

Chapter 3 Notes

Ex. 3.1: calculate function:

$$w(i) = w(i-1) - (g/l) * \theta(i) * \Delta t$$

$$\theta(i) = \theta(i-1) + w(i-1) * \Delta t$$

(repeat for the desired number of time steps)

The energy of a pendulum will increase with time for any nonzero value Δt (making the Euler Method unstable).

The Euler-Cromer method is more suitable for this problem, where the previous value of w and previous value of θ are used to calculate the new value of w .

For problems involving oscillatory motion the Euler-Cromer method conserves energy over each period.

To make the pendulum problem realistic, do not assume small angle approximation (do not expand $\sin(\theta)$), include friction (dampening) force, add a sinusoidal driving force. This is called a nonlinear, damped, driven pendulum.

With a driving force of 0 the motion is damped; the pendulum comes to rest after a few oscillations

With a low driving force, SHM.

With a large driving force, the behavior never repeats (chaotic behavior). θ increases rapidly and irregularly ($\Delta \theta = \text{Lyapunov exponent}$).

The behavior is deterministic and predictable at the same time.

Pendulums that start with nearly, but not exactly the same initial conditions will have trajectories that diverge exponentially fast.

Chaotic systems generally experience phase-space trajectories with significant structures.

Things will look simpler when we perceive them at a rate that matches the problem.

Chaotic attractors have a fractal structure.

