Design Project #3

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Introduction

The system in consideration deals with a beam, a cable, a wall, and three balls. Beam AB is pinned to a fixed point on the wall at point A. The other end of the beam is connected to the wall from point B to point C. The length of the beam is 1m. Each of the balls have a mass of 1 kg and a radius of 0.1 L_0 (where L_0 is the length of the beam). The weights of AB and BC are negligible. The challenge is develop a simulation in order to find an optimum value of α at which the tension in cable BC in minimized. This special angle is called α_{opt}.

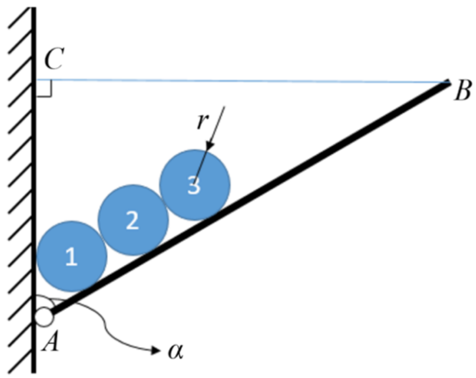


Figure 1: Figure 1 illustrates the physical system. The cable CB; beam BC; and balls 1,2, and 3 are pictured.

In part 2, the position of C is able to be move along the vertical axis. The angle it creates with the vertical axis, β, has a possible range of 20˚ to 90˚. The range of α is the same as part 1.

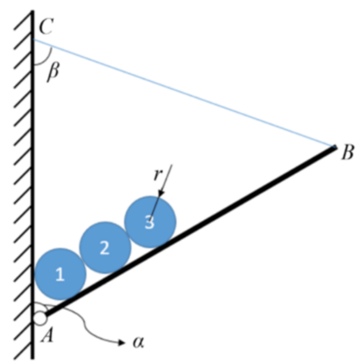


Figure 2: Figure 2 shows the set up of the system under the conditions in question 2. As shown, the new position of point C allows a new angle, β, to be formed with the vertical axis.

Constraints: The problem specifies 3 constraints as follows:

1. The angle α between the beam and the vertical wall must be between 20˚ and 70˚ from the vertical.
2. Cable BC is perpendicular to the vertical wall in the condition shown above.
3. The angle β is between 20˚ and 90˚

Methodology

To determine α_{opt} we must write tension as a function of α in the given range of possible α's. This multivariable function will be iterated using a Matlab script. First, we must derive an equation for tension as a function of α.

Variables

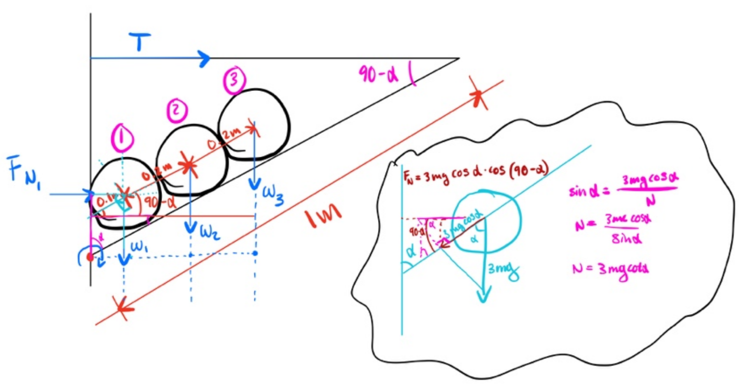
Symbol	Description	Unit
α	*Angle measure between the vertical axis and beam AB. 20˚< α<70˚*	*Degrees (˚)*
α_{opt}	*Angle measure between the vertical axis and beam AB at which the tension in cable BC is minimized. 20˚< α_{opt} <70˚*	*Degrees (˚)*
T_{CB}	*Tension in cable CB*	*Newton (N)*
r	*Radius of each ball resting on beam AB.*	*Meter (M)*
L_0	*The length of beam AB*	*Meter (M)*

To develop an equation for the tension as a function of alpha in the first problem, we must set all of the moments equal to 0 because this is the condition for static equilibrium.
We have:

$$\sum M_A = 0$$

We choose to sum the moments about the point A in order to minimize the number of unknowns in the equation. The moments about A are derived from the tension in the cable, the three balls, and the normal force of wall acting upon the three balls.

We now have:



$$\sum M_A = 0 \Longrightarrow T = \frac{-9.8(d_0\sin(\alpha) + (d_0+0.2)*\sin(\alpha) + (d_0+0.4)*\sin(\alpha)) + 3*1*-9.81*\cot(\alpha)*\frac{r}{\tan\left(\frac{\alpha}{2}\right)}}{\sin(90-\alpha)}$$

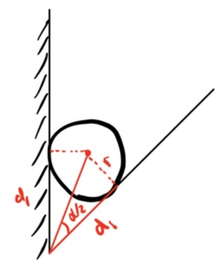
Where d_0 is the distance from the vertical axis to the point at which ball 1 contacts the beam measured parallel to beam AB.

$$d_0 = \frac{r}{tan\left(\frac{\alpha}{2}\right)}$$

$$\tan^{-1}\left(\frac{r}{d_1}\right) = a/2$$

$$\tan\left(\frac{a}{2}\right) = \frac{r}{d_1}$$

$$d_1 = \frac{r}{\tan\left(\frac{a}{2}\right)}$$

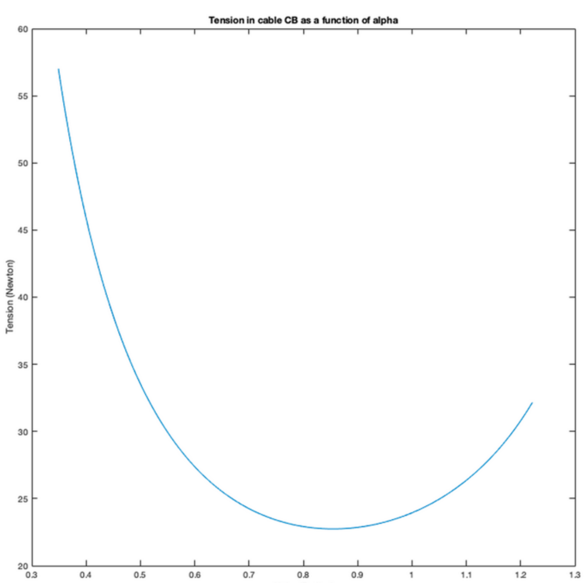


This equation was iterated in Matlab for a range of values of α from 20˚ to 70˚. The results, a vector of tensions were plotted as a function of α.

For question 2, the position of C is varied with alpha. This has an impact on our derived function above. Due to the change, tension is now in not entirely horizontal and it's component in the x direction is the only one that will be used in the moment calculation. Moreover, the distance of the moment from the point of rotation changes with Beta. The new function that we derive is as follows.

$$T = \frac{-9.8(d_0\sin(\alpha) + (d_0+0.2)*\sin(\alpha) + (d_0+0.4)*\sin(\alpha)) + 3*1*-9.81*\cot(\alpha)*\dfrac{r}{\tan\left(\frac{\alpha}{2}\right)}}{\sin(\beta)\left(\cos(\alpha)+\dfrac{\sin(\alpha)}{\sin(\beta)}\cos(\beta)\right)}$$

Similar to the simulation in part 1, we iterate this function across all possible values of α and β. The result is a matrix of tension. Because we now have two variables associated with the change in Tension, we can visualize the data in 3d with a surface plot. The plot is provided in the results section.



Results

The relationship between alpha and the tension in cable CB is roughly parabolic. Initially, as α exceeds 20 degrees and approaches αopt, the tension in the cable decreases. That's is, as alpha increases the tension decreases until it reaches a local minimum. After the function reaches a minimum, tension begins to increase with alpha, thus creating a parabolic shape. The minimum tension occurs when alpha is 48.98 degrees. The tension at this point is 22.74 N.

Figure 3: Figure 3 is a plot of tension as a function of alpha.

In part two, the surface plot has parabolic traces as functions of alpha and beta. The minimum of the plotted function is around 10 Newtons. The maximum is 80 Newtons when alpha and beta are both minimized. In general, as beta increases and alpha decreases, the magnitude of the tension increases.

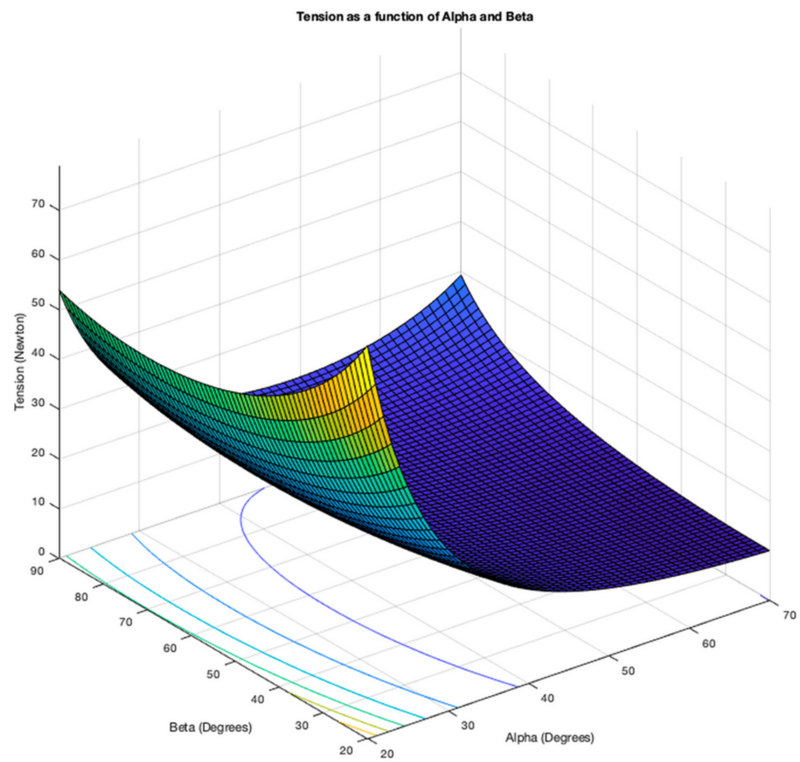


Figure 4: Figure 4 illustrates the effect of both beta and alpha in the constrained ranges on the tension in the cable.

Discussion

In question 1 we see that the tension in the cable dresses with alpha initially and then increases after alpha exceeds its optimal value. The reason for this is as follows: when alpha is less than 0.85 radians, the tension is large because the normal force of the balls against the wall produces a large moment because the balls are relatively far away from the point A. Likewise, as alpha exceeds its optimal value, the tension begins to increase respectively. This is due to the fact that the moments of each ball begin to increase in magnitude. The magnitude of each balls' moment increases because the perpendicular component of each ball's weight moves further away.

In part 2, the tension increases when alpha decreases (ceteris paribus) and increases as beta increases (ceteris paribus). The result therefore, is an inverse relationship between tension, alpha, and beta. Such that, in order to minimize the tension in the cable, alpha must be as large as possible, and beta must be as small as possible. With respect to the physical system, this means making the beam as horizontal as possible, and making point C as high as possible. This in effect, minimizes the normal force acting upon the ball and minimizes the horizontal component of the tension force.

Conclusion

In conclusion, to find the optimal angle, alpha, in order to minimize the tension in the cable, one must find a delicate balance between the normal force applied to the balls from the wall and the magnitude of the moments created by the balls.

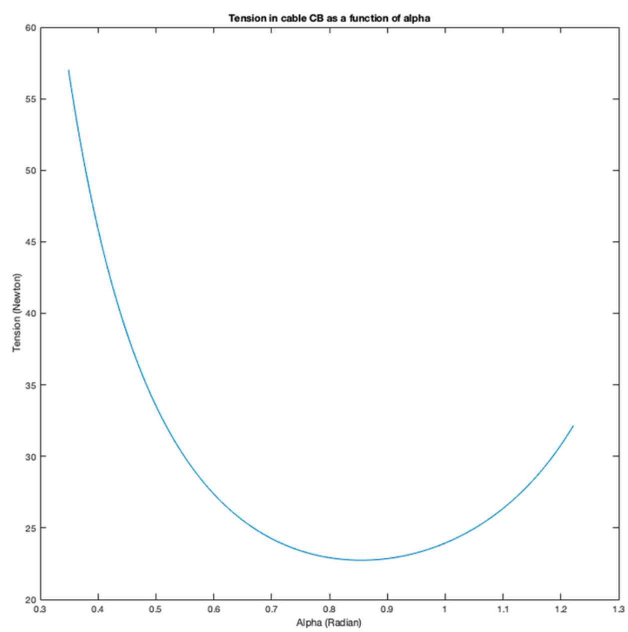
Similarly, in part 2, the minimization of the tension involves manipulating both alpha and beta to minimize the moments created by each object in the system. Matlab is an amazing tool that makes the task of performing these simulations very easy.

Appendix

Code:
Question 1:

```
a = linspace(20,70,1000);
t = [];
for ii=1:length(a)
    a(ii)=a(ii)*(pi/180);
    d0=0.1/(tan(a(ii)/2));
    t(ii)=(9.8*((d0*sin(a(ii)))+((d0+0.2)*sin(a(ii)))+((d0+0.4)*sin(a(ii)))))+((3*9.81)*cot(a(ii))*(d0)))/sin(pi/2-a(ii));
end
figure(1);clf;plot(a,t);title('Tension in cable CB as a function of alpha');
xlabel('Alpha (Radian)');
ylabel('Tension (Newton)');
j=find(t==min(min(t)));
fprintf('The optimal angle for alpha is %.2f radians or %.2f degrees.\n',a(j),a(j)*180/pi);
fprintf('The tension at this point is %.2f Newtons\n',t(j));
```

```
The optimal angle for alpha is 0.85 radians or 48.98 degrees.
The tension at this point is 22.74 Newtons
```



Question 2:

```
alphavals=20:1:70;
g=length(alphavals);
betavals=20:1:90;
h=length(betavals);
tvals2=zeros(h,g);
count2=1;
count3=1;
for jj=20:1:70
    for kk=20:1:90
        T2=(2.94+5.88*sind(jj) + (2.94*cotd(jj)/tand(jj/2)))/sind(180-jj-kk);  %solve for tension values
        tvals2(count2, count3)=T2;
        count2=count2+1;
    end
    count3=count3+1;
    count2=1;

end
figure(2);
surfc(alphavals,betavals,tvals2);
xlabel('Alpha (Degrees)');
ylabel('Beta (Degrees)');
zlabel('Tension (Newton)')
title('Tension as a function of Alpha and Beta');
```