Design Project #2

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Abstract

A system of three cables is used to support a 6.4 kg weight in 3-dimensional space. The cables are fixed at points A, C, and D. B is allowed to move in the xy plane within the range -0.6<x<1.0 m and -0.4<y<1.2 m. The first task of this lab is to identify the effect of varying B_x and B_y on the tension in the system's cables. The second task is to optimize point B's location for price. The price of the system should be minimized; the price function is proportional to the product of tension and the squared length of the cable.

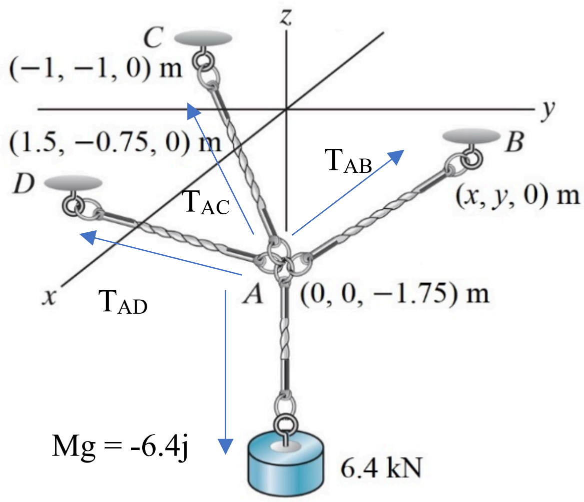


Figure 1: This figure illustrates the system. The coordinates of each of the points are provided. Note that the x and y coordinates of B are free to move in the xy plane. This figure also illustrates the forces in the system as a free body diagram.

Constraints:

1. Possible values of the x coordinates of point B are: -0.6<x<1.0 m.
2. Possible values of the y coordinates of point B are: -0.4<y<1.2 m.
3. Price must be minimized.

Methodology

The tension in all of the cables can be found by iterating solutions for each cable's tension as a function of B_x and B_y

Variables

Variable	**Description**	**Unit**
$\mathbf{B_x}$	*x coordinate of point B's fixture*	*Meter*
$\mathbf{B_y}$	*y coordinate of point B's fixture*	*Meter*
$\mathbf{T_{AC}}$	*Tension in cable spanning A to C*	*Kilo-Newton*
$\mathbf{T_{AD}}$	*Tension in cable spanning A to D*	*Kilo-Newton*
$\mathbf{T_{AB}}$	*Tension in cable spanning A to B*	*Kilo-Newton*

Derivations

It is known that the system is in static equilibrium; therefore, we hold the following laws to be true:

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0, \sum F_M = 0$$

We know that the force of the 6.4k N from the weight of the object is only in the -j direction.
By observation, we have:

$$\sum F = 0 = T_{AC} + T_{AD} + T_{AB} - 6.4\hat{\jmath}$$

We also know that the tension in each cable is equal to the tension in the vertical cable (T) multiplied by the unit vectors of each cable in the direction of its respective fixture.
Therefore, we have:

$$\sum F_x => 6.4\ kN = T_{AC}\overrightarrow{e_{AC}} + T_{AD}\overrightarrow{e_{AD}} + T_{AB}\overrightarrow{e_{AB}}$$

Where:

$$\overrightarrow{e_{AC}} = \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \frac{\begin{bmatrix}-1\\-1\\0\end{bmatrix} - \begin{bmatrix}0\\0\\-1.75\end{bmatrix}}{\sqrt{(-1)^2 + (-1)^2 + (1.75)^2}} = \begin{bmatrix}-0.\overline{4}\\-0.\overline{4}\\0.\overline{7}\end{bmatrix}$$

Likewise:

$$\overrightarrow{e_{AD}} = \begin{bmatrix}0.618853\\-0.309426\\0.721885\end{bmatrix}$$

And:

$$\overrightarrow{e_{AB}} = \frac{\begin{bmatrix}B_x\\B_y\\0\end{bmatrix} - \begin{bmatrix}0\\0\\-1.75\end{bmatrix}}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}} = \begin{bmatrix}\frac{B_x}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}}\\ \frac{B_y}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}}\\ \frac{1.75}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}}\end{bmatrix}$$

Now we can write an equation for T_{AB} in terms of B_x and B_y:

$$\begin{bmatrix}0\\0\\6.4\end{bmatrix} = T_{AC}\begin{bmatrix}-0.\overline{4}\\-0.\overline{4}\\0.\overline{7}\end{bmatrix} + T_{AD}\begin{bmatrix}0.618853\\-0.309426\\0.721885\end{bmatrix} + T_{AB}\begin{bmatrix}\frac{B_x}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}}\\ \frac{B_y}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}}\\ \frac{1.75}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}}\end{bmatrix}$$

This system can be written in standard matrix form

$$\begin{bmatrix}\frac{B_x}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}} & -0.\overline{4} & 0.618853\\ \frac{B_y}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}} & -0.\overline{4} & -0.309426\\ \frac{1.75}{\sqrt{(B_x)^2 + (B_y)^2 + (-1.75)^2}} & 0.\overline{7} & 0.721885\end{bmatrix}\begin{bmatrix}T_{AB}\\T_{AC}\\T_{AD}\end{bmatrix} = \begin{bmatrix}0\\0\\6.4\end{bmatrix}$$

Now the system is in the proper format for Matlab processing. The code used can be found in the appendix.

Price Optimization for Cable AB

In order to find the cheapest price for cable AB, the price of each possible cable configuration was computed. This was accomplished by removing all of the negative tension values computed in question 1, evaluating the price function for cable AB given by:

$$Price \propto T_{cable} * Length^2$$

And finding the minimum price in this new set of solutions. This process is performed because the negative tensions found as some solutions for cable AD and cable AC represent compressions of each respective cable. The Matlab script finds the value and index of the minimum price and prints it out as follows:

```
>> ealab2
The optimum price for cable AB is proportional to a price of $13.74.
The Bx and By coordinates of this price are (-0.07,0.73)
>> |
```

Results

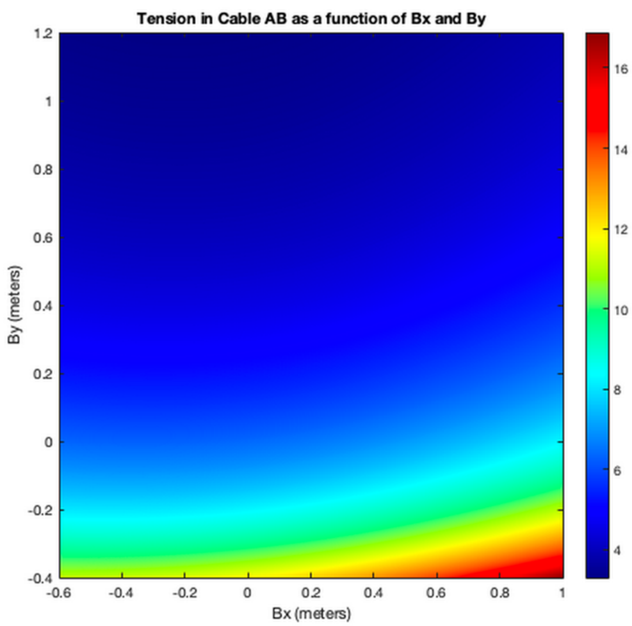


Figure 2: The first plot produced by the Matlab script illustrates the effect of varying Bx and By on the tension in cable AB. The color scale provided illustrates the tension in cable AB (kN). Potential solutions for the tension in cable AB ranged from ~2 to 17 kN. *Increasing By decreases the tension in the cable while increasing Bx increases the tension in the cable.*

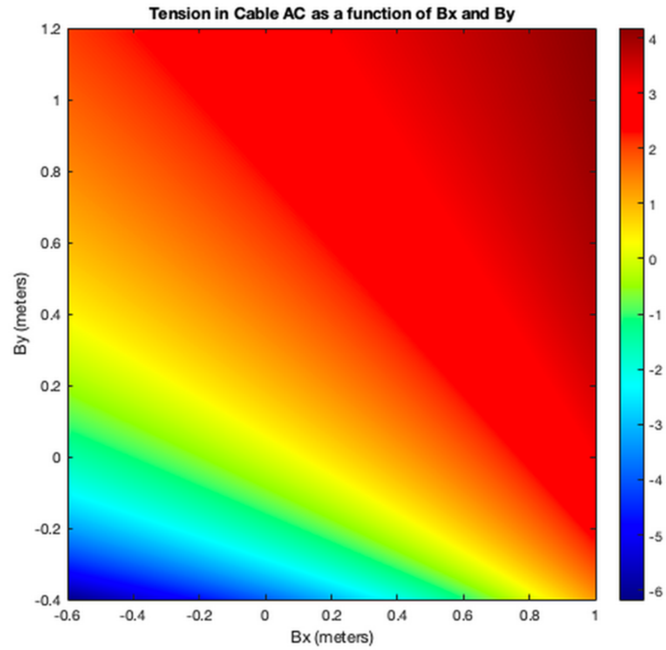


Figure 3: The next plot produced by the script illustrates the effect of varying Bx and By on the tension in cable AC. The color scale provided illustrates the tension in cable AC (kN). Potential solutions for the tension in cable AC ranged from ~ -7 to 5 kN. *Increasing By and/or Bx increases the tension in cable AC.*

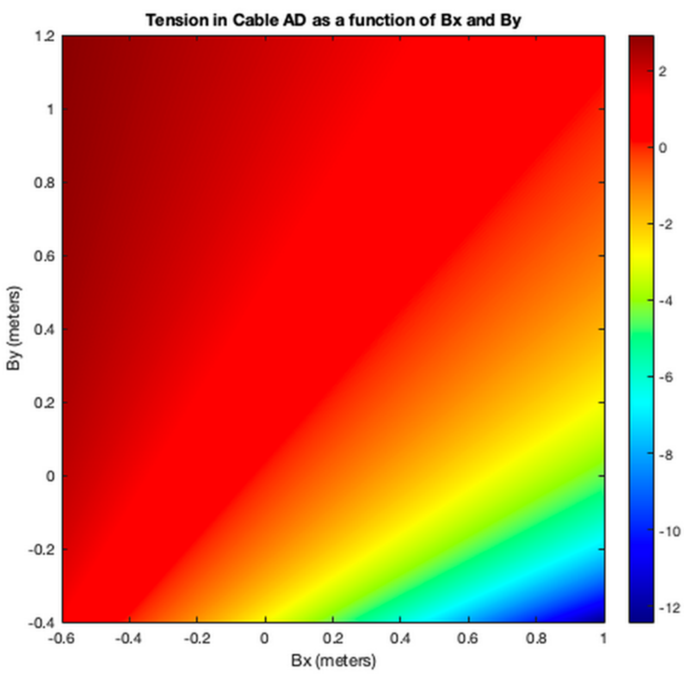


Figure 4: Plot 3 is the next plot produced by the script and represents the effect of varying Bx and By on the tension in cable AD. The color scale provided illustrates the tension in cable AD (kN). Potential solutions for the tension in cable AD ranged from ~ -13 to 3 kN. *Increasing Bx decreased the tension in cable AD while increasing By increased the tension in cable AD.*

Figure 5: The final plot produced by the script represents the optimized price for cable AB as requested in question 2. The blue region represents unfeasible solutions to the system as the at least one cable is in compression at each given value of Bx and By. The color scale illustrates the relative price of cable AB at each set of Bx and By. *From this chart, we can see that the optimized price solution is approximately Bx = 0, By = 0.7.*

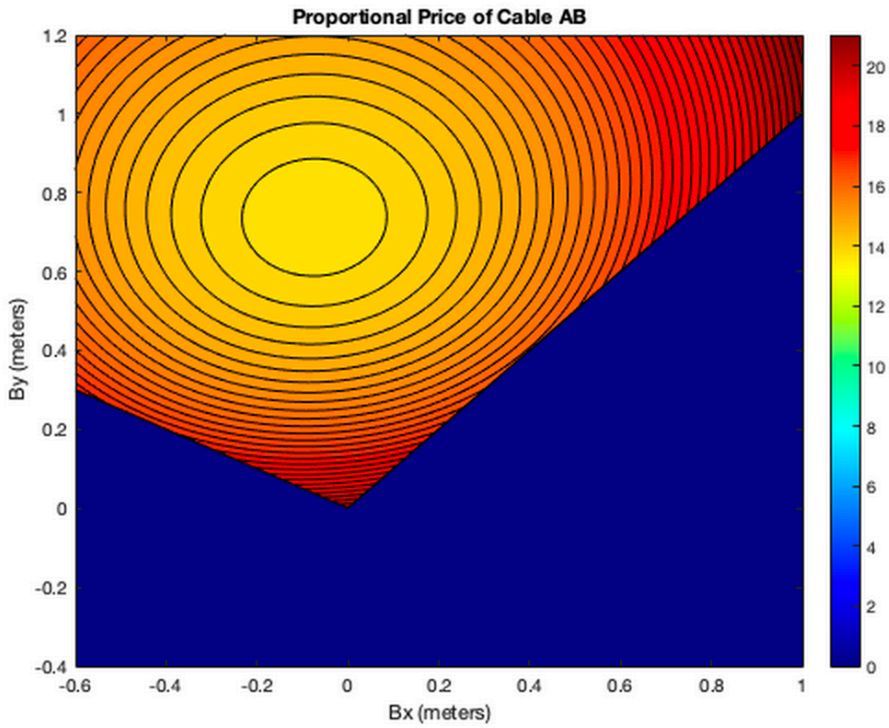


Figure 6: Finally, the script outputs the optimal price solution for cable AB. The calculated optimal location was Bx= -0.07, By = 0.73. This result is confirmed by the previous plot.

```
The optimum price for cable AB is proportional to a price of $13.74.
The Bx and By coordinates of this price are (-0.07,0.73)
```

Discussion

From the charts, the relationships between varying Bx & By on the tensions in cables can be inferred. Cumulatively, moving Bx in the positive x direction in the xy plane: **decreases the tension in cable AB, increases the tension in AC, and does not significantly change the tension in cable AD.** Likewise, moving Bx in the negative x direction will result in the inverse of the aforementioned effects. Moving By in the positive y direction in the xy plane: **increases the tension in cable AB and AC, but decreases the tension in cable AD**. Similar to Bx, movement in the negative y direction will result in the opposite of these effects. Moving point B away from the origin increases the length and tension of cable AB that is required to keep the system in equilibrium. As a result of these trends, the optimal location position of point B can easily be computed.

From visual inspection, it is possible to notice an interesting association between the three tension graphs and the final price plot. If each of the three tension plots are stacked on top of each other (summing tension values accordingly), and all possible negative solutions omitted, the resulting plot is the very similar to the price function.

Conclusion

The relationship between Bx & By and the tensions of each cable in this system is quite interesting. From the plots of the tension of each cable and an implicit analysis of the cost plot, we can see that a general movement in the xy plane of point B, away from point C results in an increase of the tension in cable AB and an increase in the tension of cable AC. This is also true for cable AD. When we move point B towards the origin, the tensions in each of the cables tends to decrease overall. The optimal position of point B represents the combination of Bx and By where the vertical components of each rope’s tension vector is maximized.

From a cost analysis perspective, if one was to construct while minimizing the cost of cable AB, the position chosen for fixture B should be (-0.07,0.73,0).

Appendix

Matlab Code:

```
res=1000;
bx = linspace(-0.6,1,res);
by = linspace(-0.4,1.2,res);
M=[bx;by;zeros(1,res)];
eac=[-1;-1;1.75]./sqrt(((-1)^2)+((-1)^2)+((1.75)^2));
ead=[1.5-0;-0.75-0;0+1.75]./sqrt(((1.5)^2)+((0.75)^2)+((1.75)^2));
Tab=[];Tac=[];Tad=[];
```

tension Calculation

```
for ii=1:res%bx, bx(ii)
   for jj=1:res%by by(jj)
       eab=[bx(ii)/(sqrt((bx(ii)^2)+(by(jj)^2)+(1.75^2)));...
           by(jj)/(sqrt((bx(ii)^2)+(by(jj)^2)+(1.75^2)));...
           1.75/(sqrt((bx(ii)^2)+(by(jj)^2)+(1.75^2)));];
       A=[eab,eac,ead];
       b=[0;0;6.4];
       Tmat=A\b;
       Tab(ii,jj) = Tmat(1);
       Tac(ii,jj) = Tmat(2);
       Tad(ii,jj) = Tmat(3);
   end
end
```

Price Calculation

```
%price =Tab*length of ab^2
lengthmat=ones(res,res);
pricemat=[];
for ii=1:res%bx,bx(ii)
    for jj=1:res%by,by(jj)
        if lengthmat(ii,jj)~=0
            lengthmat(ii,jj)=sqrt(((bx(ii)-0)^2)+((by(jj)-0)^2)+(0+1.75)^2);
        end
    end
end
lengthmat=lengthmat.^2;
pricemat=lengthmat.*Tab;
optprice = min(min(pricemat));
[xmin,ymin]=find(abs(pricemat)==min(min(abs(pricemat))));
```

tension trimming

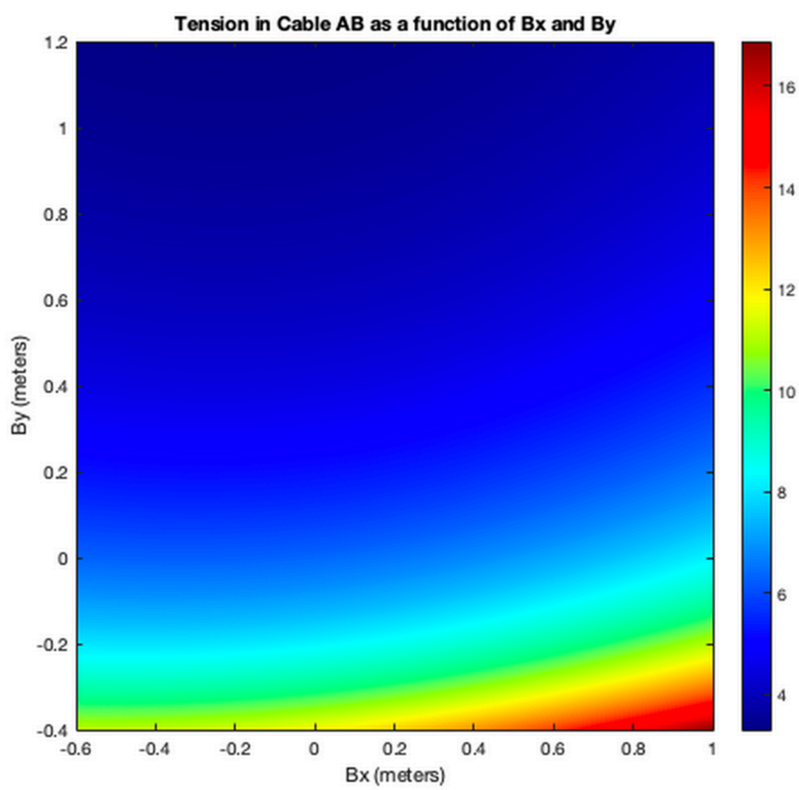
```
%this block removes solutions that contain a negative tension value after
%finding the minimum price location
for ii=1:res%x, bx(ii)
   for jj=1:res%y,by(jj)
       if Tab(ii,jj) < 0||Tac(ii,jj) < 0||Tad(ii,jj) < 0
           pricemat(ii,jj)=0;
       end
   end
end
fprintf('The optimum price for cable AB is proportional to a price of $%.2f.\n',optprice);
fprintf('The Bx and By coordinates of this price are (%.2f,%.2f)\n',bx(xmin),by(ymin));


figure(4)
contourf(bx,by,pricemat',100);%
xlabel('Bx (meters)');
ylabel('By (meters)');
title('Proportional Price of Cable AB');
colorbar;
colormap(jet(1000));
```

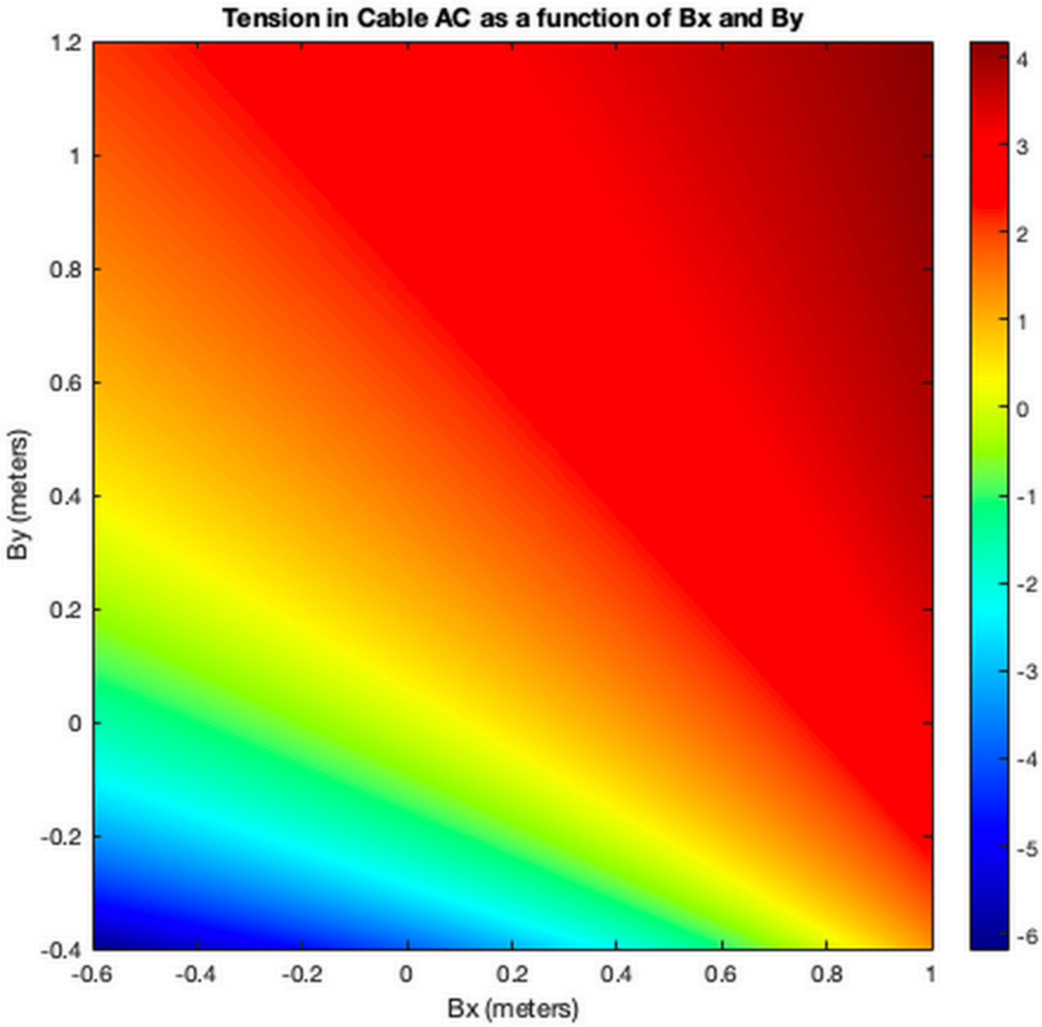
```
The optimum price for cable AB is proportional to a price of $13.74.
The Bx and By coordinates of this price are (-0.07,0.73)
```

Plotting

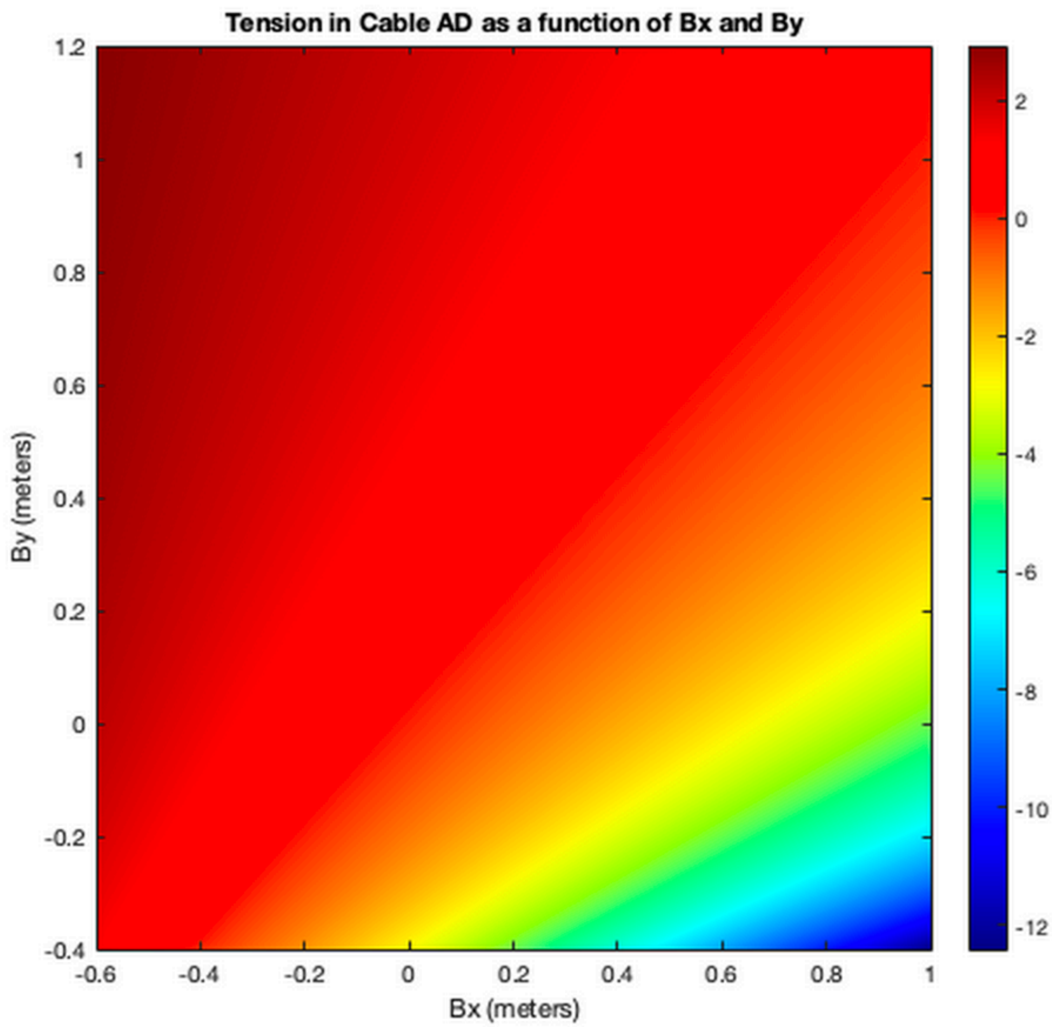
```
%Cable AB
figure(1)
contourf(bx,by,Tab',500,'EdgeColor','none','LineStyle','none');%,'EdgeColor','none','LineStyle','none'
xlabel('Bx (meters)');
ylabel('By (meters)');
zlabel('Tension in Cable AB (kN)');
title('Tension in Cable AB as a function of Bx and By');
colorbar;
colormap(jet(1000));

%Cable AC
figure(2)
contourf(bx,by,Tac',500,'EdgeColor','none','LineStyle','none');%,'EdgeColor','none','LineStyle','none'

xlabel('Bx (meters)');
ylabel('By (meters)');
zlabel('Tension in Cable AC (kN)');
title('Tension in Cable AC as a function of Bx and By');
colorbar;
colormap(jet(1000));

%Cable AD
figure(3)
contourf(bx,by,Tad',500,'EdgeColor','none','LineStyle','none');%,'EdgeColor','none','LineStyle','none'

xlabel('Bx (meters)');
ylabel('By (meters)');
zlabel('Tension in Cable AD (kN)');
title('Tension in Cable AD as a function of Bx and By');
colorbar;
colormap(jet(1000));
```

<u>Figures:</u>

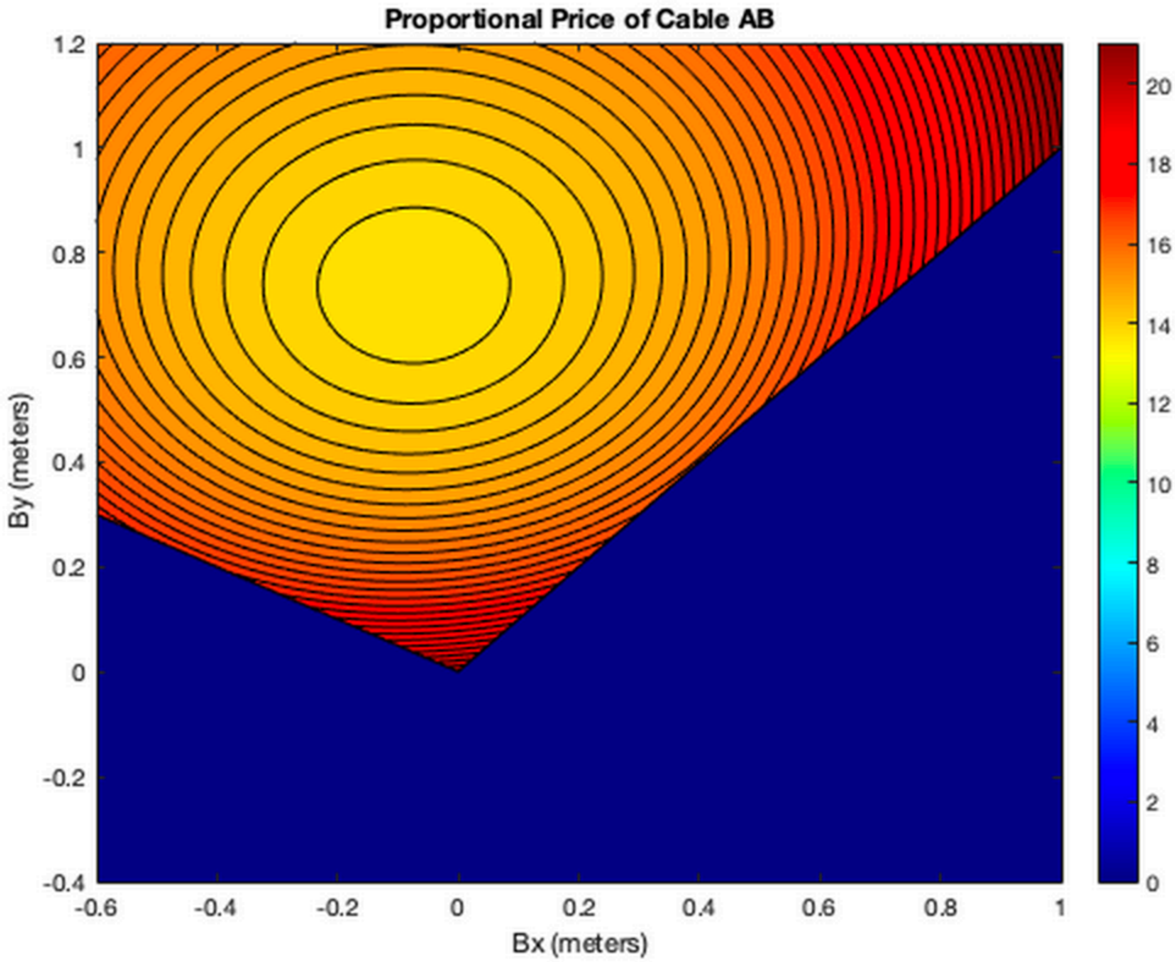




Tension in Cable AC as a function of Bx and By
By (meters)
Bx (meters)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
4
3
2
1
0
-1
-2
-3
-4
-5
-6



Tension in Cable AD as a function of Bx and By
By (meters)
Bx (meters)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
2
0
-2
-4
-6
-8
-10
-12



Proportional Price of Cable AB
By (meters)
Bx (meters)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
20
18
16
14
12
10
8
6
4
2
0