116 736 w^5 - 1 593 729 977 597 952 w^6 + 53 587 223 773 196 582 912 w^7 -$
$238 816 237 731 585 347 026 944 w^8 - 59 284 147 917 618 516 395 556 864 w^9 +$
$1 424 850 503 644 681 396 473 095 520 256 w^{10} - 1 624 923 505 226 463 735 289 624 978 784 256 w^{11} +$
$147 464 753 380 134 621 233 049 654 351 888 384 w^{12} +$
$174 972 111 111 707 636 587 930 798 595 550 216 192 w^{13} -$
$27 928 886 188 913 625 270 539 418 233 939 328 761 856 w^{14} +$
$4 996 137 403 167 453 803 279 966 420 167 564 317 425 664 w^{15} +$
$433 466 978 122 289 760 257 937 856 345 055 708 972 056 576 w^{16} -$
$14 379 940 656 774 526 352 620 975 198 817 577 764 835 557 376 w^{17} -$
$110 638 025 831 341 010 192 190 804 696 605 717 853 628 268 544 w^{18} +$
$1 875 897 419 527 822 989 964 185 181 806 730 619 843 187 310 592 w^{19} -$
$637 235 434 246 366 124 842 862 858 019 328 453 690 867 580 928 w^{20}) D_w^5 +$
$(825 573 w^3 + 543 651 368 064 w^4 - 3 194 914 913 214 464 w^5 + 52 941 779 999 824 543 744 w^6 -$
$411 402 174 983 653 830 950 912 w^7 + 411 074 552 889 039 071 414 845 440 w^8 +$
$1 850 699 655 980 481 203 752 541 356 032 w^9 - 3 466 291 859 634 598 171 101 808 451 125 248 w^{10} +$
$806 199 022 003 918 439 923 396 008 719 941 632 w^{11} +$
$399 223 576 644 517 821 184 516 057 401 498 009 600 w^{12} -$
$58 926 386 028 978 448 742 617 114 056 065 896 415 232 w^{13} +$
$12 977 055 226 355 271 739 432 713 941 058 460 206 497 792 w^{14} +$
$811 048 378 881 511 926 657 599 332 793 219 839 881 641 984 w^{15} -$
$44 483 201 514 226 778 390 002 736 654 889 553 470 611 783 680 w^{16} -$
$254 896 891 527 129 106 014 261 992 573 005 940 735 751 487 488 w^{17} +$
$5 100 399 825 683 804 414 422 028 842 339 770 408 537 075 744 768 w^{18} -$
$1 665 335 803 016 063 532 212 474 179 920 457 453 825 142 292 480 w^{19}) D_w^4 +$
$(366 681 w^2 + 194 646 676 224 w^3 - 2 311 994 468 524 032 w^4 + 13 715 171 751 880 032 256 w^5 -$
$233 583 865 679 765 731 016 704 w^6 + 629 137 739 282 553 524 900 069 376 w^7 +$
$514 561 973 198 327 935 621 178 327 040 w^8 - 2 696 397 700 711 940 448 284 763 563 753 472 w^9 +$
$1 299 546 476 139 541 119 219 198 317 826 146 304 w^{10} +$
$368 734 703 779 979 966 047 808 341 534 580 932 608 w^{11} -$
$54 443 596 398 999 119 904 376 779 685 811 403 620 352 w^{12} +$
$13 249 975 334 187 107 958 998 091 614 755 038 903 140 352 w^{13} +$
$456 992 780 501 958 347 848 682 339 887 498 663 093 600 256 w^{14} -$
$59 708 104 700 139 248 882 046 285 548 010 520 670 967 431 168 w^{15} -$
$247 256 664 876 923 896 482 699 082 298 092 801 308 625 993 728 w^{16} +$
$5 936 631 818 105 258 629 613 346 359 453 205 487 071 919 079 424 w^{17} -$
$1 841 854 111 115 667 897 875 242 940 934 157 993 938 612 387 840 w^{18}) D_w^3 +$
$(30 198 w + 7 866 004 608 w^2 - 428 016 607 567 872 w^3 + 1 196 572 192 415 416 320 w^4 -$
$24 066 296 266 656 395 034 624 w^5 + 193 700 525 364 943 463 909 425 152 w^6 -$
$146 620 831 371 548 699 838 275 125 248 w^7 - 515 913 102 855 123 977 203 808 596 918 272 w^8 +$
$600 536 992 031 939 626 911 302 972 281 454 592 w^9 + 98 455 444 321 317 684 789 766 658 474 159 112 192$
$w^{10} - 20 113 048 260 772 993 091 540 149 619 835 682 160 640 w^{11} +$
$4 145 632 777 486 513 117 058 749 012 447 381 407 399 936 w^{12} -$
$47 059 839 191 271 552 573 851 548 709 927 261 583 704 064 w^{13} -$
$28 942 491 623 962 979 315 156 545 419 545 481 049 056 215 040 w^{14} -$
$83 531 017 538 965 091 388 113 042 960 842 166 032 108 879 872 w^{15} +$
$2 423 054 515 374 000 757 391 001 755 018 818 419 605 114 978 304 w^{16} -$

$700\,460\,087\,383\,093\,691\,870\,311\,800\,861\,182\,650\,212\,490\,936\,320\,w^{17}\big)\,D_w^2 +$
$\big(42 - 62\,662\,656\,w - 6\,704\,565\,092\,352\,w^2 + 107\,741\,971\,448\,070\,144\,w^3 +$
$171\,017\,063\,690\,120\,724\,480\,w^4 + 6\,996\,891\,969\,899\,893\,478\,129\,664\,w^5 -$
$21\,555\,796\,016\,485\,230\,262\,074\,998\,784\,w^6 - 2\,355\,618\,017\,643\,102\,363\,080\,397\,422\,592\,w^7 +$
$45\,102\,963\,618\,304\,623\,721\,735\,892\,458\,536\,960\,w^8 + 665\,961\,011\,424\,762\,054\,043\,542\,434\,963\,521\,536\,w^9 -$
$2\,141\,890\,158\,941\,375\,266\,048\,659\,494\,324\,983\,037\,952\,w^{10} +$
$224\,416\,800\,413\,968\,267\,718\,516\,281\,262\,353\,460\,232\,192\,w^{11} -$
$30\,078\,361\,385\,391\,813\,998\,530\,712\,626\,639\,711\,475\,073\,024\,w^{12} -$
$3\,261\,175\,468\,951\,833\,445\,924\,493\,792\,844\,128\,559\,210\,430\,464\,w^{13} -$
$6\,303\,623\,285\,896\,079\,280\,223\,291\,968\,397\,894\,069\,235\,220\,480\,w^{14} +$
$214\,088\,454\,048\,708\,650\,675\,100\,968\,739\,345\,685\,737\,998\,647\,296\,w^{15} -$
$55\,351\,147\,482\,946\,062\,542\,145\,164\,945\,153\,825\,205\,450\,178\,560\,w^{16}\big)\,D_w +$
$\big(-1344 + 1\,639\,464\,960\,w + 114\,267\,116\,273\,664\,w^2 - 1\,211\,269\,289\,902\,866\,432\,w^3 -$
$5\,825\,052\,469\,481\,755\,901\,952\,w^4 - 84\,152\,329\,059\,287\,491\,751\,706\,624\,w^5 +$
$48\,938\,139\,253\,071\,191\,076\,992\,188\,416\,w^6 + 3\,045\,898\,181\,345\,513\,899\,617\,530\,413\,056\,w^7 -$
$78\,022\,182\,208\,697\,643\,235\,066\,215\,175\,028\,736\,w^8 - 14\,678\,268\,634\,598\,917\,861\,557\,009\,824\,329\,236\,480$
$w^9 + 991\,390\,991\,530\,383\,611\,754\,057\,315\,362\,342\,436\,864\,w^{10} -$
$247\,958\,505\,832\,498\,167\,951\,336\,010\,415\,935\,397\,560\,320\,w^{11} -$
$18\,747\,996\,529\,475\,474\,000\,600\,656\,049\,610\,020\,358\,193\,152\,w^{12} -$
$20\,499\,222\,726\,707\,352\,515\,629\,191\,626\,716\,497\,397\,678\,080\,w^{13} +$
$742\,685\,376\,897\,284\,273\,453\,811\,376\,847\,779\,469\,564\,313\,600\,w^{14} -$
$151\,026\,323\,282\,253\,922\,352\,374\,256\,330\,569\,782\,279\,536\,640\,w^{15}\big)\big\}$

In[ ]:=
```
ODENormalizedinD = ODENormalized[[1]];
ToOrePolynomial[ODENormalizedinD]
```

Out[ ]=
$\big(756\,w^7 + 658\,107\,072\,w^8 - 749\,920\,296\,960\,w^9 + 111\,850\,497\,389\,887\,488\,w^{10} -$
$166\,498\,086\,762\,886\,201\,344\,w^{11} - 781\,156\,297\,810\,381\,520\,240\,640\,w^{12} +$
$1\,637\,147\,560\,168\,901\,326\,135\,099\,392\,w^{13} - 732\,231\,540\,023\,730\,620\,969\,986\,818\,048\,w^{14} -$
$149\,697\,886\,463\,404\,317\,932\,617\,973\,366\,784\,w^{15} + 96\,719\,208\,505\,536\,419\,841\,142\,621\,892\,247\,552\,w^{16} -$
$17\,986\,272\,310\,903\,846\,816\,671\,667\,502\,362\,656\,768\,w^{17} +$
$1\,365\,121\,772\,758\,889\,406\,361\,975\,817\,513\,893\,625\,856\,w^{18} +$
$266\,982\,380\,934\,205\,139\,034\,767\,194\,888\,213\,610\,102\,784\,w^{19} -$
$3\,451\,537\,920\,763\,342\,815\,087\,344\,036\,232\,525\,206\,519\,808\,w^{20} -$
$57\,677\,198\,298\,608\,369\,079\,887\,772\,175\,160\,628\,008\,189\,952\,w^{21} +$
$655\,787\,571\,926\,373\,008\,384\,765\,243\,492\,546\,204\,588\,310\,528\,w^{22} -$
$241\,642\,117\,251\,606\,275\,763\,798\,810\,128\,911\,651\,647\,258\,624\,w^{23}\big)\,D_w^8 +$
$\big(19\,656\,w^6 + 16\,441\,947\,648\,w^7 - 27\,347\,803\,545\,600\,w^8 + 2\,574\,610\,488\,392\,417\,280\,w^9 -$
$5\,249\,125\,565\,197\,927\,317\,504\,w^{10} - 15\,499\,220\,868\,645\,516\,742\,754\,304\,w^{11} +$
$44\,547\,520\,200\,325\,176\,724\,782\,317\,568\,w^{12} - 26\,271\,062\,771\,031\,766\,027\,002\,980\,597\,760\,w^{13} -$
$2\,809\,113\,807\,217\,706\,854\,747\,701\,384\,314\,880\,w^{14} +$
$3\,067\,130\,732\,312\,127\,040\,990\,595\,373\,053\,509\,632\,w^{15} -$
$556\,735\,463\,787\,099\,621\,565\,937\,485\,671\,422\,754\,816\,w^{16} +$
$58\,257\,098\,624\,721\,575\,788\,475\,910\,177\,310\,818\,435\,072\,w^{17} +$
$8\,545\,691\,252\,953\,162\,084\,280\,545\,859\,814\,834\,133\,008\,384\,w^{18} -$
$144\,500\,592\,865\,541\,897\,790\,550\,846\,328\,407\,242\,011\,836\,416\,w^{19} -$
$1\,885\,845\,747\,781\,796\,623\,011\,396\,182\,862\,559\,944\,684\,601\,344\,w^{20} +$
$24\,148\,982\,589\,773\,082\,694\,442\,628\,638\,772\,150\,110\,779\,342\,848\,w^{21} -$
$8\,699\,116\,221\,057\,825\,927\,496\,757\,164\,640\,819\,459\,301\,310\,464\,w^{22}\big)\,D_w^7 +$
$\big(170\,457\,w^5 + 135\,320\,683\,008\,w^6 - 326\,612\,856\,774\,656\,w^7 + 19\,039\,801\,189\,877\,153\,792\,w^8 -$
$55\,385\,982\,244\,713\,425\,534\,976\,w^9 - 83\,667\,247\,293\,012\,913\,514\,610\,688\,w^{10} +$

$400\,584\,335\,738\,272\,549\,585\,146\,609\,664\,w^{11} - 319\,206\,065\,554\,628\,942\,256\,001\,341\,456\,384\,w^{12} -$
$4\,356\,271\,987\,473\,775\,255\,778\,262\,937\,239\,552\,w^{13} +$
$34\,708\,679\,504\,972\,994\,494\,049\,039\,080\,944\,041\,984\,w^{14} -$
$5\,981\,180\,449\,265\,422\,993\,307\,698\,840\,330\,303\,438\,848\,w^{15} +$
$830\,684\,467\,375\,891\,243\,937\,081\,441\,712\,575\,017\,385\,984\,w^{16} +$
$93\,975\,911\,974\,851\,192\,743\,656\,656\,015\,485\,579\,995\,119\,616\,w^{17} -$
$2\,153\,774\,728\,750\,678\,426\,000\,406\,988\,205\,673\,869\,184\,335\,872\,w^{18} -$
$21\,751\,273\,936\,265\,824\,869\,773\,226\,085\,866\,099\,655\,288\,291\,328\,w^{19} +$
$317\,992\,556\,672\,340\,793\,643\,911\,848\,429\,500\,443\,800\,325\,062\,656\,w^{20} -$
$111\,536\,295\,884\,461\,905\,077\,725\,107\,728\,044\,241\,319\,710\,687\,232\,w^{21}\Big)\,D_w^6 +$
$\Big(598\,458\,w^4 + 441\,570\,116\,736\,w^5 - 1\,593\,729\,977\,597\,952\,w^6 + 53\,587\,223\,773\,196\,582\,912\,w^7 -$
$238\,816\,237\,731\,585\,347\,026\,944\,w^8 - 59\,284\,147\,917\,618\,516\,395\,556\,864\,w^9 +$
$1\,424\,850\,503\,644\,681\,396\,473\,095\,520\,256\,w^{10} - 1\,624\,923\,505\,226\,463\,735\,289\,624\,978\,784\,256\,w^{11} +$
$147\,464\,753\,380\,134\,621\,233\,049\,654\,351\,888\,384\,w^{12} +$
$174\,972\,111\,111\,707\,636\,587\,930\,798\,595\,550\,216\,192\,w^{13} -$
$27\,928\,886\,188\,913\,625\,270\,539\,418\,233\,939\,328\,761\,856\,w^{14} +$
$4\,996\,137\,403\,167\,453\,803\,279\,966\,420\,167\,564\,317\,425\,664\,w^{15} +$
$433\,466\,978\,122\,289\,760\,257\,937\,856\,345\,055\,708\,972\,056\,576\,w^{16} -$
$14\,379\,940\,656\,774\,526\,352\,620\,975\,198\,817\,577\,764\,835\,557\,376\,w^{17} -$
$110\,638\,025\,831\,341\,010\,192\,190\,804\,696\,605\,717\,853\,628\,268\,544\,w^{18} +$
$1\,875\,897\,419\,527\,822\,989\,964\,185\,181\,806\,730\,619\,843\,187\,310\,592\,w^{19} -$
$637\,235\,434\,246\,366\,124\,842\,862\,858\,019\,328\,453\,690\,867\,580\,928\,w^{20}\Big)\,D_w^5 +$
$\Big(825\,573\,w^3 + 543\,651\,368\,064\,w^4 - 3\,194\,914\,913\,214\,464\,w^5 + 52\,941\,779\,999\,824\,543\,744\,w^6 -$
$411\,402\,174\,983\,653\,830\,950\,912\,w^7 + 411\,074\,552\,889\,039\,071\,414\,845\,440\,w^8 +$
$1\,850\,699\,655\,980\,481\,203\,752\,541\,356\,032\,w^9 - 3\,466\,291\,859\,634\,598\,171\,101\,808\,451\,125\,248\,w^{10} +$
$806\,199\,022\,003\,918\,439\,923\,396\,008\,719\,941\,632\,w^{11} +$
$399\,223\,576\,644\,517\,821\,184\,516\,057\,401\,498\,009\,600\,w^{12} -$
$58\,926\,386\,028\,978\,448\,742\,617\,114\,056\,065\,896\,415\,232\,w^{13} +$
$12\,977\,055\,226\,355\,271\,739\,432\,713\,941\,058\,460\,206\,497\,792\,w^{14} +$
$811\,048\,378\,881\,511\,926\,657\,599\,332\,793\,219\,839\,881\,641\,984\,w^{15} -$
$44\,483\,201\,514\,226\,778\,390\,002\,736\,654\,889\,553\,470\,611\,783\,680\,w^{16} -$
$254\,896\,891\,527\,129\,106\,014\,261\,992\,573\,005\,940\,735\,751\,487\,488\,w^{17} +$
$5\,100\,399\,825\,683\,804\,414\,422\,028\,842\,339\,770\,408\,537\,075\,744\,768\,w^{18} -$
$1\,665\,335\,803\,016\,063\,532\,212\,474\,179\,920\,457\,453\,825\,142\,292\,480\,w^{19}\Big)\,D_w^4 +$
$\Big(366\,681\,w^2 + 194\,646\,676\,224\,w^3 - 2\,311\,994\,468\,524\,032\,w^4 + 13\,715\,171\,751\,880\,032\,256\,w^5 -$
$233\,583\,865\,679\,765\,731\,016\,704\,w^6 + 629\,137\,739\,282\,553\,524\,900\,069\,376\,w^7 +$
$514\,561\,973\,198\,327\,935\,621\,178\,327\,040\,w^8 - 2\,696\,397\,700\,711\,940\,448\,284\,763\,563\,753\,472\,w^9 +$
$1\,299\,546\,476\,139\,541\,119\,219\,198\,317\,826\,146\,304\,w^{10} +$
$368\,734\,703\,779\,979\,966\,047\,808\,341\,534\,580\,932\,608\,w^{11} -$
$54\,443\,596\,398\,999\,119\,904\,376\,779\,685\,811\,403\,620\,352\,w^{12} +$
$13\,249\,975\,334\,187\,107\,958\,998\,091\,614\,755\,038\,903\,140\,352\,w^{13} +$
$456\,992\,780\,501\,958\,347\,848\,682\,339\,887\,498\,663\,093\,600\,256\,w^{14} -$
$59\,708\,104\,700\,139\,248\,882\,046\,285\,548\,010\,520\,670\,967\,431\,168\,w^{15} -$
$247\,256\,664\,876\,923\,896\,482\,699\,082\,298\,092\,801\,308\,625\,993\,728\,w^{16} +$
$5\,936\,631\,818\,105\,258\,629\,613\,346\,359\,453\,205\,487\,071\,919\,079\,424\,w^{17} -$
$1\,841\,854\,111\,115\,667\,897\,875\,242\,940\,934\,157\,993\,938\,612\,387\,840\,w^{18}\Big)\,D_w^3 +$
$\Big(30\,198\,w + 7\,866\,004\,608\,w^2 - 428\,016\,607\,567\,872\,w^3 + 1\,196\,572\,192\,415\,416\,320\,w^4 -$
$24\,066\,296\,266\,656\,395\,034\,624\,w^5 + 193\,700\,525\,364\,943\,463\,909\,425\,152\,w^6 -$
$146\,620\,831\,371\,548\,699\,838\,275\,125\,248\,w^7 - 515\,913\,102\,855\,123\,977\,203\,808\,596\,918\,272\,w^8 +$
$600\,536\,992\,031\,939\,626\,911\,302\,972\,281\,454\,592\,w^9 +$
$98\,455\,444\,321\,317\,684\,789\,766\,658\,474\,159\,112\,192\,w^{10} -$

$20\,113\,048\,260\,772\,993\,091\,540\,149\,619\,835\,682\,160\,640\,w^{11}+$
$4\,145\,632\,777\,486\,513\,117\,058\,749\,012\,447\,381\,407\,399\,936\,w^{12}-$
$47\,059\,839\,191\,271\,552\,573\,851\,548\,709\,927\,261\,583\,704\,064\,w^{13}-$
$28\,942\,491\,623\,962\,979\,315\,156\,545\,419\,545\,481\,049\,056\,215\,040\,w^{14}-$
$83\,531\,017\,538\,965\,091\,388\,113\,042\,960\,842\,166\,032\,108\,879\,872\,w^{15}+$
$2\,423\,054\,515\,374\,000\,757\,391\,001\,755\,018\,818\,419\,605\,114\,978\,304\,w^{16}-$
$700\,460\,087\,383\,093\,691\,870\,311\,800\,861\,182\,650\,212\,490\,936\,320\,w^{17}\big)\,D_w^2+$
$\big(42-62\,662\,656\,w-6\,704\,565\,092\,352\,w^2+107\,741\,971\,448\,070\,144\,w^3+$
$171\,017\,063\,690\,120\,724\,480\,w^4+6\,996\,891\,969\,899\,893\,478\,129\,664\,w^5-$
$21\,555\,796\,016\,485\,230\,262\,074\,998\,784\,w^6-2\,355\,618\,017\,643\,102\,363\,080\,397\,422\,592\,w^7+$
$45\,102\,963\,618\,304\,623\,721\,735\,892\,458\,536\,960\,w^8+665\,961\,011\,424\,762\,054\,043\,542\,434\,963\,521\,536\,w^9-$
$2\,141\,890\,158\,941\,375\,266\,048\,659\,494\,324\,983\,037\,952\,w^{10}+$
$224\,416\,800\,413\,968\,267\,718\,516\,281\,262\,353\,460\,232\,192\,w^{11}-$
$30\,078\,361\,385\,391\,813\,998\,530\,712\,626\,639\,711\,475\,073\,024\,w^{12}-$
$3\,261\,175\,468\,951\,833\,445\,924\,493\,792\,844\,128\,559\,210\,430\,464\,w^{13}-$
$6\,303\,623\,285\,896\,079\,280\,223\,291\,968\,397\,894\,069\,235\,220\,480\,w^{14}+$
$214\,088\,454\,048\,708\,650\,675\,100\,968\,739\,345\,685\,737\,998\,647\,296\,w^{15}-$
$55\,351\,147\,482\,946\,062\,542\,145\,164\,945\,153\,825\,205\,450\,178\,560\,w^{16}\big)\,D_w+$
$\big(-1344+1\,639\,464\,960\,w+114\,267\,116\,273\,664\,w^2-1\,211\,269\,289\,902\,866\,432\,w^3-$
$5\,825\,052\,469\,481\,755\,901\,952\,w^4-84\,152\,329\,059\,287\,491\,751\,706\,624\,w^5+$
$48\,938\,139\,253\,071\,191\,076\,992\,188\,416\,w^6+3\,045\,898\,181\,345\,513\,899\,617\,530\,413\,056\,w^7-$
$78\,022\,182\,208\,697\,643\,235\,066\,215\,175\,028\,736\,w^8-$
$14\,678\,268\,634\,598\,917\,861\,557\,009\,824\,329\,236\,480\,w^9+$
$991\,390\,991\,530\,383\,611\,754\,057\,315\,362\,342\,436\,864\,w^{10}-$
$247\,958\,505\,832\,498\,167\,951\,336\,010\,415\,935\,397\,560\,320\,w^{11}-$
$18\,747\,996\,529\,475\,474\,000\,600\,656\,049\,610\,020\,358\,193\,152\,w^{12}-$
$20\,499\,222\,726\,707\,352\,515\,629\,191\,626\,716\,497\,397\,678\,080\,w^{13}+$
$742\,685\,376\,897\,284\,273\,453\,811\,376\,847\,779\,469\,564\,313\,600\,w^{14}-$
$151\,026\,323\,282\,253\,922\,352\,374\,256\,330\,569\,782\,279\,536\,640\,w^{15}\big)$

In[∘]:=
```
ODENormalizedinTheta = ChangeOreAlgebra[w ** ODENormalizedinD, OreAlgebra[Euler[w]]];
ToOrePolynomial[ODENormalizedinTheta]
```

Out[∘]=
$\big(756+658\,107\,072\,w-749\,920\,296\,960\,w^2+111\,850\,497\,389\,887\,488\,w^3-$
$166\,498\,086\,762\,886\,201\,344\,w^4-781\,156\,297\,810\,381\,520\,240\,640\,w^5+$
$1\,637\,147\,560\,168\,901\,326\,135\,099\,392\,w^6-732\,231\,540\,023\,730\,620\,969\,986\,818\,048\,w^7-$
$149\,697\,886\,463\,404\,317\,932\,617\,973\,366\,784\,w^8+96\,719\,208\,505\,536\,419\,841\,142\,621\,892\,247\,552\,w^9-$
$17\,986\,272\,310\,903\,846\,816\,671\,667\,502\,362\,656\,768\,w^{10}+$
$1\,365\,121\,772\,758\,889\,406\,361\,975\,817\,513\,893\,625\,856\,w^{11}+$
$266\,982\,380\,934\,205\,139\,034\,767\,194\,888\,213\,610\,102\,784\,w^{12}-$
$3\,451\,537\,920\,763\,342\,815\,087\,344\,036\,232\,525\,206\,519\,808\,w^{13}-$
$57\,677\,198\,298\,608\,369\,079\,887\,772\,175\,160\,628\,008\,189\,952\,w^{14}+$
$655\,787\,571\,926\,373\,008\,384\,765\,243\,492\,546\,204\,588\,310\,528\,w^{15}-$
$241\,642\,117\,251\,606\,275\,763\,798\,810\,128\,911\,651\,647\,258\,624\,w^{16}\big)\,\theta_w^8+$
$\big(-1512-1\,985\,050\,368\,w-6\,350\,035\,230\,720\,w^2-557\,203\,438\,524\,432\,384\,w^3-$
$587\,179\,135\,837\,113\,679\,872\,w^4+6\,373\,155\,470\,045\,165\,823\,983\,616\,w^5-$
$1\,292\,611\,484\,404\,060\,407\,000\,465\,408\,w^6-5\,768\,579\,650\,367\,308\,639\,843\,349\,692\,416\,w^7+$
$1\,382\,427\,013\,757\,614\,047\,365\,601\,869\,955\,072\,w^8+358\,992\,894\,157\,107\,285\,438\,601\,960\,070\,578\,176\,w^9-$
$53\,119\,839\,081\,791\,910\,699\,130\,795\,605\,268\,365\,312\,w^{10}+$
$20\,033\,688\,987\,472\,672\,410\,340\,587\,286\,921\,796\,911\,104\,w^{11}+$
$1\,070\,184\,586\,795\,418\,191\,307\,064\,402\,944\,853\,050\,130\,432\,w^{12}-$

$$
\begin{aligned}
&47 857 531 084 168 298 968 105 213 313 896 536 229 281 792\, w^{13} - \\
&270 884 195 420 762 288 774 538 561 958 062 360 455 282 688\, w^{14} + \\
&5 786 930 575 834 638 459 669 201 820 980 856 382 306 648 064\, w^{15} - \\
&1 933 136 938 012 850 206 110 390 481 031 293 213 178 068 992\, w^{16}\Big)\, \theta_w^7 + \\
&\Big(1113 + 1 950 259 584\, w + 6 216 682 061 824\, w^2 + 988 841 093 180 162 048\, w^3 + \\
&1 233 270 686 793 691 299 840\, w^4 - 9 715 936 946 399 911 434 256 384\, w^5 - \\
&7 752 074 094 169 934 619 780 055 040\, w^6 - 3 292 303 250 603 115 641 274 504 314 880\, w^7 + \\
&6 432 398 522 881 878 319 620 478 709 268 480\, w^8 + 1 442 519 265 201 053 822 094 460 496 124 051 456\, w^9 - \\
&81 315 393 847 369 615 391 288 576 991 201 067 008\, w^{10} + \\
&46 854 607 085 100 541 227 643 541 228 521 577 775 104\, w^{11} + \\
&484 722 323 648 843 742 960 229 713 378 845 655 040 000\, w^{12} - \\
&230 657 489 060 094 958 856 963 994 975 994 903 435 149 312\, w^{13} - \\
&720 571 084 999 990 630 257 768 886 154 063 035 548 827 648\, w^{14} + \\
&22 027 520 447 398 165 760 511 055 419 885 169 351 394 852 864\, w^{15} - \\
&6 663 616 997 264 781 396 236 424 132 096 584 504 800 444 416\, w^{16}\Big)\, \theta_w^6 + \\
&\Big(-357 - 789 151 104\, w - 10 558 964 416 512\, w^2 - 679 933 490 467 176 448\, w^3 - \\
&287 227 915 264 289 931 264\, w^4 + 14 427 253 172 957 536 013 254 656\, w^5 + \\
&3 092 284 696 452 480 308 007 141 376\, w^6 + 905 311 608 923 360 926 047 701 827 584\, w^7 + \\
&14 621 774 397 415 013 636 807 083 954 274 304\, w^8 + 1 520 148 020 883 568 461 909 863 756 948 570 112 \\
&\quad w^9 - 386 791 883 303 174 384 286 527 316 852 952 006 656\, w^{10} + \\
&55 223 977 247 937 670 737 556 473 181 100 776 095 744\, w^{11} - \\
&3 961 198 864 716 838 655 960 160 400 530 693 479 727 104\, w^{12} - \\
&595 909 152 288 030 158 390 074 172 187 987 674 363 068 416\, w^{13} - \\
&1 344 613 983 895 642 776 006 711 946 247 382 448 056 827 904\, w^{14} + \\
&46 737 381 677 309 460 398 827 589 903 959 671 231 607 734 272\, w^{15} - \\
&12 917 784 851 408 785 491 873 078 058 141 402 044 309 700 608\, w^{16}\Big)\, \theta_w^5 + \\
&\Big(42 + 103 505 472\, w + 4 717 152 813 056\, w^2 + 229 951 271 138 492 416\, w^3 + \\
&33 452 654 058 350 313 472\, w^4 - 3 519 629 264 891 117 955 973 120\, w^5 - \\
&8 712 355 168 877 862 347 467 653 120\, w^6 + 3 183 462 774 294 546 535 677 280 911 360\, w^7 + \\
&12 662 024 101 532 041 005 416 571 287 371 776\, w^8 + 91 457 574 708 638 075 983 201 720 533 516 288\, w^9 - \\
&586 373 716 393 067 719 463 798 499 745 499 971 584\, w^{10} + \\
&45 402 712 266 053 628 419 392 613 379 787 913 691 136\, w^{11} - \\
&9 548 218 855 973 838 530 825 534 106 648 111 229 173 760\, w^{12} - \\
&910 171 319 762 953 713 098 938 394 074 694 947 157 573 632\, w^{13} - \\
&1 695 248 459 973 650 411 462 298 355 247 964 543 229 362 176\, w^{14} + \\
&60 316 818 440 945 087 853 828 024 483 516 860 568 289 935 360\, w^{15} - \\
&15 391 679 930 285 039 325 517 386 362 534 096 505 705 332 736\, w^{16}\Big)\, \theta_w^4 + \\
&\Big(-212 352\, w - 41 049 243 648\, w^2 + 8 757 517 736 738 816\, w^3 - 20 173 834 021 513 461 760\, w^4 + \\
&285 697 925 187 496 921 006 080\, w^5 - 5 192 831 041 959 280 753 355 259 904\, w^6 + \\
&1 854 167 396 972 337 514 541 117 079 552\, w^7 + 7 611 731 718 211 226 366 541 506 826 731 520\, w^8 - \\
&1 132 092 616 093 392 427 901 870 092 654 215 168\, w^9 - \\
&573 360 955 845 607 449 871 633 552 338 403 196 928\, w^{10} + \\
&31 196 974 018 934 147 496 719 981 299 967 906 021 376\, w^{11} - \\
&10 343 484 480 536 631 324 792 615 211 856 556 940 328 960\, w^{12} - \\
&850 871 471 160 179 197 799 997 784 539 641 065 269 886 976\, w^{13} - \\
&1 366 110 491 586 634 438 250 598 685 972 842 495 483 052 032\, w^{14} + \\
&48 462 627 453 914 171 613 580 503 834 409 896 539 840 315 392\, w^{15} - \\
&11 533 376 887 988 124 536 976 314 041 777 845 706 747 281 408\, w^{16}\Big)\, \theta_w^3 + \\
&\Big(-102 144\, w + 45 890 052 096\, w^2 + 4 372 668 181 905 408\, w^3 - 25 373 328 015 678 767 104\, w^4 - \\
&35 860 603 273 980 739 059 712\, w^5 - 2 609 042 215 039 715 330 989 490 176\, w^6 +
\end{aligned}
$$

$$
\begin{aligned}
&1\,135\,805\,724\,897\,588\,664\,940\,548\,325\,376\,w^7 + 2\,255\,387\,710\,140\,891\,706\,830\,918\,298\,632\,192\,w^8 - \\
&1\,208\,690\,199\,949\,684\,174\,443\,411\,490\,448\,867\,328\,w^9 - \\
&334\,887\,474\,030\,943\,944\,488\,261\,929\,148\,495\,167\,488\,w^{10} + \\
&17\,877\,519\,858\,996\,120\,053\,115\,971\,187\,944\,045\,150\,208\,w^{11} - \\
&6\,128\,654\,166\,961\,763\,785\,820\,170\,570\,933\,495\,910\,105\,088\,w^{12} - \\
&476\,288\,752\,718\,822\,257\,140\,265\,748\,951\,023\,617\,535\,115\,264\,w^{13} - \\
&668\,324\,955\,523\,996\,949\,091\,967\,282\,097\,867\,454\,465\,179\,648\,w^{14} + \\
&23\,673\,396\,984\,425\,987\,991\,182\,604\,909\,788\,067\,572\,173\,766\,656\,w^{15} - \\
&5\,303\,121\,535\,030\,477\,312\,378\,786\,039\,652\,035\,049\,432\,285\,184\,w^{16}\Big)\,\theta_w^2 + \\
\Big(&-18\,816\,w + 15\,679\,168\,512\,w^2 + 1\,105\,852\,812\,492\,800\,w^3 - 9\,253\,977\,260\,438\,847\,488\,w^4 - \\
&36\,631\,485\,914\,913\,630\,584\,832\,w^5 - 726\,314\,268\,655\,758\,437\,624\,315\,904\,w^6 + \\
&358\,748\,918\,263\,218\,897\,800\,795\,258\,880\,w^7 + 286\,918\,040\,829\,362\,957\,086\,349\,485\,670\,400\,w^8 - \\
&503\,093\,135\,988\,065\,408\,878\,446\,537\,502\,359\,552\,w^9 - \\
&108\,054\,624\,128\,516\,395\,031\,347\,156\,140\,800\,606\,208\,w^{10} + \\
&6\,462\,199\,176\,714\,597\,967\,385\,137\,880\,595\,550\,961\,664\,w^{11} - \\
&1\,918\,694\,308\,581\,208\,774\,434\,293\,647\,882\,766\,231\,011\,328\,w^{12} - \\
&145\,968\,214\,956\,821\,518\,855\,061\,140\,764\,657\,290\,013\,835\,264\,w^{13} - \\
&180\,192\,916\,196\,793\,417\,299\,520\,478\,544\,551\,103\,988\,498\,432\,w^{14} + \\
&6\,427\,990\,896\,954\,765\,589\,117\,223\,123\,312\,617\,887\,797\,084\,160\,w^{15} - \\
&1\,366\,788\,225\,704\,397\,997\,288\,987\,019\,791\,656\,529\,629\,806\,592\,w^{16}\Big)\,\theta_w + \\
\Big(&-1344\,w + 1\,639\,464\,960\,w^2 + 114\,267\,116\,273\,664\,w^3 - 1\,211\,269\,289\,902\,866\,432\,w^4 - \\
&5\,825\,052\,469\,481\,755\,901\,952\,w^5 - 84\,152\,329\,059\,287\,491\,751\,706\,624\,w^6 + \\
&48\,938\,139\,253\,071\,191\,076\,992\,188\,416\,w^7 + 3\,045\,898\,181\,345\,513\,899\,617\,530\,413\,056\,w^8 - \\
&78\,022\,182\,208\,697\,643\,235\,066\,215\,175\,028\,736\,w^9 - \\
&14\,678\,268\,634\,598\,917\,861\,557\,009\,824\,329\,236\,480\,w^{10} + \\
&991\,390\,991\,530\,383\,611\,754\,057\,315\,362\,342\,436\,864\,w^{11} - \\
&247\,958\,505\,832\,498\,167\,951\,336\,010\,415\,935\,397\,560\,320\,w^{12} - \\
&18\,747\,996\,529\,475\,474\,000\,600\,656\,049\,610\,020\,358\,193\,152\,w^{13} - \\
&20\,499\,222\,726\,707\,352\,515\,629\,191\,626\,716\,497\,397\,678\,080\,w^{14} + \\
&742\,685\,376\,897\,284\,273\,453\,811\,376\,847\,779\,469\,564\,313\,600\,w^{15} - \\
&151\,026\,323\,282\,253\,922\,352\,374\,256\,330\,569\,782\,279\,536\,640\,w^{16}\Big)
\end{aligned}
$$

Recurrence for {r(0), r(2), r(4), ...}

In[•]:=
```
RECNormalized = DFiniteDE2RE[ToOrePolynomial[ODENormalized], {w}, {α}];
ToOrePolynomial[RECNormalized]
```

Out[•]=

$$
\begin{aligned}
\Big\{\Big(&2\,859\,422\,318\,592 + 1\,452\,813\,680\,640\,\alpha + 322\,925\,288\,448\,\alpha^2 + 41\,014\,614\,144\,\alpha^3 + \\
&3\,255\,650\,202\,\alpha^4 + 165\,386\,235\,\alpha^5 + 5\,250\,777\,\alpha^6 + 95\,256\,\alpha^7 + 756\,\alpha^8\Big)\,S_\alpha^{16} + \\
\Big(&1\,369\,112\,633\,033\,966\,016 + 749\,960\,867\,137\,851\,264\,\alpha + 179\,694\,158\,433\,369\,216\,\alpha^2 + \\
&24\,598\,615\,258\,051\,968\,\alpha^3 + 2\,104\,205\,904\,948\,672\,\alpha^4 + 115\,177\,607\,830\,656\,\alpha^5 + \\
&3\,939\,594\,524\,544\,\alpha^6 + 76\,987\,798\,272\,\alpha^7 + 658\,107\,072\,\alpha^8\Big)\,S_\alpha^{15} + \\
\Big(&-1\,734\,792\,653\,693\,830\,004\,736 - 949\,019\,070\,255\,906\,578\,432\,\alpha - 226\,524\,572\,171\,763\,769\,344\,\alpha^2 - \\
&30\,803\,470\,801\,376\,100\,352\,\alpha^3 - 2\,608\,940\,417\,614\,004\,224\,\alpha^4 - 140\,860\,855\,192\,928\,256\,\alpha^5 - \\
&4\,731\,649\,360\,265\,216\,\alpha^6 - 90\,341\,108\,490\,240\,\alpha^7 - 749\,920\,296\,960\,\alpha^8\Big)\,S_\alpha^{14} + \\
\Big(&60\,803\,277\,620\,001\,284\,299\,948\,032 + 39\,428\,846\,812\,699\,916\,145\,000\,448\,\alpha + \\
&11\,180\,996\,921\,120\,508\,276\,899\,840\,\alpha^2 + 1\,810\,953\,673\,723\,168\,488\,947\,712\,\alpha^3 + \\
&183\,235\,931\,956\,946\,612\,781\,056\,\alpha^4 + 11\,860\,125\,063\,327\,012\,356\,096\,\alpha^5 + \\
&479\,559\,881\,836\,404\,408\,320\,\alpha^6 + 11\,075\,248\,290\,023\,866\,368\,\alpha^7 + 111\,850\,497\,389\,887\,488\,\alpha^8\Big)\,S_\alpha^{13} + \\
\Big(&-89\,019\,109\,694\,327\,271\,526\,225\,674\,240 - 58\,188\,866\,980\,847\,185\,951\,333\,548\,032\,\alpha -
\end{aligned}
$$

$16 610 124 212 723 257 407 541 608 448\ \alpha^2 -$
$2 704 024 225 155 393 409 339 359 232\ \alpha^3 - 274 541 232 572 280 640 171 409 408\ \alpha^4 -$
$17 798 808 305 108 522 215 931 904\ \alpha^5 - 719 410 062 551 481 021 628 416\ \alpha^6 -$
$16 570 995 465 074 189 008 896\ \alpha^7 - 166 498 086 762 886 201 344\ \alpha^8)\ S_\alpha^{12} +$
$(-58 193 026 228 357 038 079 242 475 143 168 - 51 098 231 803 407 605 582 884 239 310 848\ \alpha -$
$19 138 069 216 643 015 058 642 882 789 376\ \alpha^2 -$
$4 020 644 083 315 968 732 595 381 862 400\ \alpha^3 - 520 534 653 627 339 724 629 150 269 440\ \alpha^4 -$
$42 656 902 349 499 267 252 561 641 472\ \alpha^5 - 2 165 540 502 734 494 733 562 806 272\ \alpha^6 -$
$62 368 598 737 268 407 957 192 704\ \alpha^7 - 781 156 297 810 381 520 240 640\ \alpha^8)\ S_\alpha^{11} +$
$(143 253 210 914 077 584 461 892 945 482 612 736 + 117 390 434 023 857 677 801 656 082 944 229 376\ \alpha +$
$41 988 373 493 244 944 351 648 456 537 800 704\ \alpha^2 +$
$8 563 309 433 179 682 998 044 147 681 591 296\ \alpha^3 + 1 089 279 680 902 487 658 272 936 091 975 680\ \alpha^4 +$
$88 503 747 091 256 203 811 985 792 434 176\ \alpha^5 + 4 485 778 290 470 469 550 068 465 664 000\ \alpha^6 +$
$129 679 193 329 108 045 683 807 485 952\ \alpha^7 + 1 637 147 560 168 901 326 135 099 392\ \alpha^8)\ S_\alpha^{10} +$
$(-60 784 977 973 911 336 417 455 240 884 588 118 016 -$
$50 604 534 417 001 248 493 972 281 476 297 785 344\ \alpha -$
$18 364 880 128 939 667 288 013 056 370 297 274 368\ \alpha^2 -$
$3 793 122 059 759 621 935 706 090 408 477 982 720\ \alpha^3 -$
$487 433 515 130 127 581 954 429 612 206 325 760\ \alpha^4 -$
$39 881 853 439 127 215 750 555 055 217 967 104\ \alpha^5 - 2 027 413 953 997 564 608 311 335 638 269 952\ \alpha^6 -$
$58 489 250 532 075 913 349 682 400 591 872\ \alpha^7 - 732 231 540 023 730 620 969 986 818 048\ \alpha^8)\ S_\alpha^{9} +$
$(2 608 894 283 215 912 975 618 410 102 572 263 145 472 +$
$1 616 795 453 138 378 571 557 540 817 330 663 063 552\ \alpha +$
$327 616 663 513 807 904 470 525 387 480 932 286 464\ \alpha^2 -$
$873 589 559 523 801 551 554 650 493 409 034 240\ \alpha^3 -$
$11 375 650 338 286 106 535 535 280 323 773 333 504\ \alpha^4 -$
$2 110 778 990 692 750 151 126 063 557 153 652 736\ \alpha^5 -$
$184 410 301 249 112 272 763 157 224 846 524 416\ \alpha^6 - 8 198 237 719 900 262 300 321 948 425 519 104\ \alpha^7 -$
$149 697 886 463 404 317 932 617 973 366 784\ \alpha^8)\ S_\alpha^{8} +$
$(1 048 241 608 139 004 126 801 123 879 435 198 781 194 240 +$
$1 096 523 044 229 137 591 636 209 311 492 319 177 867 264\ \alpha +$
$502 483 518 474 841 486 190 896 225 121 701 233 426 432\ \alpha^2 +$
$131 841 296 561 223 271 034 561 082 320 937 371 566 080\ \alpha^3 +$
$21 678 855 366 109 987 137 873 513 889 942 248 357 888\ \alpha^4 +$
$2 289 292 202 221 334 778 014 866 102 813 074 653 184\ \alpha^5 +$
$151 731 925 148 495 278 830 633 633 775 746 023 424\ \alpha^6 +$
$5 775 268 570 467 146 796 542 588 786 036 441 088\ \alpha^7 +$
$96 719 208 505 536 419 841 142 621 892 247 552\ \alpha^8)\ S_\alpha^{7} +$
$(-52 778 235 984 846 523 329 097 656 918 526 071 020 716 032 -$
$64 501 494 488 056 900 542 629 329 788 894 135 692 820 480\ \alpha -$
$34 724 501 660 699 004 906 438 902 413 811 995 002 273 792\ \alpha^2 -$
$10 746 920 005 726 085 338 263 669 816 779 087 850 504 192\ \alpha^3 -$
$2 089 401 050 396 611 719 653 237 940 170 751 700 107 264\ \alpha^4 -$
$261 034 694 280 336 096 121 376 287 674 697 770 860 544\ \alpha^5 -$
$20 442 511 124 673 707 455 959 822 834 794 030 432 256\ \alpha^6 -$
$916 460 910 005 176 557 899 370 835 718 675 890 176\ \alpha^7 -$
$17 986 272 310 903 846 816 671 667 502 362 656 768\ \alpha^8)\ S_\alpha^{6} +$
$(3 035 817 366 536 745 176 728 786 455 945 000 034 850 832 384 +$
$4 120 711 019 306 783 344 000 865 847 316 126 147 694 034 944\ \alpha +$
$2 427 539 767 337 698 957 238 482 059 057 992 795 053 621 248\ \alpha^2 +$

$809\,015\,020\,077\,846\,132\,558\,928\,658\,581\,820\,556\,596\,740\,096\,\alpha^3 +$
$166\,367\,946\,678\,771\,551\,680\,801\,143\,800\,118\,606\,599\,028\,736\,\alpha^4 +$
$21\,534\,401\,317\,536\,332\,767\,529\,501\,758\,267\,946\,868\,670\,464\,\alpha^5 +$
$1\,703\,618\,962\,577\,866\,660\,042\,947\,168\,530\,510\,007\,762\,944\,\alpha^6 +$
$74\,638\,559\,897\,828\,248\,664\,819\,619\,987\,477\,541\,945\,344\,\alpha^7 +$
$1\,365\,121\,772\,758\,889\,406\,361\,975\,817\,513\,893\,625\,856\,\alpha^8)\; S_\alpha^5 +$
$(29\,747\,708\,350\,860\,628\,203\,102\,467\,373\,301\,142\,151\,103\,512\,576 +$
$60\,594\,267\,306\,227\,972\,555\,297\,424\,456\,256\,719\,065\,930\,792\,960\,\alpha +$
$51\,912\,211\,630\,316\,685\,165\,996\,689\,583\,949\,255\,136\,606\,420\,992\,\alpha^2 +$
$24\,722\,229\,319\,717\,714\,245\,138\,454\,282\,177\,389\,731\,990\,470\,656\,\alpha^3 +$
$7\,209\,098\,902\,288\,104\,726\,691\,278\,784\,086\,919\,701\,719\,941\,120\,\alpha^4 +$
$1\,324\,119\,011\,334\,307\,141\,754\,864\,618\,589\,389\,805\,693\,435\,904\,\alpha^5 +$
$150\,057\,997\,412\,444\,455\,387\,133\,736\,305\,754\,428\,384\,739\,328\,\alpha^6 +$
$9\,613\,620\,776\,689\,982\,640\,419\,614\,639\,367\,688\,573\,419\,520\,\alpha^7 +$
$266\,982\,380\,934\,205\,139\,034\,767\,194\,888\,213\,610\,102\,784\,\alpha^8)\; S_\alpha^4 +$
$(-541\,705\,852\,347\,998\,344\,994\,991\,067\,839\,918\,006\,018\,131\,361\,792 -$
$1\,006\,522\,647\,297\,450\,534\,105\,424\,561\,586\,982\,024\,767\,315\,378\,176\,\alpha -$
$813\,097\,476\,747\,275\,636\,497\,017\,969\,484\,824\,128\,996\,715\,266\,048\,\alpha^2 -$
$372\,604\,423\,756\,054\,312\,395\,141\,348\,687\,341\,002\,199\,152\,984\,064\,\alpha^3 -$
$105\,783\,156\,512\,463\,421\,821\,044\,857\,565\,724\,466\,683\,987\,361\,792\,\alpha^4 -$
$19\,011\,542\,666\,471\,722\,259\,199\,375\,580\,865\,919\,395\,787\,964\,416\,\alpha^5 -$
$2\,105\,453\,197\,859\,991\,626\,589\,184\,171\,698\,418\,516\,293\,058\,560\,\alpha^6 -$
$130\,694\,441\,182\,488\,526\,530\,201\,470\,183\,477\,141\,185\,757\,184\,\alpha^7 -$
$3\,451\,537\,920\,763\,342\,815\,087\,344\,036\,232\,525\,206\,519\,808\,\alpha^8)\; S_\alpha^3 +$
$(-179\,689\,780\,872\,449\,048\,689\,188\,299\,050\,106\,836\,700\,085\,878\,784 -$
$499\,831\,106\,994\,415\,306\,038\,295\,057\,512\,492\,115\,485\,259\,726\,848\,\alpha -$
$615\,448\,848\,729\,919\,042\,243\,741\,009\,674\,357\,547\,457\,759\,412\,224\,\alpha^2 -$
$439\,056\,719\,913\,933\,128\,716\,360\,967\,596\,904\,709\,697\,038\,188\,544\,\alpha^3 -$
$198\,821\,690\,211\,184\,330\,213\,340\,653\,171\,402\,935\,453\,379\,198\,976\,\alpha^4 -$
$58\,585\,124\,257\,016\,111\,943\,950\,899\,719\,045\,338\,500\,555\,603\,968\,\alpha^5 -$
$10\,972\,796\,030\,334\,800\,010\,048\,739\,237\,184\,926\,418\,840\,059\,904\,\alpha^6 -$
$1\,193\,719\,368\,198\,496\,194\,052\,742\,916\,760\,632\,408\,586\,321\,920\,\alpha^7 -$
$57\,677\,198\,298\,608\,369\,079\,887\,772\,175\,160\,628\,008\,189\,952\,\alpha^8)\; S_\alpha^2 +$
$(214\,831\,139\,425\,605\,934\,948\,554\,780\,116\,193\,465\,207\,562\,960\,896 +$
$852\,036\,786\,668\,519\,357\,669\,502\,859\,114\,101\,621\,504\,086\,114\,304\,\alpha +$
$1\,468\,636\,407\,582\,371\,972\,238\,660\,658\,710\,382\,450\,945\,346\,764\,800\,\alpha^2 +$
$1\,436\,920\,801\,120\,841\,676\,785\,358\,526\,575\,689\,998\,994\,652\,528\,640\,\alpha^3 +$
$872\,864\,233\,727\,523\,332\,930\,987\,436\,080\,400\,964\,699\,165\,818\,880\,\alpha^4 +$
$337\,152\,150\,482\,102\,751\,084\,494\,014\,299\,451\,258\,825\,361\,850\,368\,\alpha^5 +$
$80\,898\,086\,492\,179\,079\,212\,968\,894\,984\,542\,457\,756\,014\,084\,096\,\alpha^6 +$
$11\,033\,231\,151\,245\,622\,526\,747\,323\,768\,921\,226\,019\,013\,132\,288\,\alpha^7 +$
$655\,787\,571\,926\,373\,008\,384\,765\,243\,492\,546\,204\,588\,310\,528\,\alpha^8)\; S_\alpha +$
$(-151\,026\,323\,282\,253\,922\,352\,374\,256\,330\,569\,782\,279\,536\,640 -$
$1\,366\,788\,225\,704\,397\,997\,288\,987\,019\,791\,656\,529\,629\,806\,592\,\alpha -$
$5\,303\,121\,535\,030\,477\,312\,378\,786\,039\,652\,035\,049\,432\,285\,184\,\alpha^2 -$
$11\,533\,376\,887\,988\,124\,536\,976\,314\,041\,777\,845\,706\,747\,281\,408\,\alpha^3 -$
$15\,391\,679\,930\,285\,039\,325\,517\,386\,362\,534\,096\,505\,705\,332\,736\,\alpha^4 -$
$12\,917\,784\,851\,408\,785\,491\,873\,078\,058\,141\,402\,044\,309\,700\,608\,\alpha^5 -$
$6\,663\,616\,997\,264\,781\,396\,236\,424\,132\,096\,584\,504\,800\,444\,416\,\alpha^6 -$
$1\,933\,136\,938\,012\,850\,206\,110\,390\,481\,031\,293\,213\,178\,068\,992\,\alpha^7 -$
$241\,642\,117\,251\,606\,275\,763\,798\,810\,128\,911\,651\,647\,258\,624\,\alpha^8)\}$

In[ ]:=
```
RECNormalizedinS = RECNormalized[[1]];
ToOrePolynomial[RECNormalizedinS]
```

Out[ ]=

$$
\begin{aligned}
&(2\,859\,422\,318\,592 + 1\,452\,813\,680\,640\,\alpha + 322\,925\,288\,448\,\alpha^2 + 41\,014\,614\,144\,\alpha^3 + \\
&\quad 3\,255\,650\,202\,\alpha^4 + 165\,386\,235\,\alpha^5 + 5\,250\,777\,\alpha^6 + 95\,256\,\alpha^7 + 756\,\alpha^8)\ S_\alpha^{16} + \\
&(1\,369\,112\,633\,033\,966\,016 + 749\,960\,867\,137\,851\,264\,\alpha + 179\,694\,158\,433\,369\,216\,\alpha^2 + \\
&\quad 24\,598\,615\,258\,051\,968\,\alpha^3 + 2\,104\,205\,904\,948\,672\,\alpha^4 + 115\,177\,607\,830\,656\,\alpha^5 + \\
&\quad 3\,939\,594\,524\,544\,\alpha^6 + 76\,987\,798\,272\,\alpha^7 + 658\,107\,072\,\alpha^8)\ S_\alpha^{15} + \\
&(-1\,734\,792\,653\,693\,830\,004\,736 - 949\,019\,070\,255\,906\,578\,432\,\alpha - 226\,524\,572\,171\,763\,769\,344\,\alpha^2 - \\
&\quad 30\,803\,470\,801\,376\,100\,352\,\alpha^3 - 2\,608\,940\,417\,614\,004\,224\,\alpha^4 - 140\,860\,855\,192\,928\,256\,\alpha^5 - \\
&\quad 4\,731\,649\,360\,265\,216\,\alpha^6 - 90\,341\,108\,490\,240\,\alpha^7 - 749\,920\,296\,960\,\alpha^8)\ S_\alpha^{14} + \\
&(60\,803\,277\,620\,001\,284\,299\,948\,032 + 39\,428\,846\,812\,699\,916\,145\,000\,448\,\alpha + \\
&\quad 11\,180\,996\,921\,120\,508\,276\,899\,840\,\alpha^2 + 1\,810\,953\,673\,723\,168\,488\,947\,712\,\alpha^3 + \\
&\quad 183\,235\,931\,956\,946\,612\,781\,056\,\alpha^4 + 11\,860\,125\,063\,327\,012\,356\,096\,\alpha^5 + \\
&\quad 479\,559\,881\,836\,404\,408\,320\,\alpha^6 + 11\,075\,248\,290\,023\,866\,368\,\alpha^7 + 111\,850\,497\,389\,887\,488\,\alpha^8)\ S_\alpha^{13} + \\
&(-89\,019\,109\,694\,327\,271\,526\,225\,674\,240 - 58\,188\,866\,980\,847\,185\,951\,333\,548\,032\,\alpha - \\
&\quad 16\,610\,124\,212\,723\,257\,407\,541\,608\,448\,\alpha^2 - 2\,704\,024\,225\,155\,393\,409\,339\,359\,232\,\alpha^3 - \\
&\quad 274\,541\,232\,572\,280\,640\,171\,409\,408\,\alpha^4 - 17\,798\,808\,305\,108\,522\,215\,931\,904\,\alpha^5 - \\
&\quad 719\,410\,062\,551\,481\,021\,628\,416\,\alpha^6 - 16\,570\,995\,465\,074\,189\,008\,896\,\alpha^7 - 166\,498\,086\,762\,886\,201\,344\,\alpha^8) \\
&S_\alpha^{12} + (-58\,193\,026\,228\,357\,038\,079\,242\,475\,143\,168 - 51\,098\,231\,803\,407\,605\,582\,884\,239\,310\,848\,\alpha - \\
&\quad 19\,138\,069\,216\,643\,015\,058\,642\,882\,789\,376\,\alpha^2 - \\
&\quad 4\,020\,644\,083\,315\,968\,732\,595\,381\,862\,400\,\alpha^3 - 520\,534\,653\,627\,339\,724\,629\,150\,269\,440\,\alpha^4 - \\
&\quad 42\,656\,902\,349\,499\,267\,252\,561\,641\,472\,\alpha^5 - 2\,165\,540\,502\,734\,494\,733\,562\,806\,272\,\alpha^6 - \\
&\quad 62\,368\,598\,737\,268\,407\,957\,192\,704\,\alpha^7 - 781\,156\,297\,810\,381\,520\,240\,640\,\alpha^8)\ S_\alpha^{11} + \\
&(143\,253\,210\,914\,077\,584\,461\,892\,945\,482\,612\,736 + 117\,390\,434\,023\,857\,677\,801\,656\,082\,944\,229\,376\,\alpha + \\
&\quad 41\,988\,373\,493\,244\,944\,351\,648\,456\,537\,800\,704\,\alpha^2 + \\
&\quad 8\,563\,309\,433\,179\,682\,998\,044\,147\,681\,591\,296\,\alpha^3 + 1\,089\,279\,680\,902\,487\,658\,272\,936\,091\,975\,680\,\alpha^4 + \\
&\quad 88\,503\,747\,091\,256\,203\,811\,985\,792\,434\,176\,\alpha^5 + 4\,485\,778\,290\,470\,469\,550\,068\,465\,664\,000\,\alpha^6 + \\
&\quad 129\,679\,193\,329\,108\,045\,683\,807\,485\,952\,\alpha^7 + 1\,637\,147\,560\,168\,901\,326\,135\,099\,392\,\alpha^8)\ S_\alpha^{10} + \\
&(-60\,784\,977\,973\,911\,336\,417\,455\,240\,884\,588\,118\,016 - 50\,604\,534\,417\,001\,248\,493\,972\,281\,476\,297\,785\,344 \\
&\quad \alpha - 18\,364\,880\,128\,939\,667\,288\,013\,056\,370\,297\,274\,368\,\alpha^2 - \\
&\quad 3\,793\,122\,059\,759\,621\,935\,706\,090\,408\,477\,982\,720\,\alpha^3 - \\
&\quad 487\,433\,515\,130\,127\,581\,954\,429\,612\,206\,325\,760\,\alpha^4 - \\
&\quad 39\,881\,853\,439\,127\,215\,750\,555\,055\,217\,967\,104\,\alpha^5 - 2\,027\,413\,953\,997\,564\,608\,311\,335\,638\,269\,952\,\alpha^6 - \\
&\quad 58\,489\,250\,532\,075\,913\,349\,682\,400\,591\,872\,\alpha^7 - 732\,231\,540\,023\,730\,620\,969\,986\,818\,048\,\alpha^8)\ S_\alpha^{9} + \\
&(2\,608\,894\,283\,215\,912\,975\,618\,410\,102\,572\,263\,145\,472 + \\
&\quad 1\,616\,795\,453\,138\,378\,571\,557\,540\,817\,330\,663\,063\,552\,\alpha + \\
&\quad 327\,616\,663\,513\,807\,904\,470\,525\,387\,480\,932\,286\,464\,\alpha^2 - \\
&\quad 873\,589\,559\,523\,801\,551\,554\,650\,493\,409\,034\,240\,\alpha^3 - 11\,375\,650\,338\,286\,106\,535\,535\,280\,323\,773\,333\,504 \\
&\quad \alpha^4 - 2\,110\,778\,990\,692\,750\,151\,126\,063\,557\,153\,652\,736\,\alpha^5 - \\
&\quad 184\,410\,301\,249\,112\,272\,763\,157\,224\,846\,524\,416\,\alpha^6 - 8\,198\,237\,719\,900\,262\,300\,321\,948\,425\,519\,104\,\alpha^7 - \\
&\quad 149\,697\,886\,463\,404\,317\,932\,617\,973\,366\,784\,\alpha^8)\ S_\alpha^{8} + \\
&(1\,048\,241\,608\,139\,004\,126\,801\,123\,879\,435\,198\,781\,194\,240 + \\
&\quad 1\,096\,523\,044\,229\,137\,591\,636\,209\,311\,492\,319\,177\,867\,264\,\alpha + \\
&\quad 502\,483\,518\,474\,841\,486\,190\,896\,225\,121\,701\,233\,426\,432\,\alpha^2 + \\
&\quad 131\,841\,296\,561\,223\,271\,034\,561\,082\,320\,937\,371\,566\,080\,\alpha^3 + \\
&\quad 21\,678\,855\,366\,109\,987\,137\,873\,513\,889\,942\,248\,357\,888\,\alpha^4 + \\
&\quad 2\,289\,292\,202\,221\,334\,778\,014\,866\,102\,813\,074\,653\,184\,\alpha^5 + \\
&\quad 151\,731\,925\,148\,495\,278\,830\,633\,633\,775\,746\,023\,424\,\alpha^6 + \\
&\quad 5\,775\,268\,570\,467\,146\,796\,542\,588\,786\,036\,441\,088\,\alpha^7 +
\end{aligned}
$$

$$
\begin{aligned}
&96\,719\,208\,505\,536\,419\,841\,142\,621\,892\,247\,552\,\alpha^8\Big)\,S_\alpha^7 + \\
&\Big(-52\,778\,235\,984\,846\,523\,329\,097\,656\,918\,526\,071\,020\,716\,032 - \\
&64\,501\,494\,488\,056\,900\,542\,629\,329\,788\,894\,135\,692\,820\,480\,\alpha - \\
&34\,724\,501\,660\,699\,004\,906\,438\,902\,413\,811\,995\,002\,273\,792\,\alpha^2 - \\
&10\,746\,920\,005\,726\,085\,338\,263\,669\,816\,779\,087\,850\,504\,192\,\alpha^3 - \\
&2\,089\,401\,050\,396\,611\,719\,653\,237\,940\,170\,751\,700\,107\,264\,\alpha^4 - \\
&261\,034\,694\,280\,336\,096\,121\,376\,287\,674\,697\,770\,860\,544\,\alpha^5 - \\
&20\,442\,511\,124\,673\,707\,455\,959\,822\,834\,794\,030\,432\,256\,\alpha^6 - \\
&916\,460\,910\,005\,176\,557\,899\,370\,835\,718\,675\,890\,176\,\alpha^7 - \\
&17\,986\,272\,310\,903\,846\,816\,671\,667\,502\,362\,656\,768\,\alpha^8\Big)\,S_\alpha^6 + \\
&\Big(3\,035\,817\,366\,536\,745\,176\,728\,786\,455\,945\,000\,034\,850\,832\,384 + \\
&4\,120\,711\,019\,306\,783\,344\,000\,865\,847\,316\,126\,147\,694\,034\,944\,\alpha + \\
&2\,427\,539\,767\,337\,698\,957\,238\,482\,059\,057\,992\,795\,053\,621\,248\,\alpha^2 + \\
&809\,015\,020\,077\,846\,132\,558\,928\,658\,581\,820\,556\,596\,740\,096\,\alpha^3 + \\
&166\,367\,946\,678\,771\,551\,680\,801\,143\,800\,118\,606\,599\,028\,736\,\alpha^4 + \\
&21\,534\,401\,317\,536\,332\,767\,529\,501\,758\,267\,946\,868\,670\,464\,\alpha^5 + \\
&1\,703\,618\,962\,577\,866\,660\,042\,947\,168\,530\,510\,007\,762\,944\,\alpha^6 + \\
&74\,638\,559\,897\,828\,248\,664\,819\,619\,987\,477\,541\,945\,344\,\alpha^7 + \\
&1\,365\,121\,772\,758\,889\,406\,361\,975\,817\,513\,893\,625\,856\,\alpha^8\Big)\,S_\alpha^5 + \\
&\Big(29\,747\,708\,350\,860\,628\,203\,102\,467\,373\,301\,142\,151\,103\,512\,576 + \\
&60\,594\,267\,306\,227\,972\,555\,297\,424\,456\,256\,719\,065\,930\,792\,960\,\alpha + \\
&51\,912\,211\,630\,316\,685\,165\,996\,689\,583\,949\,255\,136\,606\,420\,992\,\alpha^2 + \\
&24\,722\,229\,319\,717\,714\,245\,138\,454\,282\,177\,389\,731\,990\,470\,656\,\alpha^3 + \\
&7\,209\,098\,902\,288\,104\,726\,691\,278\,784\,086\,919\,701\,719\,941\,120\,\alpha^4 + \\
&1\,324\,119\,011\,334\,307\,141\,754\,864\,618\,589\,389\,805\,693\,435\,904\,\alpha^5 + \\
&150\,057\,997\,412\,444\,455\,387\,133\,736\,305\,754\,428\,384\,739\,328\,\alpha^6 + \\
&9\,613\,620\,776\,689\,982\,640\,419\,614\,639\,367\,688\,573\,419\,520\,\alpha^7 + \\
&266\,982\,380\,934\,205\,139\,034\,767\,194\,888\,213\,610\,102\,784\,\alpha^8\Big)\,S_\alpha^4 + \\
&\Big(-541\,705\,852\,347\,998\,344\,994\,991\,067\,839\,918\,006\,018\,131\,361\,792 - \\
&1\,006\,522\,647\,297\,450\,534\,105\,424\,561\,586\,982\,024\,767\,315\,378\,176\,\alpha - \\
&813\,097\,476\,747\,275\,636\,497\,017\,969\,484\,824\,128\,996\,715\,266\,048\,\alpha^2 - \\
&372\,604\,423\,756\,054\,312\,395\,141\,348\,687\,341\,002\,199\,152\,984\,064\,\alpha^3 - \\
&105\,783\,156\,512\,463\,421\,821\,044\,857\,565\,724\,466\,683\,987\,361\,792\,\alpha^4 - \\
&19\,011\,542\,666\,471\,722\,259\,199\,375\,580\,865\,919\,395\,787\,964\,416\,\alpha^5 - \\
&2\,105\,453\,197\,859\,991\,626\,589\,184\,171\,698\,418\,516\,293\,058\,560\,\alpha^6 - \\
&130\,694\,441\,182\,488\,526\,530\,201\,470\,183\,477\,141\,185\,757\,184\,\alpha^7 - \\
&3\,451\,537\,920\,763\,342\,815\,087\,344\,036\,232\,525\,206\,519\,808\,\alpha^8\Big)\,S_\alpha^3 + \\
&\Big(-179\,689\,780\,872\,449\,048\,689\,188\,299\,050\,106\,836\,700\,085\,878\,784 - \\
&499\,831\,106\,994\,415\,306\,038\,295\,057\,512\,492\,115\,485\,259\,726\,848\,\alpha - \\
&615\,448\,848\,729\,919\,042\,243\,741\,009\,674\,357\,547\,457\,759\,412\,224\,\alpha^2 - \\
&439\,056\,719\,913\,933\,128\,716\,360\,967\,596\,904\,709\,697\,038\,188\,544\,\alpha^3 - \\
&198\,821\,690\,211\,184\,330\,213\,340\,653\,171\,402\,935\,453\,379\,198\,976\,\alpha^4 - \\
&58\,585\,124\,257\,016\,111\,943\,950\,899\,719\,045\,338\,500\,555\,603\,968\,\alpha^5 - \\
&10\,972\,796\,030\,334\,800\,010\,048\,739\,237\,184\,926\,418\,840\,059\,904\,\alpha^6 - \\
&1\,193\,719\,368\,198\,496\,194\,052\,742\,916\,760\,632\,408\,586\,321\,920\,\alpha^7 - \\
&57\,677\,198\,298\,608\,369\,079\,887\,772\,175\,160\,628\,008\,189\,952\,\alpha^8\Big)\,S_\alpha^2 + \\
&\Big(214\,831\,139\,425\,605\,934\,948\,554\,780\,116\,193\,465\,207\,562\,960\,896 + \\
&852\,036\,786\,668\,519\,357\,669\,502\,859\,114\,101\,621\,504\,086\,114\,304\,\alpha + \\
&1\,468\,636\,407\,582\,371\,972\,238\,660\,658\,710\,382\,450\,945\,346\,764\,800\,\alpha^2 + \\
&1\,436\,920\,801\,120\,841\,676\,785\,358\,526\,575\,689\,998\,994\,652\,528\,640\,\alpha^3 +
\end{aligned}
$$

$872\,864\,233\,727\,523\,332\,930\,987\,436\,080\,400\,964\,699\,165\,818\,880\,\alpha^4 +$
$337\,152\,150\,482\,102\,751\,084\,494\,014\,299\,451\,258\,825\,361\,850\,368\,\alpha^5 +$
$80\,898\,086\,492\,179\,079\,212\,968\,894\,984\,542\,457\,756\,014\,084\,096\,\alpha^6 +$
$11\,033\,231\,151\,245\,622\,526\,747\,323\,768\,921\,226\,019\,013\,132\,288\,\alpha^7 +$
$655\,787\,571\,926\,373\,008\,384\,765\,243\,492\,546\,204\,588\,310\,528\,\alpha^8)\;S_\alpha +$
$(-151\,026\,323\,282\,253\,922\,352\,374\,256\,330\,569\,782\,279\,536\,640 -$
$1\,366\,788\,225\,704\,397\,997\,288\,987\,019\,791\,656\,529\,629\,806\,592\,\alpha -$
$5\,303\,121\,535\,030\,477\,312\,378\,786\,039\,652\,035\,049\,432\,285\,184\,\alpha^2 -$
$11\,533\,376\,887\,988\,124\,536\,976\,314\,041\,777\,845\,706\,747\,281\,408\,\alpha^3 -$
$15\,391\,679\,930\,285\,039\,325\,517\,386\,362\,534\,096\,505\,705\,332\,736\,\alpha^4 -$
$12\,917\,784\,851\,408\,785\,491\,873\,078\,058\,141\,402\,044\,309\,700\,608\,\alpha^5 -$
$6\,663\,616\,997\,264\,781\,396\,236\,424\,132\,096\,584\,504\,800\,444\,416\,\alpha^6 -$
$1\,933\,136\,938\,012\,850\,206\,110\,390\,481\,031\,293\,213\,178\,068\,992\,\alpha^7 -$
$241\,642\,117\,251\,606\,275\,763\,798\,810\,128\,911\,651\,647\,258\,624\,\alpha^8)$

In[ ]:=
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RecNormalizedOrder = OrePolynomialDegree[RECNormalizedinS, S[α]]
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Out[ ]= 16

Write recurrence explicitly.

In[ ]:=
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ClearAll[Seq];
SeqNormalized = ApplyOreOperator[RECNormalizedinS, Seq[α]]
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Out[ ]=
$(-151\,026\,323\,282\,253\,922\,352\,374\,256\,330\,569\,782\,279\,536\,640 -$
$1\,366\,788\,225\,704\,397\,997\,288\,987\,019\,791\,656\,529\,629\,806\,592\,\alpha -$
$5\,303\,121\,535\,030\,477\,312\,378\,786\,039\,652\,035\,049\,432\,285\,184\,\alpha^2 -$
$11\,533\,376\,887\,988\,124\,536\,976\,314\,041\,777\,845\,706\,747\,281\,408\,\alpha^3 -$
$15\,391\,679\,930\,285\,039\,325\,517\,386\,362\,534\,096\,505\,705\,332\,736\,\alpha^4 -$
$12\,917\,784\,851\,408\,785\,491\,873\,078\,058\,141\,402\,044\,309\,700\,608\,\alpha^5 -$
$6\,663\,616\,997\,264\,781\,396\,236\,424\,132\,096\,584\,504\,800\,444\,416\,\alpha^6 -$
$1\,933\,136\,938\,012\,850\,206\,110\,390\,481\,031\,293\,213\,178\,068\,992\,\alpha^7 -$
$241\,642\,117\,251\,606\,275\,763\,798\,810\,128\,911\,651\,647\,258\,624\,\alpha^8)\;\mathrm{Seq}[\alpha] +$
$(214\,831\,139\,425\,605\,934\,948\,554\,780\,116\,193\,465\,207\,562\,960\,896 +$
$852\,036\,786\,668\,519\,357\,669\,502\,859\,114\,101\,621\,504\,086\,114\,304\,\alpha +$
$1\,468\,636\,407\,582\,371\,972\,238\,660\,658\,710\,382\,450\,945\,346\,764\,800\,\alpha^2 +$
$1\,436\,920\,801\,120\,841\,676\,785\,358\,526\,575\,689\,998\,994\,652\,528\,640\,\alpha^3 +$
$872\,864\,233\,727\,523\,332\,930\,987\,436\,080\,400\,964\,699\,165\,818\,880\,\alpha^4 +$
$337\,152\,150\,482\,102\,751\,084\,494\,014\,299\,451\,258\,825\,361\,850\,368\,\alpha^5 +$
$80\,898\,086\,492\,179\,079\,212\,968\,894\,984\,542\,457\,756\,014\,084\,096\,\alpha^6 +$
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$499\,831\,106\,994\,415\,306\,038\,295\,057\,512\,492\,115\,485\,259\,726\,848\,\alpha -$
$615\,448\,848\,729\,919\,042\,243\,741\,009\,674\,357\,547\,457\,759\,412\,224\,\alpha^2 -$
$439\,056\,719\,913\,933\,128\,716\,360\,967\,596\,904\,709\,697\,038\,188\,544\,\alpha^3 -$
$198\,821\,690\,211\,184\,330\,213\,340\,653\,171\,402\,935\,453\,379\,198\,976\,\alpha^4 -$
$58\,585\,124\,257\,016\,111\,943\,950\,899\,719\,045\,338\,500\,555\,603\,968\,\alpha^5 -$
$10\,972\,796\,030\,334\,800\,010\,048\,739\,237\,184\,926\,418\,840\,059\,904\,\alpha^6 -$
$1\,193\,719\,368\,198\,496\,194\,052\,742\,916\,760\,632\,408\,586\,321\,920\,\alpha^7 -$
$57\,677\,198\,298\,608\,369\,079\,887\,772\,175\,160\,628\,008\,189\,952\,\alpha^8)\;\mathrm{Seq}[2+\alpha] +$

$(-541\,705\,852\,347\,998\,344\,994\,991\,067\,839\,918\,006\,018\,131\,361\,792 -$
$1\,006\,522\,647\,297\,450\,534\,105\,424\,561\,586\,982\,024\,767\,315\,378\,176\,\alpha -$
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$19\,011\,542\,666\,471\,722\,259\,199\,375\,580\,865\,919\,395\,787\,964\,416\,\alpha^5 -$
$2\,105\,453\,197\,859\,991\,626\,589\,184\,171\,698\,418\,516\,293\,058\,560\,\alpha^6 -$
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$(29\,747\,708\,350\,860\,628\,203\,102\,467\,373\,301\,142\,151\,103\,512\,576 +$
$60\,594\,267\,306\,227\,972\,555\,297\,424\,456\,256\,719\,065\,930\,792\,960\,\alpha +$
$51\,912\,211\,630\,316\,685\,165\,996\,689\,583\,949\,255\,136\,606\,420\,992\,\alpha^2 +$
$24\,722\,229\,319\,717\,714\,245\,138\,454\,282\,177\,389\,731\,990\,470\,656\,\alpha^3 +$
$7\,209\,098\,902\,288\,104\,726\,691\,278\,784\,086\,919\,701\,719\,941\,120\,\alpha^4 +$
$1\,324\,119\,011\,334\,307\,141\,754\,864\,618\,589\,389\,805\,693\,435\,904\,\alpha^5 +$
$150\,057\,997\,412\,444\,455\,387\,133\,736\,305\,754\,428\,384\,739\,328\,\alpha^6 +$
$9\,613\,620\,776\,689\,982\,640\,419\,614\,639\,367\,688\,573\,419\,520\,\alpha^7 +$
$266\,982\,380\,934\,205\,139\,034\,767\,194\,888\,213\,610\,102\,784\,\alpha^8)\ \mathrm{Seq}[4+\alpha] +$
$(3\,035\,817\,366\,536\,745\,176\,728\,786\,455\,945\,000\,034\,850\,832\,384 +$
$4\,120\,711\,019\,306\,783\,344\,000\,865\,847\,316\,126\,147\,694\,034\,944\,\alpha +$
$2\,427\,539\,767\,337\,698\,957\,238\,482\,059\,057\,992\,795\,053\,621\,248\,\alpha^2 +$
$809\,015\,020\,077\,846\,132\,558\,928\,658\,581\,820\,556\,596\,740\,096\,\alpha^3 +$
$166\,367\,946\,678\,771\,551\,680\,801\,143\,800\,118\,606\,599\,028\,736\,\alpha^4 +$
$21\,534\,401\,317\,536\,332\,767\,529\,501\,758\,267\,946\,868\,670\,464\,\alpha^5 +$
$1\,703\,618\,962\,577\,866\,660\,042\,947\,168\,530\,510\,007\,762\,944\,\alpha^6 +$
$74\,638\,559\,897\,828\,248\,664\,819\,619\,987\,477\,541\,945\,344\,\alpha^7 +$
$1\,365\,121\,772\,758\,889\,406\,361\,975\,817\,513\,893\,625\,856\,\alpha^8)\ \mathrm{Seq}[5+\alpha] +$
$(-52\,778\,235\,984\,846\,523\,329\,097\,656\,918\,526\,071\,020\,716\,032 -$
$64\,501\,494\,488\,056\,900\,542\,629\,329\,788\,894\,135\,692\,820\,480\,\alpha -$
$34\,724\,501\,660\,699\,004\,906\,438\,902\,413\,811\,995\,002\,273\,792\,\alpha^2 -$
$10\,746\,920\,005\,726\,085\,338\,263\,669\,816\,779\,087\,850\,504\,192\,\alpha^3 -$
$2\,089\,401\,050\,396\,611\,719\,653\,237\,940\,170\,751\,700\,107\,264\,\alpha^4 -$
$261\,034\,694\,280\,336\,096\,121\,376\,287\,674\,697\,770\,860\,544\,\alpha^5 -$
$20\,442\,511\,124\,673\,707\,455\,959\,822\,834\,794\,030\,432\,256\,\alpha^6 -$
$916\,460\,910\,005\,176\,557\,899\,370\,835\,718\,675\,890\,176\,\alpha^7 -$
$17\,986\,272\,310\,903\,846\,816\,671\,667\,502\,362\,656\,768\,\alpha^8)\ \mathrm{Seq}[6+\alpha] +$
$(1\,048\,241\,608\,139\,004\,126\,801\,123\,879\,435\,198\,781\,194\,240 +$
$1\,096\,523\,044\,229\,137\,591\,636\,209\,311\,492\,319\,177\,867\,264\,\alpha +$
$502\,483\,518\,474\,841\,486\,190\,896\,225\,121\,701\,233\,426\,432\,\alpha^2 +$
$131\,841\,296\,561\,223\,271\,034\,561\,082\,320\,937\,371\,566\,080\,\alpha^3 +$
$21\,678\,855\,366\,109\,987\,137\,873\,513\,889\,942\,248\,357\,888\,\alpha^4 +$
$2\,289\,292\,202\,221\,334\,778\,014\,866\,102\,813\,074\,653\,184\,\alpha^5 +$
$151\,731\,925\,148\,495\,278\,830\,633\,633\,775\,746\,023\,424\,\alpha^6 +$
$5\,775\,268\,570\,467\,146\,796\,542\,588\,786\,036\,441\,088\,\alpha^7 +$
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$(2\,608\,894\,283\,215\,912\,975\,618\,410\,102\,572\,263\,145\,472 +$
$1\,616\,795\,453\,138\,378\,571\,557\,540\,817\,330\,663\,063\,552\,\alpha +$
$327\,616\,663\,513\,807\,904\,470\,525\,387\,480\,932\,286\,464\,\alpha^2 -$
$873\,589\,559\,523\,801\,551\,554\,650\,493\,409\,034\,240\,\alpha^3 -$
$11\,375\,650\,338\,286\,106\,535\,535\,280\,323\,773\,333\,504\,\alpha^4 -$

$2\,110\,778\,990\,692\,750\,151\,126\,063\,557\,153\,652\,736\,\alpha^5 -$
$184\,410\,301\,249\,112\,272\,763\,157\,224\,846\,524\,416\,\alpha^6 - 8\,198\,237\,719\,900\,262\,300\,321\,948\,425\,519\,104\,\alpha^7 -$
$149\,697\,886\,463\,404\,317\,932\,617\,973\,366\,784\,\alpha^8)$ Seq[8 + α] +
$(-60\,784\,977\,973\,911\,336\,417\,455\,240\,884\,588\,118\,016 - 50\,604\,534\,417\,001\,248\,493\,972\,281\,476\,297\,785\,344\,\alpha - 18\,364\,880\,128\,939\,667\,288\,013\,056\,370\,297\,274\,368\,\alpha^2 -$
$3\,793\,122\,059\,759\,621\,935\,706\,090\,408\,477\,982\,720\,\alpha^3 - 487\,433\,515\,130\,127\,581\,954\,429\,612\,206\,325\,760\,\alpha^4 - 39\,881\,853\,439\,127\,215\,750\,555\,055\,217\,967\,104\,\alpha^5 -$
$2\,027\,413\,953\,997\,564\,608\,311\,335\,638\,269\,952\,\alpha^6 - 58\,489\,250\,532\,075\,913\,349\,682\,400\,591\,872\,\alpha^7 -$
$732\,231\,540\,023\,730\,620\,969\,986\,818\,048\,\alpha^8)$ Seq[9 + α] +
$(143\,253\,210\,914\,077\,584\,461\,892\,945\,482\,612\,736 + 117\,390\,434\,023\,857\,677\,801\,656\,082\,944\,229\,376\,\alpha +$
$41\,988\,373\,493\,244\,944\,351\,648\,456\,537\,800\,704\,\alpha^2 + 8\,563\,309\,433\,179\,682\,998\,044\,147\,681\,591\,296\,\alpha^3 +$
$1\,089\,279\,680\,902\,487\,658\,272\,936\,091\,975\,680\,\alpha^4 + 88\,503\,747\,091\,256\,203\,811\,985\,792\,434\,176\,\alpha^5 +$
$4\,485\,778\,290\,470\,469\,550\,068\,465\,664\,000\,\alpha^6 + 129\,679\,193\,329\,108\,045\,683\,807\,485\,952\,\alpha^7 +$
$1\,637\,147\,560\,168\,901\,326\,135\,099\,392\,\alpha^8)$ Seq[10 + α] +
$(-58\,193\,026\,228\,357\,038\,079\,242\,475\,143\,168 - 51\,098\,231\,803\,407\,605\,582\,884\,239\,310\,848\,\alpha -$
$19\,138\,069\,216\,643\,015\,058\,642\,882\,789\,376\,\alpha^2 - 4\,020\,644\,083\,315\,968\,732\,595\,381\,862\,400\,\alpha^3 -$
$520\,534\,653\,627\,339\,724\,629\,150\,269\,440\,\alpha^4 - 42\,656\,902\,349\,499\,267\,252\,561\,641\,472\,\alpha^5 -$
$2\,165\,540\,502\,734\,494\,733\,562\,806\,272\,\alpha^6 - 62\,368\,598\,737\,268\,407\,957\,192\,704\,\alpha^7 -$
$781\,156\,297\,810\,381\,520\,240\,640\,\alpha^8)$ Seq[11 + α] +
$(-89\,019\,109\,694\,327\,271\,526\,225\,674\,240 - 58\,188\,866\,980\,847\,185\,951\,333\,548\,032\,\alpha -$
$16\,610\,124\,212\,723\,257\,407\,541\,608\,448\,\alpha^2 - 2\,704\,024\,225\,155\,393\,409\,339\,359\,232\,\alpha^3 -$
$274\,541\,232\,572\,280\,640\,171\,409\,408\,\alpha^4 - 17\,798\,808\,305\,108\,522\,215\,931\,904\,\alpha^5 -$
$719\,410\,062\,551\,481\,021\,628\,416\,\alpha^6 - 16\,570\,995\,465\,074\,189\,008\,896\,\alpha^7 -$
$166\,498\,086\,762\,886\,201\,344\,\alpha^8)$ Seq[12 + α] +
$(60\,803\,277\,620\,001\,284\,299\,948\,032 + 39\,428\,846\,812\,699\,916\,145\,000\,448\,\alpha +$
$11\,180\,996\,921\,120\,508\,276\,899\,840\,\alpha^2 + 1\,810\,953\,673\,723\,168\,488\,947\,712\,\alpha^3 +$
$183\,235\,931\,956\,946\,612\,781\,056\,\alpha^4 + 11\,860\,125\,063\,327\,012\,356\,096\,\alpha^5 + 479\,559\,881\,836\,404\,408\,320\,\alpha^6 + 11\,075\,248\,290\,023\,866\,368\,\alpha^7 + 111\,850\,497\,389\,887\,488\,\alpha^8)$ Seq[13 + α] +
$(-1\,734\,792\,653\,693\,830\,004\,736 - 949\,019\,070\,255\,906\,578\,432\,\alpha - 226\,524\,572\,171\,763\,769\,344\,\alpha^2 -$
$30\,803\,470\,801\,376\,100\,352\,\alpha^3 - 2\,608\,940\,417\,614\,004\,224\,\alpha^4 - 140\,860\,855\,192\,928\,256\,\alpha^5 -$
$4\,731\,649\,360\,265\,216\,\alpha^6 - 90\,341\,108\,490\,240\,\alpha^7 - 749\,920\,296\,960\,\alpha^8)$ Seq[14 + α] +
$(1\,369\,112\,633\,033\,966\,016 + 749\,960\,867\,137\,851\,264\,\alpha + 179\,694\,158\,433\,369\,216\,\alpha^2 +$
$24\,598\,615\,258\,051\,968\,\alpha^3 + 2\,104\,205\,904\,948\,672\,\alpha^4 + 115\,177\,607\,830\,656\,\alpha^5 +$
$3\,939\,594\,524\,544\,\alpha^6 + 76\,987\,798\,272\,\alpha^7 + 658\,107\,072\,\alpha^8)$ Seq[15 + α] +
$(2\,859\,422\,318\,592 + 1\,452\,813\,680\,640\,\alpha + 322\,925\,288\,448\,\alpha^2 + 41\,014\,614\,144\,\alpha^3 +$
$3\,255\,650\,202\,\alpha^4 + 165\,386\,235\,\alpha^5 + 5\,250\,777\,\alpha^6 + 95\,256\,\alpha^7 + 756\,\alpha^8)$ Seq[16 + α]

Initial values of {r(0), r(2), r(4), ...}

In[ ]:=
```
SeqListIni = {};

MAX = 20;

For[n = 0, n ≤ MAX, n++,
  coord = Select[Tuples[Table[i, {i, 0, n}], NN], Total[#] == n &];
  size = Length@coord;
  p = Sum[Multinomial[Sequence @@ (2 coord[[i]])] *
     Product[Binomial[2 n - 2 coord[[i, j]], n - coord[[i, j]]], {j, 1, NN}], {i, 1, size}];
  SeqListIni = Append[SeqListIni, p];
 ];

SeqListIni

seq[n_] := SeqListIni[[n + 1]];
```

Out[ ]=
```
{1, 32, 6048, 2 451 200, 1 391 236 000, 921 422 380 032, 663 895 856 219 904, 505 041 413 866 868 736,
 399 445 932 990 555 902 880, 325 440 143 503 901 735 429 120, 271 445 584 301 606 582 663 031 808,
 230 773 066 339 125 955 854 130 661 376, 199 326 200 240 673 646 611 787 771 995 904,
 174 478 237 021 099 598 812 491 315 604 889 600, 154 480 035 620 813 053 446 642 174 412 128 768 000,
 138 129 336 609 134 098 952 004 475 839 318 761 472 000,
 124 577 089 053 969 968 356 059 653 140 361 638 344 938 400,
 113 209 463 052 287 193 655 237 025 876 331 530 870 707 737 600,
 103 573 496 015 054 055 969 039 980 718 499 533 706 000 571 520 000,
 95 328 837 240 197 678 160 114 853 748 204 677 385 026 223 109 120 000,
 88 215 610 025 056 975 283 519 690 346 309 846 200 279 286 296 474 496 000}
```

Verify recurrence by initial values

In[ ]:=
```
Table[SeqNormalized /. {Seq → seq, α → n}, {n, 0, MAX - RecNormalizedOrder}]
```

Out[ ]=
```
{0, 0, 0, 0, 0}
```

Generate more terms in the sequence
SeqList[[n]] = r(2n)

In[ ]:=
```
Bound = 5000;

SeqList = UnrollRecurrence[SeqNormalized, Seq[α], SeqListIni, Bound];

seq[n_] := SeqList[[n + 1]];
```

Asymptotic estimation of SeqList[[n]] = r(2n)

In[ ]:=
```
<< RISC`Asymptotics`
```

Asymptotics Package version 0.3
written by Manuel Kauers
Copyright Research Institute for Symbolic Computation (RISC),

Johannes Kepler University, Linz, Austria

In[ ]:=
```
AsyList = Asymptotics[SeqNormalized, Seq[α]];
N[AsyList]
```

Out[ ]=

$$\left\{\frac{16.^{\alpha}}{\alpha^2}, \frac{256.^{\alpha}}{\alpha^2}, \frac{1024.^{\alpha}}{\alpha^3}, \frac{1024.^{\alpha}}{\alpha^2}, \frac{(-871845.)^{\alpha}}{\alpha^9}, \frac{(-2844.77)^{\alpha}}{\alpha^9}, \frac{(-376.522)^{\alpha}}{\alpha^9}, \frac{(-83.424)^{\alpha}}{\alpha^9}, \frac{(-14.7166)^{\alpha}}{\alpha^9}, \frac{0.381565^{\alpha}}{\alpha^9}, \frac{9.72218^{\alpha}}{\alpha^9}, \frac{2293.66^{\alpha}}{\alpha^9}, \frac{(-80.3841-13300.8\,i)^{\alpha}}{\alpha^9}, \frac{(-80.3841+13300.8\,i)^{\alpha}}{\alpha^9}, \frac{(94.5931-184.858\,i)^{\alpha}}{\alpha^9}, \frac{(94.5931+184.858\,i)^{\alpha}}{\alpha^9}\right\}$$

In[•]:=
```
Ind = Reverse[Table[Floor[Bound / i], {i, 1, 3}]]
Table[N[seq[Ind[[i]]] / (AsyList[[1]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
Table[N[seq[Ind[[i]]] / (AsyList[[2]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
Table[N[seq[Ind[[i]]] / (AsyList[[3]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
Table[N[seq[Ind[[i]]] / (AsyList[[4]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
Table[N[seq[Ind[[i]]] / (AsyList[[11]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
Table[N[seq[Ind[[i]]] / (AsyList[[12]] /. {α → Ind[[i]]})], {i, 1, Length@Ind}]
```

Out[•]= {1666, 2500, 5000}

Out[•]= $\{2.806687457612096 \times 10^{3007}, 6.343600724639624 \times 10^{4513}, 1.787780641892824 \times 10^{9029}\}$

Out[•]= $\{2.422718463768892 \times 10^{1001}, 3.179649140402995 \times 10^{1503}, 4.491598734476526 \times 10^{3008}\}$

Out[•]= {37.5001, 56.2783, 112.568}

Out[•]= {0.0225091, 0.0225113, 0.0225136}

... General: $\frac{1}{9.72218^{1666}}$ is too small to represent as a normalized machine number; precision may be lost.

... General: $\frac{1}{9.72218^{2500}}$ is too small to represent as a normalized machine number; precision may be lost.

... General: $\frac{1}{9.72218^{5000}}$ is too small to represent as a normalized machine number; precision may be lost.

... General: Further output of General::munfl will be suppressed during this calculation.

Out[•]= {0., 0., 0.}

... General: $\frac{1}{2293.66^{1666}}$ is too small to represent as a normalized machine number; precision may be lost.

... General: $\frac{1}{2293.66^{2500}}$ is too small to represent as a normalized machine number; precision may be lost.

... General: $\frac{1}{2293.66^{5000}}$ is too small to represent as a normalized machine number; precision may be lost.

... General: Further output of General::munfl will be suppressed during this calculation.

Out[•]= {0., 0., 0.}

Approximate Polya number

In[ ]:=
$$\text{AtOne} = N\left[\text{Sum}\left[\text{seq}[n] * \left(\frac{1}{2^{\text{MM}}\,\text{Binomial}[\text{NN}, \text{MM}]}\right)^{2\,n}, \{n, 0, \text{Bound}\}\right], 11\right]$$

$$N\left[1 - \frac{1}{\text{AtOne}}, 10\right]$$

Out[ ]= 1.0452834156

Out[ ]= 0.04332166274