Generating Artificial, BigBird like, Sparse Matrices

By Shane Donnelly

Previous work

What is BigBird?

BigBird, is a sparse-attention based transformer which extends Transformer based models, such as BERT to much longer sequences. Moreover, BigBird comes along with a theoretical understanding of the capabilities of a complete transformer that the sparse model can handle.

As a consequence of the capability to handle longer context, BigBird drastically improves performance on various NLP tasks such as question answering and summarization.

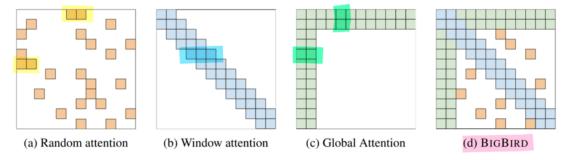


Figure 1: Building blocks of the attention mechanism used in BIGBIRD. White color indicates absence of attention. (a) random attention with r = 2, (b) sliding window attention with w = 3 (c) global attention with r = 2. (d) the combined BIGBIRD model.

Introduction

My goal was to generate python functions, using only math, numpy and pure python, with a algorithm approache, simplicity and easy to port to C. For each attention mask (a, b, c and d), i have written the following features:

- generate a boolean mask, using the corresponding parameter
- same, but based on a given sparsity (by having a function : given sparsity -> corresponding parameter)
- generate artificial matrix of only ones and zero based on the mask
- a test function which shows how to use it and the output

Code

Imports

```
In [172... import numpy as np import matplotlib.pyplot as plt import math
```

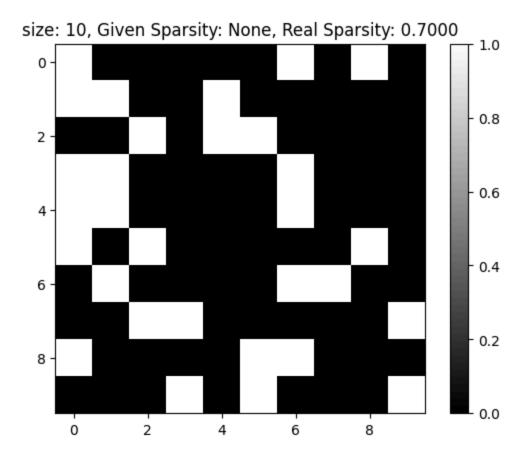
UTILS

```
def get_nb_non_zero(matrix):
In [173...
                     return np.count_nonzero(matrix)
                 def get_density(matrix, size):
In [174...
                     return float(get_nb_non_zero(matrix)) / float(size * size)
                 def get_sparsity(matrix, size):
In [175...
                     return 1.0 - get density(matrix, size)
                def show_matrix_infos(matrix, size, given_sparsity = -1.0): # -1.0 correspond to None.
In [176...
                     # conditions : shape(matrix) = (size, size)
                     real_sparsity = get_sparsity(matrix, size)
                     given_text = "None" if given_sparsity < 0 else f"{given_sparsity:.2f}"
text = f"size: {size}, Given_Sparsity: {given_text}, Real_Sparsity: {real_sparsity:.4f}"
                     plt.title(label=text)
                     plt.imshow(matrix, cmap='gray', interpolation='nearest', vmin=0, vmax=1)
                     plt.colorbar()
                     plt.show()
```

RANDOM ATTENTION

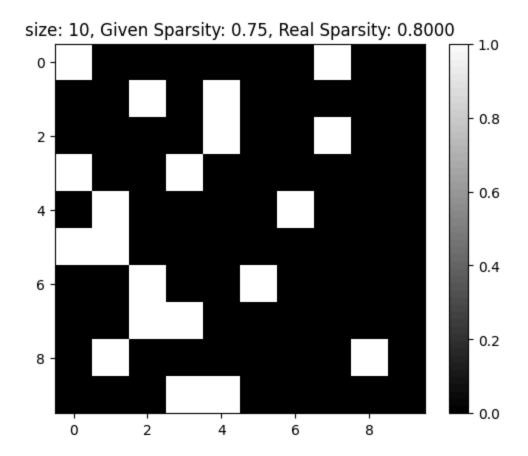
By number of non-zeros per row

```
def get random attention mask(size, nz per row):
In [177...
                    # conditions : nz per row <= size
                    rng = np.random.default_rng(121263137472525314065)
                    mask = rng.multivariate_hypergeometric([1]*size, nz_per_row, size=size).astype(bool)
                    return mask
               def generate_matrix_with_random_attention_mask(size, nz_per_row):
In [178...
                    # conditions : nz_per_row <= size
                    matrix = np.ones((size, size))
                    mask = get_random_attention_mask( size=size, nz_per_row=nz_per_row)
                    matrix[\sim mask] = 0
                    return matrix
                \textbf{def} \ \ \mathsf{test\_generate\_matrix\_with\_random\_attention\_mask()}:
In [179...
                    size = 10
                    matrix = generate_matrix_with_random_attention_mask(size=size, nz_per_row=nz_per_row)
                    show matrix infos(matrix=matrix, size=size)
                test generate matrix with random attention mask()
```

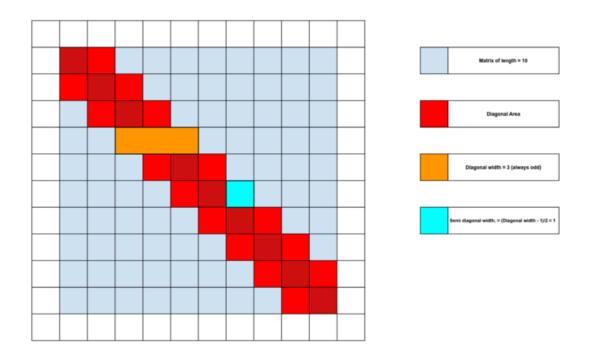


By sparsity

```
def best_nz_per_row_from_sparsity(size, sparsity):
In [180...
                      # conditions : 0 <= sparsity <= 1
return round(size * (1 - sparsity))</pre>
                  \textbf{def} \ \ \texttt{get\_random\_attention\_mask\_with\_sparsity}(\texttt{size}, \ \texttt{sparsity}):
In [181...
                      # conditions : 0 <= sparsity <=
                       nz_per_row=best_nz_per_row_from_sparsity(size=size, sparsity=sparsity)
                       return get_random_attention_mask( size=size, nz_per_row=nz_per_row)
                  \textbf{def} \ \ generate\_matrix\_with\_random\_attention\_mask\_with\_sparsity(size, \ sparsity):
In [182...
                      # conditions : 0 <= sparsity <= 1
matrix = np.ones((size, size))</pre>
                      mask = get_random_attention_mask_with_sparsity(size=size, sparsity= sparsity)
                      matrix[\sim mask] = 0
                       return matrix
                 def test_generate_matrix_with_random_attention_mask_with_sparsity():
In [194...
                       size = 10
                      sparsity = 0.75
                      \verb|matrix = generate_matrix_with_random_attention_mask_with_sparsity| (size=size, sparsity=sparsity)|
                      show_matrix_infos(matrix=matrix, size=size, given_sparsity=sparsity)
                  test\_generate\_matrix\_with\_random\_attention\_mask\_with\_sparsity()
```



WINDOW ATTENTION



Utils

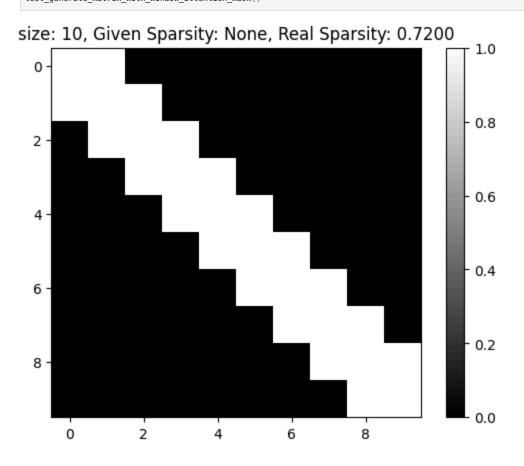
```
In [184...

def diagonal_area(size,diagonal_width):
    # conditions : size
    if(diagonal_width == 0):
        return 0

else:
    n = size
    #semi diagonal_widht
    sdw = diagonal_width // 2 # (diagonal_width // 2 - 1 because is odd)
    da = n * (1 + 2 * sdw) - sdw * (sdw + 1)
    return da
```

By diagonal width

```
def get_window_attention_mask (size, diagonal_width):
In [185...
                   # conditions : shape(matrix) = (size, size), \theta \le diagonal_width \le 2*size - 1 (cover full matrix), diagonal_width is \theta
                   mask = np.zeros(shape=(size, size), dtype=bool)
                   if (diagonal_width > 0):
                       sdw = diagonal width // 2
                       if diagonal_width == 1:
                          mask = np.fromfunction(lambda i, j: j == i,shape=(size, size), dtype=int)
                          \verb|mask| = \verb|np.fromfunction| (lambda i, j: np.abs(i - j) <= \verb|sdw|, \verb|shape=(size, size)|, dtype=int|)
              In [186...
                   matrix = np.ones((size, size))
                   mask = get_window_attention_mask( size= size, diagonal_width=diagonal_width)
                   matrix[\sim mask] = 0
                   return matrix
               \label{lem:def} \textbf{def} \ \ \textbf{test\_generate\_matrix\_with\_window\_attention\_mask():}
In [187...
                   size = 10
                   diagonal_width = 3
                   matrix = generate_matrix_with_window_attention_mask(size=size, diagonal_width=diagonal_width)
                   show_matrix_infos(matrix,size)
               test_generate_matrix_with_window_attention_mask()
```

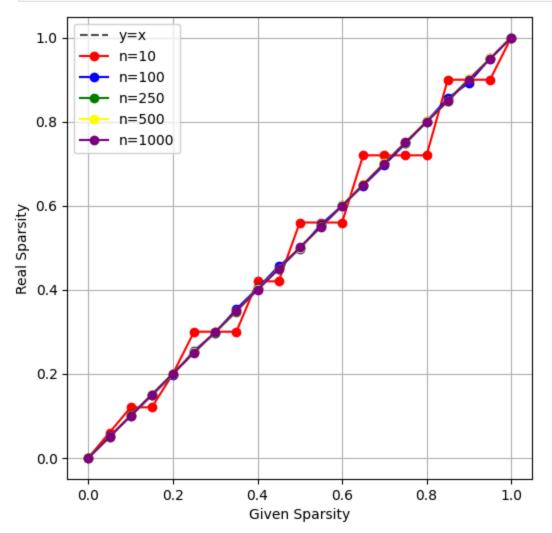


By sparsity

```
def best_diagonal_width_from_sparsity(size, sparsity):
In [188...
                     n = size
                     density = 1.0 - sparsity
                     # ideal diagonal aera
                    da = n * n * density
                    # from this point, all is explained in the related document
                    b = 2 * n - 1
                     c = n - da
                     det = b * b - 4 * a * c
                     x = (-b + math.sqrt(det))/(2 * a)
                     sdw = round(x)
                    dw = 2 * sdw + 1
                     if(dw < 0) : dw = 0
                    elif(dw > 2*n - 1): dw = 2*n - 1
# print(f"For matrix of size: {n} and given sparsity: {sparsity}, ideal semi diagonal width is : {x}, chosen dw is {dw}
                     return dw
```

In [190...

test function hided for render
test_diagonal_width_from_sparsity()



```
In [191... def get_window_attention_mask_with_sparsity( size, sparsity):
# conditions : 0 <= sparsity <= 1
dw = best_diagonal_width_from_sparsity(size, sparsity)
return get_window_attention_mask( size=size, diagonal_width=dw)
```

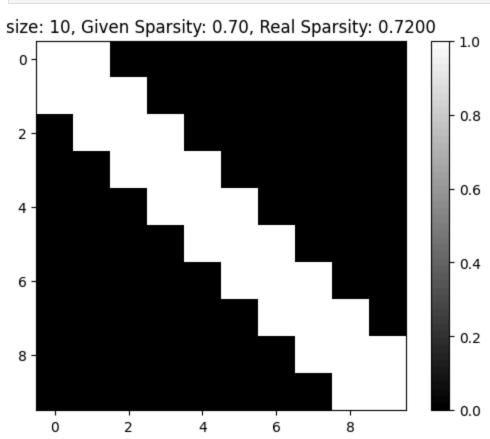
```
In [192... def generate_matrix_with_window_attention_mask_with_sparsity(size, sparsity): # conditions : \theta <= diagonal_width <= 2*size - 1 (cover full matrix), diagonal_width is odd
```

```
matrix = np.ones((size, size))
mask = get_window_attention_mask_with_sparsity( size= size, sparsity=sparsity)
matrix[~mask] = 0
return matrix
```

```
In [193...
```

```
def test_generate_matrix_with_window_attention_mask_with_sparsity():
    size = 10
    sparsity = 0.7
    matrix = generate_matrix_with_window_attention_mask_with_sparsity(size=size, sparsity=sparsity)
    show_matrix_infos(matrix,size, sparsity)

test_generate_matrix_with_window_attention_mask_with_sparsity()
```



GLOBAL ATTENTION

Utils

```
In [127...

def global_attention_aera(size,global_attention_width):
    w = global_attention_width
    n = size
    return (2 * w * n) - (w * w)

print(global_attention_aera(10,2))
```

36

```
In [128...

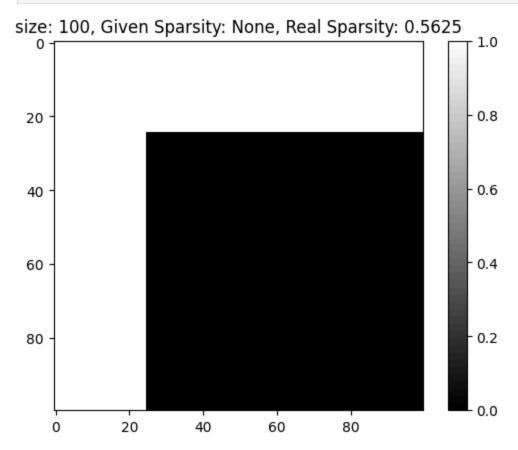
def get_global_attention_mask( size, global_width):
    mask = np.zeros(shape=(size,size), dtype=bool)
    mask[:global_width,:] = True
    mask[global_width : , : global_width] = True
    return mask
```

```
In [129...
def generate_matrix_with_global_attention_mask(size, global_width):
    matrix = np.ones((size, size))
    mask = get_global_attention_mask( size=size, global_width=global_width)
    matrix[-mask] = 0
    return matrix

In [130...

def test_generate_matrix_with_global_attention_mask():
    size = 100
    global_width = 25
    matrix = generate_matrix_with_global_attention_mask(size=size, global_width=global_width)
    show_matrix_infos(matrix=matrix, size=size)

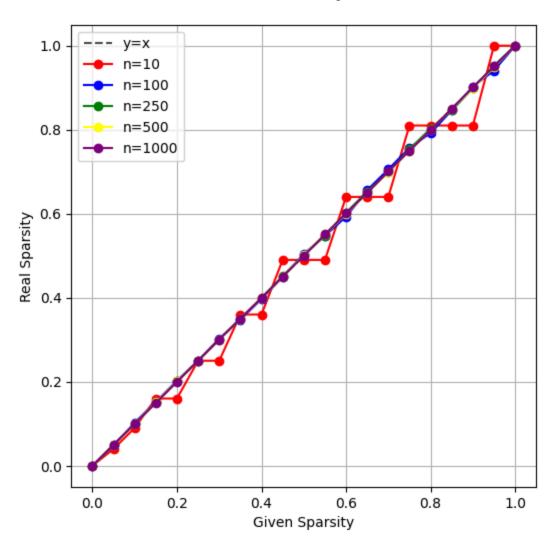
test_generate_matrix_with_global_attention_mask()
```



```
In [131...

def best_global_width_from_sparsity(size, sparsity):
    n = size
    density = 1.0 - sparsity
    # ideal diagonal aera
    ga = n * n * density
    # same as window mask but easier
    a = -1
    b = 2 * n
    c = - ga
    det = b * b - 4 * a * c
    x = (-b + math.sqrt(det))/(2 * a)
    gw = round(x)
    if(gw < 0) : gw = 0
    elif(gw > n * n ): gw = n * n
    return gw
```

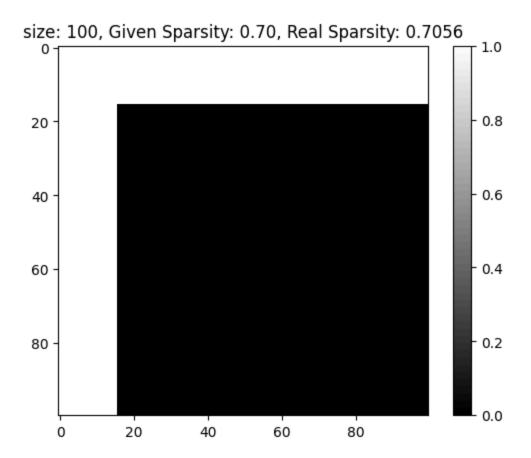
```
In [133... # test function hided for render test_global_width_from_sparsity()
```



```
In [134... def get_global_attention_mask_with_sparsity( size, sparsity):
# conditions: 0 <= sparsity <= 1
gw = best_global_width_from_sparsity(size, sparsity)
return get_global_attention_mask( size=size, global_width=gw)

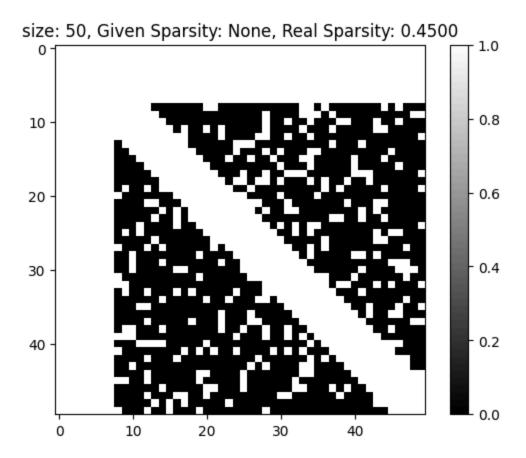
In [135... def generate_matrix_with_global_attention_mask_with_sparsity(size, sparsity):
matrix = np.ones((size, size))
mask = get_global_attention_mask_with_sparsity(size=size, sparsity=sparsity)
matrix[-mask] = 0
return matrix

In [136... def test_generate_matrix_with_global_attention_mask_with_sparsity():
size = 100
sparsity = 0.7
matrix = generate_matrix_with_global_attention_mask_with_sparsity(size=size, sparsity=sparsity)
show_matrix_infos(matrix=matrix, size=size, given_sparsity=sparsity)
test_generate_matrix_with_global_attention_mask_with_sparsity()
```

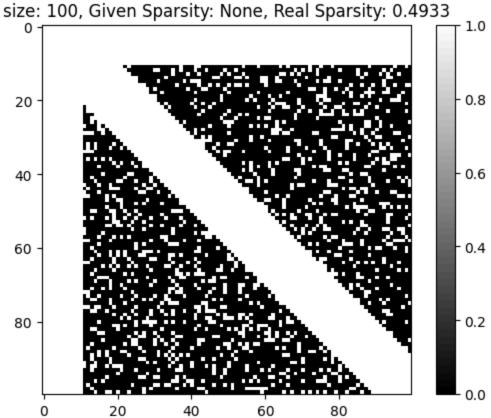


BIG BIRD (combination of all above)

```
def get_big_bird_mask(size, nz_per_row, diagonal_width, global_width):
In [137...
                   am = get_random_attention_mask( size= size, nz_per_row=nz_per_row)
                   wm = get_window_attention_mask( size= size, diagonal_width=diagonal_width)
                   gm = get_global_attention_mask( size=size, global_width= global_width)
                   total_mask = am | wm | gm
                   return total_mask
               def generate_big_bird(size, nz_per_row, diagonal_width, global_width ):
In [138...
                   matrix = np.ones((size, size))
                   \verb|mask = get_big_bird_mask( size=size, nz_per_row= nz_per_row, diagonal_width=diagonal_width, global_width= global_width)|
                   matrix[\sim mask] = 0
                   return matrix
               def test_generate_big_bird():
In [139...
                   size = 50
                   nz_per_row = 10
                   diagonal_width = 8
                   global_width = 8
                   matrix = generate_big_bird(size=size,nz_per_row=nz_per_row, diagonal_width=diagonal_width, global_width=global_width)
                   show_matrix_infos(matrix=matrix, size= size)
               test_generate_big_bird()
```



```
def get_big_bird_mask_with_sparsity( size, random_sparsity, window_sparsity, global_sparsity):
    am = get_random_attention_mask_with_sparsity( size= size, sparsity=random_sparsity)
In [140...
                      wm = get_window_attention_mask_with_sparsity( size= size,sparsity=window_sparsity )
                      gm = get_global_attention_mask_with_sparsity(size=size, sparsity=global_sparsity)
                      total_mask = am | wm | gm
                      return total_mask
                 def generate_big_bird_with_sparsity(size, random_sparsity, window_sparsity, global_sparsity):
    matrix = np.ones((size, size))
In [141...
                      mask = get_big_bird_mask_with_sparsity(size, random_sparsity, window_sparsity, global_sparsity)
                      matrix[~mask] =
                      return matrix
                 def test_generate_big_bird_with_sparsity():
    size = 100
In [142...
                      random sparsity = 0.8
                      window_sparsity = 0.8
                      global_sparsity = 0.8
                      matrix = generate_big_bird_with_sparsity(size=size,random_sparsity=random_sparsity, window_sparsity=window_sparsity, gl
                      show matrix infos(matrix=matrix, size= size)
                 test_generate_big_bird_with_sparsity()
```



```
def adjust_total_sparsity(total_sparsity):
In [143...
                    x = total_sparsity
                     # degree = 3
                    \# a = 2.61815675
                    \# b = -4.77052715
                     \# c = 2.98999146
                    \# d = 0.19945692
                    \# res = a * (x ** 3) + b * (x ** 2) + c * x
                    a = 24.08862473
                    b = -65.2963488
                    c = 64.48601296
                    d = -28.42365239
                     e = 5.98076684
                    f = 0.17082526
                    poly = a * x**5 + b * x**4 + c * x**3 + d * x**2 + e * x + f res = min(max(poly, 0.0), 1.0)
                     return res
               def get_big_bird_mask_with_total_sparsity( size, total_sparsity, adjust):
In [144...
                         total_sparsity = adjust_total_sparsity(total_sparsity)
                     random_sparsity = total_sparsity
                     window_sparsity = total_sparsity
                     global_sparsity = total_sparsity
                    total_mask = get_big_bird_mask_with_sparsity( size, random_sparsity, window_sparsity, global_sparsity) return total_mask
                def generate_big_bird_with_total_sparsity(size,total_sparsity, adjust):
In [145...
                    matrix = np.ones((size, size))
                    mask = get_big_bird_mask_with_total_sparsity(size, total_sparsity, adjust)
                    matrix[\sim mask] = 0
                     return matrix
                def find_approximation():
In [146...
                     sparsity_values = [x / 100.0 \text{ for } x \text{ in } range(0, 101, 5)]
                     size = 2000
                    given_sparsity = []
real_sparsity = []
                     for sparsity in sparsity_values:
                         matrix = generate_big_bird_with_total_sparsity(size, sparsity, adjust=False)
                         real_sp = get_sparsity(matrix, size)
```

```
given_sparsity.append(sparsity)
    real_sparsity.append(real_sp)

sort_idx = np.argsort(real_sparsity)
    real_sorted = np.array(real_sparsity)[sort_idx]
    given_sorted = np.array(given_sparsity)[sort_idx]

coeffs = np.polyfit(real_sorted, given_sorted, 5)

print(f"a = {coeffs[0]}")
    print(f"b = {coeffs[1]}")
    print(f"c = {coeffs[2]}")
    print(f"d = {coeffs[3]}")
    print(f"e = {coeffs[4]}")
    print(f"f = {coeffs[5]}")
    print(f"f = {coeffs[5]}")
    print(f"poly = a * x**5 + b * x**4 + c * x**3 + d * x**2 + e * x + f")

find_approximation()

a = 24.094159495050001

b = 65.200361207770027
```

```
a = 24.09415949505001

b = -65.30036129778037

c = 64.47572088736197

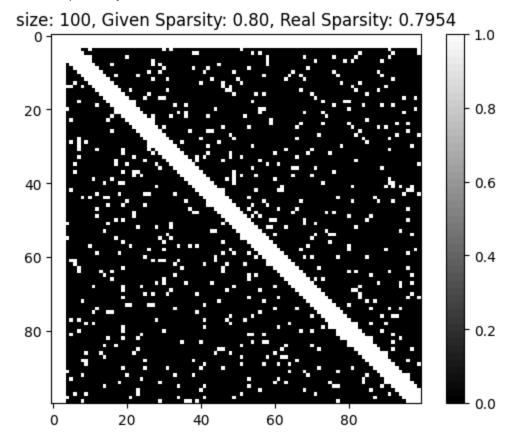
d = -28.411447168124422

e = 5.977174484231452

f = 0.17098672126501577

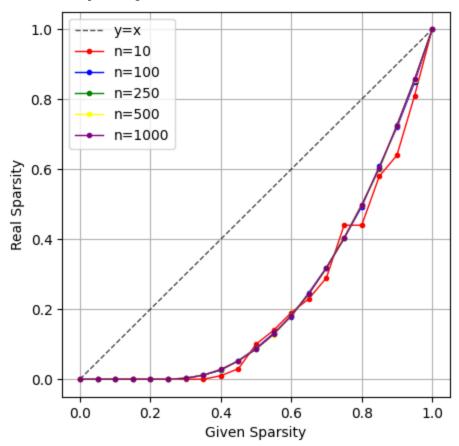
poly = a * x**5 + b * x**4 + c * x**3 + d * x**2 + e * x + f
```

Given sparsity = 0.8

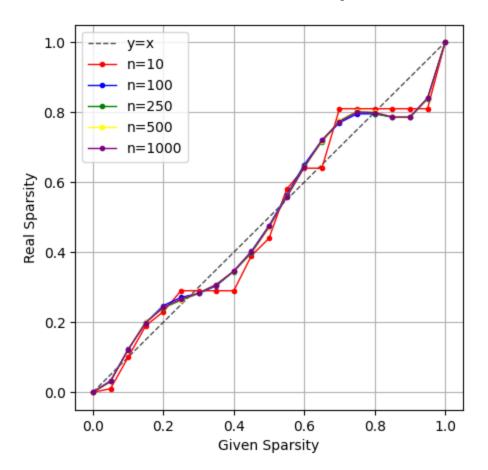


```
test_adjust_total_sparsity(False)
print("When adjusting the given sparsity : ")
test_adjust_total_sparsity(True)
```

Without adjusting:



When adjusting the given sparsity :



C code:

```
#include <omp.h>
#include <stdio.h>
#include <stdlib.h>
#include <stdbool.h>
#include <math.h>
//UTILS
static inline int min(int a, int b) {
    return a < b ? a : b;</pre>
}
static inline int max(int a, int b) {
    return a > b ? a : b;
}
void print_matrix_int(int* matrix, int size){
    int row, columns;
    for (row=0; row<size; row++)</pre>
    {
        for(columns=0; columns<size; columns++)</pre>
        {
             if(matrix[row * size + columns] == 1){
                 printf("\033[32m1 \033[0m");
```

```
} else {
                 printf("\033[31m0 \033[0m");
            }
        }
        printf("\n");
    printf("\n \n");
}
int* init matrix int( int size, int value ){
    int* res = (int *)malloc(sizeof(int) * size * size) ;
    int row, columns;
    for (row=0; row<size; row++)</pre>
        for(columns=0; columns<size; columns++)</pre>
         res[row * size + columns] = value;
    return res ;
}
void print matrix bool(bool* bmask, int size){
    int row, columns;
    for (row=0; row<size; row++)</pre>
    {
        for(columns=0; columns<size; columns++)</pre>
             if (bmask[row * size + columns]){
                 printf("\033[32mT \033[0m");
             }
            else {
                 printf("\033[31mF \033[0m");
             }
        printf("\n");
    printf("\n \n");
}
bool* init matrix bool( int size, bool value ){
    bool* res = (bool *)malloc(sizeof(bool) * size * size) ;
    int row, columns;
    for (row=0; row<size; row++)</pre>
        for(columns=0; columns<size; columns++)</pre>
         res[row * size + columns] = value;
    return res ;
}
```

```
void set boolean mask from range(bool* bmask, int size, int ymin,
int ymax, int xmin, int xmax, bool value){
    int row, columns;
    for (row=ymin; row< ymax; row++)</pre>
    {
        for(columns=xmin; columns<xmax; columns++)</pre>
         bmask[row * size + columns] = value;
        }
    }
}
void apply boolean mask(int* matrix, bool* b mask, int size){
    int row, columns;
    for (row=0; row<size; row++)</pre>
        for(columns=0; columns<size; columns++)</pre>
         if(b mask[row * size + columns] == false){
            matrix[row * size + columns] = 0 ;
         }
        }
    }
}
int* get unique random number(int count, int min, int max) {
    int range = max - min;
    int candidates[range];
    int* result = malloc(count * sizeof(int));
    int i:
    for (i = 0; i < range; i++)
    candidates[i] = i + min;
    for (i = 0; i < range - 1; i++) {
        int c = rand() / (RAND MAX / (range - i) + 1);
        int t = candidates[i];
        candidates[i] = candidates[i + c];
        candidates[i + c] = t;
    }
    for (i = 0; i < count; i++){
        result[i] = candidates[i];
    }
    return result;
}
// ACTUAL FUNCTIONS
```

```
// RANDOM ATTENTION MASK
bool* get random attention mask(int size, int nz per row){
    bool* res = init matrix bool(size, false);
    int* unique random number ;
    int row, columns;
    for (row=0; row < size; row++)</pre>
    {
        unique random number = get unique random number(nz per row,
0, size);
        for(columns = 0; columns < nz per row; columns++)</pre>
            int indice = unique random number[columns] ;
            res[row * size + indice] = true;
        free(unique random number);
    }
    return res ;
}
int best nz per row from sparsity(int size, double sparsity){
    int res = round((float)size * (1.0 - sparsity));
    return res ;
}
bool* get random attention mask with sparsity(int size, double
sparsity){
    int nz per row = best nz per row from sparsity(size, sparsity);
    return get random attention mask(size, nz per row);
}
// WINDOW ATTENTION
bool* get window attention mask(int size, int diagonal width){
    bool* res = init matrix bool(size, false);
    int row, columns;
    if(diagonal width > 0){
        int sdw = diagonal width / 2;
        for (row=0; row < size; row++)</pre>
        {
            int min c = max(0, row - sdw);
            int max_c = min(size - 1, row + sdw );
            for(columns = min c; columns <= max c ; columns++)</pre>
                res[row * size + columns] = true ;
            }
        }
    }
    return res ;
}
```

```
int best diagonal width from sparsity(int size, double sparsity){
    int n = size;
    double density = 1.0 - sparsity;
    double da = n * n * density;
    double a = -1:
    double b = 2 * n - 1;
    double c = n - da;
    double det = b * b - 4 * a * c:
    double x = (-b + sqrt(det))/(2 * a);
    int sdw = round(x);
    int dw = 2 * sdw + 1;
    if(dw < 0) dw = 0;
    else if (dw > 2*n - 1) dw = 2*n - 1;
    return dw;
}
bool* get window attention mask with sparsity(int size, double
sparsity){
    int dw = best diagonal width from sparsity(size, sparsity);
    return get window attention mask( size, dw);
}
// GLOBAL ATTENTION
bool* get global attention mask( int size, int global width){
    bool* res = init matrix bool(size, false) :
    set boolean mask from range(res, size, 0 , global width, 0,
size, true);
    set boolean mask from range(res, size, global width , size , 0,
global width, true);
    return res ;
}
int best global width from sparsity(int size, double sparsity){
    int n = size;
    double density = 1.0 - sparsity;
    double ga = n * n * density;
    double a = -1;
    double b = 2 * n;
    double c = -ga;
    double det = b * b - 4 * a * c;
    double x = (-b + sqrt(det))/(2 * a);
    int gw = round(x);
    if(gw < 0) gw = 0;
    else if(gw > n * n) gw = n * n;
    return gw;
}
```

```
bool* get global attention mask with sparsity(int size, double
sparsity){
    int gw = best global width from sparsity(size, sparsity);
    return get global attention mask(size, gw);
}
// BIG BIRD
bool* combine all mask(int size, bool* random mask, bool*
window mask, bool* global mask){
    bool* res = (bool *)malloc(sizeof(bool) * size * size);
    int row, columns;
    for (row=0; row<size; row++)</pre>
        for(columns=0; columns<size; columns++)</pre>
            res[row * size + columns] = random mask[row * size +
columns] | window mask[row * size + columns] | global mask[row *
size + columns];
        }
    return res ;
}
bool* get big bird mask(int size, int nz per row, int
diagonal width, int global width){
    bool* am = get random attention mask(size, nz per row);
    bool* wm = get window attention mask(size, diagonal width);
    bool* gm = get global attention mask(size, global width);
    bool* res = combine all mask(size, am, wm, qm);
    free(am):
    free(wm);
    free(qm);
    return res;
}
bool* get big bird mask with sparsity(int size, double
random sparsity, double window sparsity, double global sparsity){
    bool* am = get random attention mask with sparsity(size,
random sparsity);
    bool* wm = get window attention mask with sparsity(size,
window sparsity);
    bool* gm = get global attention mask with sparsity(size,
global sparsity);
    bool* res = combine all mask(size, am, wm, qm);
    free(am);
    free(wm);
    free(qm);
    return res;
}
double adjust total sparsity(double total sparsity){
```

```
double x = total sparsity;
    double a = 24.08862473;
    double b = -65.2963488;
    double c = 64.48601296:
    double d = -28.42365239;
    double e = 5.98076684;
    double f = 0.17082526;
    double poly = a * pow(x, 5) + b * pow(x, 4) + c * pow(x, 3) + d
* pow(x, 2) + e * x + f;
    if(poly < 0.0){
        poly = 0.0;
    }
    else {
        if(poly > 1.0){
            poly = 1.0;
        }
    }
    return poly;
}
bool* get big bird mask with total sparsity(int size, double
total sparsity, bool adjust){
    if(adjust){
        total sparsity = adjust total sparsity(total sparsity);
    return get big bird mask with sparsity(size, total sparsity,
total sparsity, total sparsity);
int main(int argc, char* argv[])
    int size = 25;
    double total sparsity = 0.7;
    bool* b mask = get big bird mask with total sparsity(size,
total sparsity, true);
    int* big bird matrix = init matrix int(size, 1);
    apply boolean mask(big bird matrix, b mask, size);
    print matrix int(big bird matrix, size) ;
    free(big bird matrix);
    free(b mask);
}
And execution:
make
# gcc -Wall -fopenmp -o big bird big bird.c -lm
./big bird
```

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	, - , 1	1	1
1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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