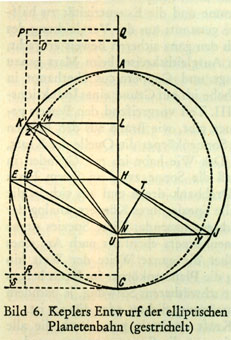
<http://www.jgiesen.de/kepler/kepler.html>

Solving Kepler's Equation of Elliptical Motion

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[**Details: series expansion, Newton's method**](http://www.jgiesen.de/kepler/kepler1.html)[**Movie of elliptical motion**](http://www.jgiesen.de/kepler/kepler2.html)[**Plots of elliptical Kepler motion**](http://www.jgiesen.de/kepler/kepler3.html)[**Circumgerence of an ellipse**](http://www.jgiesen.de/kepler/arc/ellipseArc1.html)

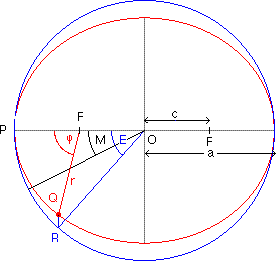
****E. Zinner: Astronomie, Alber, Freiburg/München 1951.

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| In 1609 Kepler published his work *Astronomia Nova*, containing the first (and the second) law of planetary motion: *Planets move in elliptical orbits with the sun at one focus.*  Between 1617 and 1621 Kepler wrote *Epitome Astronomiae Copernicanae*, the first astronomy textbook based on the Copernican model. Kepler introduced what is now known as *Kepler's equation* for the solution of planetary orbits, using the eccentric anomaly E, and the mean anomaly M.  The term *anomaly* (instead of *angle*), which means irregularity, is used by astronomers describing planetary positions. The term originates from the fact that the observed locations of a planet often showed small deviations from the predicted data. | http://www.jgiesen.de/kepler/img/keplerEllipse.gif |
| The mean anomaly M is the angular distance from perihelion which a (fictitious) planet would have if it moved on the circle of radius a with a *constant* angular velocity and with the same orbital period T as the real planet moving on the ellipse. By definition, M increases linearly (uniformly) with time.  Operating with radians Kepler's equation is:  **E(t) - e\*sin[E(t)] = M(t)**  or, using degrees:  **E(t) - (180°/π)\*e\*sin[E(t)] = M(t)**  The equation can be http://www.jgiesen.de/OrdnerGifs/externlink.gif[derived](http://en.wikipedia.org/wiki/Kepler%27s_equation) from Kepler's second law.  The value of M at a given time is easily found when the eccentricity e and the eccentric anomaly E are known. The problem is to find E (from which the position of the planet can be computed) when M and e are known. Kepler's equation cannot be solved algebraically. It can be treated by an iteration methods. One of them is Newton's method, finding roots of  f(E) = E - e\*sin(E) - M(t)  The true anomaly (symbol φ) is the angular distance of the planet from the perihelion of the planet, as seen from the Sun. For a circular orbit, the mean anomaly and the true anomaly are the same. The difference between the true anomaly and the mean anomaly is called the http://www.jgiesen.de/OrdnerGifs/externlink.gif[Equation of Center](http://www.astro.uu.nl/%7Estrous/AA/en/reken/zonpositie.html) C:  φ = M + C    **JavaScript using Newton's method:**  The form is preset to:  eccentricity e=0.5 mean anomaly M=27° or t/T=0.075.  The results, as shown in the figure below, are:  true anomaly phi=75.84° eccentric anomaly E=48.43° | |

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| eccentricity e (0<=e<1) |  |  |
| mean anomaly M (in °) (0<=M<=360) |  |  |
| decimal places (<15) |  |  |
| E = eccentric anomaly phi = true anomaly | **E=**  **phi=** | Iterations: |

Bottom of Form



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| An example of a series expansion is:  http://www.jgiesen.de/kepler/img/series.gif  [**Details: series expansion, Newton's method**](http://www.jgiesen.de/kepler/kepler1.html)[**Movie of elliptical motion**](http://www.jgiesen.de/kepler/kepler2.html)[**Plots of elliptical Kepler motion**](http://www.jgiesen.de/kepler/kepler3.html)[**Circumference of an ellipse**](http://www.jgiesen.de/kepler/arc/ellipseArc1.html)  For small eccentricities the mean anomaly M can be used as an initial value E0 for the iteration. In case of e>0.8 the initial value E0=π is taken.  function EccAnom(ec,m,dp) {  // arguments: // ec=eccentricity, m=mean anomaly, // dp=number of decimal places  var pi=Math.PI, K=pi/180.0;  var maxIter=30, i=0;  var delta=Math.pow(10,-dp);  var E, F;  m=m/360.0;  m=2.0\*pi\*(m-Math.floor(m));  if (ec<0.8) E=m; else E=pi;  F = E - ec\*Math.sin(m) - m;  while ((Math.abs(F)>delta) && (i<maxIter)) {  E = E - F/(1.0-ec\*Math.cos(E));  F = E - ec\*Math.sin(E) - m;  i = i + 1;  }  E=E/K;  return Math.round(E\*Math.pow(10,dp))/Math.pow(10,dp);  }  function TrueAnom(ec,E,dp) {  K=Math.PI/180.0;  S=Math.sin(E);  C=Math.cos(E);  fak=Math.sqrt(1.0-ec\*ec);  phi=Math.atan2(fak\*S,C-ec)/K;  return Math.round(phi\*Math.pow(10,dp))/Math.pow(10,dp);  }    function position(a, ec,E) {  // a=semimajor axis, ec=eccentricity, E=eccentric anomaly  // x,y = coordinates of the planet with respect to the Sun  C = Math.cos(E);  S = Math.sin(E);  x = a\*(C-ec);  y = a\*Math.sqrt(1.0-ec\*ec)\*S;  } |