Computer Science 202a
Homework #1, due in class Thursday, Sept. 17, 2015

Please include the names of anyone (including course staff) you consulted with in connection with this assignment, as well as listing any resources (including the course materials) you consulted. Partial credit will be given if the grader can easily understand enough of your answer to award it.

1. (15 points) Construct truth tables for each of the following propositional expressions, and say for each whether it is a tautology, a contradiction or a contingency.

   (a) $p \vee q \rightarrow p \vee \neg q$

   (b) $(\neg p \rightarrow \neg q) \vee (\neg p \wedge q)$

   (c) $p \wedge (q \oplus r)$

   (d) $\neg(p \rightarrow \neg q) \wedge r$

   (e) $(\neg(p \vee q) \wedge p) \vee (q \vee \neg(\neg p \rightarrow q))$

2. (20 points) Imagine that you are trying to organize a study group meeting and you have observed the following about your potential participants. If Adrienne or Bryan participates then Claire doesn't participate. If Claire participates then Deshawn and Eva both participate. If Deshawn participates then Adrienne or Claire (or both) participate. If Eva participates then Federico participates and Bryan doesn't participate.

   Using the propositional variable A to represent the truth value of "Adrienne participates", and similarly for B through F, translate the above statements into propositional logic expressions. If your observations are correct and Claire participates in a study group meeting, what can you conclude about who else participates in that meeting?

3. (20 points) Use truth tables to show the following equivalences of propositional expressions.

   (a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

   (b) $p \wedge (p \vee q) \equiv p$

   (c) $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

   (d) $(p \oplus q) \equiv \neg(p \leftrightarrow q)$

   (e) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

4. (15 points) Logician Raymond Smullyan describes the island of knights and knaves as populated by two kinds of people: those who always lie (knaves) and those who always tell the truth (knights). Imagine that you are visiting the island and you meet three of its inhabitants, conveniently named A, B and C.

   For each of the following items, determine whether A, B, and C could make the statements attributed to them. If they could make the statements, determine what you can conclude about the types (knight or knave) of A, B, and C if they did make the statements. Provide reasoning to support your conclusions.

   (a) A says, "B and C are both knights or both knaves." B says, "At least one of A and C is a knave." C says, "A and B are both knaves."

(b) A says, "B and I are of the same type." B says, "C is a knave." C says, "All of us are of the same type."

(c) A says, "Exactly two of us are knaves." B says, "Exactly two of us are knights." C says, "It is not the case that B and I are both knaves."

5. (20 points) For each of the following logical statements, give its negation in a form in which any negation symbol (¬) is applied directly to a predicate. Give an argument to show which of the two (the statement or its negation) is true in the following domain. The elements are all the natural numbers, that is, all the integers greater than or equal to zero. We define the following predicate and function symbols. $Z(x)$ stands for $x = 0$, $E(x, y)$ stands for $x = y$, $n(x)$ is the function that takes x to $x + 1$, $s(x, y)$ is the function that takes x and y to $x + y$, $p(x, y)$ is the function that takes x and y to $x \cdot y$, and $L(x, y)$ stands for $x \leq y$.

(a) $\exists x \exists y (E(n(x), y) \wedge Z(y))$

(b) $\forall x \forall y (E(s(x, x), y) \rightarrow L(x, y))$

(c) $\forall x \exists y \exists z (Z(z) \wedge E(z, s(x, y)))$

(d) $\forall a \forall b \forall c (L(a, b) \rightarrow L(p(c, a), p(c, b)))$

(e) $\exists a \forall b \forall c (\neg E(a, p(b, c)))$

6. (10 points) Translate the following arguments into statements in predicate logic with the given domain; be sure to define the predicate and function symbols you use. Taking the last statement of each argument as the conclusion, are these arguments valid or invalid? That is, does the conclusion follow strictly from the logical form of the argument? Please give reasons for your answer.

(a) (The domain is the animals in a particular zoo.)

   i. No animal has both feathers and scales.
   ii. Every bird has feathers.
   iii. An animal has wings if and only if it flies.
   iv. There is an animal that has wings and scales.
   v. There is an animal that flies but is not a bird.

(b) (The domain is some set of numbers.)

   i. For all a, b, and c, if a divides b and b divides c then a divides c.
   ii. For all d, a, and b, d is a common divisor of a and b if and only if d divides a and d divides b.
   iii. For all d, a, and b, d is a greatest common divisor of a and b if and only if d is a common divisor of a and b, and for any common divisor c of a and b, c divides d.
   iv. For all a and b, if a divides b and b divides a, then a and b are equal.
   v. For all a, b, c, and d, if c is a greatest common divisor of a and b and d is a greatest common divisor of a and b, then c and d are equal.