Design Project #1

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Due Date: January 23rd, 2019

Introduction

The objective of this project is to design a spring switch mechanism using springs and Matlab simulations. The mechanism is comprised of two spring that push against each other and collectively oppose a force *P*. The following three diagrams illustrate the path of the spring in space:

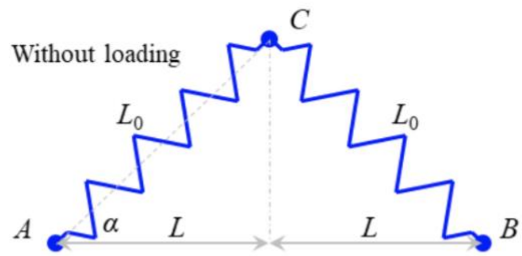


Figure 1: The first diagram illustrates the spring system with no force applied at point C. The initial lengths of the uncompressed springs are L_o and the angle between the horizontal and the springs is α degrees.

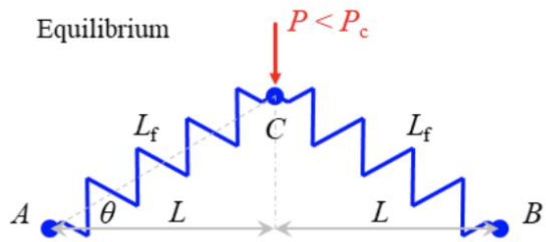


Figure 2: The second diagram shows the system in static equilibrium. That is, the force applied at point C, P, is equal to the sum of the forces applied by spring AC and BC. Notice that the spring switch in this diagram has not reached the critical angle at which $P = P_c$.

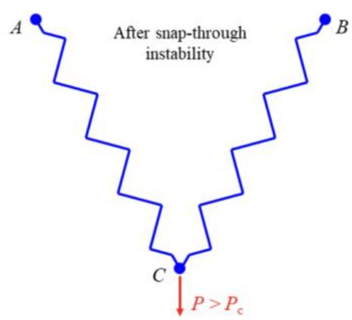


Figure 3: The final diagram shows the spring switch mechanism after the force applied, P, exceeds P_c. This leads the system to snap into a new position. The critical angle, θ_c, is the angle between the horizontal and the vertical at which $P = P_c$.

<u>Constraints:</u> The problem specifies 4 constraints as follows:

1. The horizontal projection length of each spring is fixed at $L = 1$ m.
2. The activation force P_c for the structure should be between 250 and 500 N.
3. The longest springs that the manufacturer can produce are 2 m (i.e., $L_0 \leq 2$m).
4. The stiffest spring (largest spring constant) that the manufacturer can provide is 750 N/m.

Methodology

The experiment will be carried out using Matlab simulations. The possible ranges of the design variables will be varied, and feasible solutions will be plotted for visualization.

Variables

Name	Meaning	Unit
P	*Force applied at C*	*Newton*
L_o	*Length of each spring*	*Meter*
α	*Initial angle between springs and the horizontal*	*Radian*
θ	*Angle between springs and the horizontal when P is applied*	*Radian*
θ_c	*Angle between springs and horizontal where* $P = P_c$	*Radian*
P_c	*Threshold value of P at which the system snaps through instability.*	*N*
k	*Spring constant*	*N/m*

1. Express L_0 in terms of α and L. **Equation 1:**

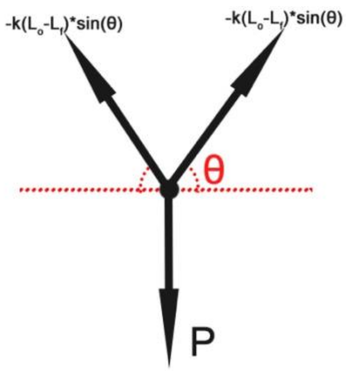
$$L_o * \cos\alpha = l$$

$$L_o = \frac{l}{\cos\alpha}$$

l is constrained to 1m (Constraint 1) so:

$$L_o(m) = \frac{1}{\cos\alpha}$$

2. Express the load *P* in terms of α, θ, k, and L. **Equation 2:**



$$P = 2(-k(L_o - L_f)\sin\theta)$$

3. Find the critical angle, θ_c, corresponding to the maximum load that can be carried before instability occurs and express it in terms of α (0 < α < π/2, 0 < θ < π/2).
   Equation 3:

Substitute expression for l into equation 2:

$$P = 2\left(-k\left(\frac{1}{\cos\alpha} - \frac{1}{\cos\theta}\right)\sin\theta\right)$$

Find derivative of *P* with respect to *θ*

$$= \frac{-2k\sin\theta}{\cos\alpha} - \frac{-2k\sin\theta}{\cos\theta}$$

$$= \frac{-2k}{\cos\alpha}\sin\theta + 2k\tan\theta$$

$$dP = \frac{-2k}{\cos\alpha}\cos\theta + 2k\sec^2\theta * d\theta$$

Set $\frac{dP}{d\theta} = 0$ to solve for θ_c

$$0 = \frac{-2k}{\cos\alpha}\cos\theta + 2k\sec^2\theta$$

Solve for θ_c (0 < α < π/2, 0 < *θ* < π/2)

$$\frac{2k}{\cos\alpha}\cos\theta_c = 2k\frac{1}{(\cos\theta_c)^2}$$

$$\frac{2k}{\cos\alpha}(\cos\theta_c)^3 = 2k$$

$$(\cos\theta_c)^3 = \cos\alpha$$

$$\cos\theta_c = \sqrt[3]{\cos\alpha}$$

$$\theta_c = \cos^{-1}\left(\sqrt[3]{\cos\alpha}\right)$$

Results

4. **Compute and plot the value of θ_c and P_c for $0 < \alpha \leq 5\pi/12$ and k = 500 N/m**

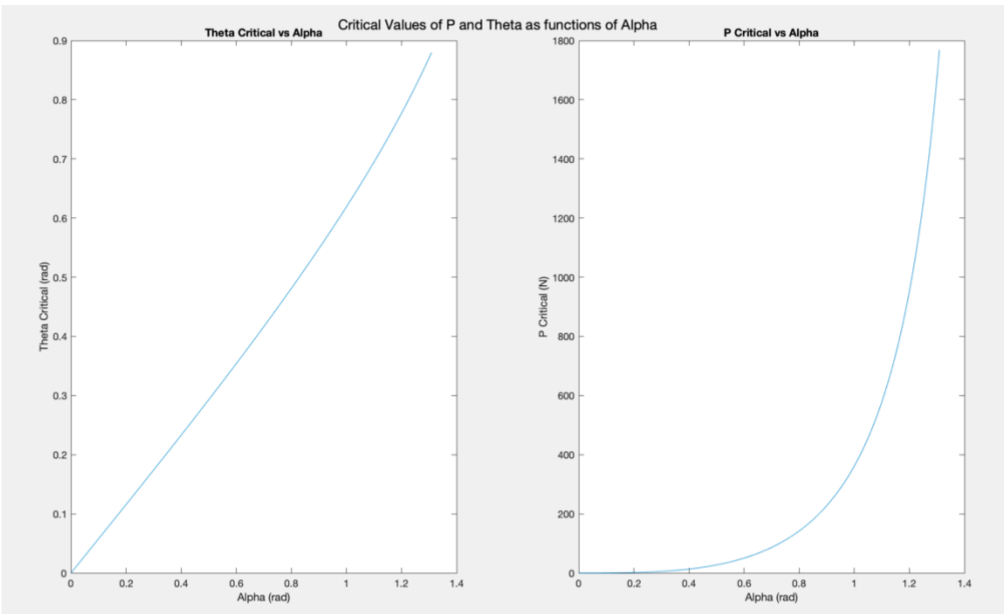
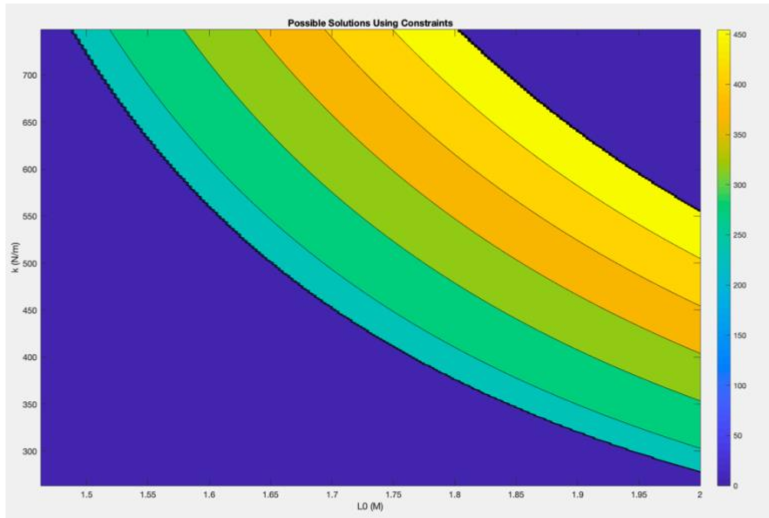


Figure 4: Computed plots of θ_c and P_c. Subplot 1 shows a positive association with a positive slope and a fairly linear correlation. Subplot 2 shows the relationship between alpha and θ_c. The relationship is positively associated and exponential.

5. **Plot the feasible range of designs in a 2D color plot of k and L_o using the inequalities arising from the design constraints.**

Figure 5: shows the contour plot representing the inequality solution to the design challenge given the constraints. It should be noted that adequate solutions contain a level-variable greater than 0. This is indicated by a color other than purple. The gradient gives an approximate value of Pc for the given combination of k and l0 at it's coordinate position. Possible solutions exist only a certain region because the values of P given as a combination of k and l0 for points outside the colored region do not meet the design criteria. The level-0 points have a value of P that is outside the range of 250-500.

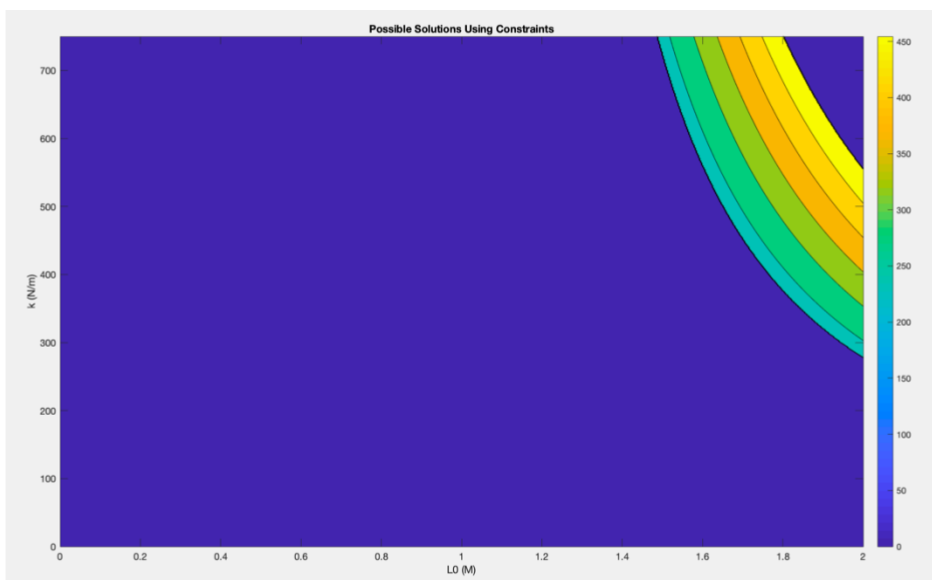


Discussion

The results from question 4 indicate that as alpha increases and approaches π/2, the critical angle increases at an increasing rate. This means that as the initial starting angle gets larger and larger, the amount of displacement that is required to snap the mechanism is less and less.

Moreover, it was discovered that as alpha increases, the critical force needed to snap the system out of it's initial position increases exponentially. This is due to the 3rd root radical in the formula derived for Pc. As this term in the denominator decreases with alpha, the force increases very rapidly and quickly exceed the criteria for the system to be designed.

Finally, in the 5 question, the results of the entire system were plotted on a contour plot. It can be inferred from the color scale next to the contour plot and by observing the shaded region that the area blued out in the bottom right corner was eliminated from the solution set because the value of Pc was less than 250. On the other hand the region excluded in the upper-right corner was omitted due to Pc being larger than 500 for the given values of k and l0. The range of accepted solutions to the parameters clearly vary in ranges of Pc as seen in the color gradient.



Conclusion

In conclusion, a sufficient spring mechanism can be created using the design constraints by choosing a point on the contour plot within the shaded region. By choosing a point in the colored region, one will obtain the spring constant and spring length that they need to construct their mechanism. From this contour plot, they are also able to ascertain the force Pc that will be needed to snap the system out of balance. If the design variable constraining switch design is Pc, one is able to choose any point in the correct color band to choose design parameters. This allows the switch builder to develop a switch with a known Pc and appropriate values of l0 and k.

Appendix

Code:

```
%the purpose of this section is to plot the value of thetac and pc for
%0<alpha<5pi/12 and k=500 N/m
k = 500;
alpha = linspace(0,5*pi/12,1000);
thetac=acos((cos(alpha).^(1/3)));
l=1./cos(alpha);
pc=2.*(k.*((1./cos(alpha))-(1./cos(thetac))).*sin(thetac));
%plot functions
figure(1);
clf
suptitle('Critical Values of P and Theta as functions of Alpha');
subplot(1,2,1);
plot(alpha,thetac);xlabel('Alpha (rad)');ylabel('Theta Critical (rad)');title('Theta Critical vs Alpha');
subplot(1,2,2);
plot(alpha,pc);xlabel('Alpha (rad)');ylabel('P Critical (N)');title('P Critical vs Alpha');

%%part 5
k = linspace(0,750,750);
l0 = linspace(0,2,750);
c = zeros(length(k),length(l0));
for ii=1:length(l0)
   for kk = 1:length(k)
       alpha = acos(1/l0(ii));
       theta = acos(cos(alpha)^(1/3));
       p = 2*k(kk)*((1/cos(alpha))-(1/cos(theta)))*sin(theta);
       if p<500 && p>250
           c(kk,ii)=p;
       end
   end
end

figure(2);
clf
contourf(l0,k,c,10)
title('Possible Solutions Using Constraints');xlabel('L0 (M)');ylabel('k (N/m)');
colorbar
```