Analog Control of Non-Lego Motors

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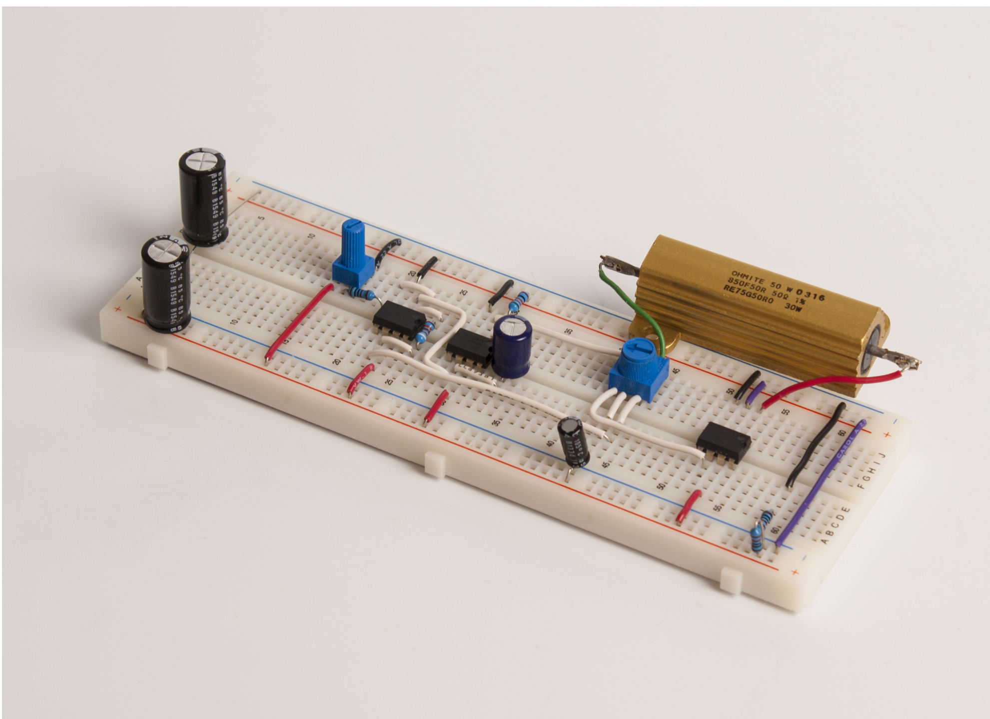
Abstract

LEGO 9V Technic Motors are known for their high torque and low friction, which makes them ideal for introducing students to motor control. Due to the rapid break down of Technic Motors and the inability to easily and inexpensively obtain new Technic Motors, we are trying to find an alternative motor to use in future iterations of Controls classes at Olin College. The motor we tested is the 23DT2R, which performs comparably well to the Technic Motors after being connected to properly tuned speed and position control circuitry.



1 Speed Control

An analog speed control circuit is composed of two basic components, a command/measure block and a proportional controller. The command/measure block has a current input that drives a motor and a voltage output proportional to the speed of the motor. The proportional controller has a voltage output that is equivalent to the input multiplied by a gain.

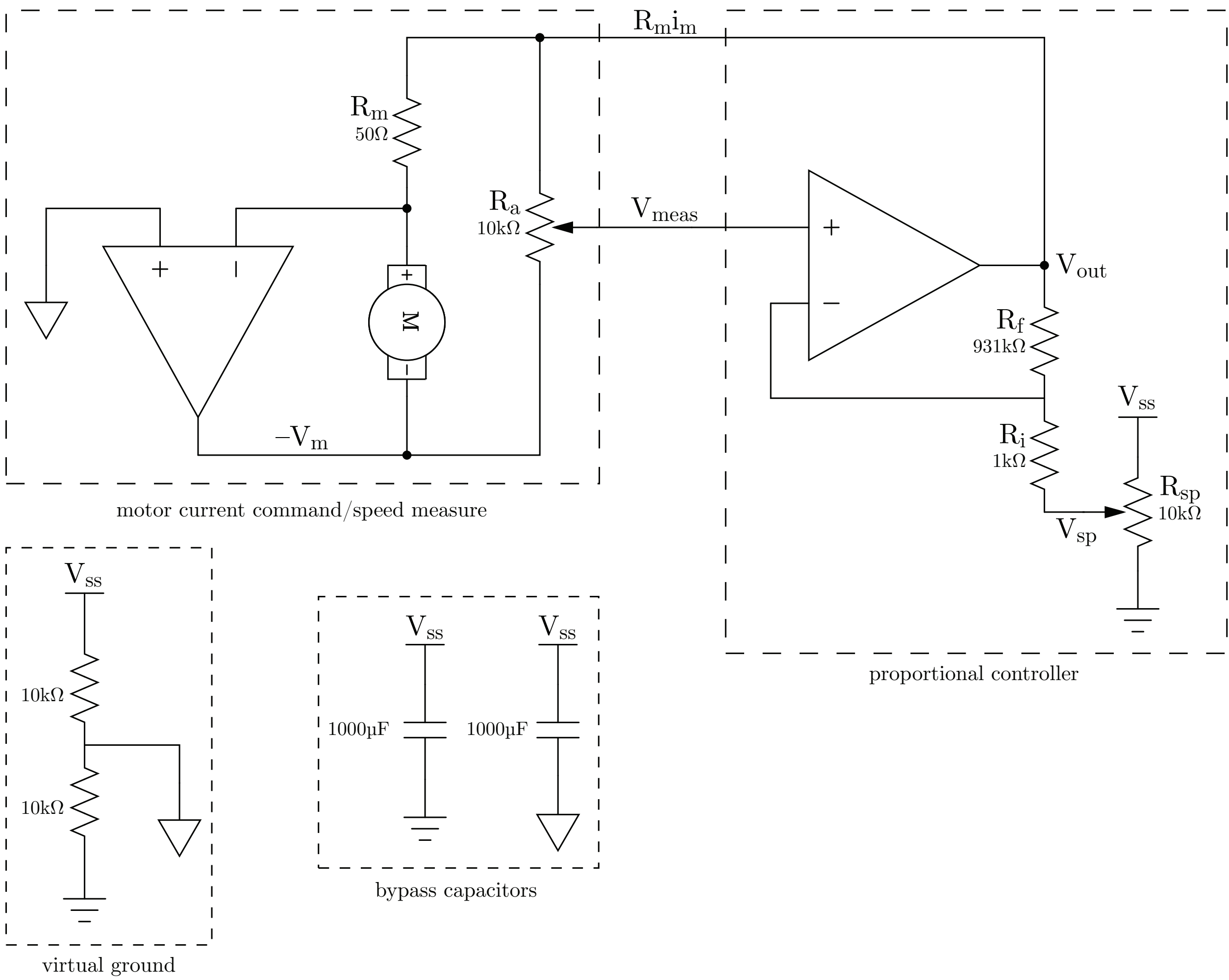


Figure 1: Circuit for controlling the motor at a set constant speed.

From Ohm's law and the behavior of potentiometers, it is known that:

$$V_{meas} = (1 - a)(-V_m) + aR_m i_m$$

where a is the percent that the dial of potentiometer R_a is turned, from 0 to 1. To simplify:

$$\begin{aligned} V_{meas} &= (1 - a)(-V_{emf} - R_s i_m) + aR_m i_m \\ &= -(1 - a)R_s i_m + aR_m i_m - (1 - a)V_{emf} \\ &= [-(1 - a)R_s + aR_m]i_m - (1 - a)V_{emf} \qquad (1) \end{aligned}$$

With the goal of making V_{meas} solely proportional to V_{emf}, we want to set the quantity $-(1 - a)R_s + aR_m$ equal to 0. In order to do this, a must be set to the correct value as follows:

$$
\begin{aligned}
-(1-a)R_s + aR_m &= 0 \\
R_s - aR_s &= aR_m \\
-aR_s - aR_m &= -R_s \\
a &= \frac{R_s}{R_s + R_m} \\
1 - a &= \frac{R_m}{R_s + R_m}
\end{aligned}
$$

Equation (1) on the preceding page can now be redefined to find that V_{meas} is proportional to V_{emf}, where $\beta = \frac{R_m}{R_s+R_m}$:

$$
\begin{aligned}
V_{meas} &= [0]i_m - \frac{R_m}{R_s + R_m} V_{emf} \\
&= -\frac{R_m}{R_s + R_m} V_{emf} \\
&= -\beta V_{emf}
\end{aligned}
$$

With the input to the proportional controller defined, the output of the proportional controller, V_{out}, can be calculated:

$$
\begin{aligned}
V_{out} &= -\beta V_{emf} - (V_{sp} + \beta V_{emf}) \frac{R_f}{R_i} \\
V_{out} &= -\beta V_{emf} \left(1 + \frac{R_f}{R_i}\right) - \frac{R_f}{R_i} V_{sp} \\
V_{out} &= -\beta V_{emf} - \frac{R_f}{R_i} \beta V_{emf} - \frac{R_f}{R_i} V_{sp} \\
V_{out} &= -\beta V_{emf} - \frac{R_f}{R_i} \left(\beta V_{emf} + V_{sp}\right)
\end{aligned}
$$

Implementing the circuit shown in Figure 1 on the previous page with a 23DT2R motor results in constant speed control, as shown in Figure 2 on the following page.

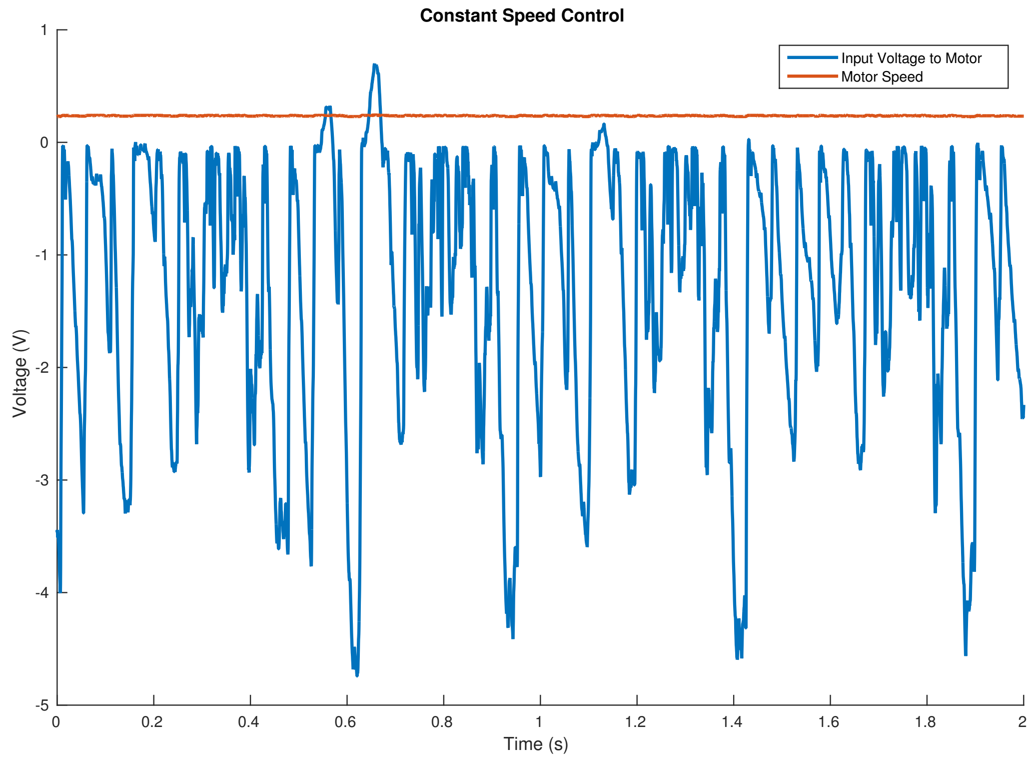


Figure 2: Controlled input and resulting speed of a motor with an unbalanced pendulum attached.

Motor speed is the orange line, which remains relatively constant. The blue line is the input to the command/measure circuit from the proportional controller, which rapidly oscillates in order to maintain constant speed of the motor when an unbalanced pendulum is attached.

2 Position Control

With functioning speed control, a non-inverting integrator can be prepended to the proportional controller to achieve position control, as depicted in Figure 3.

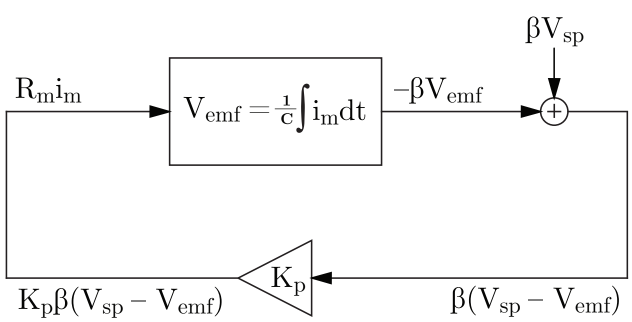


Figure 3: How an integrator functions when prepended to a proportional controller.

Knowledge of integrators and the aforementioned speed control circuit can be used to characterize the feedback for position control as follows:

$$
\begin{aligned}
V_{emf} &= \frac{1}{C}\int i_m dt \\
i_m &= \frac{K_p\beta}{R_m}(V_{sp} - V_{emf}) \\
V_{emf} &= \frac{1}{C}\int \frac{K_p\beta}{R_m}(V_{sp} - V_{emf})dt \\
\dot{V_{emf}} &= \frac{K_p\beta}{R_m C}(V_{sp} - V_{emf}) \\
\dot{V_{emf}} + \frac{K_p\beta}{R_m C}V_{emf} &= \frac{K_p\beta}{R_m C}V_{sp}
\end{aligned}
$$

Position control with an integral controller prepended to a proportional controller can be implemented as shown in Figure 4.

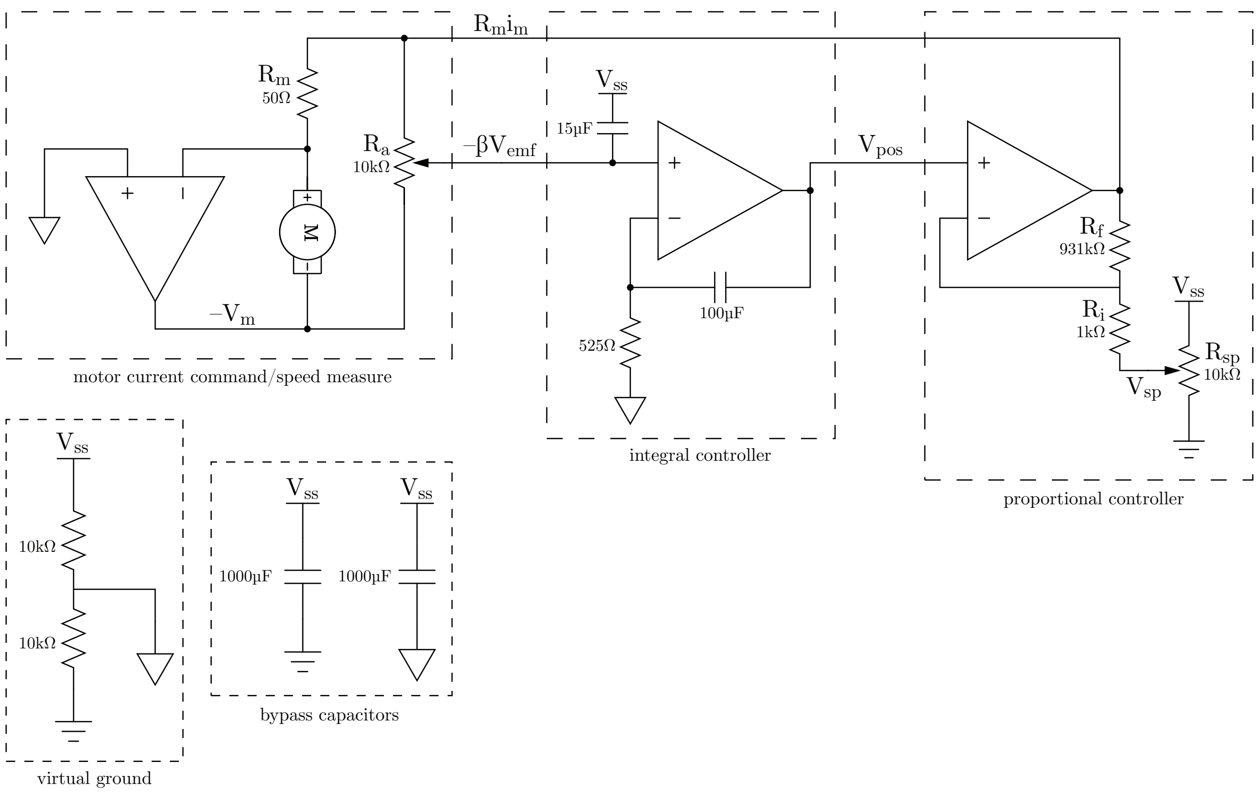


Figure 4: Circuit for controlling the position of a motor.

After implementing the circuit shown in Figure 4, the position of the 23DT2R motor can be controlled. This is shown in Figure 5 on the following page.

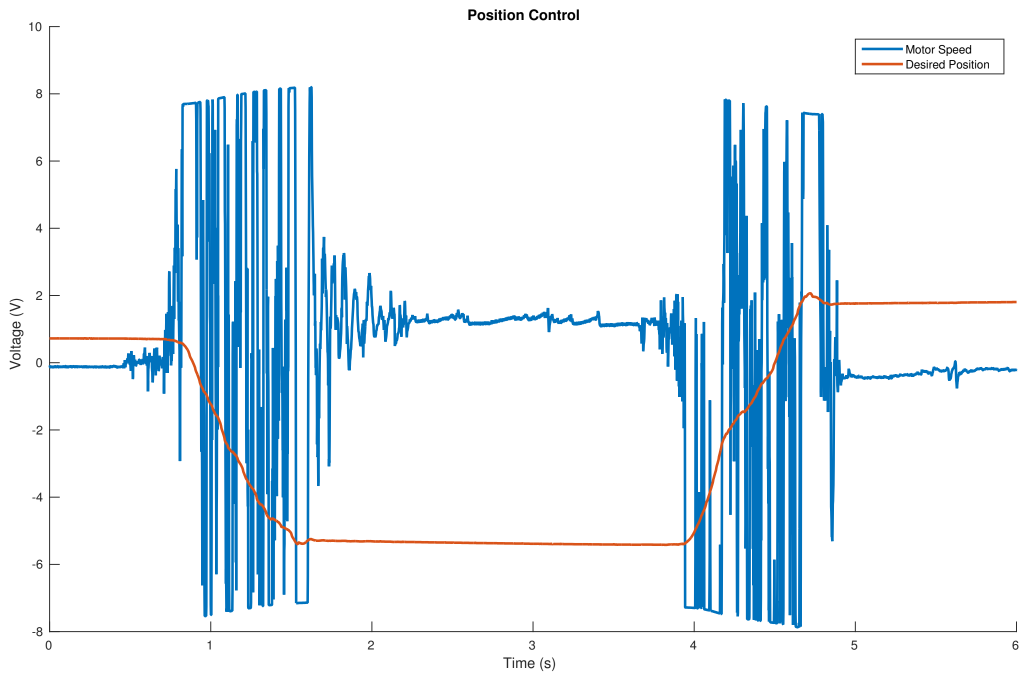


Figure 5: Desired position, set by R_{sp}, and the resulting motor speed.

The desired position of the motor is the orange line, which is changed by rotating potentiometer R_{sp}. The motor's speed, in blue, is then controlled to move to the new position desired position. Video of position control with the 23DT2R motor can be found here: `https://youtu.be/2HeKoDAQzFE`.